MCDM Techniques Using R
Prof. Gaurav Dixit
Department of Electrical Engineering
Indian Institute of Technology- Roorkee

Lecture – 11
ELECTRE-Part IV

Welcome to the course MCDM techniques using R so in previous lecture you know we were discussing ELECTRE and specifically we started one exercise in R studio environment and R environment so let us go back to this place where we you know left in the previous lecture

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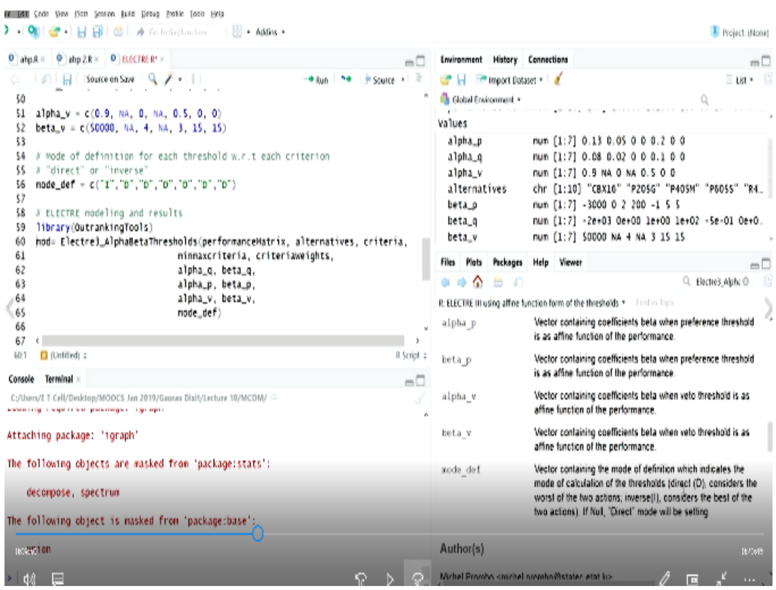

So to you know give you a recap of what we have done you know what we had done in the previous lecture so this was the decision problem ranking of French cars and we have seven criteria to consider so you can see the names here in the script and the alternatives so we have 10 alternatives to consider you know names of these alternatives are also coded here you can see in the comments section.

And then we created a few I know variables for example the character vector for the criteria names if you use the C function that is combined function for combining values then we also you know initialized another variable criteria weights again using the C function combined function where we have the numeric values indicating the weights of the criteria and then we created another variable min max criteria again we use the c.

And these you know character values they were you know combined using this function so whether the criteria is to minimize or maximize that was indicated using this particular vector or variable. So in the let me also give you a note on R studio so typically the you know in the R studio environment or R environment they call you know they were referred variables as vectors.

So for our discussion point of view when we say variable or vector we since we are meaning the same thing it could be character vector or you know it could be in a numeric vector but essentially all of them are variables for us. So then we also created another character vector that is you know alternatives so this variable was also created using the function combine C and then we created a matrix for the performance so performance table that we wanted to have.

So because we have 10 alternatives 7 criteria so 10*7 matrix was created and for this we used these two function matrix which was the main function to create the matrix and for the first argument we use the combined function that is C to combine the values that we would like to pass on the first argument and the other you know arguments were specified appropriately as you can see in the code as well.

Then we you know we were discussing at the last part of the lecture we were discussing the thresholds so as we have talked about that there are a number of results that we need to specify under this method ELECTRE. So we talked about you know our indifference difference and VIKOR thresholds and all of them I know we are going to use in this particular exercise. So then again there are two things two variable that we need to that we need to define.

For each of these thresholds so when its called alpha Q then otherwise beta Q. Beta Q is actually having the you know the actual value of these results whether it is beta Q or Beta P or beta V then alpha Q and alpha P and alpha V they are having you know another information which is to be used if the thresholds are you know there is also being specified using the relative you know relative mechanism.

If they are absolute than it is going to be fixed value if they are relatives or relatives is typically back to some performance so 10% of best performance or 10% of worst performance kind of scenario so this is the best and you know worst is actually referred to as mode of definition, I know for each threshold with respect to each criterion so it could be direct or inverse direct indicating you know worst and the inverse indicating the best.

