

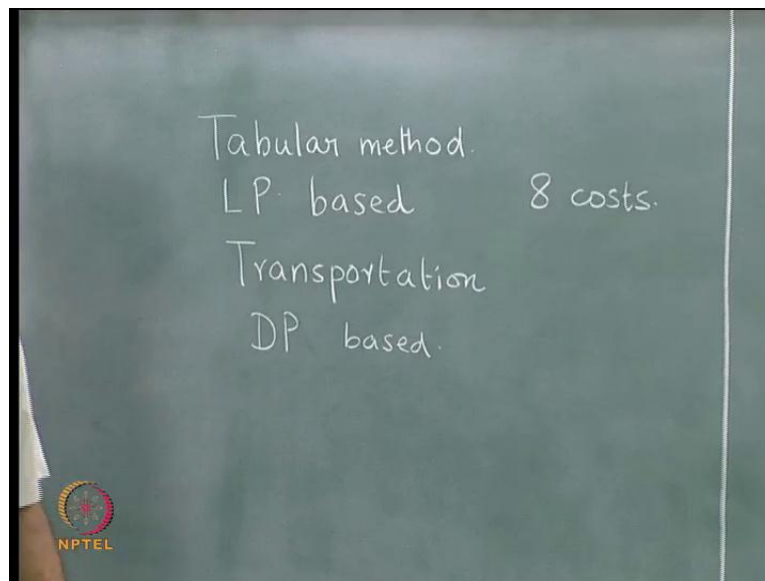
Operations and Supply Chain Management
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Module - 01

Lecture - 08

Aggregate Planning, Quadratic Model, Demand and Capacity Planning

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So far, we have seen several models for aggregate planning. We started with the tabular method. Then, we looked at the linear programming or LP based algorithm, we then did the transportation based algorithm. We also did a dynamic programming based algorithm. The tabular method was an evaluative algorithm, where the user inputs the values of the production quantities, and the workforce as relevant and the tabular method would evaluate a total cost function.

The user could vary these values and try and locally optimize the total cost. The linear programming based formulation, actually considered a maximum of eight different costs. This kind of occurred in four pairs regular time and overtime costs, inventory and shortage costs, hiring and laying off costs, outsourcing and under utilization costs and then there were a variety of constraints.

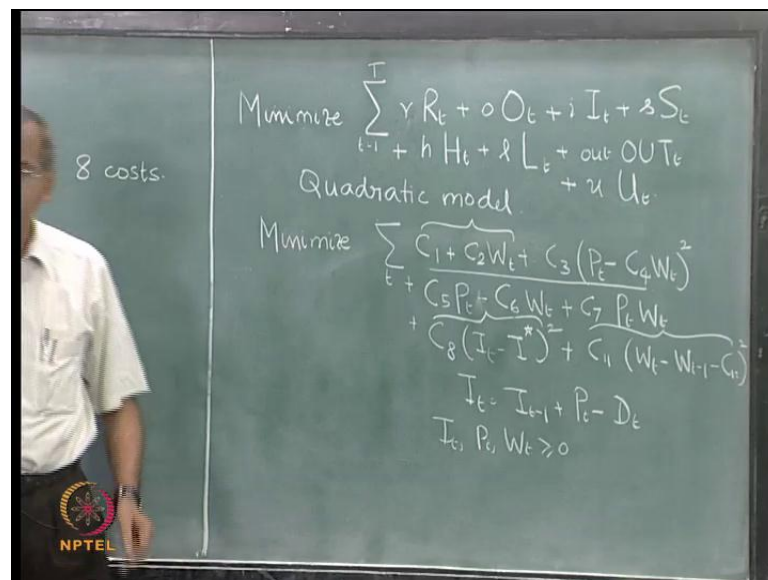
The transportation based algorithms essentially solving a certain restricted version of the linear programming, the transportation model did not consider variable workforce, but

we showed that it could consider the six remaining costs. Then, we solve the dynamic programming based algorithm. In the D P algorithm, we introduced a new idea that if there is production, then there is a set up. So, there was a definite set up function or a set of costs whenever there was a production. Now, all four of them have something in common which is that all four in all four models, the costs was linear or proportional.

