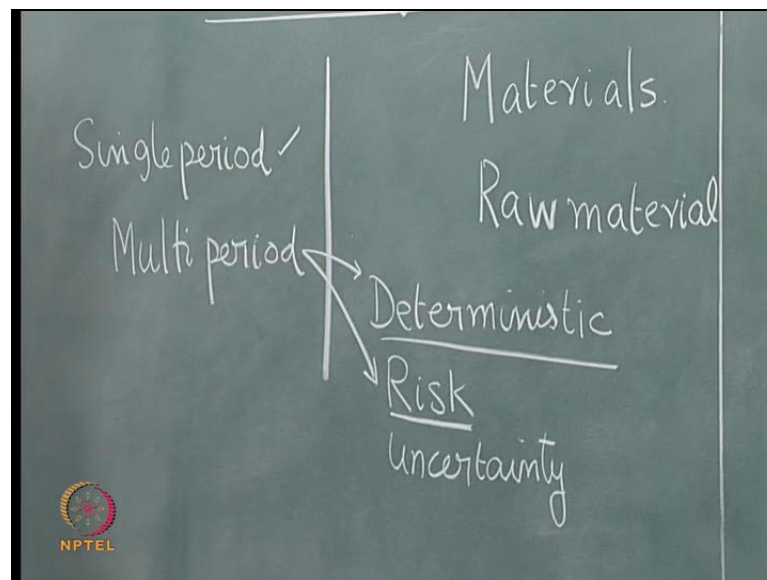


Operations and Supply Chain Management
Prof. G. Srinivasan
Department of Management Studies
Indian Institute of Technology, Madras

Module - 01
Lecture - 09
Inventory Models - Costs, EOQ Model

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In today's lecture we study inventory models. Inventory essentially deals with materials the procurement of materials. Materials management is to how we store the material for subsequent use in every manufacturing organization or manufacturing situation. We buy material and these material are usually bought as raw material. Sometimes these raw material undergo manufacturing and become semi finished or finished products. Some of the bought out items go directly into the assembly of the product. Earlier we have seen aggregate production planning where the output of the aggregate planning is telling us how much of production capacity is available for every period.

Later we would also be studying disaggregation models, where the production capacity that is available in each period is, now allocated to the various products that are made. So, at the end of disaggregation we would know what are the products that are going to be made. In order to make these products we need to buy material. So, we are now going to look at how we buy material. So, inventory essentially answers two very important

questions. If there is an item or material that we are going to buy there are two important decisions that are associated with the purchase of the material.

And these two questions are how much to order and when to order. So, obviously when the production planning is complete and the allocation of time to the various products are made one has an idea of how much or how many products of a certain type we are going to produce. So, the final product, whatever we are going to produce of the final product of each of the products.

Now, requires certain raw materials or bought out items in a certain number. So, if the demand of the various products are known. Now, this demand can be suitably multiplied to find out the requirement of each of these items or material that goes into the final product. Once we know the annual demand of each of the item that goes into the final product we need to make decisions on what quantity we are going to order every time we place an order and when exactly we are going order.

So, inventory control or inventory management essentially deals with these two aspects. Let us also spend some time on the classification of inventory models. Broadly two types of classification. One is called single period inventory models and multiple period inventory models.

Single period inventory models assume that the planning horizon is a single period and the decision is made only once. Multi period models assume that there are several planning periods or multiple periods. And decisions are made more than once. Sometimes these are also called static and dynamic inventory problems. Now, the important thing that drives the inventory management is the actual demand for the item. The moment we know that demand then only we can answer these two questions how much to order every time we order and when to order.

So, now depending on the nature of the demand we can have broadly three classifications one is called deterministic demand the other is probabilistic demand and within the probabilistic demand we have two classifications. One is called inventory under risk the other is called inventory under uncertainty. Deterministic means the annual demand is known and known with certainty. Risk would mean that the demand is known, but the demand is not deterministic it follows a certain distribution and the entire distribution is known. If the distribution of demand is known then it is called problems under risk.

