MCDM Techniques using R
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Lecture – 9
ELECTRE – Part II

Welcome to the course MCDM Techniques using R. So in previous lecture, we started our discussion on another method that is ELECTRE. So let us continue that part of the discussion. So before we can move ahead, let us do a quick recap of what we discussed in the previous lecture. So we talked about ELECTRE, the meaning of ELECTRE being that elimination and choice expressing the reality, developed by Roy, referred as ELECTRE in brief, belongs to the category of outranking school of thought.

Slightly complex because of the number of technical parameters that are involved and the steps that we have to perform while implementing the ELECTRE technique. So again in this technique also just like AHP, we use pairwise comparisons. So these aspects we were able to discuss in the previous lecture. We talked about the advantage of ELECTRE as well that if we want to avoid the compensation effect, the trade-off between criteria, then this is the suitable technique.

If we want to avoid the normalization process which can distort the original data, then again ELECTRE can be suitable. We also talked about that different methods had been developed for solving different types of decision problems, so that has been discussed as well. When to use ELECTRE, some scenarios were also discussed in the previous lecture. The choice problems, there are different methods ELECTRE I, ELECTRE Iv, ELECTRE Is. Then for ranking problems, there are different ELECTRE methods ELECTRE II, ELECTRE III and ELECTRE IV.

Then for sorting problems, we have different methods ELECTRE-Tri, ELECTRE-Tri-B, ELECTRE-Tri-C. So we talked about all these aspects in the previous lecture. We also talked about certain points related to the inputs that are required from the decision makers and how that can be done. We talked about the automatic elicitation and where we talked about that we need to get a clear ranking from the decision makers and then the criteria weights and threshold are actually inferred from there.

The problem with ELECTRE was also discussed that certain parameters or inputs that are taken from decision makers, they are prone to the inconsistencies right, that are there with the decision makers, so that particular aspect we talked about. Then we started our discussion on ELECTRE III in more detail. We talked about two particular phases that is first phase, in the first phase we do construction of outranking relations, in the second phase we exploited the output of the first phase, whatever outranking relation we have discussed that are used to produce a preference ordering.

We also said that how compensation effect is avoided because preference direction is generally taken for the increasing side for the all the criteria. Then at the last part of the lecture, we were discussing outranking relation. We talked about a few terms how things are denoted in ELECTRE. So if there are two alternatives a and b then a S b denotes that a outranks b or in other words, we can say a is at least as good as b.

Then how do we measure outranking relation, how do we measure the strength of this particular assertion, a particular outranking relation. So for that, we have a concept called outranking degree, so that is used to actually measure the strength of the outranking relation. So this is denoted as S (a, b) value typically lies between 0 and 1. Now let us talk about the outranking degree in more detail in this particular lecture.

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ELECTRE

- Outranking degree
  - Considers two perspectives
    - Concordance
    - Discordance

    of the statement that a outranks b
  - Measurement of these perspectives
    - Incorporate the decision maker's preferences on criteria
    - Concordance degree
      - Indifference and preference thresholds
    - Discordance degree
      - Veto threshold

So there are two perspectives which are considered in outranking degree and both these perspective are later aggregated to create a global perspective on the outranking degree. So let us talk about them. So first one is called concordance and the second one is called discordance. So both of concordance and discordance, they are measured for a particular statement for a particular outranking relation like a outranks b or a is at least as good as b. So all those of kind of outranking relation we do our measurements for concordance and discordance perspectives.

So what do we require to perform these measurements, so let us talk about that particular aspect. So measurement of these perspective we incorporate the decision maker's preferences on criteria so that is something going to be part of this process as we have discussed before as well. Then in the concordance degree, we typically incorporate these two threshold indifference and preference.

So indifference and preference threshold are actually incorporated or actually involved while measuring the concordance degree. When we talk about the discordance degree, then the third threshold that we use in ELECTRE is veto threshold, so veto threshold is also going to be the part of the measurement related to discordance degree. So, now let us talk about. So we will take both these perspectives, concordance and discordance perspective one by one.