So that also we are indicating here mode and code def is the variable that we are going to create for this so again are going to use the combined function which will indicate the mode I or the inverse and D for direct. So let us compute these variables so alpha Q beta Q alpha P beta P alpha V beta V and then the mode now once all this part is done and then we come to the you know the important part of the selector modelling.

So for this we need to have the sub package installed that is outranking tools so using the install dot package function you can always install this you know in this part of the package and then we need to use the library function which is to actually load this particular package into the current running you know instance of this R studio and therefore all the functions which are part of this package would be available for us for the execution purpose.

So the function that we are going to use the out of this package ELECTRE 3 alpha beta thresholds function so this is the function which had implemented I you know ELECTRE 3rd method you know which we discussed in the previous lectures as well. So let us you know look at the details of this function in the help section. So in the in the help section we will just type this function.

So before we can even access this particular function we need to load this library because once this is library is loaded only then we would be able to actually I know look at the help section here help part okay so let us again okay now you would see suggestions are coming there it seems the you know libraries not loaded because the pack is not installed here so what we are going to do is well faster install this package here.

So okay let us type install dot so I think so let us type this function install dot packages okay and you would see the moment we start typing few characters and a suggestion comes in there so we can always select from these suggestions install dot packages is there and within the doubt quote we are going to specify the name of you know the packers so which we can always copy from here.

So outranking tools and we can copy and you know paste it within the double quotes ad once we enter so this package would be installed. So just like we did in the previous technique that is ASP so here also we require certain packages to be installed. So this is the package that we are going to be outranking tools. Now this is installed here now we are going to you know load this library so now this is loaded so now we can go to the help section.

And you would see these suggestions already coming there ELECTRE 3 underscore alpha beta threshold so this is the function that we are going to be using so let us look at the help section of this function say as you can see in the description part this is you know ELECTRE 3rd has been implemented and you can have a look at the you know the function and different arguments which are part of this.

So you can see here performance matrix is the matrix that we created which contains the performance table that is the first argument that we need to pass on. So then the next one is the alternatives so which is actually names of the alternatives so this can be passed on a second element then the criteria again names of the criteria then minmax criteria to indicate the preference direction of each of each criteria.

Then we have the criteria weights so this vector having a numeric value of you know weights for each of the criteria then you can see alpha q and beta q so these are you know as we talked about or related to a different threshold indifference or reference or beta or beta thresholds these six arguments are to be specified then we are mode def so mode of definition which indicates sign of direct.

So you can see here in the help action also it did mention worst of the two actions. Actions actually meaning alternative and then the I is for the inverse that is you know and the best top two actions meaning hear alternative. So all this part we have done now we have already defined all the arguments so we can just call this function pass on these arguments and the output is going to be the code and the mod variable.

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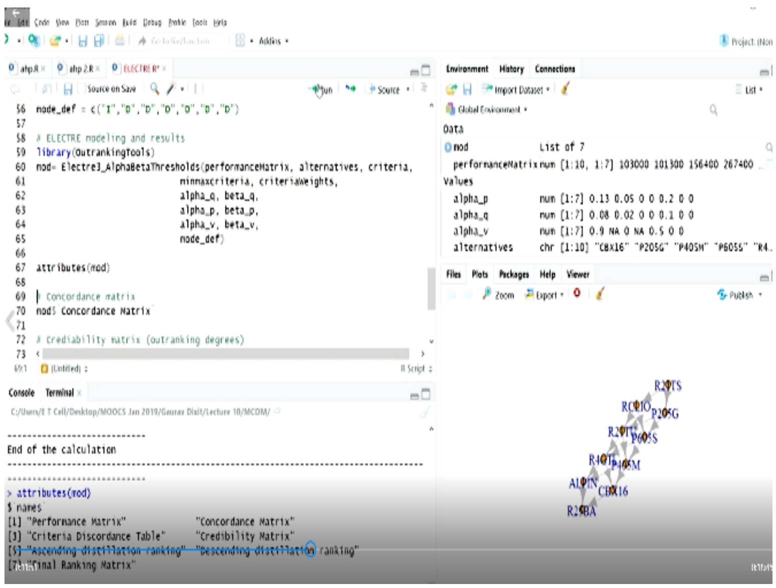

So let us run this now we can have it so you can see in the environment section a mod variable which is the model output in this case has been created and it contains lists top seven so it has seven elements within the so mod is a list know variable list data type as you know typically defined in you know in our environment so it has 7 you know elements. So let us look at what are these seven elements we can use the attributes function and it will give us the names.