If it is assumed that the demand is not deterministic the distribution of demand is not known but, certain important parameters such as mean and variance of the demand are known, then it is said to be inventory models under uncertainty. And in this lecture series on operations and supply chain management we will be looking at multiple period deterministic models and multiple period with risk.

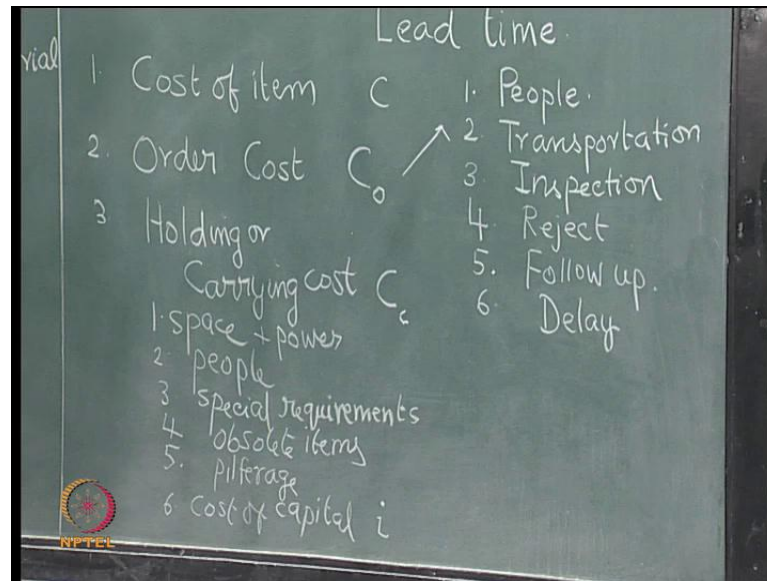
We would also try and look at some one or two single period models with nondeterministic demand. So, we will first take the case of the deterministic demand and then look at some models where the demand is known and deterministic. Then we will move into probabilistic inventory models particularly covering situations of inventory under risk. Then as mentioned in inventory problems we basically answer these two questions which are how much to order and when to order.

One of the reasons we have to answer these two questions is the material that we buy or the items that we buy for manufacturing are special items which somebody else is producing and selling to the organization - that is manufacturing this product. So, the vendor or the person or the supplier who actually provides this material also requires some time to make this and then sell it.

Therefore, it is necessary to fix on a certain order quantity which is optimal such that the person who makes it and sells it has the time to do. So, organizations do not for example buy items the same item everyday traditionally they would. But, a slightly larger quantity than what was required every day. So, the question of how much to buy or how much to order came about more recently organizations have moved towards just in time manufacturing.

And just in time purchasing where organizations tend to buy exactly what is demanded or they buy for the demand of a very short period of time. Unlike in the older models where you actually end up fixing an order quantity which is optimized which may result in the quantity being the demand of a larger time period. We will first look at some traditional models where we actually try and optimize and try and find out what is the quantity that we have to buy.

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Usually this thing how much to order is called the order quantity and is denoted by the letter Q and when to order is called reorder level and is denoted by r . Now, the second question is also important because when we place an order to the supplier the supplier has to make that item and then give it. So, there is a time that the supplier needs to make it and the variable called when to order is chosen such that when we place the order. And by the time the order arrives we should have enough material with us to meet the demand during the period between placing an order and arrival of the order.

Now, this period between placing an order and arrival of the order is called the lead time. Very broadly defined, lead time is the time between placing an order and receiving it. Lead time also actually has several components. Sometimes we look at identifying that an order has to be placed and placing an order between these two certain time elapses and then the order is placed and the order is met or supplied. And then by the time the item is supplied and the time the item is put to use certain amount of time elapses. Sometimes we consider all of them as the lead time, but for the purpose of our discussion we would assume that the lead time is the time between placing an order and receiving it.

