MCDM Techniques Using R
Prof. Gaurav Dixit
Department of Electrical Engineering
Indian Institute of Technology- Roorkee

Lecture –15
VIKOR Part – I

Welcome to the course and MCDM techniques using our previous few lecture we completed our discussion on TOPSIS. So, in todays lecture we are going to start our discussion on another technique that is called the VIKOR. This technique is also quite similar to TOPSIS income soft procedure. So, let us start our discussion so what a VIKOR stands for as you can see in the slide.
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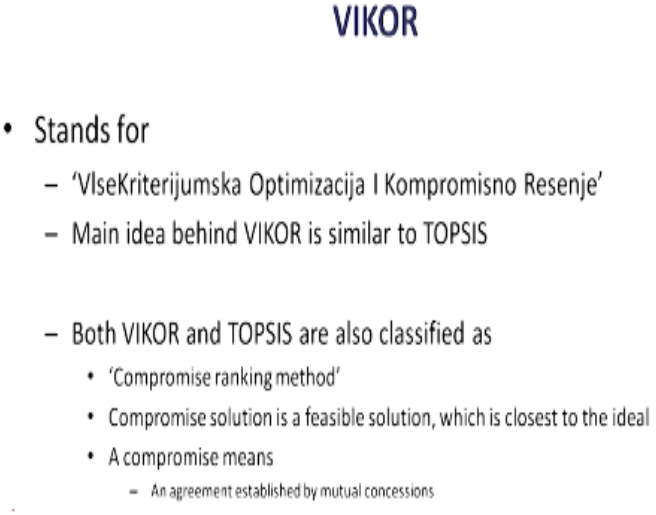

Then the main idea behind the VIKOR is quite similar to what we discussed in TOPSIS both VIKOR and TOPSIS. They are also classified as compromised ranking method in previous lecture on TOPSIS we talked about that TOPSIS is also categorized under you know distance based method or reference based approaches. A VIKOR can also be classified similarly under the distance with a method and a reference based approach.

Also you know compromise ranking VIKOR is also classified as a compromise ranking method. So, what do we mean by this? so compromised solution is a feasible solution which is close to the ideal so just like in a you know TOPSIS we were looking for a solution which were you

know which were having shortest distance from the ideal point and farthest away from the anti-ideal point.

Similarly, in VIKOR also we are looking for a solution which is close to the ideal point so the compromise word that is done that is being used here for these two techniques so it also essentially means a similar thing that compromise solution is a feasible solution which is closest to the ideal and what do you mean by a compromise here? Then an agreement established by mutual concessions.

So, that is the additional part you know in a VIKOR so VIKOR typically it is a use for 02.36 of optimization of complex systems so what we you know once we do our VIKOR modelling. So, the kind of output that we get is the compromise ranking list the compromise solution and the weight stability intervals for preferences stability of the compromise solutions obtained with the given weights.

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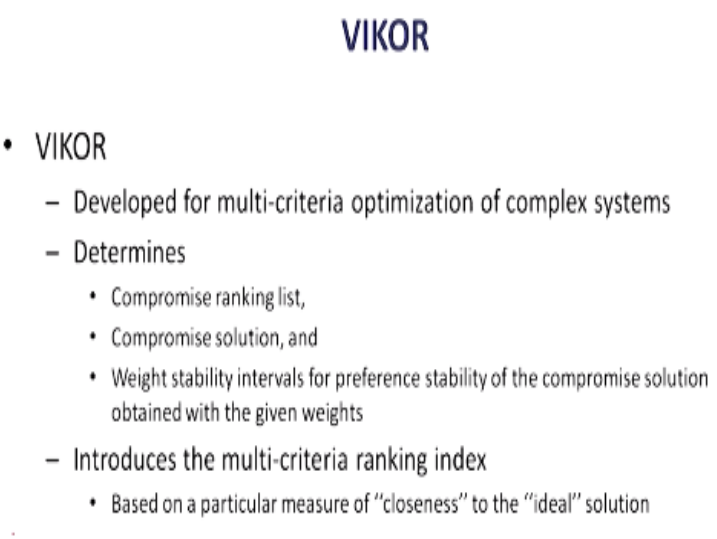

So, this more like a doing a sensitivity analysis of the solution with respect to weights so if you make certain changes certain delta changes into your criteria weights. So, how the reserves how the compromise ranking list or the compromise solution is going to change so that is also you know can be easily determined in VIKOR. Weight stability / weight stability intervals what we mean is the range off criteria weights.

