Introduction to Data Analytics

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Module – 06
Lecture – 34
Artificial Neural Networks

Hello and welcome to this module on Artificial Neural Networks.

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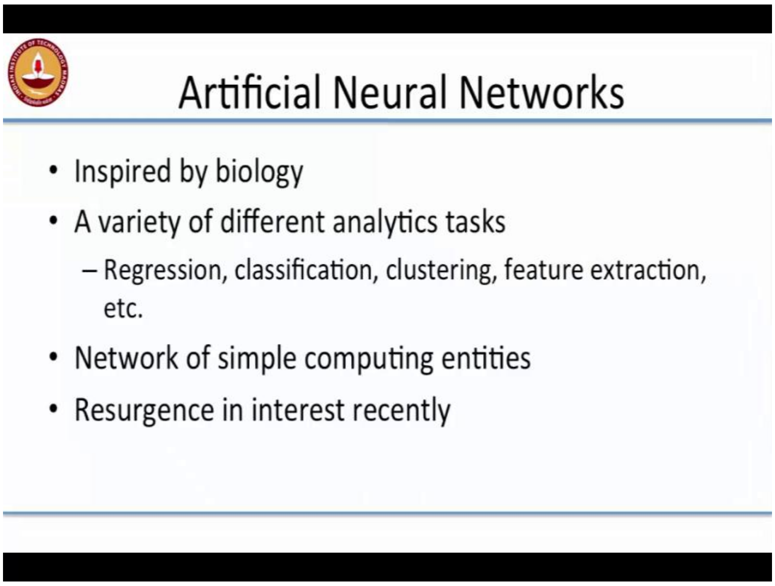

Artificial Neural Networks

- Inspired by biology
- A variety of different analytics tasks
  - Regression, classification, clustering, feature extraction, etc.
- Network of simple computing entities
- Resurgence in interest recently

So, artificial neural networks, where a computing module in spite by biology. So, we have neural network architectures that have been proposed for a variety of different analytics tasks like regression, classification, clustering, feature extraction, etc. So, these architectures essentially are networks of simple computing entities and so this is like a very simple threshold entities that are connected together in the specific network architecture that give raise to complex computing functionality.

Now, oscillate there has been significant resurgence in interest in artificial neural networks, especially under the domain of t networks about which we will see in one of the later modules. So, for this module and the discussion about artificial neural networks is concerned in this course, we will look at only the classification task and many of the ideas we talk about here for classification or generalizable to regression, like for while for the other kinds of analytics task we need different architectures and, but we are not going to cover that in this course.

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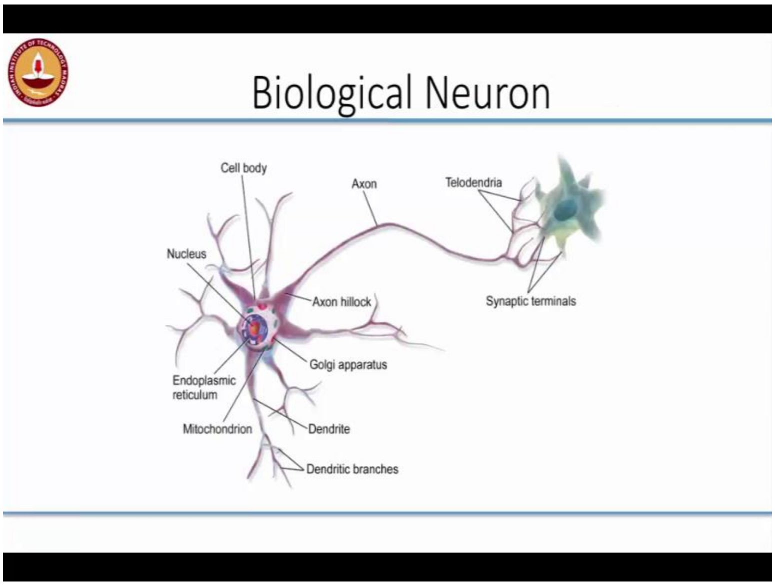

Biological Neuron

So, the inspiration comes from biological neuron. So, let us not worry about the complete complex structure of a neuron, what we really have to focus here is on the input and the output. So, the neuron receive inputs from the dendrites or from the dendrite branches from other neurons and when the input signals is above a certain threshold, it is going to produce an output, that is going to be transmitted via the synopses to neurons that are further down the line.

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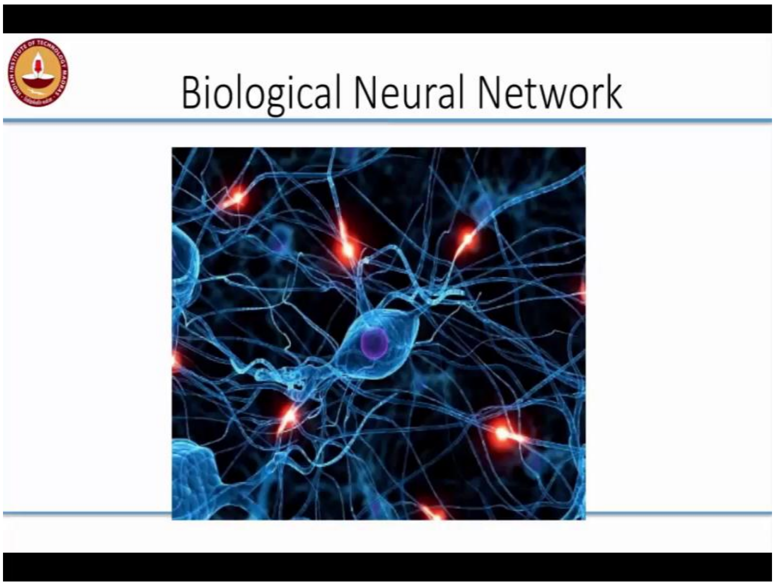

Biological Neural Network

So, these connections for the dendrites and synopses are going to be result in a very complex network and even though, the computing done by each element is very, very simple summation and thresholding. The sum total of this taken across the entire network and give raise to daily complex computations, which we will see.