So, the when to order question becomes important because we need to place an order such that by the time the material arrives or for the period taken between placing the order. And the arrival of the material which is the lead time we should have sufficient stock to meet the demand for the item during the lead time. So, generally reorder level or

the point at which we place the order is equal to the demand during lead time. So, that that quantity which is called the reorder level we should be able to meet the demand during lead time which is called lead time demand.

When the demand is deterministic and the lead time is also deterministic and known with certainty, then the lead time demand can be computed as the product of lead time and daily demand. But if one of them is nondeterministic, then we have to look at expected value of the lead time demand and then try and see whether the reorder level is about the expected value of the lead time demand or sometimes more than that. We will see those aspects as we move along as we move towards probabilistic inventory models later.

Now, we will restrict ourselves to deterministic inventory models where the demand is known with certainty we go back to the other question how much to order quantity there one other reason. That we do not order the same item everyday for consumption is that there are other costs associated with inventory. So, obviously when items are bought there is a price or which is called the cost of the item. So, when we buy a single item we there is a unit cost associated with that item.

So, if we order a quantity Q and if the unit cost is C then Q into C is the actual worth or cost of the item that we have to incur when we buy that item. If we assume that the annual demand is known and this capital d then D into c is the worth of the item or the money spent on buying the item per year under the assumption of course. That the cost of the item does not change during the planning period we normally assume that C remains as C . And it does not increase or there is no effect of inflation on the cost of the item and so on. But, in addition to the cost of the item there are few other costs that we incur. Now those are three other types of costs that we incur.

One is called order cost or ordering cost which is denoted by the notation C_{naught} or C_0 or C_o order cost which is incurred every time an order is placed for an item. Now, this cost is different from the actual cost of the item this does not depend or is independent of the quantity that is being ordered. But, every time there is a decision to place an order or make an order a certain order cost is incurred order cost has several components some of which I will list here now. First one is the cost of people or payroll money given to the people who work in the purchasing areas their salaries have to be factored in. And they become part of the ordering cost because they order several items day by day.

So, there is a part of the order cost that goes into meeting the salaries of the people who are involved. Second is there is a transportation cost for the items to come from the supplier to the organization which has been ordered which is included in with cost of order. We can assume that there is a onetime transportation cost for bringing the items from the supplier point to the organization third is there could be inspection costs because these items that come in have to be inspected.

Sometimes we could have hundred percent inspection. Sometimes there could be random sampling. But, in some form these incoming goods are inspected and there is a cost associated with the inspection. Next is however undesirable, we could have a situation where at the end of the inspection there is a reject. So, the rejected items have to go back and then have to be replaced, so there is a delay there is a time element as well as there is a cost element.

So, there could be cost of reject and rework with respect to the supplier side. Fifth one and the perhaps another important one is called the cost of follow up. For example, an order is placed and sometimes there could be delays and then the retime is estimated and known. And just about the time we are going to reach the lead time which means the item has to come then there is a lot of follow up activity going on. And some of these follow up activities could even be senior people from this organization visiting the supplier and making sure that the critical items that are required come in time.

So, there is a cost associated with follow up. Follow up could be over the phone it could be over other forms which are expensive it would also mean a visit of a person going to this place and then may be inspecting it then and there and bringing it here. So, there are these costs that are associated with the cost of ordering. Of course, there is any delay then each of these delays is going to into result in a higher cost.

So, all these are the components of what is called order cost or ordering cost and this cost does not depend on – it is assumed that it does not depend on the quantity that is ordered. For example, if you look at these components one could say that at least this component could be could depend on the quantity that is ordered. But, then right now we assume that the order cost is not dependent on the quantity that is ordered and is fixed for every order.

Now, the other important cost that comes - that the organization has to incur after the item arrives is called inventory holding cost or carrying cost - holding or carrying cost. Now, this is denoted by C_c inventory carrying cost of carrying inventory which is C_c . This also has many components. Now the material arrives and it arrives in a certain quantity which is Q which we have to find out.