Where a particular compromise ranking lists our compromise solution is going to remain the same so within that range a particular solution a particular ranking list so it is valid if you make you know if you change the criteria where it is slightly you know outside of that particular range then the list and the solution might change. So, that is what it means by you know in the 3rd point that weight stability intervals.

We also obtain in VIKOR which is actually indicating the preference stability of the compromise solution obtain with the given weights introduces the multi criteria ranking index. So, just like in TOPSIS also we get a rating index here also the similar thing. So, based on the particular measure of closeness to the ideal solution so they are also in TOPSIS when we discuss there also we had developed we have discussed the closeness ratio.

Which was used to finally in order to remind you know the closeness from the ideal and the ideal points similarly here we are going to use we are going to discuss a particular measure which is to be used to determine the closeness to the ideal solution. Next move forward so the major that we just talked about so how this measure is formulated. So, we are going to discuss that aspect now so this a multi criteria measure for VIKOR.

And for compromise ranking that we obtain in VIKOR is developed from LP metric the same metric is also used in developing the matrix for TOPSIS. But in this particular measure we are going to discuss this in more detail so there we did not refer to LP matrix aspect and TOPSIS lecture but here VIKOR becomes more important. So, the multi criteria measure for ranking in VIKOR that we use is based on LP metric.

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VIKOR

- VIKOR
  - The multi-criteria measure for compromise ranking is developed from 'L_p-metric'

Which is used as an aggregating function in a compromise programming method

It is defined as:

$$L_{p,k} = \left\{ \sum_{j=1}^{m} \left[w_j \left(f_j^* - f_{kj} \right) / \left(f_j^* - f_j^- \right) \right]^p \right\}^{1/p}$$

Where $1 \leq p \leq \infty$

$L_{p,k}$ represents the distance of the alternative a_k to the ideal solution

So, this is typically used in an aggregating functioning in another method referred as compromise programming method so this metric has been taken from there so how this multi metric is defined you can see in this slide so LPK where p is the parameter ranging 1 to infinity and you know the expression can see that within the curly braces a power of 1/ p and you know the inside of it.

We have this summation ranging from j= 1 to m where j is for criteria so the number of criteria that we have is known m criteria so we have total number of m criteria so the summation is ranging from 1 to m and then within the brackets what we have the power is p and again the parameter is p and within this bracket. What we have is kind of a weighted sum of something so the weighted sum of the ratio.

This Wj is actually standing for the criteria weight of j is indicating the criteria and then we have this ratio expression here where this is fj* - fkj where k is actually standing for the alternative. So, alternative or you know we are considering that they are n alternatives so k is ranging from 1 to n so this ratio is fj*-fkj /fj*-fj- in super step. So, we look at these expressions this looks like a minmax kind of a normalisation.

If you really look at it and then a way, it is also there so therefore it looks like a weighted normalization of max kind of you know formulation. So, that is being used as this kind of

formulation is being used to define what we call as LP metric so you know in VIKOR values of you know this parameter p 2 specific values are taken therefore to a specific forms of this Lp. metrics. Lp metrics is actually you know are taken for VIKOR so we will talk about those two forms. So, you know as I have said that in the previous so you know that we talked about.

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VIKOR

- VIKOR
  - Alternatives: k = 1, 2, ..., n
  - Criteria: j = 1, 2, ..., m
  - f_{kj} is the rating score of k^{th} alternative w.r.t j^{th} criterion
  - f_j^* is the best rating score w.r.t criterion j
  - f_j^- is the worst rating score w.r.t criterion j
  - w_j is the weight of the criterion j

K stands for you know another alternative so 1 to n and the j stands for m criteria you can see here. Here you can see j summation is over j 1 to m and this is for the alternative you can see alternative k 1 to n and criteria j 1 to m and as i said fkj this is the rating is score so just like in TOPSIS we talked about that we got to have the common stable and the common score of various alternative with respect to different criteria.