So you can see performance metric which was actually the one of the argument that we passed on same we can also access through the output you know list as well and we have the concordance matrix so we have discussed in you know previous lectures the how the concordance is to be you know computed so that the matrix is part of the output then we have the criteria concordance table.

So you know you know the discordance values can be displayed here with respect to you know each you know criteria then we have the credibility matrix. So we will talk about what we mean by credibility matrix later in this lecture and coming lectures as well then we have the ascending distillation ranking. So we talked about the distillation procedures second phase so there are two sub procedure within that.

So ascending distillation descending distillations of these rankings are to be produced and then the final ranking which is the intersection these two so all these are part of the output so we can look at the concordance matrix so for this we need to we can use this notation mod is a listed structure and we can use dollar notation and to access the element that is part of it so mod dollar and concordance matrix.

So once you start typing mod and dollar it will says the number of you know elements which are part of that structure and you can pick and you can run that so let us run this so this is the concordance matrix you know that is there so all these values they indicate the you know concordance degree there then similarly creditability matrix is there which will actually have the outranking degrees.

We talked about in the previous lecture that for every outranking relation to measure it we are required to compute the outranking degrees so the credibility matrix is actually consisting of all the pairwise outranking degrees that are there. So during this particular code mod dollar credibility matrix we can see what was the credibility matrix that was computed so you can see here again 10/10 matrix.

So both concordance matrix and creditability matrix they are 10/10 matrix because you are having 10 alternatives. Now we can always have a look at the ranking that we finally get so here I catered a data frames so data frame is another type of a structure that is there. So this is quite similar to the way we use data in the in the in the statistical analysis where columns indicate variables and the rows indicate records.

So similar kind of thing can be done in R studio environment using data by creating data frames so for that we have this data dot frame function. So these are three vectors mod dollar i know descending distillation ranking model mod dollar ascending distillation ranking and the final ranking mod dollar final ranking matrix. So these three variables we can combine them column wise and we will create this data you know frame.

So that will give us an output which would look something like this so here you can see the you know different rankings the pre order rankings descending and ascending and the final rankings here so you can see here. So how these rankings are computers what is the underlying procedure so all that we are going to discuss now so let us go back to our discussion.

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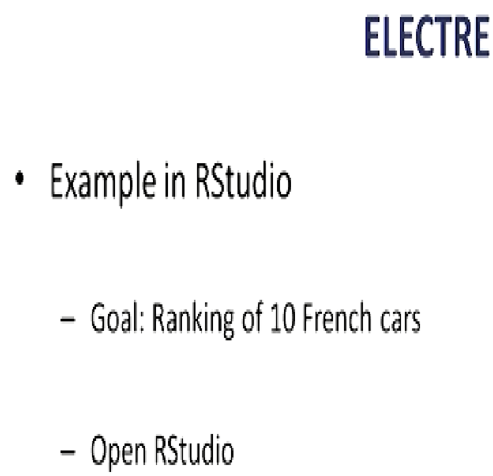

So now we are going to cover the underlying mathematics the underlying steps procedures of how these you know computations are actually performed. So let us first start without ranking relations so this as we have talked about this is two major and outranking relation that is denoted using a S b to measure this we computer outranking degree.