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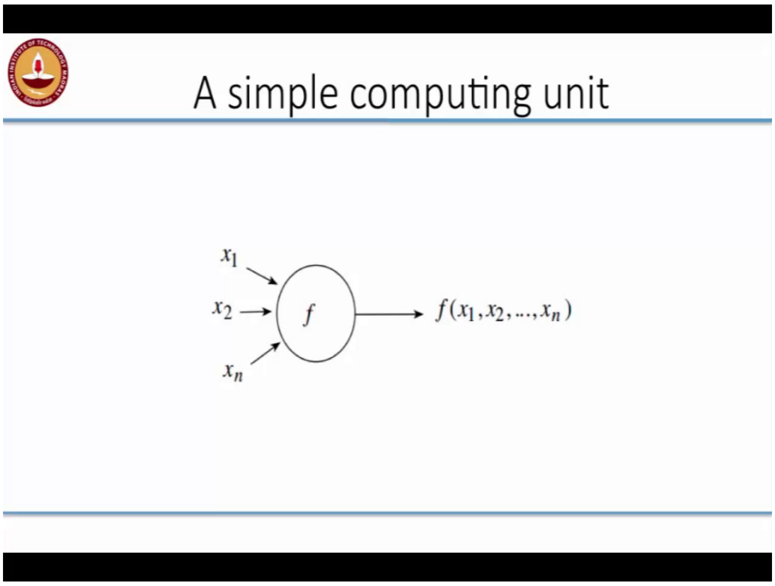

A simple computing unit

So, the completing unit is something that is very simple. So, it is going to take a set of inputs x 1 to x n and it is going to compute some functional arm, it is function is very simply incredible and then it will produce an output. So, we will look at what this function is going to be in detail in the next few slides.

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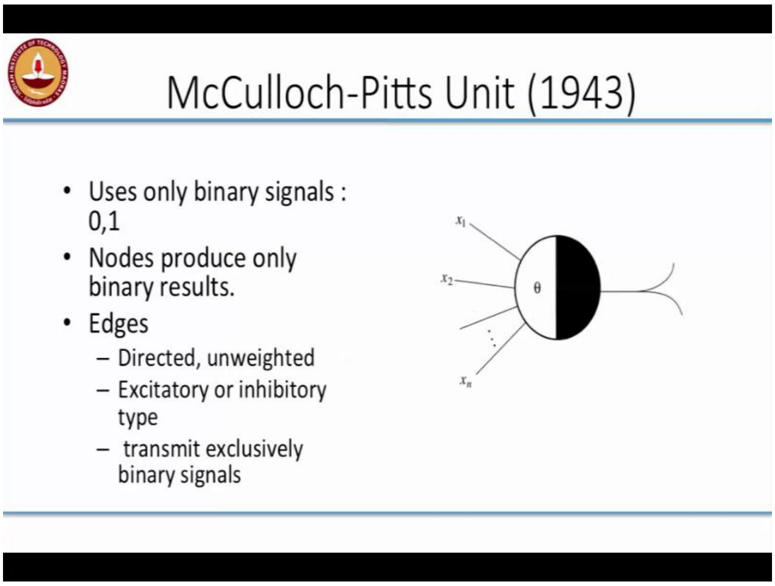

McCulloch-Pitts Unit (1943)

- Uses only binary signals : 0,1
- Nodes produce only binary results.
- Edges
  - Directed, unweighted
  - Excitatory or inhibitory type
  - transmit exclusively binary signals

So, the initial module for this for a biological neuron was proposed by McCulloch-Pitts in 1943 it is called the McCulloch-Pitts unit, it is only binary signals, so 0s and 1s. So, either an input is active, then these cases are represented by 1 or if it is not active, in this case it is represented by 0 and the nodes also produced only binary results. So, the outputs could be either 0s or 1. So, the edges between these different nodes where directed, unweighted, they could be of two types that could be excitatory or inhibitory and again I can mentioned earlier, the transmit binary signals.

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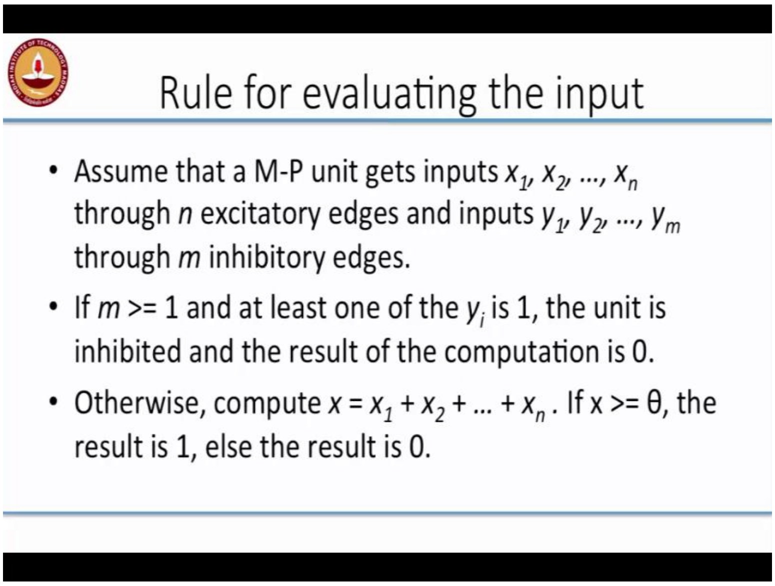

Rule for evaluating the input

- Assume that a M-P unit gets inputs $x_1, x_2, \ldots, x_n$ through n excitatory edges and inputs $y_1, y_2, \ldots, y_m$ through m inhibitory edges.
- If $m >= 1$ and at least one of the y_i is 1, the unit is inhibited and the result of the computation is 0.
- Otherwise, compute $x = x_1 + x_2 + \ldots + x_n$. If $x >= \theta$, the result is 1, else the result is 0.

