MCDM Techniques Using R

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Lecture – 12
Electre - Part V

Welcome to the course MCDM techniques using R. So in previous few lectures, we have been discussing electre. So let us do a quick recap of what we discussed in for electre in the last lecture. So this part has been covered. So we will just quickly browse through to the part which we were discussing in the last particular lecture. So we will just reach there.

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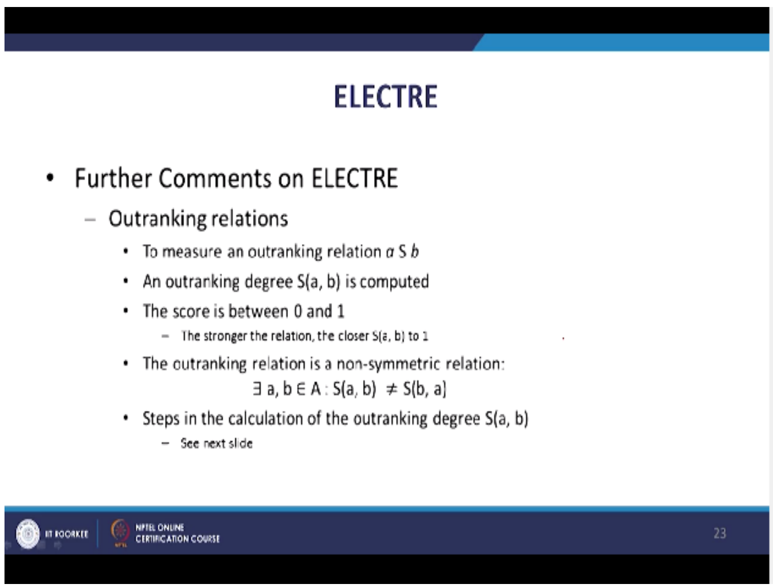

So we were actually discussing the underlying mathematics of electre. So we talked about outranking relations and we also talked about that this is a non-symmetric relation. The outranking relation is a non-symmetric relation. So therefore, S of a, b is not going to be equal to S of b, a. And then because of this, when we are going to derive the preference relation between 2 alternatives, we are going to use both of these outranking relation, S of a, b and S of b, a while deciding on the preference between a and b. So let us move forward.

So we also in the previous lecture, we also talked about the steps that are going to be that we can actually follow to compute the outranking degree or credibility matrix. So we talked about these particular steps in the previous lecture, the partial concordance degree and partial discordance degree that is the 2 computations that we need to perform first.

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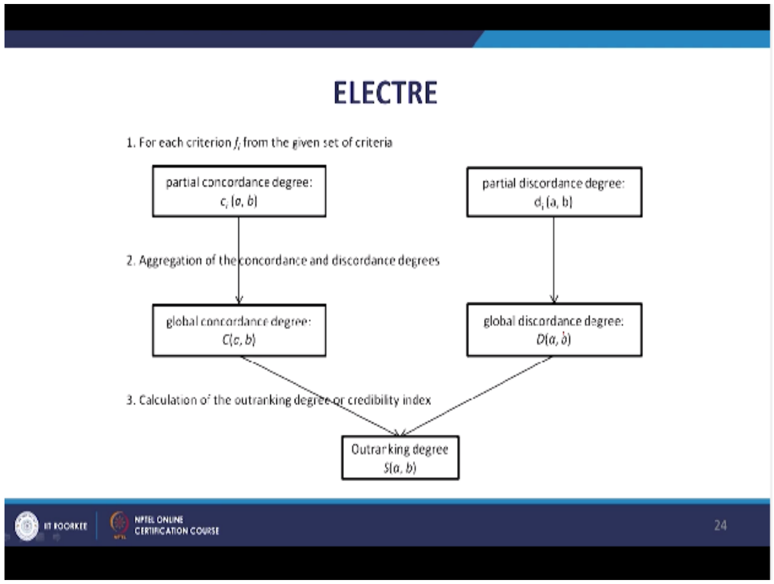

Then we are supposed to do aggregation of these 2 degrees, concordance and discordance degrees. And we will arrive at the global concordance degree and global discordance degree. Then we calculate the outranking degree combining these 2. We talked about the partial discordance degree and the underlying mathematics.

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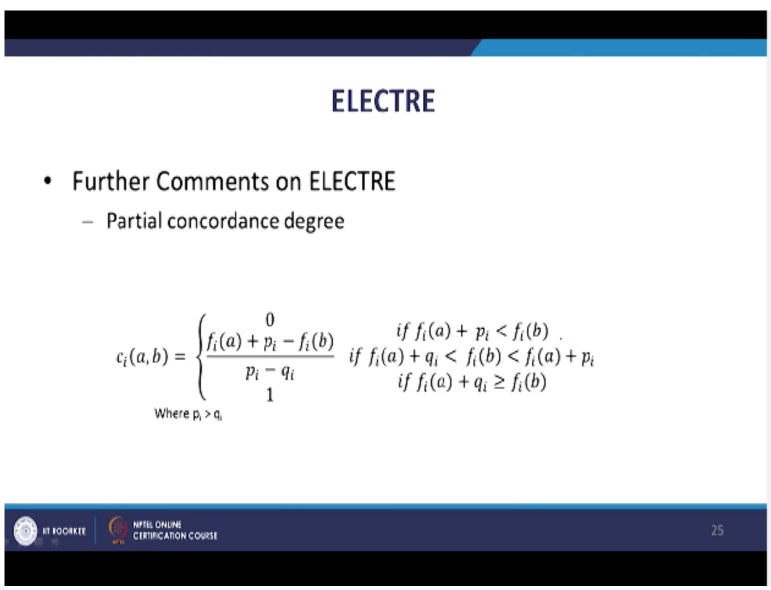

So we can see here in this slide itself, the different scenarios for computation of partial concordance degree and the values that could be there that we can see in this slide.