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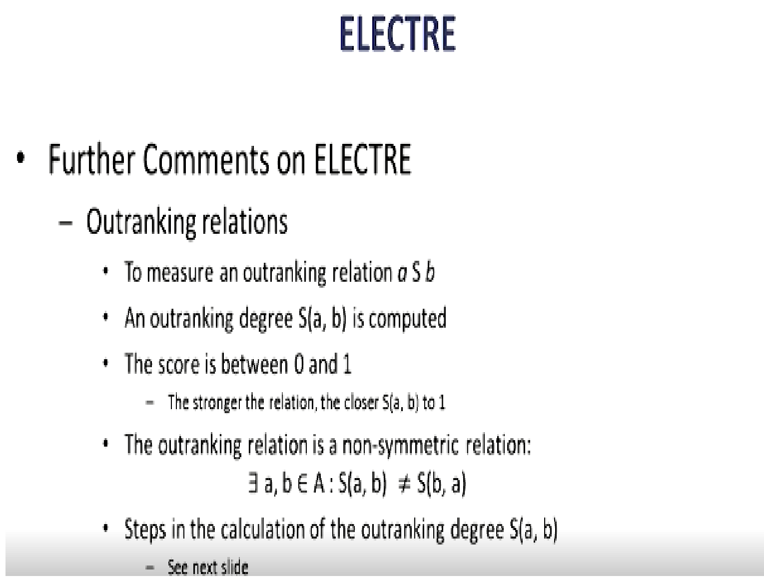

Which is the denoted during S and a.b the score is directly between 0 and 1. So these you know things we have already discussed the stronger the relation the closer is S ab is to 1. Now the next important aspect of this outranking degrees that this is non-symmetric relation so what does this mean is that for every you know for every a,b belonging to A which is the set of alternatives so for every a and b alternatives belonging to A the set of alternatives.

The S a,b is not= S b,a so non-symmetric relation so the outranking degree that we compute for a and b is not going to be the same as the outranking degree that we are going to compute for b and a. So given that what that we need to do for you know our you know computations under ELECTRE that we will see in this lecture coming slides. So now let us talk about the steps in the calculation of the outranking degree.

So you know quite a part of this theoretically we have covered in the previous lectures now we will more focused on the steps and the underlying mathematics. So this particular chart you know flow chart actually covers the steps so the first step is going to be for each criterion fi from the given set of criteria so for example the exercise that we did we had a you know a seven criteria there.

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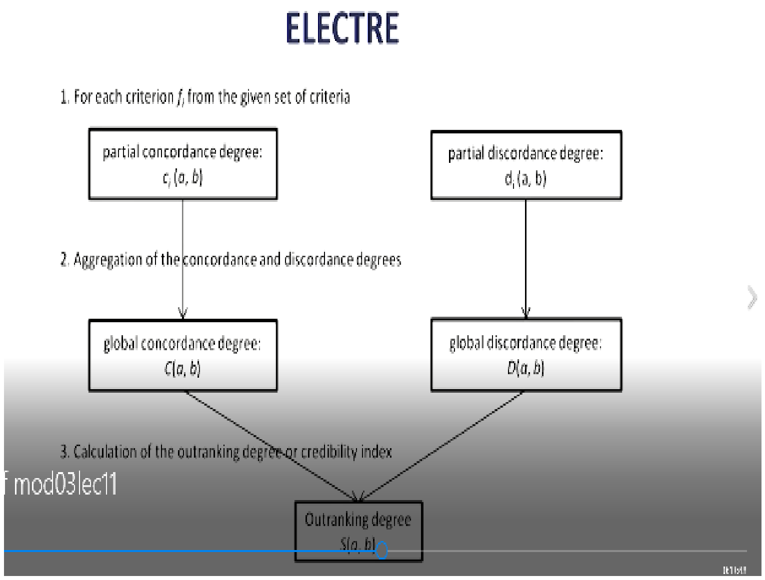

So for each of these 7 criterias we will compute the partial concordance degree that is ci a and b a and b they are alternatives belonging to this set of alternative that is A. So with the respect to each criterion a we are going to compute partial a concordance degree pairwise so any pair of alternatives and then we will also consider then we also compute the you know partial discordance degree.

So partial concordance degree and the partial discordance degree di a of b that is the partial concordance degree for each criteria this has to be done then the next step the aggregation has to be done. So aggregation is of the concordance and discordance degree degrees that a we might have computed in the previous steps so this is so we will get the global concordance degree that is C you know a,b and the global discordance degree

Then calculation of outranking degree or credibility index is done so outranking degree or we also refer it you know as credibility index so this is a S a, b that is outranking degree. So this is how so these are the typical steps that we need to do to you know compute to do ELECTRE modelling. Now let us look at each one of them how the computations are performed some part of it we have already covered in you know in previous lectures. So let us look at it in more structure fashion so partial concordance degrees.