So, what is the computation that happens here? So, I let assume that the McCulloch-Pitts unit gets inputs x 1 to x n through n excitatory edges. So, these are positive edges and inputs y 1 to y m through inhibitory edges. So; that means, these are edges that could produce the depression in the functional arm could actually stop the functioning of the neuron. So, the assumption that was made is, if m is greater than or equal to 1 that is at least one inhibitory edge and if any one of the inhibitory edges is 1, so if there is a one inhibitory input then the unit as a whole does not produce any output, regardless of what the inputs x 1 to x n are.

If none of the inhibitory inputs are 1 or if there are no inhibitory inputs at all, the unit computes the summation of x 1 to x n, let us call it x and if x is greater than the threshold that is specified for each unit, if it is greater than the threshold theta then the result of the computation is 1 as the result is 0. So, it is very simple, so essentially you can think of it as adding up all the inputs that come to the neuron and if the summation is greater than our threshold theta, your output 1; otherwise, your output 0. So, the inhibitory edges in some sense here acting act as a gating signal. So, if it is 1, the output is always 0, if the inhibitory is only 0 then the output is the result of the computation.

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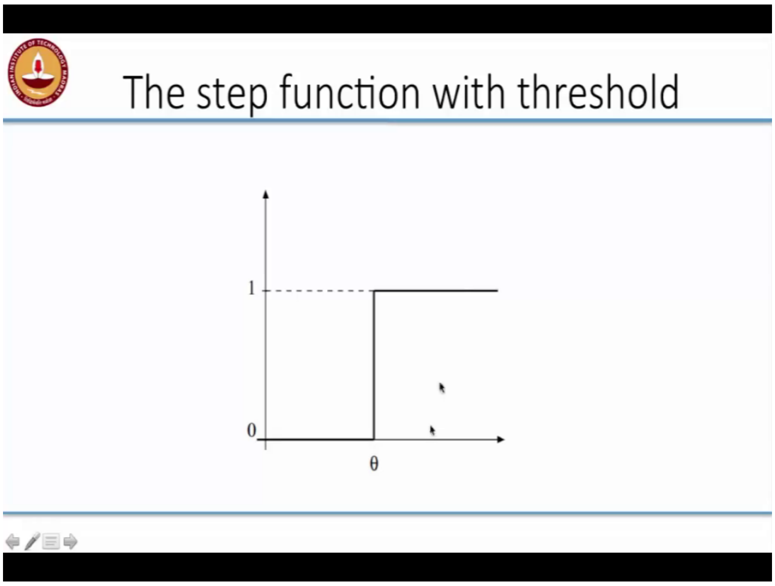

The step function with threshold

So, it is essentially the McCulloch-Pitts unit, it is implementing just threshold function. If the input is below theta you are going to see a output of 0, if the input is above theta you are going to see a output of 1, that this is essentially a step function.

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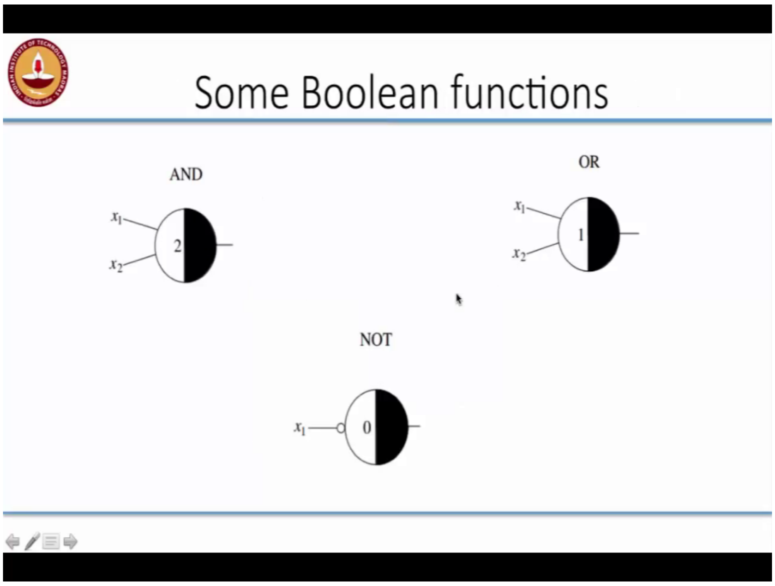

Some Boolean functions

So, what kind of computations can you do with this? So, you can actually do bulb of solve your familiar Boolean operations with the McCulloch-Pitts neuron. So, you can think of doing an AND operation, you have two inputs x 1 and x 2 and the threshold is set a 2. So, if only both x 1 and x 2 are one, so it will be greater than or equal to the threshold and therefore, the output will be 1 and for implement in a OR you can set the threshold that one. So, if either x 1 or x 2 is 1 to the output will be 1 after complimenting a NOT unit it can implement the NOT unit by having x 1 act as an inhibitory input. So, this circle here indicates an inhibitory input.

So, if x 1 is 1; that means, they neuron and inhibitory output will be 0 on the other hand x 1 is 0 then the output will be whatever according to the result of the computation. But, we can see here that the threshold for this neuron set as 0 and that is for the output will be always 1 as long as there is no inhibitory input. So, if x 1 is 1 then the output will be 0, x 1 is 0 output will be 1, that how we have implemented NOT function. Now, once we unable to implement this kinds of AND, OR and NOT then you know that you can connect neurons together and then implement any Boolean function that we want and is this what really we are interested in.