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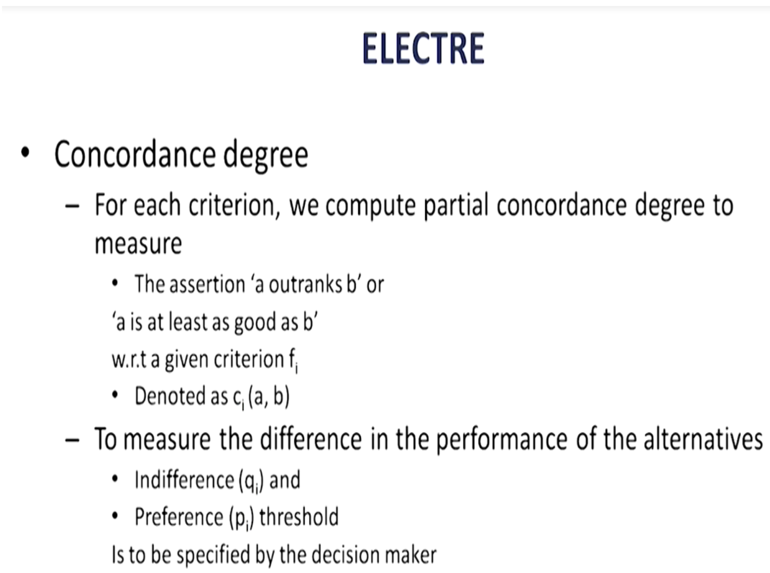

So let us start with concordance degree. So what happens in this particular perspective is that for each criterion, we compute partial concordance degree to measure what a particular outranking relation, so the assertion that a outranks b that is an outranking relation or a is at least as good as b. This is actually measured with respect to a given criterion fi and this is actually denoted as ci, c is standing for concordance degree ci (a, b) a and b both are alternatives and we are denoting the concordance degree measurement as ci (a, b).

So if you look at this partial concordance degree, this is with respect to a criterion, something similar to what we do in AHP when we say that each level in the hierarchical structure of AHP is actually evaluated, the pairwise comparisons are performed with respect to the immediate upper level right. Similarly here the outranking relation that is a is at least as good as b that is performed with respect to a given criterion right and here in this case, so in AHP we used to perform the pairwise comparisons even for the criteria, which is not done here for the criteria.

We take the input from decision makers and it is actually the alternatives among which we compute these outranking relation and do our measurement. These outranking relation each of them are actually measured with respect to each criterion and this is something which is referred as partial concordance degree. So from the word partial concordance degree itself, you will get the idea that later on we will be aggregating the results and we would be computing the global concordance degree.

So next, let us talk about the next point about concordance degree now to measure the difference in the performance of alternatives, so we use these two thresholds that we have talked about indifference that we are denoting using qi and preference we are denoting it by pi threshold. So these two thresholds, these two parameters, they are to be specified by decision maker something that we have discussed before also. We also talked about automatic elicitation method, how these parameters can be inferred from decision makers.

So we need these two thresholds so that we can move ahead and compute the partial concordance degrees and later on global concordance degree. So let us talk about these two thresholds, till now we have talked about briefly what we mean by these threshold, so now let us define these threshold before we can move ahead.

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ELECTRE

- Concordance degree
  - Indifference threshold
    - Indicates the largest difference between the performances of the alternatives w.r.t a given criterion such that they remain indifferent for the decision maker
  - Preference threshold
    - Indicates the largest difference between the performances of the alternatives w.r.t a given criterion such that one is preferred over the other
  - Partial concordance degree is deduced by linear interpolations involving these two thresholds

So what do we mean by indifference threshold. So this particular threshold indicates the largest difference between the performances of the alternatives with respect to a given criterion such that they remain indifferent for the decision maker. So, through the indifference threshold, we are actually indicating the delta, difference in the performance of two alternatives a and b, that difference delta that is going to be there.

So the largest such difference, the largest such delta, which is not going to change the decision of the preference of decision maker for these two alternatives. So, the main idea being is that if the performance of a is let us say 4.2 and the performance of b is let us say 4.4, so the difference being just a 0.2. So the indifference threshold is 0.5, and if just the difference is 0.2, then since this difference is less than the indifference threshold, therefore this difference is going to be ignored by the decision maker.

Decision maker as per their preference, as per this ELECTRE process and given their preferences, the outranking relation is going to be constructed. So now that we have defined indifference threshold, now you will get the sense that these particular threshold are very much important part of how we are going to construct the outranking relations.

So to come again, indifference threshold we mean that largest difference between the performances of two alternatives with respect to a given criteria that will not change the preference of decision maker. Now let us talk about the second threshold that is preference threshold. So what do we mean by this particular threshold. So this preference threshold indicates the largest difference between the performances of two alternatives with respect to a given criteria such that one is preferred over the other.

