MCDM Techniques using R
Prof. Gaurav Dixit
Department of Management Studies
Indian Institute of Technology – Roorkee

Lecture – 5
Analytic Hierarchy Process (AHP) – Part II

Welcome to the course MCDM Techniques using R. So in the previous lecture, we started our discussion on AHP that is Analytic Hierarchy Process, so let us continue that discussion. So before we move ahead, let us do a quick recap of what we discussed in the previous lecture. So as we discussed AHP was developed by Saaty and particularly useful when we are not able to construct the utility function. So all these aspects we have talked about.

The different steps that are involved in AHP that is problem structuring, priority calculation, consistency check and sensitivity analysis that we have discussed in previous lecture. So problem is structuring. This part we have discussed that we need to create a hierarchical structure, divide and conquer is the main strategy here. Breakdown of the problem that we have discussed in terms of three minimum levels, top element, second level for criteria and the lowest level for alternatives.

If we have sub-criteria, then we can certainly add additional levels. Then we talked about this specific example on shop location problem. So, we will be discussing this problem in this lecture as well. This is an example that we talked about the hierarchical structure, the sample example for shop location problem. Then in AHP, further important steps we covered in the previous lecture where we talked about how each level is to be prioritized, so that is typically with respect to immediate upper level, so this particular aspect also we discussed.

We also discussed for criteria and alternatives, how the prioritization has to be done, so that we discussed in the context of shop location problem. We also talked about what we mean by priority and the different types of priorities that we need to calculate. For example; criteria priorities, alternative priorities and global priorities that we need to compute. So these particular priorities also we covered.

The technique to calculate these priorities we also talked about, that is pairwise comparisons, so this also we discussed. We also talked about the scale that is used. So we talked about that

the fundamental 1 to 9 scale is used; however, decision makers are asked to use the verbal scale that is then mapped to a numerical scale. So, typically we construct the comparison matrices based on comparison among criteria and also among alternatives with respect to criteria and sub-criteria.

So that particular aspect we talked about in the previous lecture. So, this is one example of a comparison matrix how it is going to look like. So for example, shop location case that we discussed we had 4 criteria and you can see this is 4/4 matrix and how the pairwise comparisons they have been displayed here. Then at the last end of that particular lecture, we started about discussion on comparison matrix.

We talked about in terms of how we can find out the necessary number of comparisons that we need to perform to construct a comparison matrix, so that is n square- n/2. So how we arrive at this formula also something that we discussed in the previous lecture. Then we started our discussion on consistency check. So, let us start from this step again.

(Refer Slide Time: 04:21)

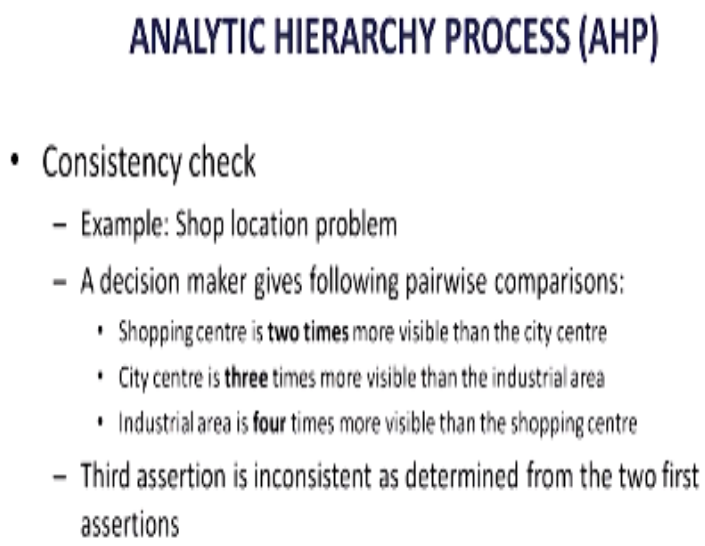
ANALYTIC HIERARCHY PROCESS (AHP)

- Consistency check
  - When several successive pairwise comparisons are done, they may end up contradicting each other
    - Vaguely defined problem,
    - Lack of sufficient information (known as bounded rationality),
    - Uncertain information, or
    - Lack of concentration

So as we talked about when we are supposed to perform successive several comparisons, pairwise comparisons, then some of these comparisons might contradict each other. So, there are few reasons which can contribute to this. For example, vaguely defined problem, the problem is not well defined and therefore decision maker find it difficult to make their pairwise comparisons.