And then as I mentioned when we do the optimization Q , let us say, is a little more than the daily demand which means that the material that comes is going to be used for a certain number of days. And then an order is going to be placed the next order is going to arrive and it is going to be used. So, when this quantity Q comes it has to be saved it has to be stored it has to be held in control and there is a cost associated with that, now that cost is our inventory holding cost or inventory carrying cost.

So, the first cost that we can think of is the cost of space to keep these items. So, they have to be kept in some warehouse or some storage area and stored and kept safe and secure and the moment we want safety and security. We need people who will guard these items from theft and keep it safe and secured. So, the salary associated with - salary of the people who are associated with storing the inventory is another component.

Sometimes along with space you could also have power and other accessories which are required. So, there will be cost of power that is there in the storage area and other accessories. Sometimes some of these items may require very specific environment such as a dust proof environment or sometimes certain temperature which has to be maintained for some of these items.

These are not very common, but not uncommon either that there could be some items which require a certain temperature conditions, dust proof conditions and so on. So, we need cost to do certain other things such as other special requirements. Now, the next thing that can happen is suppose we order a very large quantity Q . And before we consume this entire quantity which would say run into several days of consumption for some reason either the technology changes or there is a better product that comes in and so on and this item that we have bought can become obsolete and unusable.

So, if such a situation occurs there could be cost of obsolescence. So, I write this as cost of obsolete items again however undesirable it is, there are times when the items get

stolen. And there is a certain pilferage cost that the organization has to occur particularly when they are metals tend to be easily stolen if they are not properly watched.

So, however small and undesirable it is there could be a cost of pilferage. Now, over and above all these, is an extremely important cost which is a very integral and a large component of the cost of carrying. Now, when we buy this item say Q quantity of Q of a particular item which costs C then Q into C is the cost of the item and the organization has to spend this money in buying it.

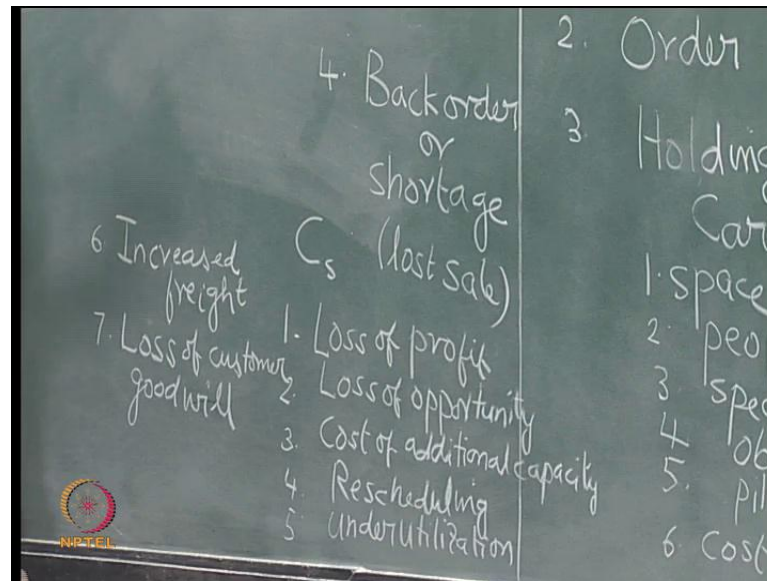
Now, the organization is going to recover the money much later after all these items are put to use the product is manufactured the product is sold and then the money is recovered. So, when the organization spends this Q into C to, but this item the organization actually borrows this money Q into C from some source. So, for the borrowed money the organization has to pay interest which is called the cost of capital.

So, the most dominant cost of holding the inventory is the cost of capital though I have written it as the sixth item or the last item this is indeed the most important of the costs. So, this is also defined as some kind of interest rate that the organization spends which is given by small I and therefore this C c if we consider all these costs. Then this C c is the cost of holding the item it comes in. So, many rupees per unit per year or per period many times it is customary and if we actually workout the calculations for the various components one would see that this takes a very large proportion of the C c and the rest of them are actually very small compared to the contribution of this.