So if the difference between the performance of two alternatives it grows, it is higher, and this difference is higher than a given value which is preference threshold, then we are clearly going to say that alternative a is preferred over alternative b or one particular alternative is preferred over the other alternative. So indifference threshold in a sense is telling us to ignore small difference in the performances of alternatives and preference threshold is telling us if the difference goes bigger than a particular value, then we clearly specify our preference.

So these two thresholds and the way they are defined will also give you an idea, will also give you a sense about when we say that imprecise or uncertain data if that is there, even in that situation ELECTRE method can be suitable. So you can see that if the values related to the performances of these alternatives is here and there, but they remain within a limited range, then these concept of indifference and preference threshold will still produce the same result right. So that idea you will immediately get from these two definitions.

Now, partial concordance degree is deduced by linear and interpolations involving these two thresholds. So if you look at we are discussing concordance degree, we talked about two perspective, concordance and discordance, and we talked about those two are going to be combined. Now within concordance, we are going to compute the partial concordance degrees and how even the partial concordance degree is now going to be deduced. So now the partial concordance degree is going to be deduced using these two thresholds.

So the basic level of computation or basic level of measurement that we do in ELECTRE method, there also these thresholds play a major role. So let us move forward and talk about how these partial concordance degrees they are computed.

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ELECTRE

- Concordance degree
  - If $f_i(b) > f_i(a) + p_i$
    - Then $c_i(a, b) = 0$
  - If $f_i(a) + q_i < f_i(b) < f_i(a) + p_i$
    - Then $0 < c_i(a, b) < 1$
  - If $f_i(b) <= f_i(a) + q_i$
    - Then $c_i(a, b) = 1$

So few terms you can see here. So fi(b), this is for alternative b. So fi(b) performance with respect to performance of alternative b with respect to criterion fi. So what we mean by fi(b) is performance of alternative b with respect to criterion fi and what we mean by fi(a) is performance of alternative a with respect to the given criterion that is fi and by pi as we discussed we mean the preference degree.

So if fi(b) is greater than fi(a) + pi right, so if this performance of alternative b is greater than the performance of alternative a plus the preference threshold, then the partial concordance degree that is ci(a,b) that is going to be defined as 0. So to visualize this and understand in much better sense, we will look at this chart here which is about this partial concordance degree ci(a,b).

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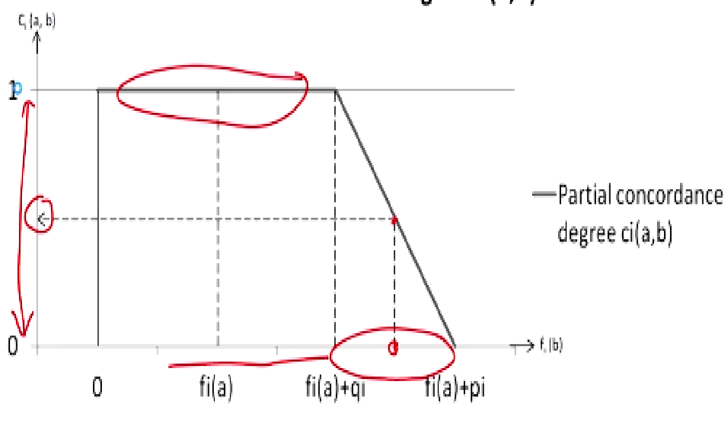
ELECTRE

Partial concordance degree ci(a,b)



So here you can see that on the Y axis we have ci(a,b) which is partial concordance degree that we want to measure for these two alternatives a, b. So outranking relations is that a outranks b. So you can see here on the Y axis the range that we have is between 0 and 1, so values are going to be lie between 0 and 1. So here, you would see that in the X axis we have fi(b). **(Video Starts: 17:30)** So if fi(b) which is being measured along the X axis if this fi(b) is greater than fi(a) + pi. Let us go back to the previous slide.

You can see the RHS side of first point fi(a) + pi, you can see here fi(a) + pi. So if the fi(b) is greater than this value here, then you can see the value of ci(a,b) is going to be zero so which is what is mentioned here. So if fi(b) is greater than fi(a) + pi that means performance of alternative b is greater than performance of alternative a and that difference is even higher than what is indicated in the preference degree, then the partial concordance degree of ci(a,b) is going to be defined as zero because b is more preferred because the value of b is higher than a and even after adding the preference threshold.

Now let us move to the second point. If the value of fi(b) is in between fi(a) + qi and fi(a) + pi that means the performance of alternative a with respect to criterion fi and if we add the indifference threshold there so the value of b is higher than this value fi(a) + qi but lower than fi(a) + pi. So if we add the preference threshold on the value of a then the value of fi(b) is lower than this, then the partial concordance degree ci(a,b) going to lie between 0 and 1.