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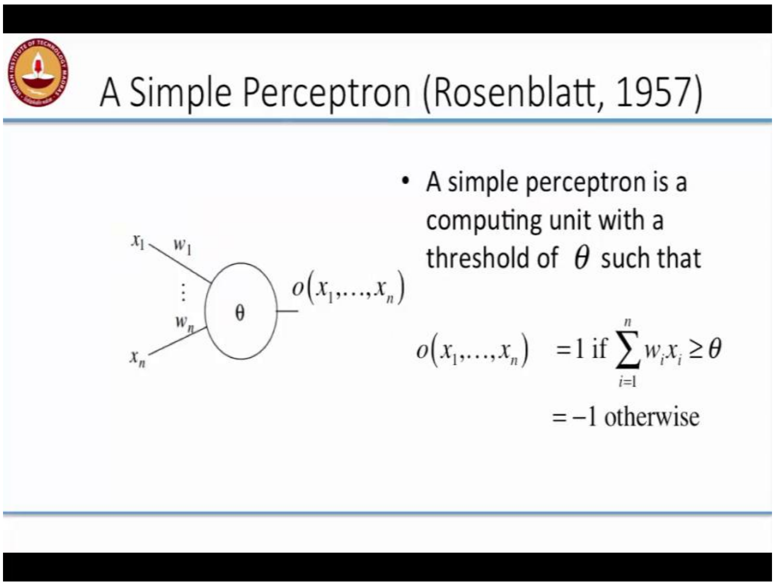

A Simple Perceptron (Rosenblatt, 1957)

- A simple perceptron is a computing unit with a threshold of θ such that

$$o(x_1,\ldots,x_n) = 1 \text{ if } \sum_{i=1}^{n} w_i x_i \geq \theta$$
$$= -1 \text{ otherwise}$$

So, we are not really interested in that because we want to be able to do more complex classification problems, then we would like learn simple things like linear surfaces or more complex surfaces that is separate two classes. So, that is ((Refer Time: 06:45)) goal of classification we are looked at so far. So, in 1957 rosenblatt proposed a very simple extension to the McCulloch-Pitts module which we called the perceptron, the more crucial thing what the perceptron is that a it introduced weights at the inputs, crucial differences from the perceptron from the McCulloch-Pitts module is that the perceptron introduced weights at the input.

And then it the output could be either a one or a minus one depending on whether the weight at some of the inputs is greater than threshold that one that is the computing unit with a threshold theta ((Refer Time: 07:23)) it. So, the output of the be around is one if the weight at some of the inputs is greater than or equal to theta is equal to minus 1 other wise.

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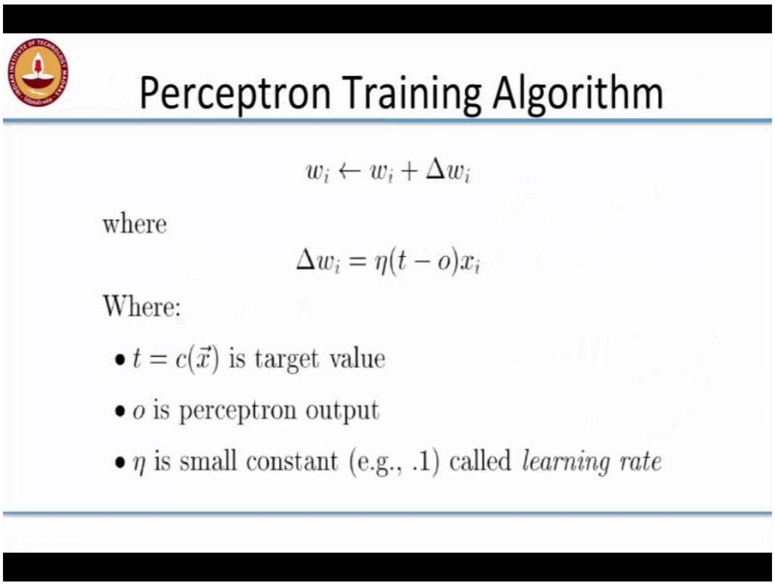

Perceptron Training Algorithm

$$w_i \leftarrow w_i + \Delta w_i$$

where

$$\Delta w_i = \eta(t - o)x_i$$

Where:

- $t = c(\vec{x})$ is target value
- o is perceptron output
- η is small constant (e.g., .1) called *learning rate*

So, what is the goal here in perceptron learning, when perceptron learning we are essentially trying to learn a hyper plane, trying to learn a separating surface as we have done in the past in the other classification problems, we are trying to learn the separating surface that can separate one class from the other. So, what would the classes we in our case, classes in our case would be plus 1 and minus 1. So, this essentially means if w i x i is greater than equal to theta, it essentially defines the equation of a hyper plane as we have seen at the previous modules.

So, if this you can take the theta to the other side. So, we like w i x i minus theta is greater than or equal to 0. So, we have seen that was greater than 0 to some one side of the hyper plane if it is lesser than 0 it is on other side of the hyper plane and we are going to say that data points to one side of the hyper plane belong to class 1 data points other side of the hyper plane belongs to class minus 1. So, now, the question is given a set of training data that gives you the x x the vector x and the decide output y.

How would we find these weights w i's such that the perceptron is actually implementing that hyper plane, implementing the right separating hyper plane. So, the weighted all this is follows, you start of the randomly initializing the weights to some value and then we look at the prediction that is made by the way. So, the prediction that is made by the current setting of the weights, let us called it o and the target is the two class of the data point x. So, with this, it will be plus 1 or minus 1 and likewise o is also plus 1 or minus 1. So, your goal is to make sure that here perceptron output maxes the target value.

So, the perceptron training algorithm has a very simple rule. So, at every presentation of an input point, we change the weights by an amount that is proportional to difference between the target value and the actual output produce times that the input on the particular it. So, w i changes by an amount that is proportional to t minus o times x i. So, eta here is a small constant may be 0.1 or 0.01 as called the learning data.

