**Lecture – 4**
**Analytic Hierarchy Process (AHP) – Part I**

Welcome to the course MCDM Techniques using R. So in previous lecture, we completed our discussion on the introductory part of this course. Now in this particular lecture, we are going to start our discussion on the first technique of this course, first MCDM technique that is AHP that is Analytic Hierarchy Process. So let us start.

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# ANALYTIC HIERARCHY PROCESS (AHP)

- AHP was developed by Saaty (1977, 1980)
  - Particularly useful when unable to construct a utility function
  - Four steps to obtain the ranking of alternatives
    - Problem structuring
    - Priority calculation
      - Based on the pairwise comparisons
      - A relative verbal appreciation is used instead of a numerical judgement
    - Consistency check (optional but recommended)
    - Sensitivity analysis (optional but recommended)

So AHP, Analytic Hierarchy Process, was actually developed by Saaty during 1977 and 80. So as we know that MAUT method if sometimes is difficult for decision makers to construct utility function, then AHP could be a suitable MCDM method that can be used to solve decision making problems. So that is the first point, particularly useful when unable to construct a utility function. Now if we look at AHP in terms of steps, so generally the steps to actually follow in an MCDM and MADM method, we have covered into the introductory lecture of the course.

Specifically for AHP, we have four steps that are required to obtain ranking of alternatives. Typically AHP is used to solve ranking decision problems. So let us talk about these four steps. So first one is problem is structuring. So in this step, we structure our problem in the sense the goal, the criteria to be considered, and the alternatives to be evaluated and the

framing and formulation of the problem and decision makers who are going to respond. All those things will come under this step that we need to decide.

The second step is on priority calculation so once we have done our problem structuring, so we know the decision makers, we understand the decision problem, goal, criteria and alternatives. We can take responses from decision makers with respect to our goal for criteria and with respect to our criteria for alternatives, and these responses are taken using pairwise comparisons. So that is what you see here the priority calculation this step is about is based on pairwise comparisons, and instead of using a numerical scale, we typically use a verbal scale.

So a relative verbal appreciation is used instead of a numerical judgment because as we have talked about in the introductory lecture, it might be difficult for decision makers to use a numerical scale while comparing criteria or alternatives. So a verbal scale is used and then it is mapped to a numeric scale. Now problem is structuring and priority calculation. These are the two main steps of AHP. So once we structure the problem and we do our computation for priority, then AHP will get the ranking so the method and implementation would be over.

However, two recommended steps, two additional steps are also there which can also be completed. So the third step is consistency check, so this is optional but recommended. So what might happen is that because of the number of pairwise comparisons that decision makers have to perform, there might be some contradiction that could creep in the pairwise comparisons that are to be performed and which might lead to inconsistency in the results, the final ranking that we produce.

So how do we take care of this problem, how do we do a consistency check to find out whether our pairwise comparisons are okay or not. So that is part of this particular step that is which we refer as consistency check. Then fourth step that is sensitivity analysis, this is optional but recommended. So once we have got our results, so how do we confidently say that this ranking is final and is a robust ranking that we have got after applying AHP and small changes in input values or other variables are not going to change the results.

So robustness of results, how do we find out this aspect. So all this comes under sensitivity analysis. So this is the fourth step, optional but recommended. So this will give us an idea

about how good our results are. Let us move forward. So now, we will start discussing each of these steps one by one. So let us start with the first one that is problem structuring.

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## ANALYTIC HIERARCHY PROCESS (AHP)

- Problem structuring
  - Main idea is *divide and conquer*
  - Breakdown the problem
    - By structuring according to a hierarchy: minimum three levels
    - Top element is the goal of the decision problem
    - Second level of the hierarchy represents the criteria
    - lowest level represents the alternatives
    - Additional levels representing the sub-criteria can be added

So the main idea in this step of problem structuring is that divide and conquer. So as you would see in the name of this particular method AHP that is Analytic Hierarchy Process, we actually use hierarchical structure there. So in any typical hierarchical structure, you would see one element and then it is divided into in the next hierarchical level you have two elements, then later on third list, you will have many more number of elements.