For example, in the tabular method, if we said that it is going to cost 100 rupees to produce an item on regular time, then we said it would cost 200 rupees for 2 items. If certain x number of items or man hours is utilized and it cost 100, then the total cost will be 100 into x. So, the same linearity or proportionality was used across all four of them even in the D P based algorithm.

We said that if there is an inventory cost of carrying a unit from one period to another if x units are carried then the total inventory cost will be x into the number of the x into the cost of carrying a unit from one period to another. So, there was this linearity or proportionality in all of these, let me just write only the objective function of the L P algorithm and also try and show the linearity there.

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So, in the linear programming formulation, the objective was to minimize sum of - we used a certain notation we used r into R t plus o into O t plus i into I t plus s into S t plus h into h t plus l into L t plus into outsource of t plus u into U t. We had used eight costs in the objective function and the objective function was to minimize the sum of eight

costs the lower case alphabet represents the unit costs. The upper case represents the variable, so if capital R_t is the number of regular time man hours used and r is the cost per man hour per period.

Then, the total cost of regular time is equal into r into R_t summation over t equal to 1 to t for all periods we could consider that this r is constant across all periods. We could have this r changing across all periods and if it changes then we would add a small subscript R_t which says this is the regular time costing period t , but if this cost is going to be same across all periods.

Then, we will use this, similarly, we use all these values, now obviously when we have to solve this linear programming problem, it is necessary to know the values of R , O and I , S , H , L , OUT and U . These will have to be provided by the organization for whom or for which an aggregate plan is being made. Now, let us ask ourselves two things. Let us first try and question the linearity assumption in the objective function. Then, let us also ask a question as to how does one go about finding out or computing the actual values of these coefficients.

It is only customary to say that if we want to find out this r which is the cost of regular time production, which means cost per man hour of regular time, how does an organization compute this value? There are many ways by which they can do it, but the most common way to find out what are all the components of the regular time cost. Now, in this model we have not explicitly used the payroll, so we assume that pay roll as also a part of this r the salary to the operator who is working plus it will have certain raw material cost plus it will have some cost of consumable and so on.

So, all these components of this cost are now listed and then one goes back to earlier period, say the last six months or the last one year, and then we can compute total money spent on regular time production on each of the components of the regular time cost. Then, we add them to get the total regular time production cost incurred over the last five or six periods, and then when we divide it by the actual quantity or man hours employed in regular time production. Then, we would get this r , similarly we would get this o where all the components of the overtime costs are listed.

Then, they are evaluated or computed and that total cost divided by number of man hours used in overtime would give us this o which is this per period per man hour cost for

overtime. Thus one can go about trying to compute all these costs. May be, the easiest would be the outsourcing cost assuming that the entire product is outsourced. So, finding the outsourcing cost would be relatively easy, inventory cost would be reasonably easy. If the inventory cost is dominated by the interest on borrowed capital if we are going to use the other components of inventory cost such as cost of space cost of power electricity and other resources cost of man power cost of special facility.

Then, this it would have be largely dominated by the interest on borrowed capital or the interest rate. Then, the other cost associated with space people facility etcetera will have to be added. Now, computing this S is not easy because any shortage or back order cost will have an important component which is eventual loss of customer good will by not delivering things in time and by back order. So, that cost is not easy to compute, similarly the cost of underutilization though computable is relatively difficult because this is the cost of keeping facility idle or keeping a man hour of resource idle.

So, it also has to be computed carefully H and L are cost hiring and lay off which can be calculated reasonably well. At the end of it, I wish to say is that it usually for any of these eight costs, the common practice is to go back in time and find out what was actually spent under this cost. Whatever expenditure could qualify under this head of say either hiring cost or a outsourcing cost or a overtime cost and then divide it by the number of man hours that is been used under that to get this. Therefore, it would be difficult to have an R t and say that if we are looking at twelve periods, it would be difficult actually to give twelve different values.