So let us again go back to this particular chart of partial concordance degree. So here on X axis if fi(b) is going to lie somewhere between fi(a) + qi and fi(a) + pi, then you would see if this is the point you can see the value of partial concordance degree ci(a,b) is going to lie between 0 and 1, so this is how we can deduce. So these are two scenarios that we talked about to deduce the partial concordance degree.

Now the third scenario is if the fi(b) is less than or equal to fi(a) + qi. In that case, the ci(a,b) is going to be defined as 1. So let us go back to the chart. So here on the along the X axis, we are talking about fi(b). If fi(b) is less than fi(a) + qi that means here if the fi(b) is less than this value that means we are talking about this particular zone. So in this particular zone, the value you can see here is 1, so the same thing is mentioned here that if fi(b) is less than fi(a) + qi, then the partial concordance degree of ci(a,b) is going to be 1 right, so a is going to be clearly preferred over b. **(Video Ends: 21:27)**

So in the first scenario when the value of fi(b) is greater than fi(a0)+ pi, then of course b is clearly preferred over a. In the second scenario when the value of fi(b) is lying between fi(a) + qi and fi(a) + pi, then the value is going to be 0 and 1 and so we can make that if a is at least as good as so the value is going to be lie in between. Then we talk about third scenario. So in the third scenario fi(b) is less than or equal to fi(a) + qi, so here a is very clearly preferred over b.

Two scenarios very clear preference and the one scenario lies in between where the partial concordance degree can be between 0 and 1. So let us move ahead. So this was the discussion on the partial concordance degree. Now these partial concordance degrees are calculated with respect to each criterion.

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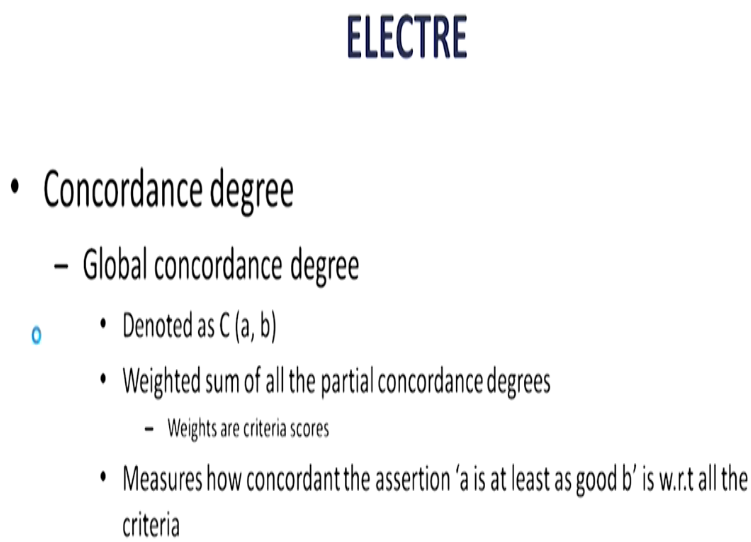

Now let us talk about the global concordance degree. So this is denoted as C(a,b), so this is actually weighted sum of all the partial concordance degrees. So all the partial concordance degree that we might have computed for all the outranking relations, so all those partial concordance degrees are going to be used exploited here and a weighted sum is going to be taken.

Now where these weights are coming from, so these weights are actually going to be the criteria scores, so something we talked about that even the criteria scores, they are going to be based on the inputs from decision maker. So those inputs, we will infer the criteria scores and those are going to be used as weights to compute the global concordance degree when we take the weighted sum of partial concordance degrees.

So once we compute the global concordance degree, then we can actually make our assessment and this global concordance degree in that assessment is going to measure how concordant the assertion a is at least as good as b is with respect to all the criteria. So this global concordance degree would be a measurement with respect to the all the criteria. So let us move forward. So till now what we have talked about the first perspective that is concordance degree. Then now, we will talk about the discordance degree.