So, it is like a tree like diagram and starts from the root note. We keep on dividing and keep on moving towards more number of elements. So it is like divide and conquer is the main idea here. So we are looking to divide, we are looking to structure the problem using the hierarchical process. So how do we typically breakdown the problem. So we structure according to hierarchy as I talked above. So, there are going to be minimum three levels, so there could be more than three levels depending on the decision problem, depending on the criteria that are going to be used.

If there are just criteria, sometimes there might sub-criteria so within four with respect to each criteria, there could be number of sub-criteria as well. So that might lead to increase in the number of levels here. However, minimum three levels are required. The top level there is going to be just one element that is top element that is goal of the decision problem. So when you think about tree and a hierarchical structure, the top element just one element is going to be there.

This element is going to represent the goal of the decision problem. Then, we will come down to the second level in the hierarchy and the second level of the hierarchy represents the criteria. So the criteria that we might have criteria 1, criteria 2, criteria 3, criteria 4. So depending on the number of criteria, we will have those numbers of branches coming out from the top element of this structure, this hierarchical structure.

So now as I said, in the second level which is about displaying structuring criteria if there are sub-criteria within each criteria in the second level, then of course number of levels will increase accordingly. On the lowest level which typically is the third level as well when there are just minimum three levels, then the lower level represents the alternatives. So as I have talked about that there are going to be pairwise comparisons among alternatives with respect to each criterion.

So therefore the second level branches that we have with respect to each criterion, we are going to have the alternatives which are going to be compared. So in the third level, we will have the alternatives and the pairwise comparisons would be with respect to the immediate upper level, that is criteria. So as I said additional levels representing the sub-criteria can also be added. So this is the main breakdown of the problem.

So we have to figure out the goal, formulate the goal, and then we have to find out the relevant criteria to solve that particular decision problem and then the alternatives that are to be evaluated based on those criteria and given the goal that we have. So using these elements, we can structure our decision problem

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# ANALYTIC HIERARCHY PROCESS (AHP)

- **Example: Shop location problem**
  - A new sports shop in one of three different locations (alternatives)
    - A shopping centre
    - The city centre
    - A new industrial area
  - Criteria
    - Visibility
    - Competition
    - Frequency
    - Rental cost

So let us discuss further with the help of this example which is about a shop location problem. So if we are looking to open a new sports shop and we are looking to identify locations so we have a number of location, so those are actually the number of alternatives, where we can actually open our sports shop. So these locations are shopping centre, city centre and a new industrial area. So we have these three locations and we are looking to open a new sports shop. So how do we arrive at the best location for our new sports shop.

So if we look at the criteria that could be useful for this decision problem, these criteria could be visibility, competition, frequency and rental cost. So we are looking to characterize these alternatives, so what these alternatives are about. So typically shopping center is the place where there are more number of people, more number of shops are there, and a generally kind of customers and people would be coming to this shopping center locations.

If we talk about the city center, this is more like a happening place where typically youth and other customers might be arriving and some events might be organized there. So if we think about the sports shop, so typically the targeted customer would be youth. So therefore, city centre might be more suitable in that sense. However, shopping center because it is attracting all customers across the segments, so therefore shopping center is also there.

New industrial area, so these industrial areas are typically outside the city in the suburban area of the city, so it is going to be a new area. So in terms of customers visiting the area so that frequency is going to be on the lower side and mainly people who are working around the area they would be visiting the place. So this is the main characterization of these three

alternatives and the characterization itself indicates about what kind of criteria could be useful in solving this decision problem.

So visibility as we talked about city centre, any new shop that you open in a city center, it will have good visibility. Shopping center, higher visibility because of the number of people that will come there for shopping. Then industrial area, probably it will have low visibility because it is outside the city, so only people in the nearby areas will visit. Then if we talk about the competition, this could be another criteria that we can consider. In the shopping center, there would be a number of shops there, same goes for city center.