So, in a situation where this dominates the rest of them it is also customary to write C c as i into C where i is the interest percent per year and C is the unit cost of the item. So, carrying cost can be written in two ways one is it is called C c cost of carrying where all these costs are included. And there are situations where cost C c is written as i into C where the interest on the borrowed capital dominates and is taken and the rest of them are assumed to be negligible compared to the interest rate.

Now there is a fourth cost.

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Let me write the fourth cost here and the fourth cost is called cost of back order or cost of shortage. Now, let us assume that we are buying a certain item and based on the availability of this item. Let us say that we have scheduled the production of a certain product or a component let us say on a Friday. And let us assume that this item is expected to come on Thursday. So, that on Friday morning when the production happens the material is available for production.

And let us assume for some reason we are not getting on Thursday or Friday and when the Friday morning when the production is about to begin the item has not come it is not available. Therefore, we will not be able to produce it on Friday say now if the due date for the item or for the product that is to be made on Friday, which means we will not be able to deliver it to the customer on Friday.

So, there will be a delay now that delay is ordinarily called as shortage. We have not been able to meet the due date or not able to meet the customer requirement for want of material. Now, this can be handled in two ways one is to say that well I am not able to deliver it to the customer on Friday. At the same time I will carry the unmet demand to the next day which is say Saturday produce it and give it.

Now, such a situation is called back order where the unmet demand is carried to the next period produced and then the demand is met. The other is called shortage or also called lost sale which means if I am not able to meet a certain demand on Friday I have lost it

and I will not try to recover it by back ordering it. Very loosely both of them are called shortage. Shortage has to be specified whether we are looking at a back ordering situation or on a last sale situation usually when it is not specified it is assumed that it is back ordering situation. So, there is a cost associated with this if the material does not come in time, now such a cost is given by the notation C_s cost of shortage.

Now, this also has several components the first component particularly if it is a lost sale is the loss of profit associated with the sale because the sale is not complete because for want of material. So, there is a loss of profit associated with that one could even go to the extent of adding something like loss of opportunity. For example through the sale I might get further business which is lost because I have not met that deadline or due date.

If it is a back ordering which means I carry the demand to the next period I need some more capacity in the next period to meet the additional demand. So, cost of that capacity. So, cost additional capacity to make this item in either case whether it is a back order or a shortage I do not have the material. So, I would not be able to produce, so either I have to do some rescheduling and produce some other things for which material is available. So, if we do that then there is a cost of reschedule if we are unable to produce some other item during the time that we have specified for this item. Then the resource is not utilized for this period is idle for this period. So, cost of idleness of resource or underutilization of resource.

Now, let us go back to the situation where we have not been able to produce it on Friday. But, we try and give it on the next day we produce it on the next day, now let us assume for a moment that we had we had actually decided scheduled it for Friday. Now, let us assume that if we were able to make it on Friday then let us say we would have used a truck to take the finished goods to the customer.

Now, there is a back order we are not able to produce and meet it on Friday. Let us say we are able to do it only on Saturday, but then we do not want that delay to be carried to the customer we do not want it to go much later. So, one of the ways we can actually speed it up is by instead of sending it by road or by truck one could air lift the item and send it to the customer. So, that would that would result in increased freight.

If we choose to take faster modes of transport which are expensive. So, there could be an increased freight if in order to meet the due date we try and speed it up by sending it by

air instead of sending it by road. Now, we have seen six different aspects of C_s , but there is one most important one just as cost of capital is the most important one in C_c there is very important component of this which is called loss of customer goodwill.

Now, if there is a delay when the customer is not going to be happy with the performance of the manufacturing organization. And if this delay continues then definitely there is a point at which the organization would lose the customer or the organization starts getting worried. That they are going to lose the customer and there is a goodwill which is extremely important for the survival of any organization now that gets hurt and there is a loss of good will.