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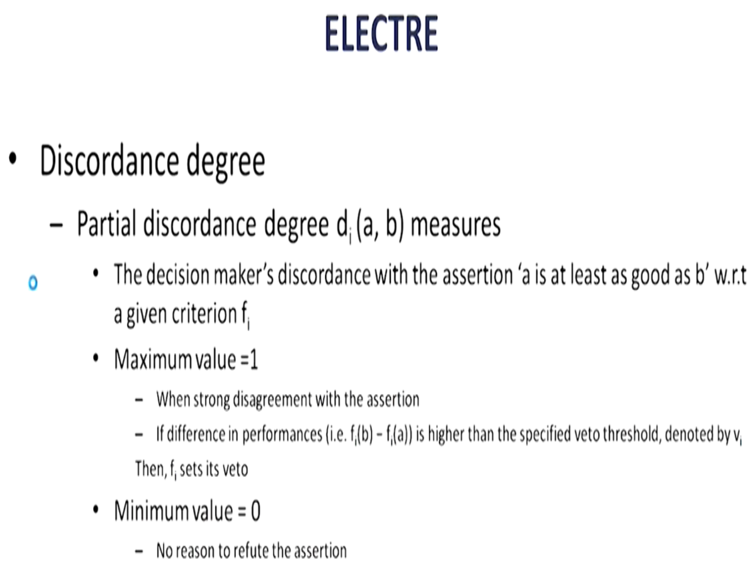

So what we mean by discordance degree is that here also to compute the discordance degree we are going to compute partial discordance degree with respect to a given criteria, so the similar kinds of statements you are going to see here in the slide as well. So a partial discordance degree denoted as di(a,b) where a and b being two alternatives, so this measures the decision maker's discordance with the assertion that a is at least as good as b with respect to a given criteria fi.

So just like in the concordance degree it was the concordance which was being measured and first we measured the partial concordance. Similarly here also in the discordance degree, first we measure the partial discordance and each assertion, each outranking relation, is going to be measured with respect to a criterion that is going to be called partial discordance degree. So this partial discordance degree is going to attain its maximum value of 1 where there is a strong disagreement with the assertion, the assertion being a is at least as good as b.

So if there is a strong disagreement with this, then the discordance value is going to be 1, so if difference in performance, that is fi(b)-fi(a), this is higher than the specified veto threshold. So till now, we haven't talked about veto threshold much. In the previous lecture, we talked about the veto threshold that is something that is used here in the ELECTRE. So in the veto threshold if fi(b)-fi(a), if this difference in performance is higher than this value, then this value is denoted by vi v indicating for veto, then this fi the criterion the sets its veto.

So the veto concept is applied if the difference is higher than the given threshold level. So this particular difference and its comparison involving the veto threshold that is going to give us value of this partial discordance degree, and based on that, we will make further computations. So for the maximum value of this partial discordance degree as I said if there is a strong disagreement, then this is going to be 1.

If there is no reason to refute the assertion that a is at least as good as b, then this partial discordance degree is going to attain its minimum value that is zero. So now let us consider the three scenarios. So just like for the concordance degree, here also the similar kind of scenarios are there, but now this time in the case of discordance, they involve the threshold.

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ELECTRE

- Discordance degree
  - If $f_i(b) > f_i(a) + v_i$
    - Then $d_i (a, b) = 1$
  - If $f_i(a) + p_i < f_i(b) < f_i(a) + v_i$
    - Then $0 < d_i (a, b) < 1$
  - If $f_i(b) <= f_i(a) + p_i$
    - Then $d_i (a, b) = 0$

So first one is that if fi(b) is greater than fi(a) + vi where vi is the veto threshold, then partial discordance degree di(a,b) is going to be 1.

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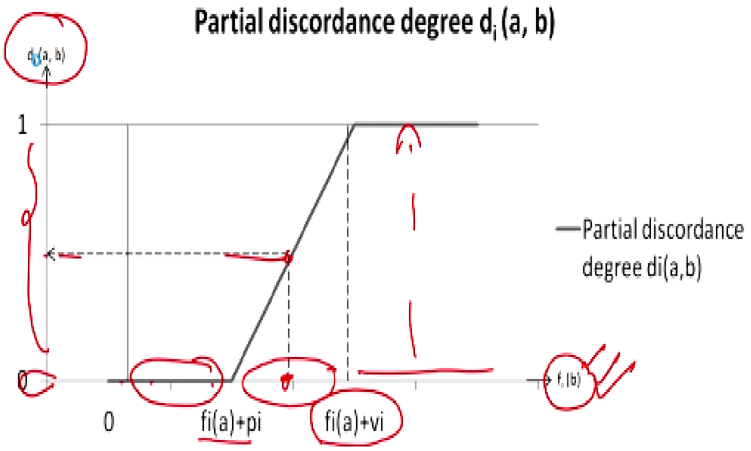
ELECTRE



(Video Starts: 28:01) So let us understand the same thing through this chart. So let us notice the RHS value in the first point that is fi(a) + vi. So you can see fi(a) + vi here and in the X axis we have fi(b) here. So if fi(b) is greater than this value, so we are talking about this zone, so you can see the value here this is 1, so di(a b) that is partial discordance degree this is going to attain its maximum value that is 1.