Similarly, here also we are supposed to have dj scores where fkj is denoting the rating score of k as alternative with respect to j criterion. So, we go back to the previous mathematical equation we are talking about here so this ratio overall this ratio expression that we have in this and you know this is fkj so that is what I meant by you know max kind of normalization. So, here * is best and – is worst rating score with respect to the criterion j.

We go back to the previous equation it looks like you know max - the value divided by max - min. So, that is why I know when I said that it looks like maximum when kind of you know

normalisation and then Wj is weight of the criterion j so this is about the Lp metric that is actually used to determine the major and VIKOR.

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VIKOR

- VIKOR
  - In VIKOR, $L_{1,k}$ (denoted as S_k) and $L_{\infty,k}$ (denoted as R_k) are used as boundary measures to formulate the ranking measure

$$S_k = \sum_{j=1}^{m} w_j \left(f_j^* - f_{kj}\right)/\left(f_j^* - f_j^-\right)$$

$$R_k = \max_j \left[w_j \left(f_j^* - f_{kj}\right)/\left(f_j^* - f_j^-\right)\right]$$

Now what parameters so what values of a parameter p is actually taken for the VIKOR method so L1k so p = 1 and L infinite k so p being infinite so these are actually used as boundary measures to formulate the ranking measure. So, here you can see L1K is also denoted by SK and also L infinity k also denoted by RK so these 2 measures SK and RK will become important part of the VIKOR steps that we are going to talk about.

So, both these when we take the values of 1 infinite to define these 2majors you can see how the stations are going to be SK will be something like this so if you go back to Lp metric formulation you can see 1/p and p here so when you take the value of p1 and the infinite so the expression will actually change so this is what we get so as case now summation over the criteria and this is the expression Wj weights of criteria.

And then multiplied by this expression of fj*- fkj/fj*-fj - super script so this is SK so here we are taking summation and so here we you notice that SK where k is standing for alternative so this is standing for alternative so for all the alternatives we are going to compute this some as we will discuss in the steps of VIKOR. You know this is for the criteria so as we have as I have mentioned before as well so you can see.

This is weighted normalised score kind of expression and summed over and criteria this weighed sum is actually taken for all the alternatives you know individually so this is what we have the major as SK then we have RK so here if you see the same expression weighed in a score then we are just trying to compute the maximum value so this is with respect to j so out of all the criteria that we might have we are going to find out the maximum value.

That is going to be defined as another measure RK so let me take you back to the Lp metric expressions. So, one important aspect of this is what this expression is actually representing so here you can at the last line of the slide you can see this that this Lp metric is actually the presenting the distance of alternative Lk to the ideal solution. All right so this is the whole formulation is in a way it is a distance metric.

So, that is when I said at the start of this lecture that even VIKOR is also categorized as classified as a distance based method. So, here the metric Lp metric that here that we are using is also a kind of representation of distance of an alternative k Lk to the ideal solution so now the same representation is being used here also for SK and RK also.

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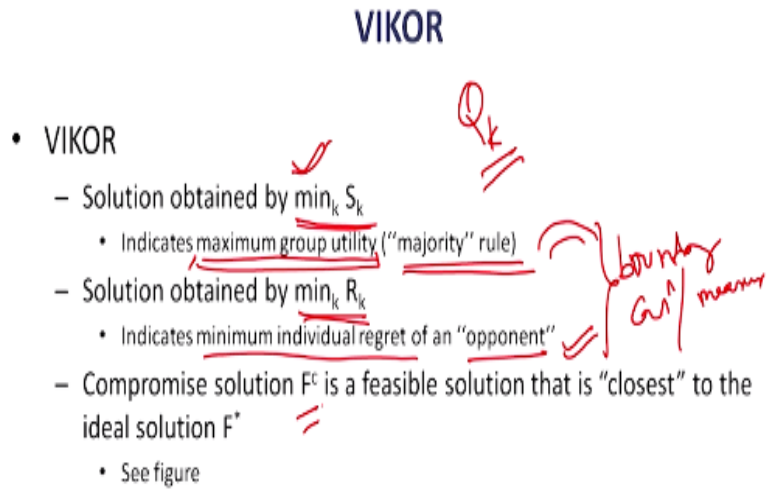

So, let us move forward. So if we obtain a VIKOR solution using min that means all the as values for different alternatives that we did mine and find the minimum of a minimum as well.