So, one thing to note here if for a particular input x I will produce the correct output. So, the class is minus 1 and they produce minus 1, the class is plus 1 and they produce plus 1 this expression evaluates to 0. You can see that this expression evaluates to 0 and therefore no changes in the weights will happen. So, essentially what happens here is you change the weights only whenever you make a mistake and that to you change the weight proportional to the input variable. So, if x i is say a small value say 0.1 or 0.2 then will be changes in the weight will be small and as for as the poster when x i is the large value let us say 1 or 0.95 and things like that then the change it be next will be large.

So, this essentially because the larger the input variable the more important it is going to be in the production of the output at least the way we are set up this perceptron. So, that is essentially the simple training rule. So, whenever you make a mistake, you take the vector for which we have made a mistake add some small fraction of that vector to the weights.

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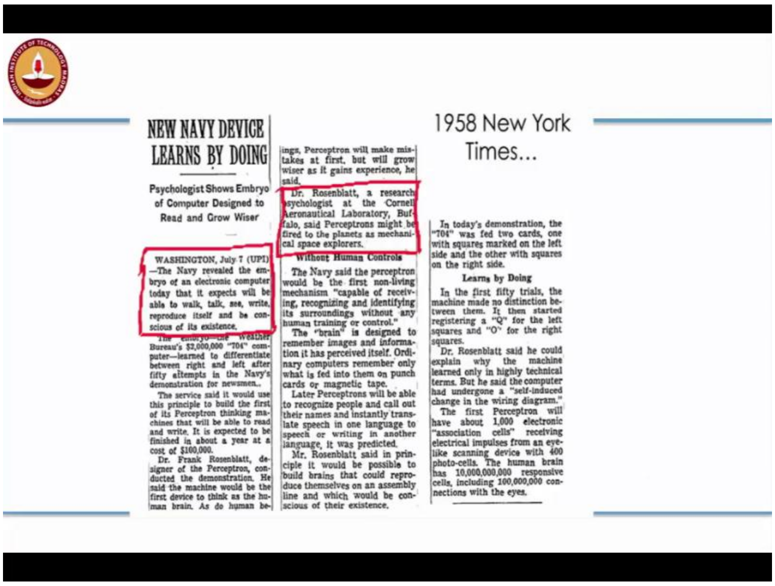

NEW NAVY DEVICE LEARNS BY DOING

Psychologist Shows Embryo of Computer Designed to Read and Grow Wiser

WASHINGTON, July 7 (UPI)—The Navy revealed the embryo of an electronic computer today that it expects will be able to walk, talk, see, write, reproduce itself and be conscious of its existence.

The embryo—the Weather Bureau's $2,000,000 "704" computer—learned to differentiate between right and left after fifty attempts in the Navy's demonstration for newsmen..

The service said it would use this principle to build the first of its Perceptron thinking machines that will be able to read and write. It is expected to be finished in about a year at a cost of $100,000.

Dr. Frank Rosenblatt, designer of the Perceptron, conducted the demonstration. He said the machine would be the first device to think as the human brain. As do human beings, Perceptron will make mistakes at first, but will grow wiser as it gains experience, he said.

Dr. Rosenblatt, a research psychologist at the Cornell Aeronautical Laboratory, Buffalo, said Perceptrons might be fired to the planets as mechanical space explorers.

Without Human Controls

The Navy said the perceptron would be the first non-living mechanism "capable of receiving, recognizing and identifying its surroundings without any human training or control."

The "brain" is designed to remember images and information it has perceived itself. Ordinary computers remember only what is fed into them on punch cards or magnetic tape.

Later Perceptrons will be able to recognize people and call out their names and instantly translate speech in one language to speech or writing in another language, it was predicted.

Mr. Rosenblatt said in principle it would be possible to build brains that could reproduce themselves on an assembly line and which would be conscious of their existence.

1958 New York Times...

In today's demonstration, the "704" was fed two cards, one with squares marked on the left side and the other with squares on the right side.

Learns by Doing

In the first fifty trials, the machine made no distinction between them. It then started registering a "Q" for the left squares and "O" for the right squares.

Dr. Rosenblatt said he could explain why the machine learned only in highly technical terms. But he said the computer had undergone a "self-induced change in the wiring diagram."

The first Perceptron will have about 1,000 electronic "association cells" receiving electrical impulses from an eye-like scanning device with 400 photo-cells. The human brain has 10,000,000,000 responsive cells, including 100,000,000 connections with the eyes.

So, this looks like a very simple rule, but then pack in 50's this perceptron's created a lot of human cried the people saw that the perceptron's by the able to learn from scratch trying to solve something which are considered hard learning problems and then they used the perceptron's they were able to solve that, so much so you can see here the height was that they are going to build the computer that expects to be able to walk, talk, see, write, weight reduce itself and be conscious of it is existence, such the significant amount of height and it is always hard to live up to any height that this proportionate and to the actual effect that was achieve that point.

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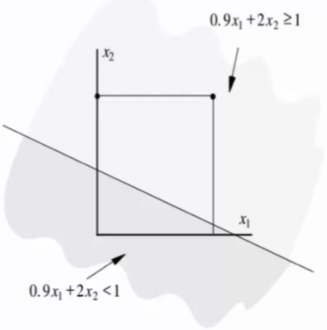
What can a perceptron learn?



So, let us take a look let us just back and take look at what can of perceptron learning is a news paper article really true or what are the limits to the perceptron’s learning ability. So, here is a very simple perceptron here, so it has a two input variable x 1 and x 2 and that is the threshold of one and the weight w 1 is 0.9 and w 2's 2. So, if you look at it essentially it implements this straight line here, so everything above the straight line this light color regions belong to one class and the dark color regions belong to another class.

So, we know that these are data which are linearly separable, you saw this in the case with SVM's. So, these are data that are separated by a linear hyper plane or the linear separating surface. So, all data points for which the w transpose x evaluate, so greater than x is 1 will get a class of plus 1 all those that evaluate to lesser than 1 and get a class of minus 1.