Let us go back and talk about the second scenario wherein if fi(b) is going to lie between fi(a) + pi, remember pi is the preference threshold. So in the case of discordance degree, we typically involve two thresholds, one is the preference threshold, the another one being the veto threshold. So in the second scenario the fi(b), the performance of alternative b with respect to given criterion fi, if it is lying between fi(a) + pi and fi(a) + vi then the value of partial discordance degree di(a,b) b is also going to lie between 0 and 1.

So let us understand it through this chart. So we are talking about this zone. So if the value of fi(b) is lying in this zone, so you can see if this being the point you would see that here the value is going to lie between 0 and 1 in this scenario. Let us go back. The third scenario is when fi(b) is less than or equal to fi(a) + pi, so if you look at the RHS side, it is fi(a) + pi that means addition of preference threshold, so if fi(b) value is less than or equal to. Then let us understand the same thing through this chart. So what we are talking about is the scenario of this particular zone.

So if the value of fi(b) is in this zone, that means less than fi(a) + pi, then this value is going to be in this zone and which comes out to be zero. So the value of di(a,b) is 0, so in this case the partial discordance degree is going to attain its minimum value. **(Video Ends: 30:37).** Let us move forward.

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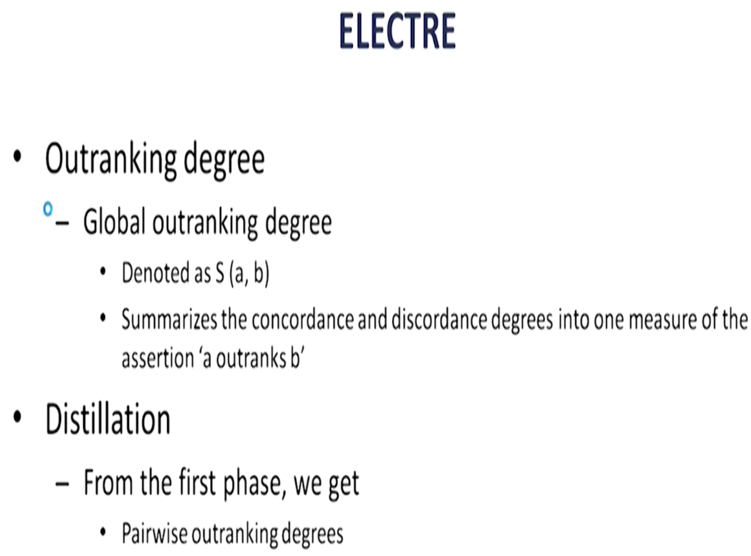

Now how do we compute the outranking degree and mainly global outranking degree. So we have talked about the concordance degree. We have also talked about the discordance degree. In both these degree concordance and discordance, we compute the partial concordance degrees with respect to all the criteria and then partial discordance degrees with respect to all the criteria and the global concordance degree and global discordance degrees are computed.

Then for the global outranking degree, we then use these concordance and discordance degrees to summarize the global outranking degree. So one measure is computed and this particular measure is going to tell us about the strength of the assertion a outranks b. So denoted as S(a b), so this is something that we will understand more clearly when we do an exercise in R. After this, the another important aspect which is actually referring to the second phase of ELECTRE method is distillation.

So from the first phase, we get pairwise outranking degrees and the different steps which are involved in this, we have just discussed. So once we get these pairwise outranking degrees, so there within the next phase where we produce the preference order, also referred as distillation phase.

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ELECTRE

- Distillation
  - Done through
    - Ascending and descending distillation procedures
      - Lead to a transitive pre-order from each procedure
      - These pre-orders might be different from each other
    - A final ranking is generated as the intersection of the two pre-orders

So here, we have these two procedures, ascending and descending distillation procedures. So these two procedures are actually going to be used to produce a final ranking. So each of these procedures, ascending and descending distillation procedures, they are going to produce their own transitive pre-orders which might be different from each other. Therefore, we will take intersection of these two pre-orders and produce a final ranking.

So these are the steps that we have to perform to produce our rankings to solve our decision problems using ELECTRE methods. So we have been able to cover these steps now. In the next class, we will understand these concepts in more details through an exercise in R. So we conclude this lecture here. Thank you.