However, in industrial area, there might not be enough number of shops or similar shops. So that is one characterization with respect to competition. If we talk about frequency, as I said frequency typically would be more on the higher side for city center and shopping center and on the lower side for industrial area. We talk about the rental cost, then industrial area scores in this aspect because the renal cost would be on the lower side because this is outside the city. So not many people would be looking to rent a shop, so therefore the renting cost will come down.

Shopping center, there shops of all kinds would be looking to rent a place there in the shopping center, so the renting cost would be on the higher side. Now city center, it is going to be the happening area, so of course the renting cost is going to be much higher in this. So given the characterization that I have given and the criteria that I have talked about and these three alternatives, now we will talk about how we can apply AHP technique to solve this problem. So problem is structuring the step we have talked about and in that we talked about three levels; goal, criteria and alternatives.

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## ANALYTIC HIERARCHY PROCESS (AHP)

Traditional representation of the hierarchy
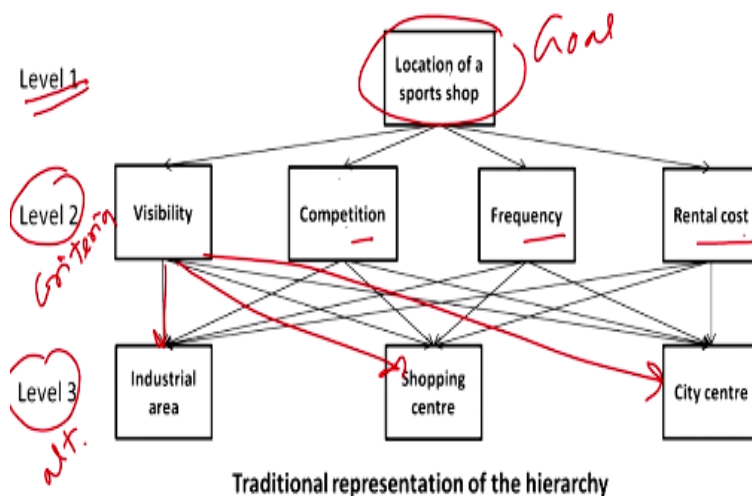
So here new sports shop, for this the traditional representation of the hierarchy the way it is done is as you can see in the slide. So you can see level 1 here and in the level 1, we have the goal which is location of the sports shop, so this being our goal, so this is the top most element. So as I said from the top element, there are going to be number of branches you can see level 2. Now level 2 is for criteria. So you can see 4 criteria that we talked about; visibility, competition, frequency and rental cost, so you can see them here in the second level.

So these criteria when we talked about pairwise comparisons that decision makers have to perform, so they would be comparing these criteria with respect to the immediate upper level that is location of a sports shop that is the goal of the decision problem. Now in the third level, we have the alternatives, so here you would see three alternatives that we have named here, but if we look at in terms of number of arrows that are emanating from criteria to alternatives you can see.

Visibility 3 arrows are going out and for these 3 alternatives so 1 and then 2 and then 3, similarly for competition, similarly for frequency and similarly for rental cost. So, these three alternatives are going to be compared with respect to each of these criterion. So this is how we typically structure a decision problem if we are looking to apply AHP. So top level goal, then we have criteria and in case we have sub-criteria then another level is going to be added just below level 2, and the last level is typically always going to be the level for alternatives and then arrows I have already explained.

So let us move forward. So as I said each lower level is prioritized according to its immediate upper level, so that is criteria with respect to goal and alternatives with respect to criterion.

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## ANALYTIC HIERARCHY PROCESS (AHP)

- In AHP, each lower level is prioritized

  - According to its **immediate** upper level

  - Appropriate question to ask with regard to prioritization depends on
    - The context and
    - Sometimes on the decision maker

Appropriate question to ask with regard to prioritization. So when we are looking to take responses from decision makers while doing these pairwise comparisons among criteria or among alternatives, then what are going to be the appropriate question that we can ask, so that depends on the context of the decision problem that we have and also the decision makers. So based on that, we can actually frame our questions. So let us see a few examples.