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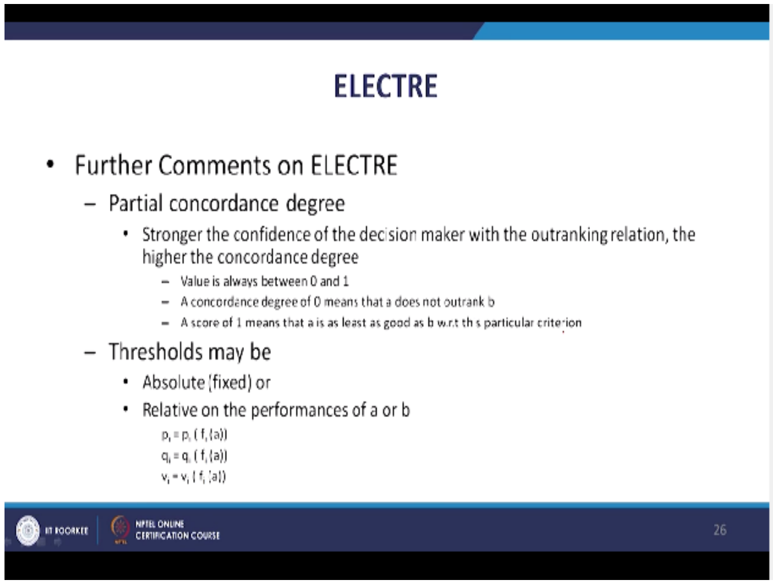

And it was discussed in the previous lecture. Similarly we also talked about this absolute or relative performances of a or b.

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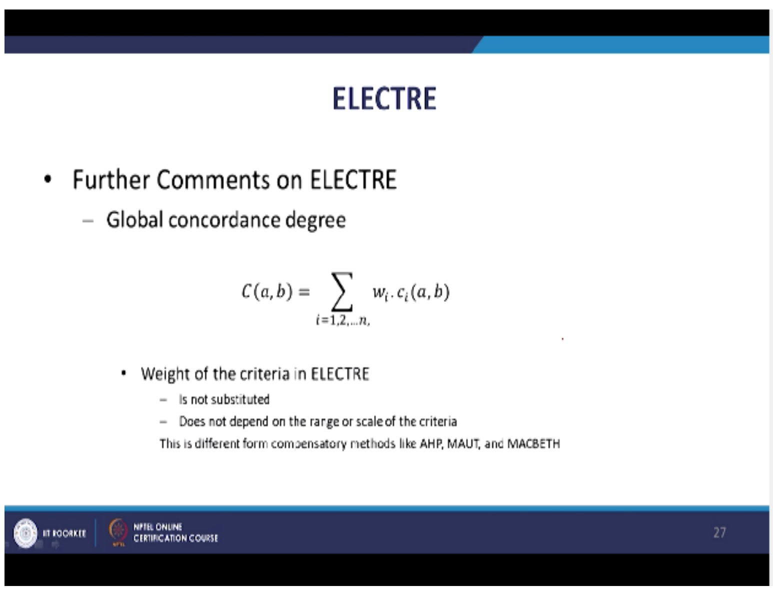

We talked about the global concordance degree, how it is summation of weighted concordance, partial concordance degrees where weights are actually the criteria weights. So this part we had covered. We also talked about the partial discordance degree.

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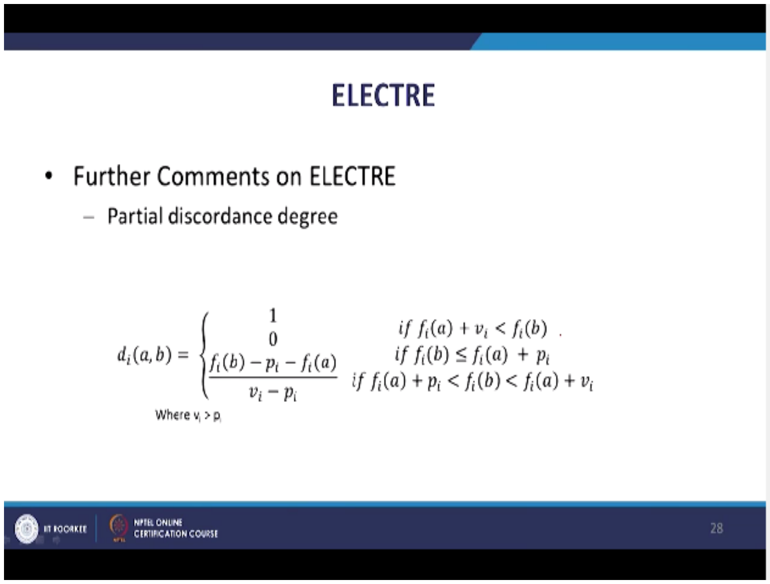

And you can see the formula in the slide and in different scenarios and the values that could be there.