So, organizations are very concerned about the loss of goodwill that could happen because of delays and because of material not being available. So, the cost associated with loosing customer goodwill is an integral part of C_s . The only difficulty there is it is extremely hard to give a numerical value to this one cannot say that it might cost goodwill so, many rupees per day delayed. Every organization knows that this cost is high and every organization wishes to treat this cost as high. So, that we do not compromise on that or we do not take that for granted at the same time almost no organization can accurately estimate what is the cost of loss of customer goodwill.

So, these are all the components that go into the inventory, now four important costs the unit cost of the item that is bought the ordering cost the carrying cost. And the back order or shortage cost which are described using notation C for the cost of the item C_n or C_o which is the ordering cost C_c or C_i into C which is the carrying cost and C_s which is the shortage cost.

Now, the other three costs are different and they are additional costs they are over and above the actual cost of the item. So, when an organization maintains - buys maintains inventory the organization actually incurs all these four costs depending on certain situations. So, the inventory management which talks about how much to order which is Q which is the first question that we will look at we will, now have to try and optimize and keep all the four costs to the minimum.

So, the objective will be to try and minimize the sum of all the associated costs. All the four costs which are there depending on the setting or depending on the assumptions the decision variable is how much to order which is Q . So, with this we will try and look at

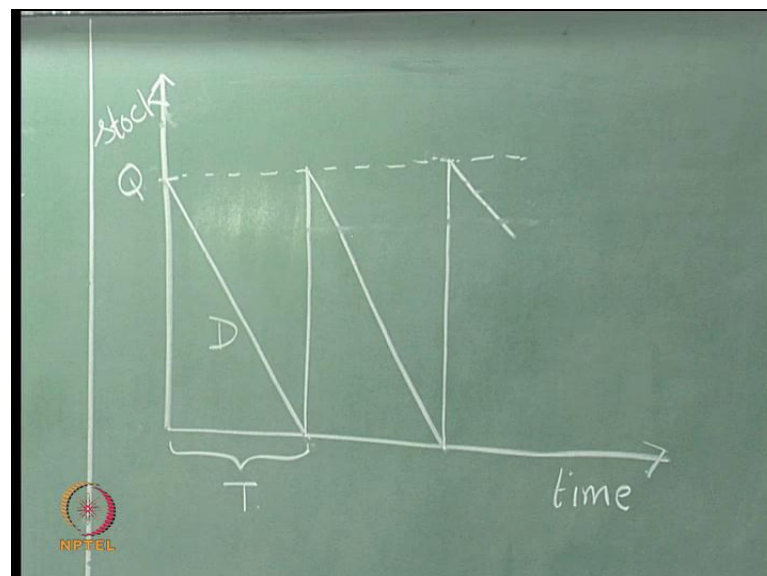
some very basic inventory models and then extend these inventory models to slightly more advanced situations.

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Annual demand = D / year
 no. of orders / year = $\frac{D}{Q}$
 Total order cost = $\frac{D}{Q} C_o$ Rs/year
 Average inventory = $Q/2$ units
 Total holding cost = $\frac{Q}{2} C_c$ Rs/year
 Cost of the item = DC Rs/year
 $TC = \frac{D}{Q} C_o + \frac{Q}{2} C_c + DC$
 $\frac{dTC}{dQ} = 0 \Rightarrow -\frac{D}{Q^2} C_o + \frac{C_c}{2} = 0$
 $Q = \sqrt{\frac{2DC_o}{C_c}}$
 Economic Order Quantity
 EOQ

So, the first inventory model that we will look at which we call as model one is called single item continuous demand and instantaneous replenishment. Now, let us assume that the annual demand for this item is D per year and is known. Now, let Q be the ordering quantity that we decide to order every time we place an order. So, let us look at how the inventory for this item behaves with respect to time.