Lack of sufficient information. So about the decision problem certain things certain domain knowledge that we are supposed to have that is not there, so it might also lead to faulty pairwise comparisons. Then uncertain information or lack of concentration, so these are few more reason which can lead to this contradictory comparisons. So if we have already collected data about pairwise comparisons from the decision makers, then how do we find out whether consistent is there or not.

(Refer Slide Time: 05:50)



So we discuss this shop location problem. So one scenario that we have given here to make it more clear what we are talking about, what we mean by consistency check, that is in this example we had three alternatives; shopping centre, city centre and industrial area and as you can see three statements three assertions are written here. Shopping centre is two times more visible than city centre, so visible is the criteria with respect to which we are asking for pairwise comparisons.

So first one shopping centre two times more visible than the city centre, then the city centre is three times more visible than the industrial area, then industrial area is four times more visible than the shopping centre. So we look at first two assertions and try to derive a relevant assertion similar to third one, then we will see there is inconsistency there because if shopping centre is two times more visible than city centre and in turn city centre is three times more visible than industrial area.

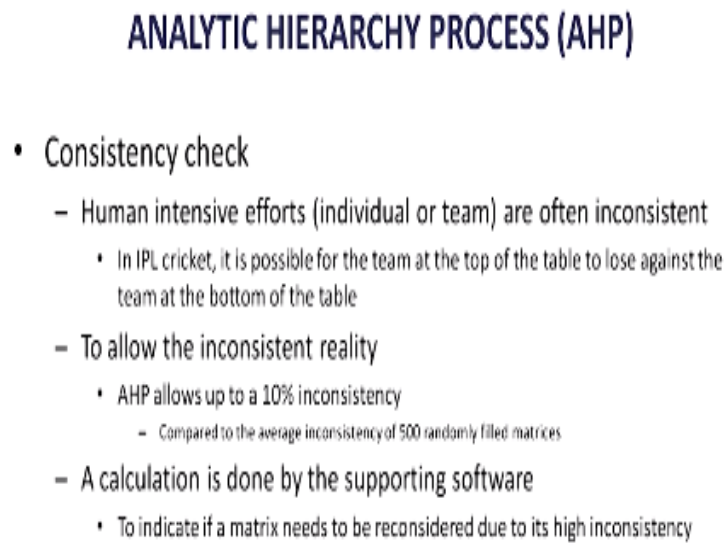
Then therefore by the rule of transitivity and we just use these two assertions, then we will come to know that shopping centre should be six times more visible than industrial area.

However if we look the third assertion, it says that industrial area is four times more visible than the shopping centre. So therefore, the third assist assertion is actually inconsistent as determined from the first two assertions, so this is what we mean by consistency check.

Especially if in a decision problem if we are dealing with a large pool of criteria and a big set of alternatives, then in that case this kind of scenario can be more common. Therefore, we hope we can move ahead and we need to find out whether this consistency is there or not. So this particular aspect is checked in the third step, that is consistency check. Now few more important points about consistency check, human intensive efforts.

So typically what we do in decision making MCDM techniques in general decision problems, that is the decision makers who are supposed to provide us with the pairwise comparisons in the case of AHP and therefore that requires a lot of intensive human effort and human efforts are typically, they can be often sometimes they are inconsistent.