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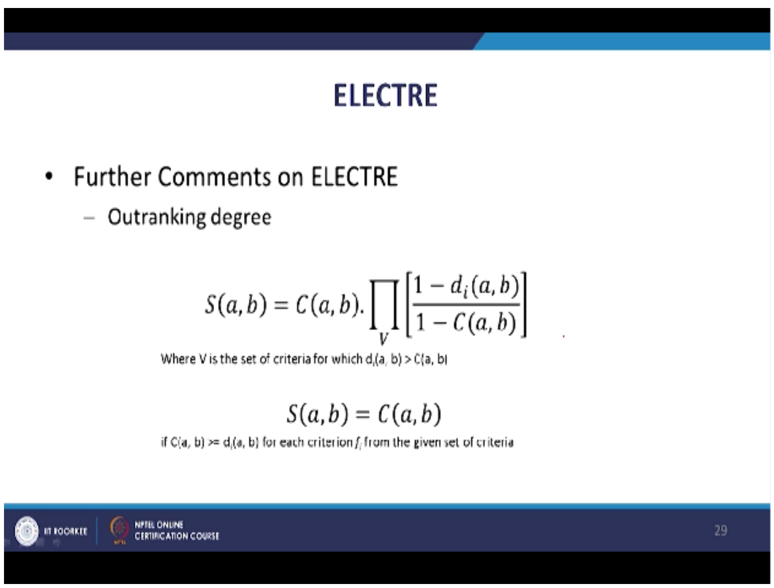

Then we talked about the outranking degree, how it is finally computed. So we can see here the S of a, b and you can see the concordance matrix is the one which more often they now determines this outranking degree, the credibility matrix. However, you can see how the partial concordance degree is incorporated in the overall expression and that in a way provides us the global discordance degree kind of expression.

So you can see here, we talked about this particular aspect that if C of a, b is global concordance degree is greater than or equal to partial concordance degree that is d of a, b for each criterion given in the set of criteria. Then probably the S of a, b is going to be equal to C of a, b. So therefore, in that scenario, in that limited scenario, corner case scenario, this credibility matrix is going to be nothing but the concordance matrix.

So all these we have talked about in the previous lecture as well. So preference relation can be denoted as aPb, so where it means that a is preferred over b.

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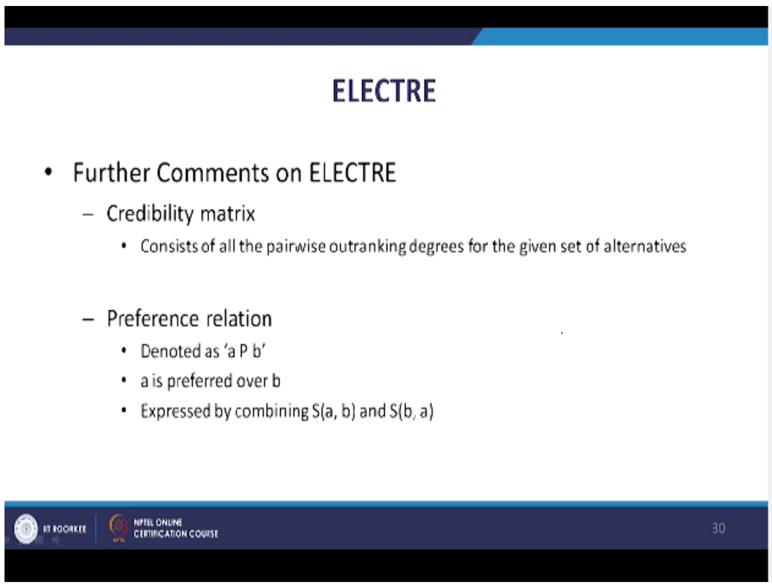

So as we talked about that S of a, b and S of b, a, both of these values, they are going to be part of the way we are going to be, the part of the calculations and the steps the way we are going to compute the preference relation. So in the last part of the previous lecture, we actually talked about some of these equations which are actually used to determine the preference relation.

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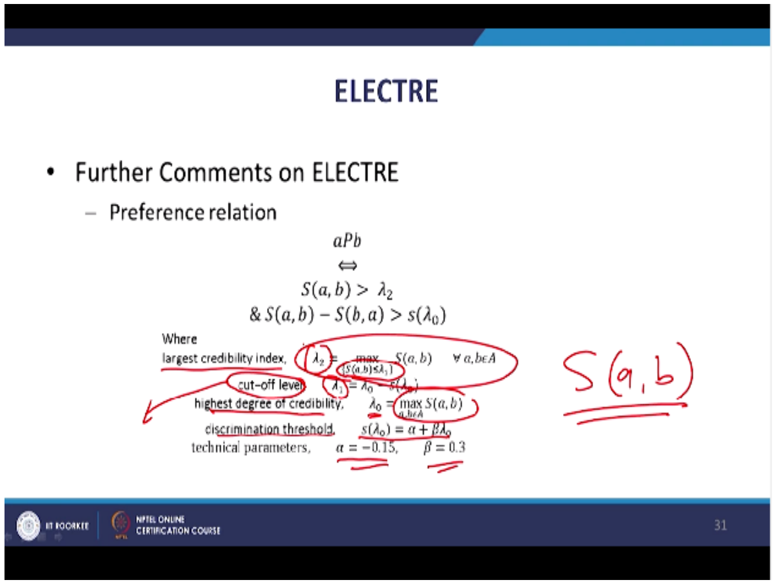

So aPb, so preference of a over b, so this is going to be determined based on these 2 equations whether these relations, whether S of a, b>lambda 2 where lambda 2 you can see here which is largest credibility index here, you can see. And this is the expression to compute it. And then we need another computation that is S of a, b-S of b, a. Now this difference would be greater than s of lambda 0.

So what do we mean by s lambda 0 and lambda 0, specifically. So we can see here, lambda 0 is highest degree of credibility. So if we look at this matrix S a, b, so the value which is the maximum value which is, element having the maximum value from this matrix is actually assigned as lambda 0.

And from there, we can compute discrimination threshold which is this given by this particular equation s of lambda 0=alpha + beta lambda 0, where these alpha and beta are technical parameters and typically different permutation and combination can be tried out for these parameters. But typically these are the values which are, so these values have been derived based on number of experimentation and these values seem to be giving the better outputs.

So therefore, using these technical parameters, we can compute this discrimination threshold. And given this discrimination threshold and highest degree of credibility, we can compute the cut-off level. This is important. So this cut-off level actually determines how easy for us to compare 2 alternatives in terms of determining the preference relation. So this cut-off level lambda 1 is actually determined using lambda 0-s lambda 0.