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So, let us draw a simple graph which is like this is time and let us assume that we have just placed an order for Q . And we have just instantly received it instantaneous replenishment would mean that as soon as you place the order the item arrives which also means that lead time is 0. So, the moment the order is placed the items are received.

So, let us assume that we have received Q and we are consuming or using this Q . Now, when we say continuous demand we assume that the demand is there the same amount of demand is there at every instance of time. Therefore, as time goes on the demand D is met is there for every instance of time it is met and the stock of the item comes from Q . It is consumed and comes till it reaches 0, now this slope is D which is actually the demand for the item to put it in simple terms more understandable terms. It will be like saying if the annual demand is ten thousand and let us say if the company works for two hundred and fifty days.

Then the daily demand is forty if the company works for eight hours a day then the hourly demand is five if the hourly demand is five. The demand per minute is five by sixty and demand per second is that divided by sixty and so on and the demand is the same. That is that is how we should look at this continuous demand which means at every instance of time there is a demand. And since that demand is same which is D per year, now this will come as a straight line from Q to 0.

Now, the moment this stock position reaches zero because we have instantaneous replenishment and we also assume no shortage. We do not want to model shortage situations no shortage since we do not want to model shortage as soon as the demand reaches as soon as the inventory position reaches zero. I will call this as stock or inventory position on the y axis time on the x axis. So, as soon as the inventory position reaches zero I place an order I place an order for another Q and there is an instantaneous replenishment of that. So, as soon as I place an order here my stock position goes up to this Q .

Now once again the cycle continues and then I start consuming this till I reach this. Then again I place an order and then I start consuming. So, this is how the inventory position will behave please note that it is the same Q which is here as well as here and. So, one now let us call time period as capital T , now let us derive a very simple formula to find out Q , now every time we place an order we order a quantity Q . So, number of orders per

year will be D/Q because every time we place an order we order for Q number of orders per year is d/Q total order cost is equal to D/Q into C_o .

Now, C_o is this order cost. So, every time we place an order we incur a C_o and this C_o has a unit of rupees per order. So, D/Q is the number of orders per year C_o is rupees per order. So, D/Q into C_o will have a unit of rupees per year spent in order cost, so this cost we have considered. Now, we have to consider this cost holding cost or inventory cost now holding cost or inventory cost is defined as C_c and this has a unit called rupees per unit per year.

So, either year, or the same time period that we are taking about since we are looking at year here we would also define this as year rupees per unit per year. So, this is the cost of holding one unit rupees per unit per year. So, what is the average inventory that we have at any point in time multiplied by C_c will give us the holding cost per year and the average inventory should be in terms of number of units that we have.

Now, there are two ways of calculating the average inventory. So, average inventory is like saying if I start this cycle with Q then it comes down it comes down comes down to zero at time t . So, in a particular cycle the beginning inventory is Q the ending inventory is zero the average inventory is $Q/2$ because you actually have a line here that is one way of saying that the average inventory is $Q/2$. Other way is to say that to take any period of time find out the total inventory that you have divide it by time to get the average inventory.

So, if you take one particular cycle the total inventory that we have held during this cycle is the area of this triangle which is half into base into height, so half into Q into T . Now, that much inventory is held for a period T , so half into Q into T divided by T which gives us half into Q . So, average inventory either way is $Q/2$ units average inventory is the kind of inventory that we have at any point in time averaged over a reasonable value. Now, the cost of holding this inventory is called total holding cost is equal to $Q/2$ into C_c cost of holding this also has rupees per year because this is rupees per unit per year this is units. So, it becomes rupees per year the units get cancelled and you get rupees per year.

Now, we look at this cost which is back order or shortage cost one of the assumptions is that there is no shortage. So, we do not incur shortage in this particular model which

means the inventory position does not become negative. So, there is no shortage assumption in this model therefore, we will not consider back order or shortage cost in our total cost.