(Refer Slide Time: 08:59)



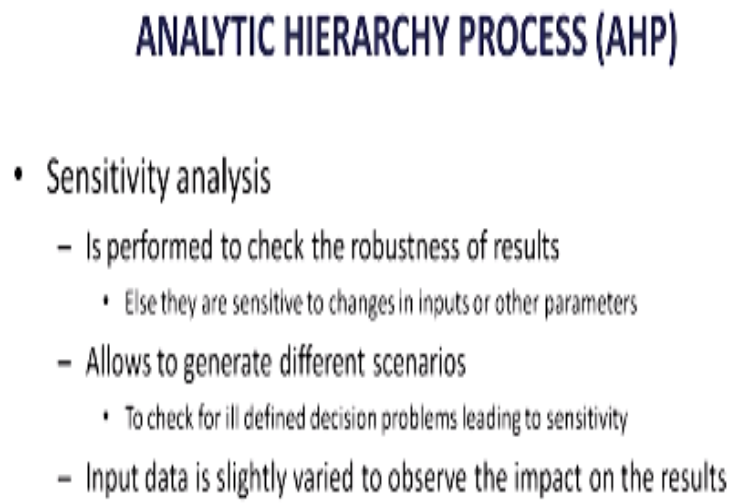
One example we can think of an analogy that I can give you to explain this is that in IPL cricket it is possible for the team which is sitting at the top of the table to lose against the team which is sitting at the bottom of the table, so that inconsistency is part of human efforts and therefore we need to check against it. Now how much inconsistency is allowed. These comparison matrices that we need to construct, do they need to be 100% or some lower level of inconsistency is allowed.

So for this, there are certain things that had been suggested. For example, certain low levels of inconsistent reality has been allowed. AHP specifically allows up to a 10% inconsistency. So compared to the average inconsistency of 500 randomly filled matrices if this scenario can be created and we compute inconsistency level, so if it is below 10%, then that is typically allowed in AHP. Now for this, how do we do this consistency check.

So typically, most of the software that can be used for AHP, they allow this, they have this calculation and it will actually indicate whether a matrix needs to be reconsidered due to its high inconsistency. So for a particular comparison matrix if the consistency ratio or any other matric that we are using for consistency check if that comes out to be greater than the allowable limits, then of course we need to reconsider that particular comparison matrix.

Now, this brings us to the next step of AHP that is fourth step, sensitivity analysis. So as we have talked about, the first two steps are mandatory steps for us to be able to perform or complete the AHP analysis; however, the third and fourth step that is consistency check and sensitivity analysis, they are optional but recommended.

(Refer Slide Time: 10:20)



So what we do in sensitivity analysis. So, this is typically performed to check the robustness of results, whether they are sensitive to changes in inputs or other parameters. So for example, in the specific case of AHP if the decision makers would like to change their subjective preferences in terms of pairwise comparisons to by a small amount, whether that is going to change the ultimate ranking or ultimate results that we get, so small changes should not be able to change the final results.

Otherwise the usefulness of the model, the repeatability of model, the generalized ability of the AHP technique and method would be under question. So therefore, this sensitivity analysis will help us in the sense when we apply AHP to solve a particular decision problem whether the results can be trusted, in the sense minute smaller changes in the input variables, inputs or other parameter, they should not be changing the final results.

If it happens the otherwise, if certain changes in inputs or other parameters if they lead to changes in the results, then we call it that the results are sensitive. So another advantage of sensitivity analysis is that it allows to generate different scenarios. So under sensitivity analysis, we are talking about making certain small changes into input parameters, therefore we can create different scenarios depending on different input parameters, for example criteria.

So if we do small changes in criteria 1, if we do small changes in criteria 2, and if we do small changes in criteria 3, how the changes are going to impact the final results. So different scenarios in this fashion can be created and we can always check for the robustness of results, whether the results are still coming out to be the same, or if we just change few values here and there whether in the criteria or pairwise comparisons, whether the results are changing or not.

So different scenarios can be created and that creates us a better picture in terms of understanding the reliability of our results, that is why typically sensitivity analysis is recommended. Now input data, so what we typically do in sensitivity analysis is that input data is slightly varied to observe the impact on the results. So given the techniques or model that we are using in our calculation under a particular method, so in this case AHP, so given how the priorities are going to be created if we change the input data.

If we vary the input data, how that is going to impact the results. So analyzing this particular aspect comes under the sensitivity analysis.