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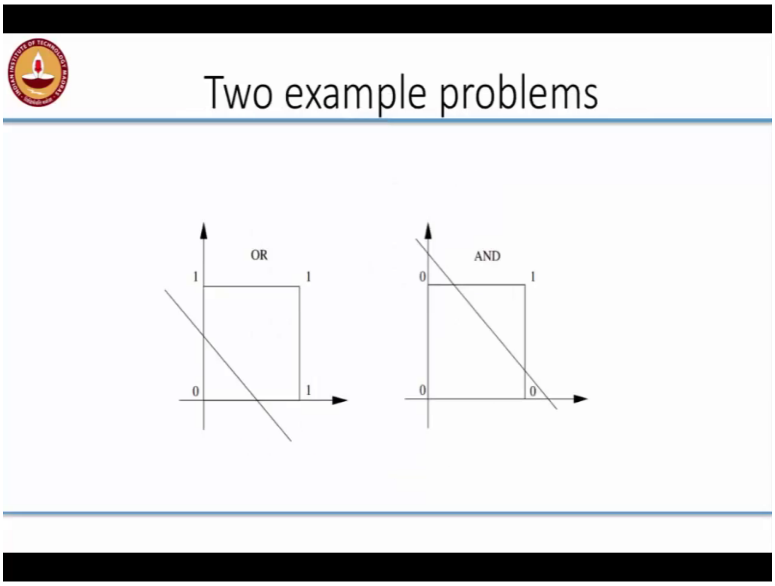

Two example problems

So, again let us go back and look at the simple logic function that we saw earlier. So, it can implement that OR. So, essentially OR requires you to have a hyper plane and this passing cares. So, everything to this side this become plus 1 everything to this side becomes minus 1 and likewise you can implement and so you can draw a simple hyper plane. So, everything to this side become plus 1 and everything this side becomes minus 1 or 0, I mean depending on how you wanted to predict the output.

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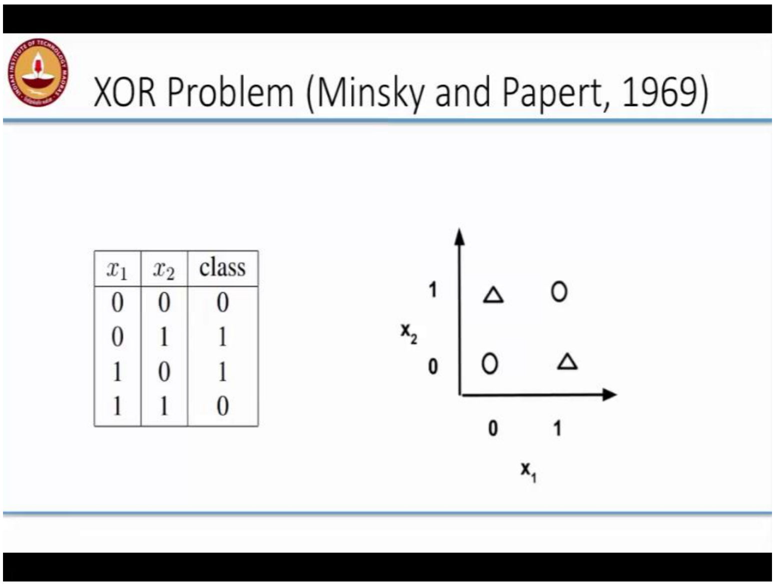

XOR Problem (Minsky and Papert, 1969)

x_1	x_2	class
0	0	0
0	1	1
1	0	1
1	1	0

And let us look at another one, look at simple problem just like OR and AND they solve problem. So, Minsky and Papert in 1969 in a famous monograph called the perceptron’s showed that well a simple problem like XOR. So, where the truth table is given here is the inputs of the same output is 0, if the inputs are differently output of 1, the simple problem like XOR is not linearly separable, you cannot draw a hyper plane that separates these two classes.

So, forget about walking, forget about talking and doing all those wonderful things that was claimed to news paper article perceptron’s cannot even solve this as simple problem as XOR is essentially says that two things are same, the output is 0, two things are different the output are 1 that we cannot recognize the similarity between this simple inputs like 0's and 1's that kind it do complex computations. So, once Minsky and Papert showed this, it is a kind of you know ((Refer Time: 14:36)) research into neural networks for a long time until there was revival much later.

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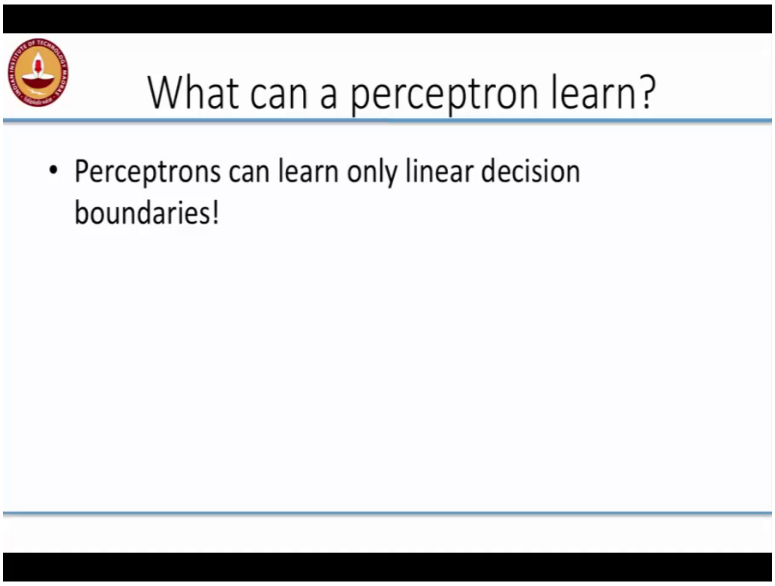

What can a perceptron learn?

- Perceptrons can learn only linear decision boundaries!