Now, there is this cost of the item, so cost of the item is equal to D into C D is the annual demand C is the unit cost of the item. So, D into C rupees per year, so total cost total annual cost $T C$ which is the sum of the order cost the holding cost as well as the cost of the item is given by d by $Q C$ naught plus Q by $2 C c$ plus D into C . Now, in this total cost expression as I said there are three costs cost of the item cost of ordering cost of holding the inventory. There is no shortage or back order cost when we want to find the variable as Q rest are all the things are known they are known parameters. So, only thing that we need to do is to find out Q , now this total cost is a non-linear function of Q because you find Q in the denominator here you find Q in the numerator here.

It is a single variable non-linear function, so differentiating with respect to Q and setting it to zero will give the first derivative equal to zero will give the optimum value of Q . Whether it is maximized or minimized will depend on the second derivative. Now, our objective is to minimize this $T C$, so setting it first derivative to zero would give us $d T C$ by $d Q$ equal to zero will give us $\text{minus } D \text{ by } Q \text{ square } C \text{ naught plus } C c \text{ by } 2$ equal to 0. The third term is D into C which is a constant it does not contain Q . Therefore, its derivative will be 0, so a first derivative equal to 0 would give us $\text{minus } D \text{ by } Q \text{ square } C \text{ naught plus } C c \text{ by } 2$ equal to 0 which on simplification would give us Q equal to root of $2 D C \text{ naught by } C c$.

Now, actually speaking Q is a square root of something, so Q should be plus value or minus value mathematically. But, since Q is the quantity that is ordered it cannot take a negative value, therefore it is customary to write Q equal to root of $2 D C \text{ naught by } C c$. And write only the positive value we do not write plus minus root of $2 D C \text{ naught by } C c$ only the positive value is taken; now we have to find out whether at this value of Q the original function is a maximum or a minimum. Now, we are interested in minimizing the total cost, so a second derivative will be this is $\text{minus } D \text{ by } Q \text{ square } C \text{ naught}$. This will become a constant, now there is a negative term and a Q square here.

So, on further differentiation we will get a positive term here and Q^3 will come in the denominator. So, for a positive value of $\sqrt{2DC}$ naught by Cc the second derivative will be positive indicating that this quantity is actually the minimum quantity.

Now, this quantity Q is called the economic order quantity which also are called EOQ for short. Now, the formula that we have derived is called the EOQ formula for a single item and this formula is also called the Wilson's EOQ formula attributed to Wilson who derived it somewhere I think in the early forties or thirties so on. Sometimes this formula is also credited the credit goes to Harris it is also called Harris inventory formula. And the literature talks about the contributions of both Wilson and Harris who supposedly have independently derived or thought about the use of such a formula, to find out the economic order quantity. Or the order quantity that minimizes the sum of the ordering cost as well as in the carrying cost.

So, out of the four costs as I said we did not consider the back order or shortage cost we also observe that the three remaining costs that we have got this is the order cost component this is the inventory holding cost component. This is the item cost component we now realise that the item cost actually does not play a part in the economic order quantity. So, it is also a customary to leave out the item cost and derive Q simply by considering the order cost. As well as the carrying cost alone and not including the item cost because item cost being a constant is not going to contribute to the derivation of the economic order quantity. So, there are only two costs which are optimized which are the order cost as well as the carrying cost.

So, in this formula of Q equal to plus root of $2DC$ naught by Cc . D is known which is the annual demand. C naught is known which is rupees per order Cc is rupees per unit per year what is very important is that D is annual demand which is demand per year. Then Cc should be defined as rupees per unit per year if D is defined as month then Cc should also be defined as, so many rupees per unit per month. So, D and Cc should have the same time period in their unit of measurement. So, if DC naught and Cc are known then the economic order quantity can be calculated. And then we can substitute this Q into the total cost function to find out the minimum total cost that the organization would incur. In the next lecture we will consider numerical examples and we explore a little further on inventory models.