(Refer Slide Time: 14:25)

ANALYTIC HIERARCHY PROCESS (AHP)

- Example in RStudio
  - For installation steps refer my previous course
    - 'Business Analytics & Data Mining Modeling Using R'
  - Goal: Choice of a small car
  - Open RStudio

So what we will now do is we will take an example and do an exercise where we will take this choice of a small car, this is our goal. So this is an example that we are going to take forward and we will do an AHP modeling in RStudio. Now if you are not familiar with RStudio and R environment, then you can refer to my previous course Business Analytics and Data Mining Modeling using R where you can find the relevant lecture where you can actually look at the steps which are to be used for the installation purpose.

So typically, first we need to install the latest version of R depending on the operating system you have. So if you are running a 64 bit operating system, then probably you would like to install the 64 bit version of R, and once you have installed R, then you can download the RStudio binary or setup file and depending on the operating system again if it is 64 bit, then you download the same and install it. So first, R is to be installed and then RStudio is to be installed.

So if you want more detailed step, you can obviously refer to my previous course and follow those instructions to setup your environment. The example that we are going to take in this particular lecture is choice of a small car. So let us open RStudio. **(Video Starts: 15:52)** So here let us open this file AHP.R. If you are not familiar with an RStudio environment, then again you can refer to my previous course where I have talked about this RStudio environment and different panels that you can see here and what is the role of these panels.

Briefly, I will tell you the role of these different panels that you can see in this interface. So, this is the typical interface of RStudio that you are seeing right now. This top left panel in this

interface this is for R code, this is R script. So, R is a programming language written for statistical and data mining and also for decision making problems. So this programming language can actually be used to write your own code and build your model, test your models and do everything that is required.

So that code is typically written in this particular panel top left panel and you can see R script is written here. So typically if you want to write a new script then you can go to file and when you go to file you can see new file R script. So if you click here, then a new file would be opened just like this and you can of course rename that file and start writing your code. For this exercise, I have already written the code that is required for our decision problem, choice of a small car. The second panel that you see at the bottom left corner of this interface, this is console.

So any particular R code that I execute here, the results and the whole execution process would actually be displayed in this console. So if I take the first line of the code, so this is here so I need to take my cursor here into this top left panel that is a scripting panel and I need to select the first line that I want to execute and you would see a small run button here. So you would just have to click this run and you can see look at the console panel. So this particular line of code has been executed and the results are also displayed.

So this is the console panel where any code any line is executed and the results of that execution is displayed in this panel. Then we have the third panel here which is the top right part of the interface and this typically has number of sub tabs where you have environment, history and connections. The important one for us right now is the environment, which is the default tab that would be opened in this panel, and under this tab, you would see the variable that are going to be created after our execution of the code.

So those variables are going to be created and would be displayed here in this environment section. So at any time, we would be able to see what all variables had been created till now and the values of those variables and few brief details, descriptive details about those variables can always be seen here in this particular top right panel. Then, there are other sub tabs in this interface. So there is sub tab history. So in this, all the commands that had been executed, so the history of previous commands that is going to be there.

So at any point of time if you would like to run a particular command again, then you can use this sub history tab. So these are two important tabs in this particular panel. If we come to the last particular panel of this RStudio interface, so this is the bottom right panel and here you would see in the first tab that is files different file names which are there in the current directory that is opened by the RStudio. So you can see number of files here. Then there is another important tab that is here plots.

So if I happen to create any graph in my code that particular graphic is going to be displayed in this particular tab. Then we have the third tab that is packages. So the list of packages which are actually installed in this particular instance of R and RStudio that you can see here. Then we have the help section. So this help section is an important part of this interface. So just like any other programming environment, you would like to see the manual pages of any function that you would like to use.

So in this help section, yon can always go there and in this small search box that we have here you can always type and that particular page is going to be displayed. For example if I type ahp here in the search box and I enter, then since there is no package right now installed or loaded so no results have been displayed, but once we install the appropriate packages, then those results would be displayed here . So we will see this in a moment now, we will come back to this help tab again.

Now in this help tab we will see later on that the details of the function, what all the arguments that are required to be passed in, few examples for that particular function all those details are going to be available. This is just like manual of different functions that can be used in R environment. Then, we have another important tab here. So if we want to have a look at the data, any file that is there, then any other details, it is always going to be opened in this particular tab.