Then that would actually indicate a maximum group utility you know majority rule so what do we mean by this? This in the sense if you go back to the expression this is the summation over criteria right.

So, this is like you know summation of weighted scores so when we take the minimum values so it is like common denominator the common you know out of all the you know criteria that are there. So, that value is being taken so this is the maximum this is in a way indicating a kind of rule majority rule and the maximum it is indicating the maximum group utility so we take the minimum SK.

So, at least that much of you know group utilities going to be represented by this particular solution so as I said previously that the major that we are to use in VIKOR is actually based on these SK and RK as boundary measures. So, the actual measure that we are going to use in VIKOR for ranking is we will discuss later. But this SK and RK will actually serve as a boundary condition for that major as a boundary measures.

So, here when we say that you know we are trying to you know as min k SK actually indicating the maximum group utility the utility which is you know a minimum you know in a sense because this meanwhile is being taken. So, this is the best representation of the group realty that is going to be shared across all scenario so this will become a kind of a boundary measure you are all VIKOR implementation.

Then solution obtained by min RK here this indicates minimum individual regret of an opponent so if you go back to see how the RK was formalized so here we are taking the maximum value of this a way to this score and again out of all the values that we have for you know or alternatives. So, we take you know the minimum of them that is actually indicating in a way you know minimum in the individual regret of an opponent.

So, this is also in a way indicating a boundary condition so these are like a boundary measures so indicating some you know boundary conditions or boundary measures which overall how this is to be used in the final measure we will see that in this lecture. So, these are 2 scenarios the SK

and RK that we talked about and how they are going to be incorporated in the particular major that we are going to use in VIKOR.

So, we take min k of SK and the min k of RK and these are used as a boundary measures so we would actually in the final measure Qk that we are going to talk about in the coming slide this is actually going to be a you know taking a balance between these 2. That means maximum group utility and minimum individual VIKOR regrets so we are going to you know discuss a formulation a particular expression.

That is going to take a balance between these two boundary measures and that particular measure is going to be finely used for producing the obtaining the y ranking through VIKOR. Now, at this point let us also talk about the compromise solution Fc. So, what do we mean by compromise solution FC? So, we have already talked about what we do we mean by a compromised solution. so we are here indicating it using a Fc. So, this is a feasible solution that is closest to the ideal solution F* so let us understand this.

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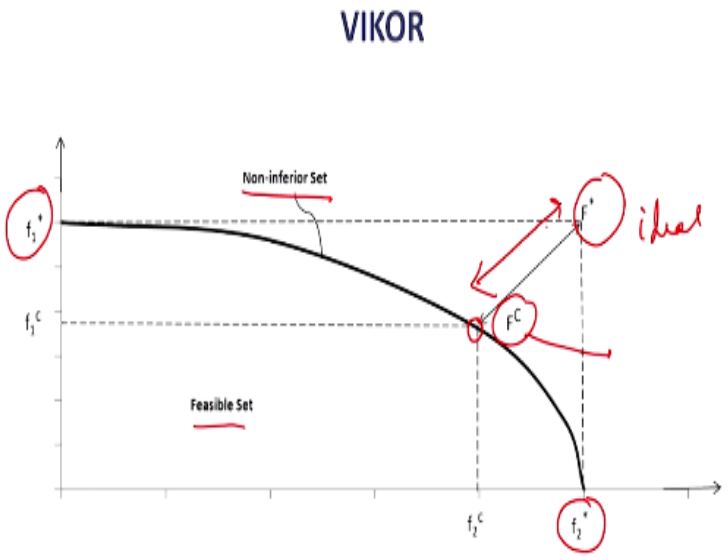

So, let us understand this through a graphic here so just like in TOPSIS you know we there also we talked about a particular you know, a number of alternative time. We talked about ideal and anti-ideal points and we were looking for you know shorter distance point ideal point and the

farthest distance from the end yodel point for a particular alternative to be the best alternative so here also similar kind of graphic we can see here.

So, here you can see that this is our ideal point so 1 difference that you can notice by now that is there in VIKOR in comparison to a TOPSIS that we are just comparing with the ideal point. So, we are there is no anti-ideal point in case of VIKOR. So, this is our ideal point so that we talked about and this is our compromise solution so these are 2 criteria 1 is f1 along y-axis then other F2 along the x axis.