So as you can see here. So all these equations play their part in terms of determining whether a particular alternative a is going to be preferred over alternative b. So this particular aspect we had discussed in the previous lecture. So let us move forward. To indicate again to come back to this expression to tell you how this lambda 1 cut-off level is determining ease of computing the relations, you can see lambda 1.

And you can see in the lambda 1, this is maximum of where this particular, we are looking for the maximum value where S a, b, this expression you can see here, S a, b is less than or equal to lambda 1, right. So the comparison is all the elements that are part of the; this S matrix, right outrank this credibility matrix. So they are compared with this lambda 1 value. So for the elements for which this, elements which are having less than this cut-off level value that is lambda 1.

So out of those elements, we find the maximum value and that is assigned as lambda 2. So we can see it is the cut-off level that can in a way determines how easily the preference relations are derived. So let us move forward. So now, that was the part that we had discussed in the previous lecture that was more about the first phase of electre, first stage of electre where we compute the outranking degree and the preference relation, outranking relation and preference relation.

Then we have the second part, the distillation phase that we talked about in previous few lectures that ascending distillation procedure and descending distillation procedures. These are the 2 procedures that are part of the distillation phase wherein we get 2 pre-orders of partial ranking, right. So and then if we call them O1 and O2, then we will have to take an interaction to find the final ranking. So we are going to understand the underlying steps that are required to perform, to go through all those computation under this distillation procedure. So let us start.

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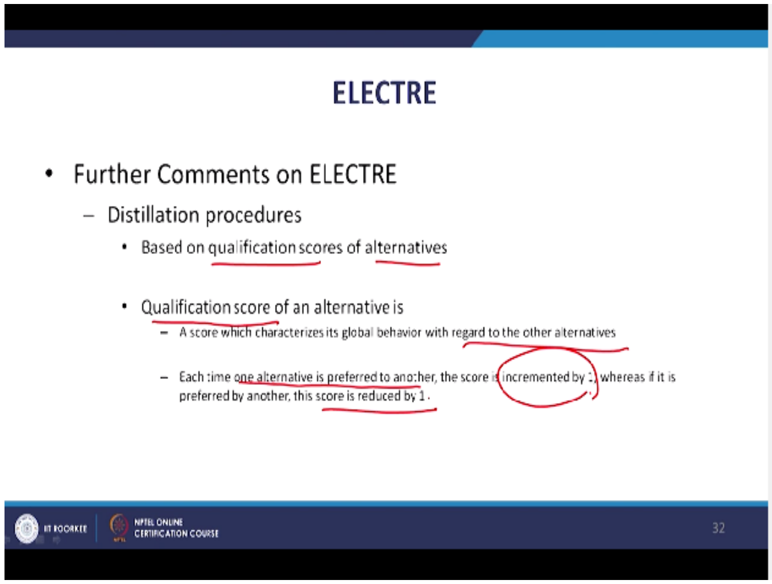

So distillation procedures, they are based on qualification scores of alternatives as we can see here in slide. So for each of the alternatives that we have, we need to compute the qualification score. And what do we mean by these qualification scores? So as you can understand that in the stage 1 of electre, we have already computed these outranking relation degrees and preference relations.

Of course, they are going to be part of the calculation in the second stage. So you would expect that qualification score would have something to do with our previous computation. So qualification score of an alternative is defined as a score which characterizes its global behaviour with regard to other alternatives. So when we compute as we saw in this lecture also, that when we are talking about preference of a over b, that is aPb, so aPb, that preference when we are talking about that preference, it incorporates in both the pair of outranking degrees where it is S a, b and S b, a.

But what about the comparisons with other alternatives? So that actually is done in the distillation procedure. So we compute, typically compute qualification scores. So that is why in the definition itself you see that a score which characterizes its global behaviour with regard to other alternatives.

Now in this particular stage, we are going to do comparisons with other alternatives and determine the global behaviour of a particular relation. So specifically, each time one alternative is preferred to another, the score is incremented by 1, whereas if it is preferred by another, this score is reduced by 1.

So based on these scores that we have already computed in previous stage, we see that how an alternative is preferred over another and the score is either incremented or reduced by 1. And finally, we get the qualification scores and that is actually used to get the preorders. So now as we talked about that there are 2 distillation procedures that we have, the descending distillation procedure and ascending distillation procedure.

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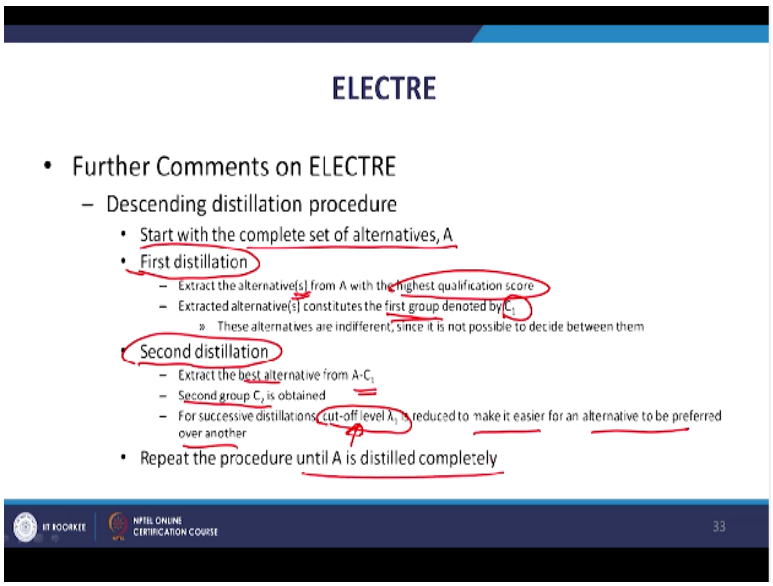

So let us understand the steps of the descending distillation procedure first. So what we do in descending distillation procedure is that in the first step we start with the complete set of alternatives, A. So all the alternatives, so whatever computations that have happened in the previous stage. Having those scores in mind, now those scores are going to be part of this computation now.