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## ANALYTIC HIERARCHY PROCESS (AHP)

- Example: Shop location problem
  - Prioritize criteria of level 2
    - With regard to the goal
    - An appropriate question would be:
      - 'Which criterion is most important for choosing the location of the sports shop and to what extent?'
  - Prioritize alternatives in level 3
    - With regard to each criterion in level 2
      - 'Which alternative is preferable to fulfil the given criterion and to what extent?'
  - Total **five** different prioritizations are required
    - One for criteria and four for alternatives

So shop allocation problem that we have discussed. If you are looking to prioritize criteria of level 2, so as I said this is going to be done with regard to the goal and an appropriate question would be which criterion is most important for selecting the location of a sports

shop and to what extent. So as you would see that importance of criteria that is being asked through this questions, which criterion is most important for selecting the location of the sports shop and to what extent.

So the decision maker would be responding in terms of verbal scale and we will talk about what verbal scale is typically used. So that verbal scale will be used by the decision maker and they will express their response using that scale that would reflect because we are doing pairwise comparison so between 2 criteria, we will get a response and that would be reflective of importance and the extent of that importance.

Similarly if we talk about prioritizing alternatives in level 3, so this is going to be with regard to each criterion in level 2 as you can see in slide and we have talked about this so many times. Now an appropriate question for this kind of pairwise comparison could be which alternative is preferable to fulfill the given criteria and to what extent, so with respect to criterion which alternative is fulfilling that particular criterion and what extent. So again same verbal scale we can use and the decision makers can express their responses using the scale.

So if we talk about this problem, shop location problem, so we have just one level for criteria there is no level for sub-criteria. So one prioritization would be among criteria so that is 1 and since we have 4 criteria and the comparisons between alternatives have to be performed with respect to each one of them, so therefore 4 prioritization would be there for 4 criteria, so total 5 prioritization would be required; so 1 for criteria and 4 for alternative, this 4 is coming because we have 4 criteria there.

So, total 5 prioritization. So this was about the first step that problem structuring and now let us talk about the second step priority calculation. So before we move ahead, let us talk about what is priority.

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# ANALYTIC HIERARCHY PROCESS (AHP)

- Priority calculation
  - A priority is
    - A score that ranks the importance of the alternative or criterion in the decision problem
  - Three types of priorities have to be calculated:
    - Criteria priorities
    - Alternative priorities
    - Global priorities

So by priority what we mean is a score that ranks the importance of the alternative or criterion in the decision problem. So when we are asking for pairwise comparisons, then it is actually we are looking for importance to be reflected. So this particular score is going to rank that importance when we are comparing criterion to criterion or we are comparing alternative to another alternative. So this is a priority. So in this step, we will talk about the steps that are required to compute this.

So typically, there are three types of priorities that we have to calculate. The first one is criteria priorities. So first among the criteria, we have to do pairwise comparisons. So based on that, we will be computing criteria priorities. Then alternative priorities is there. So with respect to each criterion, we are going to have pairwise comparison so that information is to be used while we are computing alternative priorities.

Then global priorities where we typically combine the criteria priorities and the alternative priorities that we have got in the previous steps and then we compute the global priorities. So these three types of priorities are there. So let us talk about each one of them.

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## ANALYTIC HIERARCHY PROCESS (AHP)

* Criteria priorities
  - Importance of each criterion w.r.t the top goal
* Alternative priorities
  - Importance of an alternative with respect to one specific criterion
* Global priorities
  - Calculated using the criteria & alternative priorities
  - Rank alternatives w.r.t. all criteria and consequently the overall goal

So criteria priorities as I said importance of each criterion with respect to the top goal that is reflected. In the alternative priorities, importance of an alternative with respect to one specific criterion so that is reflected. In the global priorities, calculated using the criteria and alternative priorities as I have said, so this global priority is actually going to indicate the rank. It is going to rank alternatives with respect to criteria and consequently the overall goal. So this is about the different types of priorities that we need to calculate