And you can see here so these are probably the best values so if we take a car here so along these 2 criteria this Fc somewhere here so this is in ways representing you know a feasible solution which is closest to the ideal. So, this in a way it also gives us the feasible set here and the other part become the non- inferior set so out of all the points which are part of this line so they would become non-inferior set and this would we ever feasible solution.

And we are feasible solution as we talked about it should have the shortest distance from the ideal point. So, this is what we do actually in VIKOR modelling. So, let us talk about this in more detail.

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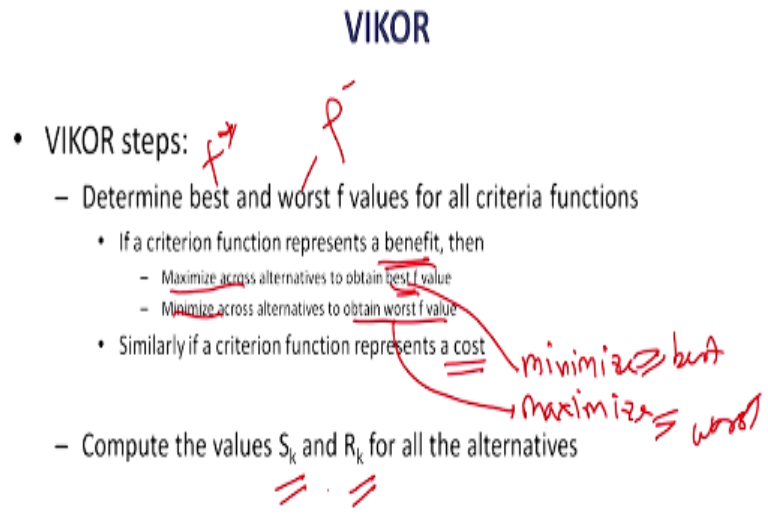

So, now we are going to do is we will discuss the steps of VIKOR and whatever we have discussed till now about different metrics like S and R. We will talk about them and then we will also discuss the final metric UK that is actually used for reducing the ranking. So, VIKOR steps first one is that determine best and worst f values for all criteria function. So, why this is the first step because if you go back and understand this you know SK and RK.

You know mathematical expression here so you can see at f*j here and you can also see fj- so fj* and fj - so these 2 values are going to be used you know for you know computation of SK and RK values for all the alternatives. So, it is better that we do this computation first so best and worst f values best indicating the f* and worst indicating alpha – so if we have for all criteria we compute these 2 values only then we can move here.

Because SK values and RK value which are actually with respect to alternatives so first step we need to compute these values. Now how this is to be done how we are going to determine what is best and worst f values of criteria so if criteria function represents a benefit. So, just like in previous techniques that we have discussed in this particular course whenever we talk about the preference direction of, you know in criteria.

So, whenever a criteria is to be maximized whenever the criteria is you know function which represents a benefit. So, we need to maximize you know across alternators to obtain best value and minimize to obtain worst value however if the criteria function is this preference direction for criteria function is actually cost then the situation will change in terms of determining the best and worst f values.

So, they are we will have to, you know you do the reverse because there is a cost function so therefore we would like to minimize the value for best score. So, the best of value would be here obtained through minimization. So, we will minimize and the worst values which actually be through maximize. So, if the cost is on the higher side so that would be representing a worst scenario and if the cost is on the lower side so that would be the presenting the best scenario.

So, this is what we mean so depending on whether the criteria benefit or cost we would be doing it accordingly. Now let us talk about, you know the next step so once we have these best and worst values where * or - these are being computed as part of the 1st system then we can go ahead and you know plug in these values in the expression that we saw for as SK and RK so we are going to compute the values SK and RK for all the alternatives.

So, here k is in the case of standing for alternative as we talked about so for all the alternatives we would be you know computing these values.