So now let us come back to our R script code. So this is the most important panel. So always we are going to start from this top left panel where we have written our R script. So let us go through this this R script that I have written for this particular exercise. So this particular R script has been written using MCDA package that is available. So one important aspect here is that we need to check whether the packages that we are going to use whether they are installed or not.

So first what we are going to do is, this is the part that we are going to do manually here. So we will check whether this package is installed or not. So directly, we will run the installation function. We will call the installation function and if the package is not installed, it will be installed, otherwise we will get the message. So before we move ahead into executing our R script, we need to check whether a particular package is installed or not. For example as you can see that we are going to use MCDA package in this particular script.

So, we need to find out whether this package is installed or not. So directly, we will use the installation function, if the package is installed, it would be installed, otherwise we will get a message. So this is the name of the function install.packages and then within the double quotes we can specify the name of the package. So in this case, this is MCDA. So we will just hit enter and so this package is being installed right now. So this is how you can proceed with installation of any packages within the RStudio environment.

So we just saw one example with the MCDA package. Similarly another example, I will try with another package that we would be using later on. So again as you can see here once you start typing the name of the function, suggestions are already inbuilt into RStudio interface and you will see install.packages being displayed here. So we just need to click here and go to the parenthesis and use double quotes and name of the package. So the name of the package has to be exactly entered the way it is, so it is case sensitive.

So therefore, the name of the package has to be installed you should be aware about exact name of the package. So again, I will hit enter and you would see that installation process has started. So any error that you might get while you are executing your R code which is referring to that particular package is not installed, then you can always go to console, type install. packages function and just pass the argument, name of the package as the argument and hit enter and that package is going to be installed.

So you can see that now AHP package is being installed. So both MCDA and AHP, these are two important packages that we are going to use in this course and MCDA also has a functionality, a specific function for AHP technique, so that also we are going to use and we will use AHP package itself. So we will see what are the differences between these two packages, and depending on our requirements and needs, we can always select which

package to use. So let it install and then we will start the execution of the R script and solve our decision problem.

In meantime while this installation takes place, you can see that in this example in the R script, the goal is choice of a small car. I have 3 criteria that is the style, reliability and fuel and then 4 alternatives are there. So these are the names of small cars; Corsa, then you have Clio, then Fiesta and Sandero. So these are 4 alternatives that we have. We have 3 criteria and the goal. So 3 levels that we talked about, the minimum 3 level that we require in AHP, so that we can see here.

Then we need to get responses from decisions makers in terms of comparing these criteria, so which criteria they would like to give more importance. So importance of the criteria has to be reflected in pairwise comparisons, so this we will see. So we have already talked about the scale that we typically use in AHP that is 1 to 9 fundamental scale. So here we can always express whether particular criteria or alternative A how many times it is more preferable in comparison to criteria or alternative B or it could be the reverse.

So the values are typically going to be like 1, 2, 3 or 4 times or 1/2, 1/3, 1/4 which means the reverse, that means criteria B is more preferable. So in that sense, the decision makers have to give their response. so typically as we have talked about, they will give their verbal response and we will be converting them into numerical scales. So in this particular exercise, we are going to create a comparison matrix based on hypothetical responses. So the installation is still going on. So let it complete.

So in the meantime, we can look at the first line of the code in the R script. So you can see is style. So style is the first criteria that we have mentioned here. So style, reliability and fuel 3 criteria. So first we will see for the style, the pairwise comparisons of alternative for style criterion. So with respect to style criterion, how alternatives had been compared so that are mentioned here. So you can see matrix is the function that can be used to create this matrix, number of rows 4, number of columns 4, so this is 4/4 matrix.

Therefore, we need to input 16 entries. So you can see 16 values had been used here; 1, 0.25, 4 and 1/6 and so on. So once this matrix is considered, we can always name the columns and rows. So appropriate names for columns and names have been given because we are

comparing alternatives right now. So there is going to be in the rows also we will have the name of the alternatives and the columns also we will have the name of the alternatives. So these are the entries that we need to give to construct the comparison matrix. **(Video Ends: 31:30)**

So we will start with the execution of this particular code in the next lecture and see how the AHP modeling can actually be implemented in RStudio environment for this particular example. Thank you.