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ELECTRE

- Further Comments on ELECTRE
  - Partial concordance degree

$$c_i(a,b) = \begin{cases} 0 & \text{if } f_i(a) + p_i < f_i(b) \\ \dfrac{f_i(a) + p_i - f_i(b)}{p_i - q_i} & \text{if } f_i(a) + q_i < f_i(b) < f_i(a) + p_i \\ 1 & \text{if } f_i(a) + q_i \geq f_i(b) \end{cases}$$

Where $p_i > q_i$

This is how it can be computed so you can see ci a,b so remember in the previous lectures we had a graphic and we are we talked about how you know the partial concordance degrees to be computed. So the same whatever we discussed using that particular graphic and the equations that we saw there the same as summarized in this format here as you can see so ci ab partial concordance degree is going to be 0 if fi a+ pi <fi b.

So if that is the case the value is going to 0 if fi a+qi this particular point if fi a+qi is >fib then the value is going to be 1 so you know and f the fi b if in between these two fi a+qi and fi a+pi then the value is going to be computed using this particular expression that is first we take the difference because fib in between these two values so you would actually see it we are taking the difference between fie+ pi and fib.

So the same is in the numerator is indicated here then you would see the interval a you know that is there that pi and qi so that is that was the denominator pi+qi so the ratio value is being taken if the fi b has been is lying within these two values right. So in this fashion we compute the partial concordance degree now let us move forward. Now let us talk about the you know a you know a few other aspects so within the partial concurrence degree.

We can say stronger the confidence of the decision maker with the outranking relation the higher the concordance degree. So that is how for us if we want to interpret depending on the value that we get for the concordance degree how to be how it can we you know interpret on the point of the decision maker if the higher the higher the confidence of the decision maker then values are always going to be between 0 and 1.

And means that a does to outline b and score of 1 means that a is at least as good or as b or a outranks b So with respect to that particular criteria so you know thresholds as we talked about in the R exercised that we did that it could be absolute or relative. So when we say absolute the value is I actually fixed.

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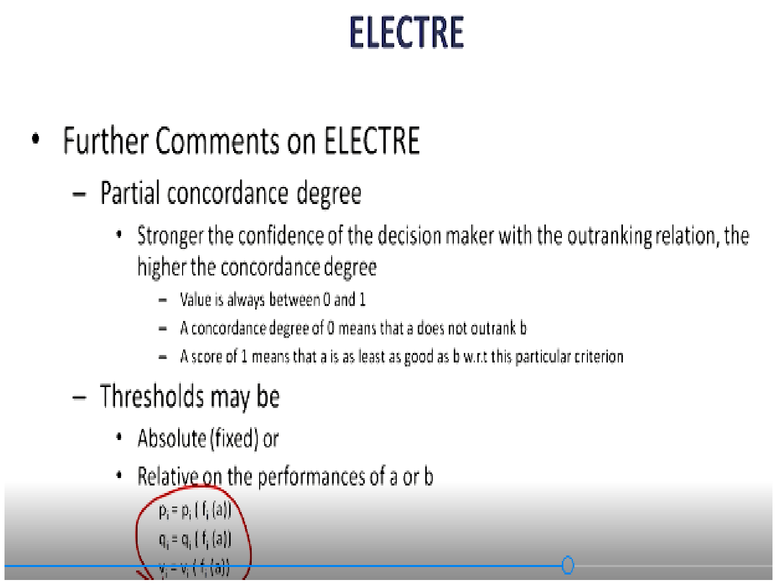

so for each of the you know the different resources that we have with preference indifference and veto so there the value is going to be a fixed value if the if the thresholds are relative so they are going to be the relative on the performance or performance of a or b and ab being the two alternatives so they can be expressed in this kind of format where pi is pi*fi a so fi a is indicating the performance of alternative way and the pi and then qi.

So this fi a is this is what in our exercised where we had alpha p alpha q alpha and alpha v so this kind of relative performance was actually you know that is what we actually wanted to compute it there. Let us move forward once we have all the partial concordance degrees how do we go ahead and compute the global concordance degree which is the second step.