So, perceptron’s can learn only linear decision boundaries that is the take away message here. So, that is make that is whole idea of neural networks completely useless, because they can learn only linear decision boundaries in case of SVM's we saw that we could get it to do all linear boundaries by going into carnal expansion this has something similar that we can do here.

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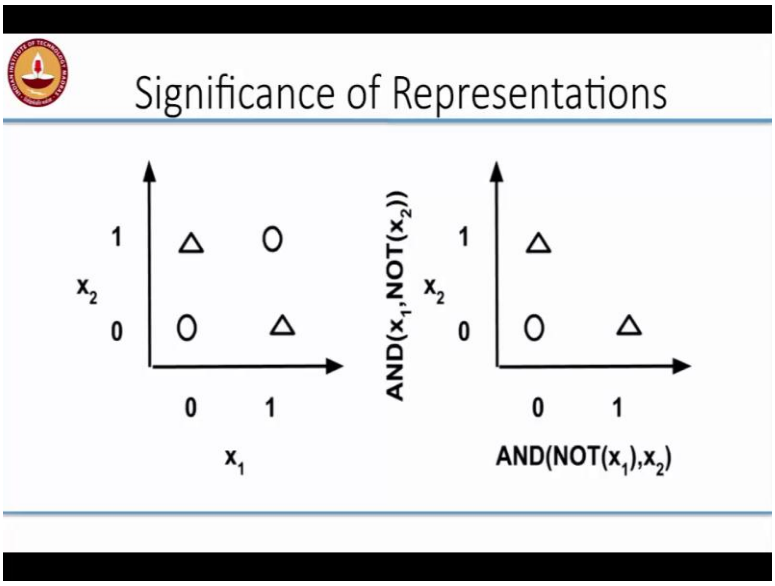

Significance of Representations

Let us look at how we can change the representations and try to do something more clever. So, if you look at the original problem the XOR problem, so I have my inputs x 1 and I have my input x 2 and now we can see that in this space the problem is not separable. But, let us look to do a simple transformation on my data points, so instead of looking at x 1 I will define my first variable as NOT x 1 and x 2 and similarly I will define my second variable as x 1 and NOT x 2.

So, if you think about it, so we can now plugging different values of x 1 and x 2 here and see what the outputs will be and then you can see that when x 1 is 0 and x 2 is 0, the output is going to be 0, when x 1 is 1 and x 2 is 0. So, the output here will be x 1 is 1 and x 2 is 0, the output here will again be 0 and x 1 is 1 and x 2 0 the out here will be 1. And we know that 0 1 the output has to be 1, so that we get it here and likewise for the symmetric case this will be the output and so you can see that this is again going to be 1 and when x 2 x 1 x 2 both are 1 again the output will be 0 0 and therefore, this is the resulting point.

Now; obviously, this representation the data points are linearly separable. So, now, the task becomes one of finding the right representation, such that the data becomes linearly separable for the next level, next stage of computation. So, people realized this very quickly, so even though a single perceptron cannot solve complex problems like XOR which not linearly separable, he could actually stack layers of neural neurons and then have the first layer compute something that is simple.

So, you can always compute NOT of x 1 we saw that earlier and also can be computed by a single neuron. So, you can this get have layers of neuron that exactly compute your features and of NOT x 1 comma x 2 and then have another neuron, which takes the output of this neurons combine same together and produces the output that you want. So, people very quickly realize that stacking this kinds of neurons into layers allows you to do more complex computation.

In fact, it is easy to show that stacking this neurons into layers actually builds a universal function representation that learns to a represent any Boolean function, you see a combination of neurons. So, what is a problem, now we know how to solve this more complex problems, why did the research in neural networks pick up again.

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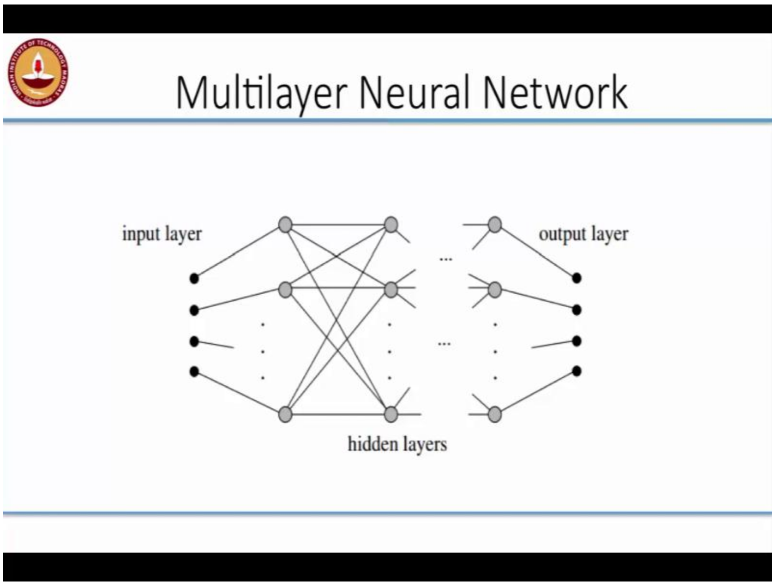

Multilayer Neural Network

So, the question here is when I start connecting all of these neurons into layers. How do I find the weights? So, perceptron learning algorithm make no longer work in this case actually does not work in this case and people were struggling to come up with the mechanism for training all these weights. So, you can see that the way of started putting these things into layer. So, that is one input layer and one output layer, so there is one input layer, there is one output layer and in between this you could have many layers of neurons, they are typically called hidden layers because you do not observe their outputs directly.

So, now, we have this many, many hidden layers of weights and it is little hard to find out what this weight should be and so in the mid 80's around 83 an algorithm was proposed called back propagation which allow you to learn the weights of this we solve this hidden layers.