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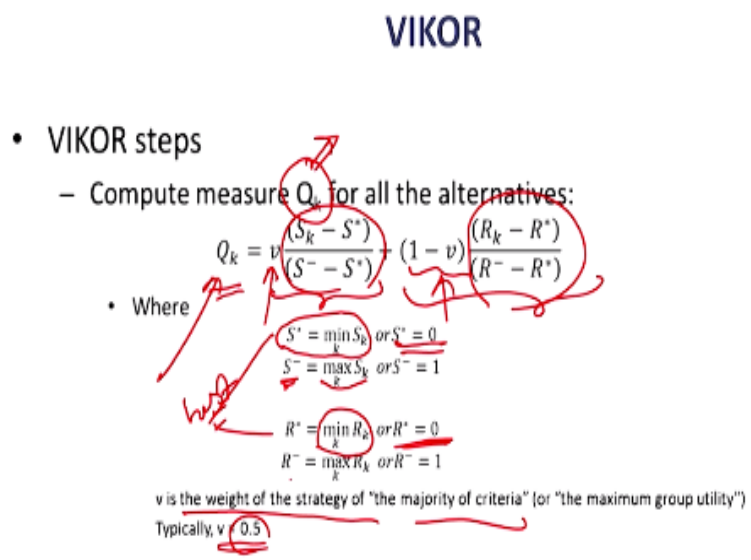

Once this is done once we have these values then now we are going to talk about in the next step our major QK so this is the measure that is actually use to produce or obtain the ranking in VIKOR and so if you look at this particular you know measure here so this measure is to be computed for all the alternatives just like SK and RK. Here if you look at this measure so we have introduced a new parameter here that this is v.

And here we have 1-v so what is this v is the weight of the strategy of majority of the criteria so this v you know typically it is representing the you know the weight of this strategy and this is strategy can be you know there could be different scenario for the strategy weight can be accordingly set. However, the default values that is you know value that is typically taken is 0.5. so that is actually indicates a scenario of consensus.

So, something that is being obtained through consensus so if you see these two expression of Q and K which have been some so they also look like you know a weighted scores here so v and 1-v are representing a weight in a sense and then we have to raise your expression here look at the ratio expression 1 is SK - as * which is defined like this as * is minimum of you know min k SK so when we talked about previously the solution.

We talked about min k SK and we talked about min k RK so they are representing the best solution here. So, these values to be plugged in here and in this expression and again get a score for Qk so you can see when we talked about that SK and RK are boundary measures and to be used later on for the metric that we are going to use to produce the ranking. So, we say Qk so here you can see SK and RK are being used.

So, we are trying to balance between you know the utility and the 2 particular aspect that we talked about. We are trying to balance here a maximum group utility versus this maximum group utility versus minimum individual regret. So, this is exactly is being represented in this occupation so through this weight you know which is in a way you know a data mining the relative importance of maximum group utility and a minimum individual regret.

We are trying to balance this out and trying to produce a metric here now if these values you know so we can take mean values or otherwise, we can take as a * as 0 and R * as 0 as – is the opposite of this because this is a blending the worst scenario. So, max k SK and then similarly R – is also representing the worst scenario then max k value is to be taken so this is about the metric Qk.

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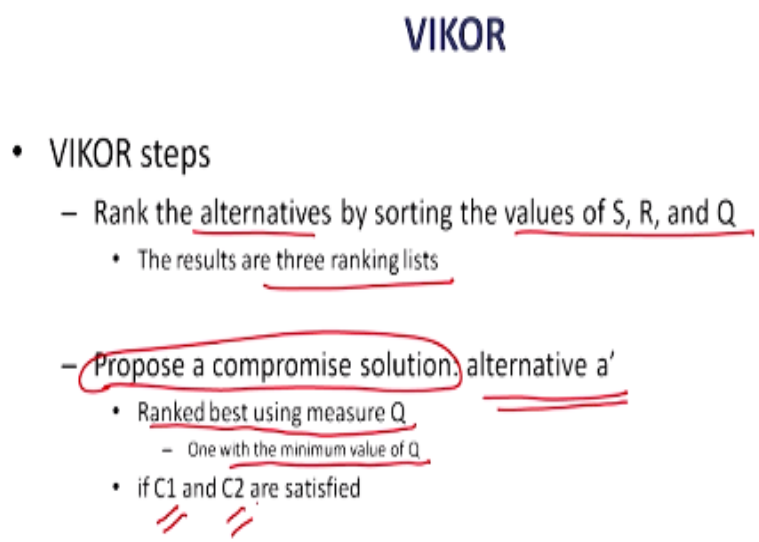

Now once we have a computer the values for all the alternatives for these 3 metrics S R and Q then we are in the next step what we are going to do is we can sort by alternatives by these values S R and Q. So, what we will get is 3 ranking lists so at the end of this results we will get 3 ranking list. Using this 3 ranking list we are finally going to remain out a compromise solution so this so therefore then once this computation part has been completed.