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## ANALYTIC HIERARCHY PROCESS (AHP)

* Same technique is used
  - To calculate criteria & alternative priorities
    * Pairwise comparisons
      - Easier and more accurate to express a preference between only two elements than simultaneously among all the elements
      - Generally evaluated on the fundamental 1-9 scale
      - In practice, other scales are also in use
    * Conversion from verbal to numerical scale
      - See table
    * Comparisons are collected in a matrix
      - See matrix for Shop location problem

Now how this computation is actually performed. So the same technique is used whether for criteria prioritization or for the alternative prioritization. So, this is pairwise comparisons that we use. So why pairwise comparisons are used. So it is understood that typically if we are looking to express a preference, so we are asked to give preference among the number of

elements, number of items, then it might be difficult for us to give a certain importance, indicate a certain importance, if there are more number of items.

However, if we are just competing just two elements, then it might be easier for us to compare these two and indicate the importance. So that is why the pairwise comparisons are considered to be easier and more accurate to express a preference, so that is why it is preferred. In terms of scale, so typically we use a fundamental 1 to 9 scale. So there are other scales also which are available which have been used also.

For example, 1 to 5 scale can also be used, but sometimes analysts and researchers might feel that the level of detail that is to be captured might not be adequately captured by this 1 to 5 smaller scale, so probably they will need a higher scale 1 to 9 scale. So that is why 1 to 9 scale is the most popular scale, mostly used scale. Some people have also suggested 1 to 100 kind of a scale, much larger scale that can also be used, but of course, it is going to be very difficult for decision maker.

The decision maker might get lost while expressing their preferences when using such a large scale, I know 1 to 100 scale or other scales. So other scales are there, they are in use as well, but typically most popular being the fundamental 1 to 9 scale.

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## ANALYTIC HIERARCHY PROCESS (AHP)

| Table: The 1–9 fundamental scale | |
|---|---|
| Degree of importance | Definition |
| 1 | Equal importance |
| 2 | Weak |
| 3 | Moderate importance |
| 4 | Moderate plus |
| 5 | Strong importance |
| 6 | Strong plus |
| 7 | Very strong or demonstrated importance |
| 8 | Very, very strong |
| 9 | Extreme importance |

So let us now see the conversion from verbal to numerical scales. As we talked about that we take responses from decision makers using this verbal scale, you can see in the slide. So first column is reflecting the degree of importance here. You can see 1, 2, 3 up to 9 and what we

mean by 1, so it is indicating equal importance, 2 is indication weak importance, then 3 moderate importance, then 4 a moderate plus, 5 a strong importance, 6 strong plus, 7 very strong or demonstrated importance.

Then we have 8 which is very, very strong and then 9 extreme importance. So, we would see it is like increasing scale, 1 to 9 scale, and given two criteria or two alternatives, decision makers are supposed to express their preference using the verbal scale and then it can easily be converting to a numerical scale as you can see in this table.

Now let us talk about comparisons. So comparisons are typically collected in a matrix. So as we talked about in the hierarchical structure that we talked about because first the comparisons would be among criteria so four criteria that we have four shop location problem, so it is going to be 4 versus 4. Each criterion is going to be compared with some other criterion in the set.

Similarly, for alternatives, we are going to collect the data again in a matrix format. So each alternatives, there we have three alternatives, so we are going to have 3 by 3 matrix. So each alternative is going to be compared with the another alternative of course with respect to a specific criterion.

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## ANALYTIC HIERARCHY PROCESS (AHP)

| | Visibility | Competition | Frequency | Rental costs |
|---|---|---|---|---|
| Visibility | 1 | ¼ | 1/5 | 2 |
| Competition | 4 | 1 | ½ | 1 |
| Frequency | 5 | 2 | 1 | 4 |
| Rental costs | ½ | 1 | ¼ | 1 |

Comparison matrix for Shop location problem

So one example you can see here in the slide, so this is a comparison matrix for the shop location problem that we are discussing. So we have four criteria here as you can see. So we have four rows corresponding to each criterion and four columns corresponding to again each

criterion. Now visibility compared to visibility is going to be 1, so the diagonal elements as you can see they are all 1, So the main comparison are actually either in the upper triangular part of it or the lower triangular part of this matrix.