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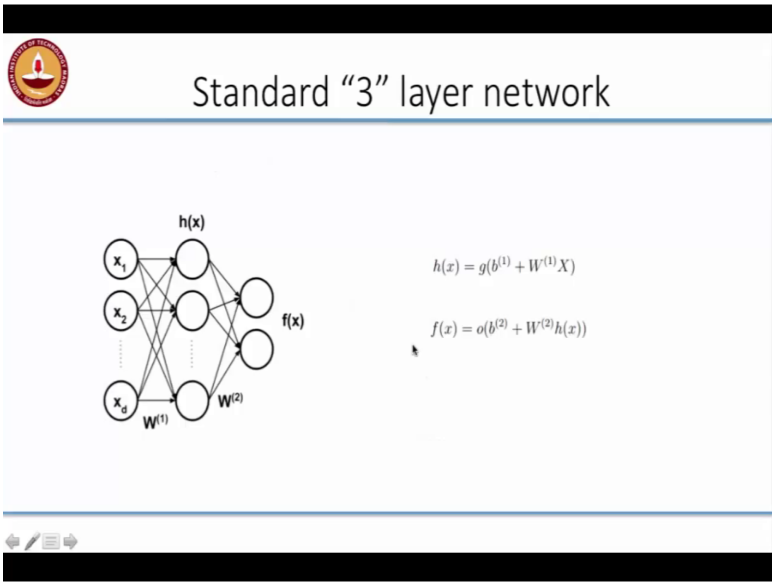

Standard “3” layer network

$$h(x) = g(b^{(1)} + W^{(1)}X)$$

$$f(x) = o(b^{(2)} + W^{(2)}h(x))$$

So, for the rest of the presentation, we will be looking at the standard three layer network. So, there is an input layer x 1 to x d and the output layer which will denote by f of x and one hidden layer of neurons. So, these take the inputs from the input layer do the weight at some do your thresh holding function and then produce an output and then the neuron and output layer will take all this outputs of the hidden layers take their weight at some and take the threshold or not and that produce the output, instead of using hard threshold we use a kind of a soft threshold like in order to do this competitions this is needed, so that you can derive more efficient training algorithms later.

So, the output of a hidden units and it given by g of the bios term, this is the theta that we had earlier. So, instead of theta so it is going to call it b 1 and plus w 1 times x and the output of this will be note by h of x and the output of the final layer of neurons is given by some function o of b 2 plus w 2 times h of x. And so now, the goal here is to figure out what this w 1 and w 2 are going to be. So, this is called the three layer network, even though there are only two sets of weights that we have to learn.

So, the layers here talk about the neurons here, so we use for each input variable we are same that there is separate neuron that is activating the hidden units. So, this is called the standard three layer network structure.

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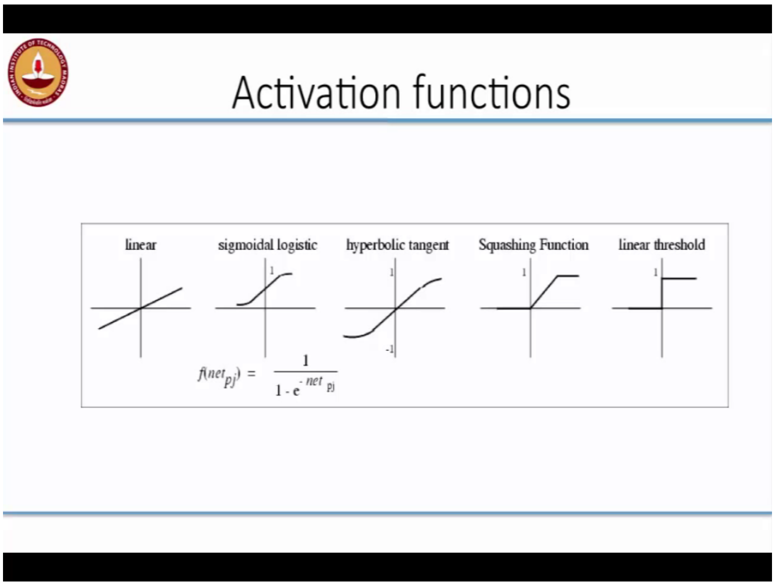

Activation functions

$$f(net_{pj}) = \frac{1}{1 - e^{-net_{pj}}}$$

So, what are the different activation functions you can use? So, we already looked at one which is the threshold function, we can also have just a linear activation function that basically takes the summation of weighted summation of all the inputs and outputs as it is. We can also look at the sigmoidal function, sigmoid logistic function which takes the summation input and then squashes the input. So, that get lays between 0 and 1 and then there is a steep raised somewhere around the threshold. So, that it transitions rapidly from 0 to 1.

When if you are interested in having signed outputs then you can think of using a hyperbolic tangent, where the outputs are going to taxation between minus 1 and plus 1 and again around the threshold. So, there are parameters at control where the threshold would be and how steep the price would be. So, another transition function some time gives is this squashing function, which is 0 before the threshold and one at a certain distance higher than the threshold and in between you have a... So, linear approximation adds to the step function, this called the squashing function.

So, typically in most of the neural network architectures that we look at will be looking at either the hyperbolic tangent or this, the logistic sigmoid or the linear activation, because these are different shape and this allows as to derive efficient training algorithms for the same. So, if you are doing a classification problem then the output layer could be the hyperbolic or a logistic sigmoid and if your solving a regression problem, the output neuron could be a in linear neuron. So, that you can do appropriate regression fit.

The hidden layer almost always has to be a non-linear function and where the little bit of what you can show that if the hidden units have a linear activation, like you might as for not have them at all. And what is the function that is implemented is something which can be as well implemented by a single layer of neurons. And the next module we look at how you the exactly find out these weights given the assumption that they are working with the sigmoidal logistic function.

So, the function for the sigmoidal logistic thing is given by f of net is 1 by 1 minus e to the power of minus net. So, that is the function and look at, given that this is the activation function how we are going to derive the weights of the two layer standard three layer you will get. So, that is in the next class.