And we have got the values for S R and Q for all the alternatives then next step becomes more important in terms of what are the rules that we follow to determine the compromise solution or I know compromise ranking list. So, in the next few steps we are going to actually talk about this so now the next step is about proposing a compromise solution that is alternative a‘ so this is something that we are going to propose.

So, what steps we follow so this alternative a‘ is actually ranked best using measure Q and how do we determine what is the best you know using measure Q. So, 1 with the minimum value Q so as we talked about in the previous steps so far all the alternatives we will be sorting them you know we will get three ranking list by sorting S R and Q values and therefore it will be easy to spot values.

I know for S R and Q we are having the alternative in the minimum value so for you know a first we will focus on the list that has been produced using Q. So, we will look on that list and the

alternative business having the minimum Q value that can we consider that is going to be first considered as the a compromise solution. I know ranked best, alright however 2 conditions are to be satisfied C1 and C2,

So, these are the 2 conditions that would be satisfied before this alternative having the minimum value in the Q list is actually can be ranked best.

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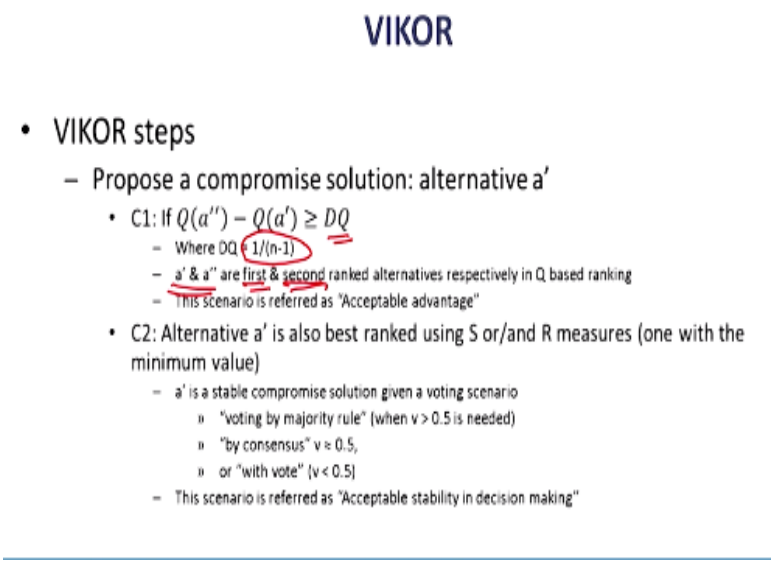

So, what are these 2 conditions C1 and C2 so to propose a compromise solution alternative a‘ and 1st condition is C1 if Qa you know Q a“ so this a“ is actually the second ranked alternatives a dash is the first rank alternatives. And I have already told you that it is based on the minimum, Q value so the alternative with minimum Q value would be a dash and the alternative you know with the 2nd you know 2nd best or 2nd best minimum value is going to be a double dash.

So, once we have identified a dash and a double dash we can check for this expression if Qa double dash – Qa dash >= DQ where DQ is this value 1/n-1 where n is the number of alternatives. So, this particular differences being checked so that there is an acceptable difference between the best alternative and the second best alternative. So, only then best alternatives should be you know accepted as you know best in the list.

If the difference between you know I know 1st and 2nd length alternative it is quiet close then probably acceptability of I know considering the 1st rank alternative as best a might not be that higher. So, therefore that is being checked through this mechanism where if Q a double dash – Qa dash this is going to be if this is a > or = DQ where DQ is 1/n-1 so this is the first condition that should be met before we can say that alternative a double dash is actually the best alternative.

So, then there is this condition 2 this should also be satisfied so at this point we would like to stop here in this lecture and in the next lecture continue our discussion on this part. We will talk about what the condition C2 is and what happens if 1 of these conditions is not satisfied and how do we determine the compromise solution so we will discuss all this in the next lecture. Thank you.