Let us say that regular time cost was so much was in the first period, so much in the second period and so on. So, it is customary to use the same value of r across all twelve periods and because of the way r is used and computed we have we are forced to use the linearity or proportionality saying that if this is a per hour per man hour regular time cost. If R t man hours are being used then r into R t is my total regular time cost, but in practice we also know that every cost particularly in production related situations, there is a fixed component.

Then, there is a variable component, so if we start writing this r itself as having two components something of the form say r naught plus r 1 into r of t if this r has is of this component which means there is a fixed cost r 0. Then, there is a variable cost per man

hour used, then which is which a very common occurrence because the organization has certain fixed cost which can be apportioned to each of these costs. So, if we start defining this r itself as a fixed cost having a fixed component and a variable component then this will become r naught plus r 1 into R t multiplied by R t . So, this will become quadratic in nature, so if each of this cost has a fixed component and a variable component, then the linearity assumption is questionable.

We may have to use a model use which is quadratic in nature. Perhaps the other disadvantage of the linear formulation the advantage of the linear formulation lies in its simplicity. It is easier to formulate proportionality constraint is accepted and additively is also accepted, methodology wise the simplex algorithm is well known and it is easy to solve the linear programming problem. Then, when we computed this R and O and I and S if in reality this R had a fixed component and a variable component, but for the ease of computation, we have simply used one particular R which represents that R naught plus R 1.

Then, it is likely that there could be an error in the computation of r and then the linear programming solution can sometimes give even a higher cost. Then, what is actually incurred by the organization because the cost that is actually incurred by the organization has a fixed component and the variable component. Then, we have approximated it to a single number r . The advantage of linear programming technique is its simplicity ease of modeling and ease of solution. The disadvantage is perhaps its inability to actually capture the reality because in reality costs have a fixed component and a variable component.

So, considering this in mind some authors proposed a quadratic model. So a quadratic model for linear program for aggregate planning is quite famous. It is also called the HMMS model named after four researchers who are the authors of this work stands for Holt, Muth, Modigliani and Simon. So, it is called the HMMS model, we will not get into the full details of the HMMS model, but I would possibly write down only the objective function and then explain how the HMMS model actually works. So, the HMMS model the objective function would be minimized, so the objective function has several costs, first the variables W t is the workforce employed in period t .

Here, P_t is the number of hours of production in man hours in period t , W_t also can be written in terms of man hours production man hours workforce man hours and I_t is the inventory at the end of period t which could also be written in terms of man hours. So, there are three sets of decision variables a certain variables for the workforce certain variables for the production and variables for inventory. Now, the objective function is made out of several costs. Now, one is it has regular time payroll cost. This is like regular time payroll cost. There is a fixed component and a variable component, now there is an overtime cost, there is an inventory cost and then there is a workforce changeover cost.

So, this is the inventory cost, this is the workforce changeover cost and this is the overtime cost, this is the overtime cost expanded in a certain form. Now, we have realized that this portion is called regular time payroll cost, there is a fixed cost and a variable cost, this is the typical inventory cost. In this model shortages or back orders are not allowed. So, $I_t - I^*$, there is an ideal inventory that is defined and $I_t - I^*$ the whole square is the penalty and that is multiplied by a C_8 . Now, here again there is a workforce changeover cost between W_t and W_{t-1} and C_{12} whole square.

Now, these are other costs that relate production and workforce, so they would represent the overtime payroll cost and so on. Now, we are not going to go into too much of detail. One form of the objective function is kept here. It is not absolutely necessary that the objective function has to be this. This is one form of the objective function. This objective function shows a quadratic objective function, the quadratic comes from this, comes from here as well as comes from here. It is not linear anymore now there is a quadratic objective function subject to a set of linear constraints.