Now we will start with complete set of alternatives A. And the first distillation is performed. So what happens in first distillation is actually we extract the alternative from A with the highest qualification score. So qualification scores are supposed to be computed. And based on the qualification scores for each of the alternatives, so as we discussed qualification scores are going to be based on the computation that we have done before, like we have determined the preference relation already.

So based on that we can compute the qualification scores. Now in the first distillation from the set of alternatives that A, we are going to extract the alternative which is having the highest qualification score. Now this extracted alternative, sometimes 2 alternatives might have the same score. So therefore, that is why you would see that it could be 1 alternative or 2 alternatives, or even more.

So therefore, these alternatives are going to be extracted which are having highest qualification score, right. So now these alternatives which have been extracted, they are referred to as first group denoted by C1. So this is actually the first extracted group from the set of alternatives A. So this is what happens in first distillation. Now we again do a similar kind of thing in the second distillation.

So the remaining alternatives that are there after removing the first group, after extracting the first group that is C1, that means now the set can be referred as A-C1. So here C1 is subtracted from the set A. So now we have this subset A-C1. Now again we extract the best alternative from this particular subset. So now again in the second distillation also, the best alternative that we extract, so there could be more than 1 having same scores.

So therefore these extracted alternatives, they are referred as second group, C2. So this second group is obtained. So this is our second distillation. Now for successive distillation, this cut-off level lambda 1 that we talked about, right, so this can be reduced to make it easier for an alternative to be preferred over another. Because given the number of alternatives and the kind of scores that they might have, initially it might be easier for us to extract the alternatives with the highest scores.

But as we go down the list and try to extract other alternatives, given the scores that might be there, it might be difficult for us to extract. So therefore, cut-off level lambda 1 can play an important role. And it can be reduced to make it easier for an alternative to be preferred because you change the lambda 1 and the qualification scores will change because of the change in the preference relation that we might have.

So this is what happens in the second distillation. And then we keep on, we can keep on repeating this process until A is distilled completely. So that is why this is referred as the distillation procedure or distillation phase wherein best alternatives are being extracted one by one. So let us understand the same thing using this particular graphics.

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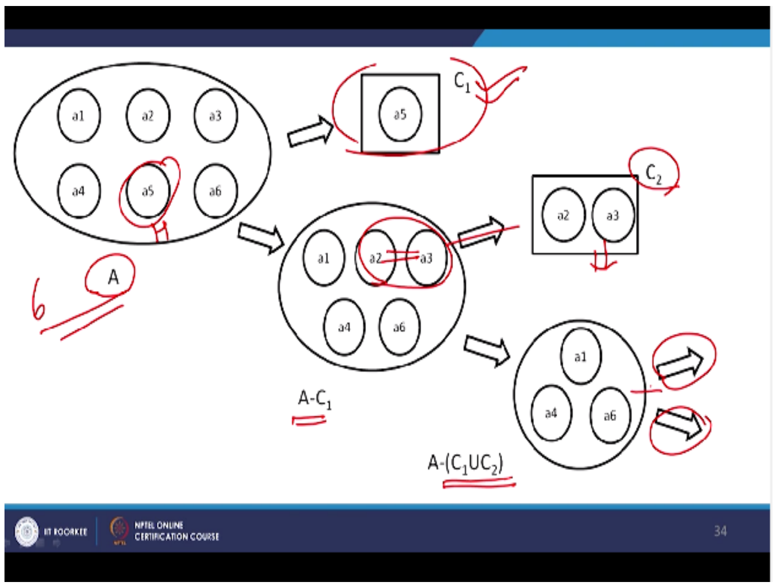

So here you can see we start with the complete set of alternatives that is A. So we have 6 alternatives here. You can see, 1, 2, 3, 4, 5, 6. So these 6 alternatives we have and we start with this particular set. Now let us assume that out of these 6 alternatives, a5 is having the highest score. So therefore, this is going to be extracted and this will become part of first group. You can see here C1.

So this is what is going to happen. So we would be extracting this first group C1 having the highest, having the alternative with the highest qualification score. Now the remaining set is going to be A-C1. Now you can see here we just have the 5 alternatives, a1, a2, a3, a4 and a6. Now if we are going to repeat the, we are going to perform the second distillation, so again we will extract the best alternative with the highest, based on the qualification scores.

So this time a2 and a3, they are both having the same score. So both of them have been extracted and they form the second group that is C2. You can see here, a2 and a3, they have been extracted and they form the second group. Now this process is going to be repeated. Now what we are left with is this particular set that is A-(C1UC2). So this is the set that we have and we have now just 3 alternatives, a1, a4, a6.

And now this process based extraction, this process, this will continue. So in this fashion this particular distillation procedure is actually performed. Now let us move forward. So the steps that we just talked about descending distillation procedure, now the similar steps are there in the ascending distillation procedure. The only difference or the main difference being that the alternatives with worst qualification scores are considered now.