You would also see that if you are comparing visibility with competition and when we are comparing competition with visibility, so this value, so would see that this is kind of repetition here. So actually, the number of actual comparisons can be reflected either by the upper rectangular matrix or the lower one. So this is in a sense reciprocal matrix. So let us talk about the comparison matrix in more detail, but before we go there one important aspect here is in terms numbers as you can see.

The scale we saw 1 to 9. So if we are comparing visibility with competition, so here you would see a value of 1/4 so this is actually indicating that competition is being given more preference in terms of visibility right. So similarly if we look at frequency and visibility, so this value is 5. So you can see frequency is being given more importance in comparison with visibility.

So what we mean by these numbers actually 5, 4 and all those number that we have seen in this scale itself. However, when we say 1/4 so it is you can take it though this is for visibility with competition, but value of it being 1/4, it actually means that competition is being given more importance with respect to visibility.

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# ANALYTIC HIERARCHY PROCESS (AHP)

- Comparison matrix
  - All comparisons are positive
  - Comparisons on the main diagonal are 1
    - A criterion is compared with itself
  - The matrix is reciprocal
    - The upper triangle is the reverse of the lower triangle
  - No. of necessary comparisons for each comparison matrix is

  $$\frac{n^2 - n}{2}$$

  Where n is the no. of elements

Now let us talk a bit more about comparison matrix. So all comparisons are positive as you can see, we also saw the scale, so comparisons all values are going to be positive. Comparisons on the main diagonal are 1 as I said a criterion if it is compared with itself that value is going to be 1. The matrix is reciprocal, as I discussed the upper triangle is the reverse of the lower triangle because it is the same comparison that is just the different way of representing the same information, so that is there.

Then if we want to find out how many necessary comparisons one has to perform. So that we can derive using this formula n square-n/2 where n is the number of elements. So we look at the previous matrix this was for criteria. This is 4 by 4 matrix. So if we look at this, then we had 16 values in this matrix so which is reflected by n square. So we had 16 here and then there are diagonal elements so which are comparison of element with itself, so if we subtract this right, and then since the upper triangle and the lower triangle those values is just the reverse, so we are just dividing this number by 2.

So this will give us this formula in the sense will give us the necessary comparisons that are required in each comparison matrix. So required number of comparisons can be very high if you increase the value of n, if there are more criteria to be considered, then of course the required number of comparisons will go up. Similarly if there are more alternative to be evaluated, then of course the required number of comparisons will go up.

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# ANALYTIC HIERARCHY PROCESS (AHP)

- Comparison matrix
  - Required no. of comparisons can be very high
    - As the value of n increases
  - Software will use these comparison matrices
    - To calculate criteria & alternative priorities

So these comparison matrices that we talked about whether for criteria or for alternatives, so these are actually used in the software to calculate the priorities for criteria and alternative. So

in that sense, comparison matrices become input for different functions that we use in software for MCDM and then output is produced in terms of priorities and weights**.**

So in this course as I have talked about we are going to use R platform, so we will see in coming lectures what functions and how the input is used to compute priorities and the ranking. So at this step, we would like to stop here.

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## ANALYTIC HIERARCHY PROCESS (AHP)

- Consistency check
  - When several successive pairwise comparisons are done, they may end up contradicting each other
    - Vaguely defined problem,
    - Lack of sufficient information (known as bounded rationality),
    - Uncertain information, or
    - Lack of concentration

In the next lecture, we will start our discussion on the third step of Analytic Hierarchy Process which is on consistency check. So as I have talked about why we need to perform consistency check and what are the different reasons because of these contradictions might appear there in the comparison matrices and how do we ensure how do we find out how much minimum consistency is required. So all this, we will discuss in the next lecture. Thank you.