This is a standard constraint that relates production inventory and demand. D_t is the demand in period t in terms of man hours. Here, I_t is the inventory at the end of period t I_{t-1} is inventory at the end of period $t-1$, which is inventory at the beginning of period t , this is production in period t , this is demand in period t . So, this is typically the inventory balance constraint, so now we have a quadratic objective function. Let me again repeat and indicate that it is not absolutely necessary that his has to be the objective function. A quadratic model suitably defined for a specific organization may include additional costs also or may leave out certain costs.

Also, this may include backorder which is not included in this function. It may include outsourcing which is not included in this function and so on. This is only a representative quadratic function where some of these costs are shown as quadratic functions of the decision variables which are essentially P_t , W_t and I_t . So, one may look at other objective functions also, but the objective function is quadratic, the constraints are linear, and then we have a non negativity restriction on the variables. So, this becomes a quadratic programming problem which can be solved in many ways. A popular way is to actually one could think in terms of taking this constraint into the objective function using a Lagrangean multiplier.

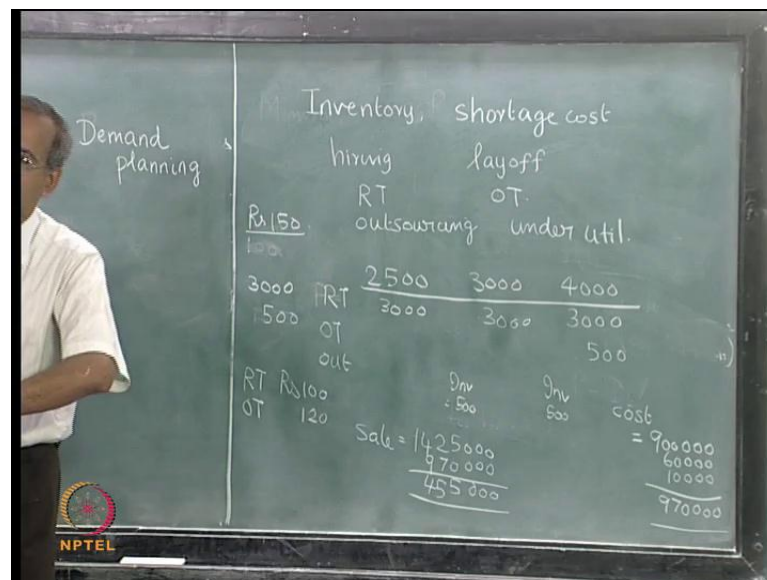
Then, one could solve the other one that is now associated here is quite simple in the sense that we have a quadratic function. So, the fundamental principle of a quadratic optimization problem is that the first derivative of a quadratic function is a linear function. So, one can differentiate this with respect to the variables at the same time one should be able to capture the dependency of the variables, the dependency of the variables are here. So, even though we have P_t , W_t and I_t as variables there is a relationship between P_t and I_t which is given here.

So, it is possible to eliminate one of the variables through this constraint and then differentiate with respect to the other variables and solve it. So, when we differentiate we will get a linear function involving P_t and W_t , we also have equations here differentiation would also give us a set of equations. So, finally, what will happen is on differentiation and inclusion of this constraint we will have a set of linear equations to solve. Now, as we proceed we may have some more variables than the number of equations the reason being these I_t minus 1 and I_t as well as W_t minus 1 and W_t would sometimes create more variables than the number of equations.

In such cases, we define some initial conditions as well as some final conditions that we want, so we make the number of variables equal to the number of equations. Then, we solve a certain equations to get the optimum values of P_t , W_t and then I_t from them using this relationship. So, the primary purpose of doing the quadratic model is only to inform or indicate the need to consider a quadratic objective function because though linear optimization and linear programming and linear model is very simple to model. It is easy to solve, it has the difficulty that these coefficients, there can be some kind of an approximations in these coefficients.

Even though we solve the L P optimally, a solution may not reflect what actually happens because in reality the costs are not linear. So, if you want to capture that, then we can bring it into quadratic model with a suitably defined objective function and I have explained broadly the methodology to solve a quadratic optimization problem. So, in terms of solution methodology, this is a little difficult compared to this, but this can give results, which are little closer to what is happening because in practice costs have a fixed component and a variable component. Therefore, can result in quadratic optimization problems, now let us look at some more aspects of aggregate planning before we go to the next topic if we look at the costs that we wrote, we wrote eight different costs.