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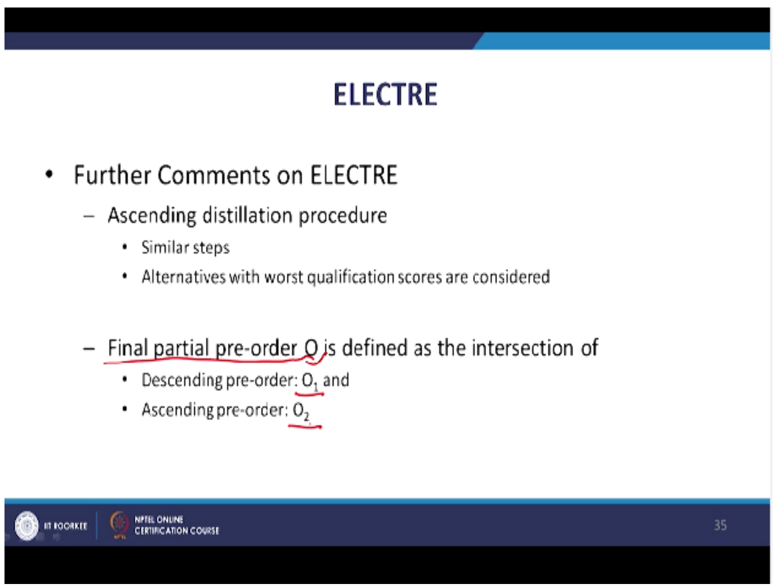

So in the 2 procedures that we have descending distillation procedure and the ascending distillation procedure. In the one, in the descending one, we look for the alternative with the best score, the highest qualification score. And in the ascending distillation procedure, we look for the alternative with the worst qualification scores. And using these 2 procedures, 2 pre-orders are computed which are referred as O1 and O2.

So you can see in the next one slide, final partial pre-order O is actually defined as the intersection of O1 and O2, where O1 has come out of this ascending procedure. So it is referred as descending pre-order and then we have O2, that is ascending pre-order. So these 2 are partial pre-order that will get, they are then used to arrive at the final partial pre-order, that is O. Now how do we perform these steps?

How do we determine the final partial pre-order? So we are going to talk about the steps which can be used to determine this, to define global relations. So now once we have O1 and O2, those 2 pre-orders, given the steps that we are going to discuss now, we can always define global relations using these steps. So we have these 2 alternatives a and b.

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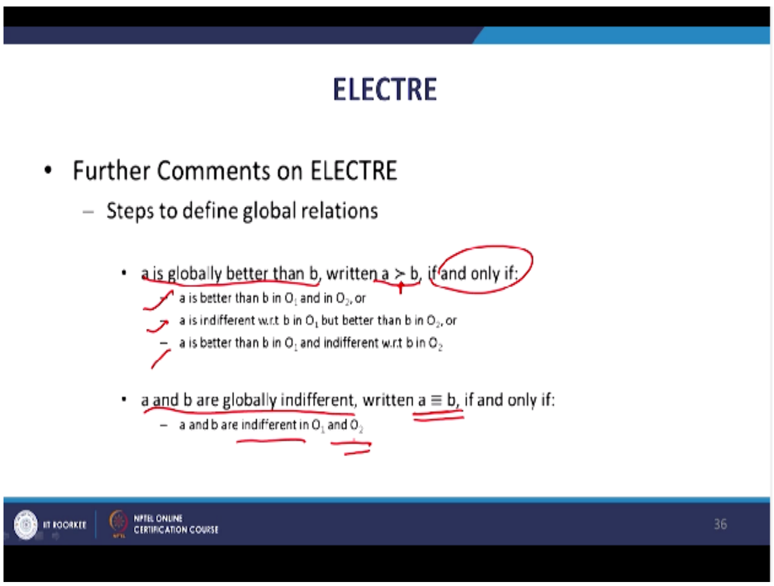

And now a is globally better than b, written as a and you can see these notation similar to greater than notation. So this a>b, that is a is globally better than b if and only if these 3 scenarios are there. So any of these scenarios, if any of these scenarios are true, then a is going to be considered as better than b. And it can be written as a>b in this particular fashion. So now each of these scenario you can see that we are considering both the pre-orders that we have just computed.

So a is better than b in O1 and in O2. So if alternative a is better than b in both the pre-orders, then of course we can say that a is globally better than b. Now if a is indifferent with respect to b in O1 but better than b in O2, can also we will say that at least in one of the pre-orders, a is better than b; therefore, we can consider that a is globally better than b. So again we can arrive at the same conclusion.

If a is better than b in O1 and indifferent with respect to b in O2. So that is essentially the same kind of thing. Now instead of O2, a is better than b in O1 and indifferent with respect to b in O2. So if theoretically there are just 2 scenarios, if a is better than b in both the pre-orders, O1 and O2 or a is better than b in at least one of the pre-orders and indifferent with respect to b in the other pre-order.

So in that case, we can deduce that a is globally better than b and they are written as a>b. So this is how we can define this particular global relation. Now in some other scenarios, this situation can be totally different. For example, a and b are globally indifferent. So when we can arrive at this particular outcome, when a and b are globally indifferent, written as a and you can see like equivalent kind of operator.

So a=b if and only if a and b are indifferent in O1 and O2. So we have 2 pre-orders. One is O1, the other one O2. So if in O1 as well, a is indifferent with respect to b and in O2 as well, a is indifferent with respect to b, right. So in that case, a and b can be clearly said to be indifferent in O1 and O2. And therefore, they can be termed as globally indifferent as well. So this could be another scenario where a and b could be globally indifferent. So let us move forward. Now what is the third scenario?