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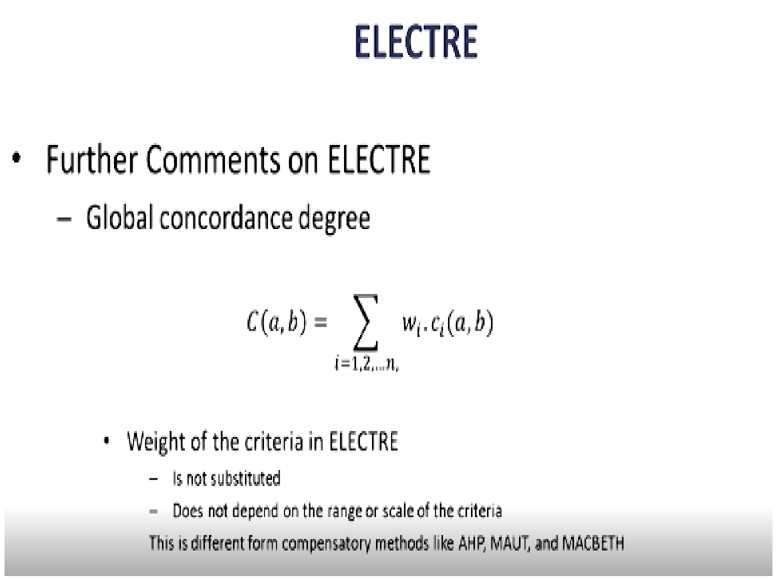

So this is the formula so global concordance degree as we have talked about in the previous lectures as well this is nothing but the weighted sum of all the partial concordance degrees where weights are actually the you know the criteria weights so you can see here weight of the criteria so this is wi so this is what we mean by you know in wi. So for computing global concordance degree we take the weighted sum where weights are the criteria weights.

Now in other techniques like AHP, MAUT and MACBETH which are more like compensatory methods. Here in this case of ELECTRE criteria are not substituted so they are specified from the decision makers and then the same are used for the computations and you know the weight of the criteria also not depend on range of the scale of the criteria. So in that sense ELECTRE gives us more flexibility.

More in a few more advantages is we talked about in the you know starting lectures of ELECTRE. Now let us move forward I know I will talk about the next two computation that is partial discordance degree how do we compute so remember in the previous lecture we had one graphic and through that graphic we understood the different scenarios and how this partial discordance degree is computed. The same thing is now we are you know depicting in this performance.

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ELECTRE

- Further Comments on ELECTRE
  - Partial discordance degree

$$d_i(a,b) = \begin{cases} 1 & \text{if } f_i(a) + v_i < f_i(b) \\ 0 & \text{if } f_i(b) \le f_i(a) + p_i \\ \dfrac{f_i(b) - p_i - f_i(a)}{v_i - p_i} & \text{if } f_i(a) + p_i < f_i(b) < f_i(a) + v_i \end{cases}$$

Where $v_i > p_i$

So di ab if it is going to be 1 if you know fib is >fi a+vi vi mean the veto threshold for the respective criterion so and it is going to be 0 if fi b is <=fi a+pi where pi is the preference threshold if it is in between these two values then this is the you know formula that we have so here again you see that fib in the numerator part it is nothing but you know of difference of fib here and this one.

So we take the difference you know in the numerator and vi-pi this is again these two threshold we take the difference and this is the ratio value. So if fib is lying between these two values we take this ratio as the partial discordance degree. Here you will also notice that vi is assumed to be >pi. Similarly, in the partial concordance degree vi is assumed to be >qi so this is how we compute the partial discordance degree.

Now let us talk about the outranking degrees so once we have the partial concordance degree and partial discordance degree now in the third step of you know the ELECTRE the third that we talked about the outranking degrees to be computed by combining these two you know concordance degree and the discordance degree.

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ELECTRE

- Further Comments on ELECTRE
  - Outranking degree

$$S(a,b) = C(a,b).\prod_{V}\left[\frac{1-d_i(a,b)}{1-C(a,b)}\right]$$

Where V is the set of criteria for which $d_i(a, b) > C(a, b)$

$$S(a,b) = C(a,b)$$

if $C(a, b) >= d_i(a, b)$ for each criterion f_i from the given set of criteria

So here you can see the formula that we have S a,b so this is going to be C a,b C a,b is the global concordance degree that we which is weighted sum of partial concordance degree then in the second explanation we have this multiplication of you know this particular expression where in the numerator we have 1-di of a/b which is partial discordance degree so and then divided by in the denominator.