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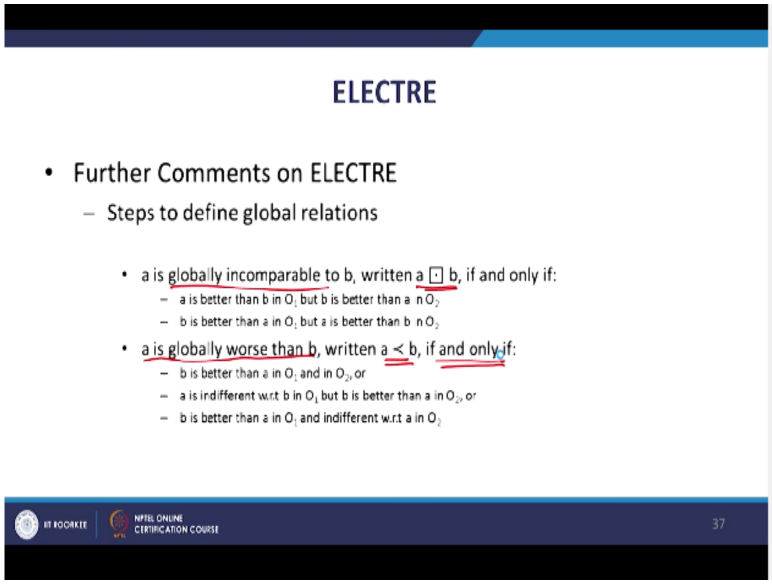

It could be that a is globally incomparable to b. You can see here, a is globally incomparable to b. So it is typically denoted by a square operator. So a square b and this is going to be if and only if a is better than b in O1 but b is better than a in O2 or b is better than a in O1 and a is better than b in O2. So one particular pre-order, a is better than b and the another pre-order, b is better than a.

So therefore, we will reach an impasse and we will not be able to determine. So therefore, we can term this particular, in this particular scenario as that a is globally incomparable to b. So given the output that we have, we cannot compare these 2. So this is how we are going to terminate. Now that a is globally incomparable to b and it can be written as a square b. Then comes the last scenario here.

There can be scenario when a is globally worse than b. So this is written as a<b. And when this is going to happen? So this is going to happen if and only if b is better than a in O1 and in O2 or a is indifferent with respect to b in O1 but b is better than a in O2 or b is better than a in O1 and indifferent with respect to a in O2. So if you look at these 3 sub-scenarios, they are very similar to what we discussed in the first one wherein we were talking about a is globally than b.

So now when we say a is globally worse than b, it is similar to saying that b is globally better than b. So the same kind of scenario and the same kind of sub-steps, sub-scenarios, or comparisons are to be performed. So we can see here, this is how we can actually go about determining the global relation between 2 alternatives.

So this is going to be performed for all the alternatives, the pairwise comparisons between all the alternatives. And this can be done. And finally, we get the final pre-order. Now let us talk about some of the drawbacks of electre III. So in this particular topic that we are discussing electre, we had mainly covered the third version that is electre III.

There are several limitation of this method just like for AHP that we discussed, there were also several limitations related to AHP, several shortcomings of, drawbacks of AHP. Similarly, electre III also has a number of drawbacks. So let us discuss them one by one. So in electre III, we typically cannot always clearly differentiate between indifference and incomparability between 2 alternatives in the final ranking.

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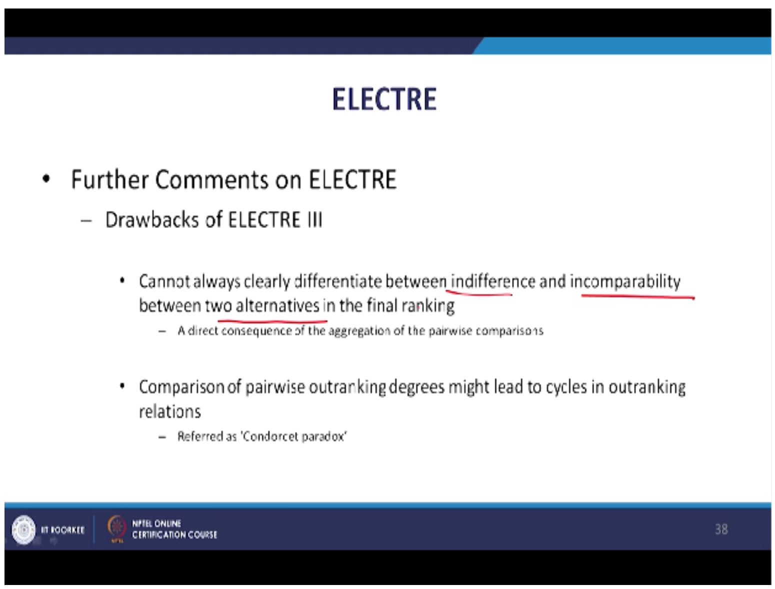

So this might be the situation in many scenarios where the local comparison might be indicating that 2 particular alternatives are indifferent to each other. However, when we compute the global relation because of the aggregation procedure that we perform, the aggregation that we do, we might conclude that particular alternatives, they are incomparable. So this is typically a direct consequence of aggregation.

Because this particular algorithm just like AHP, this also comes under the category of aggregation methods. So AHP and electre, they both in some of their steps, they perform aggregation. And this is the consequence of this aggregation steps. Now second drawback of electre III is that comparison of pairwise outranking degrees might lead to cycles in outranking relations. So this is referred as Condorcet paradox.

So this might be there because we are comparing, when we are comparing the outranking relation degrees in a pairwise fashion for 2 particular alternatives, a with b and then b with c and then it might lead to c with a, so there could be a cycle. So there could be certain cycles that could be created. So these cycles of outranking relation that could be there. a might be outranking relation b, b might be outranking relation c and c might be outranking relation a.

So this is referred as Condorcet paradox. So this is also one of the drawback of electre III. Then the another drawback is that pairwise rank reversal might occur.