We have 1- C of a b which is global you know concordance degree. So if you look at this expression this is more like you know till now of what we have discussed we have not talked about the global discordance degree. So this particular is like equivalent of computing global discordance degree. So this is the global concordance degree this is like a you know equivalent of global discordance degree.

However, there this is conditional so computation of this express and therefore the computation of the expression and therefore the computational of the outranking degree is conditional. Conditional on what this multiplication is in this with respect to the set elements of the set V which is the set of criteria for which di there is a di that is partial discordance degree is > C that is the global discordance degree.

So for the you know for the you know criterion for the criteria where the partial discordance degree is >the global discordance degree this is the expression that is to be used to compute the outranking degree. However, if you know for you know if all the criteria for all the criteria this is a global discordance degree is greater or equal to the partial discordance degree then the outranking degree is nothing but the concordance degree.

So there is a scenario where the concordance degree you know degree is going to be taken as the outranking degree given that the you know global you know concordance degree is >=partial discordance degrees. However, that is not the case for a number of criteria this partial discordance degree is >the global concordance degree now this is the expression that is to be used to compute the outranking degree.

So this is about how we can compute outranking degree so we go back to the steps that we talked about started we talked about here and this pack of flow chart so we talked about the partial concordance degree how that is to be computed we understood the mathematics underlying it and the partial discordance degree then the aggregation of concordance and discordance which is global concordance degree.

However, we do not actually go for a global discordance degree but that is actually part of the outranking degree that we finally compute. This outranking degree that we computer is also referred as credibility index and the matrix that we can actually compute so once we have computed all the outranking degrees for all the pairwise combinations of alternatives so given the set of alternatives that is A all the pair wise combination that we can have.

If we compute if we have computed the outranking dd follow all of them so and the put it into a matrix format so if there are for example that we had the you know ranking of French cars we had 10 alternatives. So we can have 10*10 matrix so that 10*10 matrix we will have the pairwise comparisons of each alternative with other alternatives. So therefore we will have the credibility matrix.

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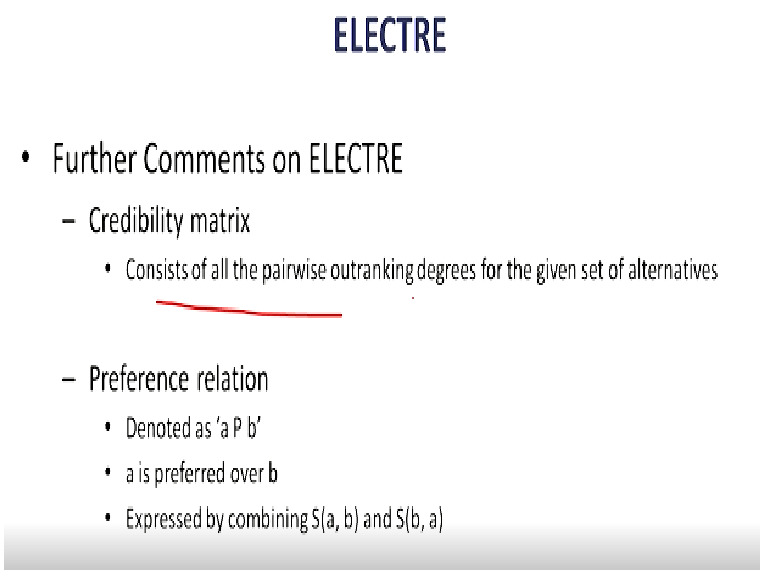

So this is what we mean by credibility matrix consist of all the pairwise outranking degrees for the given set of alternatives now this credibility matrix is going to have you know a S b and also v S a so but we have talked about that a S b you know is not going to be to be = v S a because this is non-symmetric relation. So you know how do we determine the preference relation finally from the credibility matrix.

So there are a few more steps that are part of this exercise so wherein we actually combine S ab and S b a as you can see here ow do we you know finalise preference relation. So preference relation is denoted by a P b that will actually indicate that a is preferred over b so how this is done so we combine S a b and S b a. So how this part is actually done so that is something that we will discuss in the next lecture. So at this point we would like to stop here thank you.