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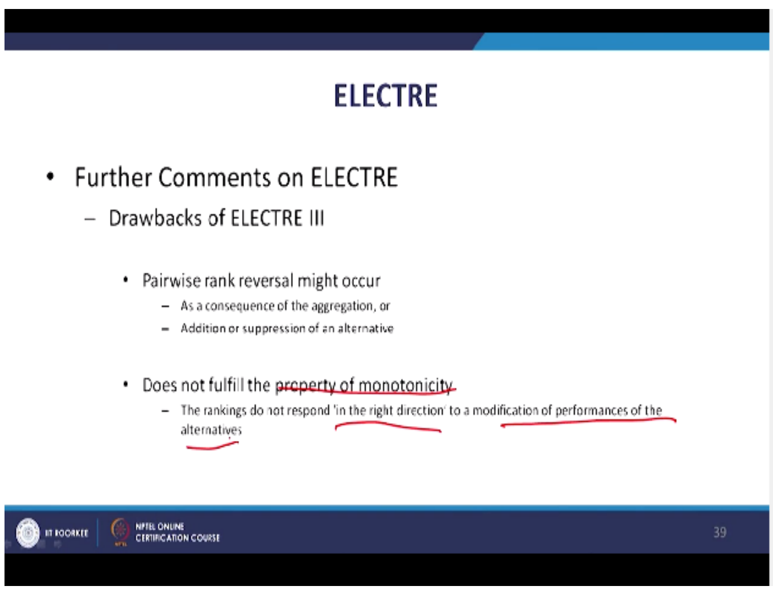

So this could be as a consequence of aggregation that a local pairwise comparison might be indicating one thing. But when we aggregate while deriving the global relations, the rank reversal might occur. And then addition or subtraction of an alternative, so that can also change this because local comparisons might change or it might change, it might get changed at the global level.

So addition or subtraction might also change the final ranking, might lead to a rank reversal. And then the aggregation itself can lead to rank reversal. Then the another drawback of electre III is that it does not fulfil the property of monotonicity. So the rankings do not respond in the right direction to a modification of performances of the alternatives. So as you remember in the electre procedure, in the electre III method that we have discussed in previous few lectures, performances of alternatives are considered.

So if the performance are changed, some modification is done in the performance, so whether that is reflected in the same fashion in the final ranking, so that might not be the case. So that is what we mean by property of monotonicity that if we change a certain value, then the ranking might change. So this is another drawback of electre. Now we go back to the exercise that we had done in RStudio.

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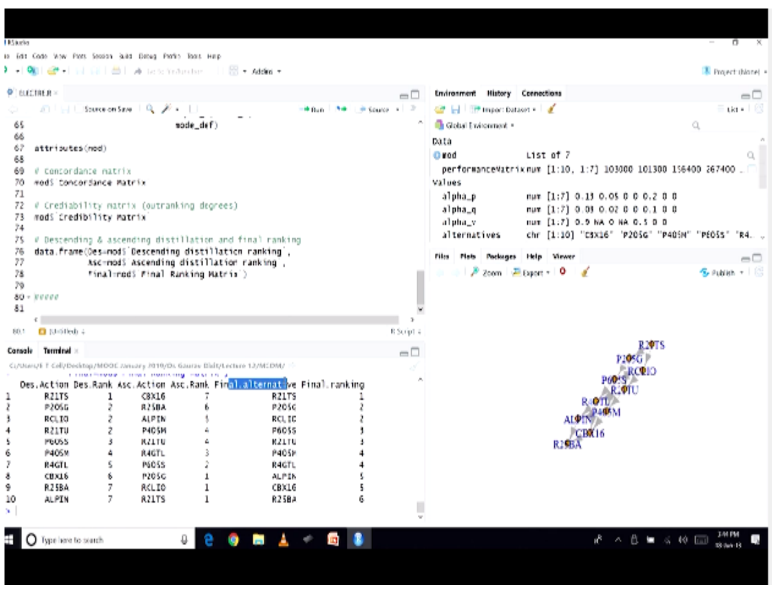

Now whatever the output that we had got, you would be able to understand it a bit better with more clarity and now that we understand the underlying mathematics. So let us quickly run through this particular exercise that we had done. So in this exercise, we were ranking French cars and we have 7 criterias and 10 alternatives. So let us quickly execute some of these lines. So we have created criteria as you can see in the environment section also and in the console also.

Then criteria weights, then the direction of criteria, then the names of alternatives, then we are going to compute this performance matrix. So let us run this. So this is created. So let us move forward. Then these thresholds or indifference preference and veto thresholds, so let us compute them one by one. So once this is done, we also need to specify the mode of difference. So let us run this. This is also done. Now we are going to use this library outranking tools.

So let us run this. Now we are going to call this function and all the arguments we are going to pass on here and we will get the output. Test the scan, it seems that beta v did not run, yes. Now you see certain computations had been performed. You can see and a plot has also been generated here which you can see here. Now attributes like the last time we saw that these are some of the attributes.

Now the important one is the, as we have discussed in the presentation slides as well, the concordance matrix is one important intermediate result that we would like to have a look at. So we can always refer to this concordance matrix. So you can see here how this has been computed. So weighted some of partial concordance degrees. So for each pair of alternative, you can see the values here.

Then let us talk about the, let us see the output for credibility matrix. So this have the, now the outranking relation degrees had been incorporated in creating this credibility matrix. So let us run this. So this is matrix that we get. This is the outranking matrix or credibility matrix. Now let us have a look at some of the distillation procedures. So if you run this code, we will get this output. So you can see descending distillation ranking, you can see and the ascending distillation ranking you can see here.

So this is ascending, this is descending. So these are the 2 pre-orders that we can have. So based on the preference relation, which are in turn based on the outranking or credibility matrix. Then the final ranking you can see here. This is the final ranking. So in this, now you would be able to see what this operator does and the different things that are part of the computations and different output that can be seen here. So one good thing in this RStudio and R interfaces in this R environment is.

That because we are using the functions to compute, to perform certain computations and different intermediate results can be accessed because the returned value, it is typically a data frame and different intermediate results all are available there. So if we need them for our research purpose or education purpose for businesses, then we can always refer to the intermediate results.

And we just, of course, we will have the final output as well, but the intermediate how if you want to verify manually, so we can always do that. So with this we would like to conclude electre and in the next lecture, we will start our discussion on another MCDM technique. Till then, thank you.