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So, let me consider two of those costs here one is the inventory cost and the other is a shortage cost the other pair is hiring and lay off and then of course, regular time overtime and outsourcing and underutilization. Now, let us consider a simple situation, where the demands are 2500, 3000, 4000 in three periods. Now, in a typical aggregate planning problem, we are now going to find out what is this p how much of production man hours are we going to use in these three periods such that we minimize the total cost. Now, p actually has two components which is regular time production and overtime production and we could outsourcing there are three things by which we can actually produce. Now, the total demand is 9500 over three periods, now we could do two things.

One is we can produce exactly 2,500 in this month produce 3,000 in this month produce 4,000 in this month. If we are able to do that it means there is going to be no inventory or shortage at the end of this period, alternately we could do for example, the total is 9,500. So, let us say we could produce about 3,200 here and 3,200 here and 3,100 here or 3100, 3200, 3200 which means we are trying to keep the production quantity same which is roughly the average of the demand. If we do that the production, quantities are the same, but there will be inventory at the end of this period and inventory at the end of the other period sometimes there could even be shortages.

For example, if the 4,000 were here and 2,500 were there, then initially there will be a shortage or a back order. So, by producing, by making the production quantities equal in all the periods, we would incur inventory and shortage and therefore, inventory and shortage costs will dominate in the cost of the aggregate plan. So, that is one type of strategy to keep the production constant, now to keep the production constant implies that the number of people who are employed the workforce is also kept constant. On the other hand, if we produce 2,500, 3,000 and 4,000 every month, which means we are varying the production quantity, which means we are varying the workforce.

This means we will incur hiring and laying off costs, but we will not incur inventory and shortage costs that is another strategy. So, organizations can use either of these pure strategies which is one of the pure strategy is to produce an equal quantity incur inventory and shortage keep the workforce fixed. The other pure strategy would be to produce the demand vary the workforce and do not incur inventory or shortage costs. Otherwise, the organization can also look at what is called a mixed strategy, where we have a combination of pure strategy as both the pure strategies. Sometimes, we can adjust or increase the workforce produce more hold inventory and so on.

So, the problem which included all the eight costs the linear programming problem was essentially trying to identify the best mixed strategy. So, one is the strategy implied a mixed strategy can be used a mixed strategy and solve it. The other is sometimes there can be limits or constraints of these which would also make a pure strategy not doable. For example, in this month the demand is 4,000, now if we are following a pure strategy where we vary the workforce and we want to produce a 4,000 here. Suppose, the regular time production capacity is only 2,000 or 3,000, let us say the overtime production capacity is 500.

Then, it would force us to go for outsourcing for another 500, here suppose the regular time production capacity is 2,000, there is a thousand capacity there is a 500 capacity. Then, only 3,500 is the maximum that we can reach here. So, we cannot follow the pure strategy, it also means that somewhere we have to produce store the inventory and then use it for subsequent periods so that we can meet the demand of the subsequent periods. So, if we look at this things that we want to do are or things that we can play around are one is to consider the pure strategies.

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500	500	500	3000
500	1000	500	1500
900000			
180000			
260000			
<u>1340000</u>			
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	472500		
			RT Rs 10
			OT 10

The other is to adjust the capacities, now somewhere in the discussion I mentioned that the regular time capacity if it is 2,000. Let us say then it will be 2000 in all these months, but can we adjust this regular time capacity during peak periods by hiring additional workforce one of the models that we saw in the tabular method did exactly that. So, one is adjust the capacities by adjusting the workforce adjust the capacities by overtime and outsourcing. So, all these are strategies that organizations used to meet the demand particularly if the demand shows a lot of variation. Otherwise, organizations will have to go towards outsourcing and it might be difficult to get the entire product outsourced certain critical operations. Sometimes, it is outsourced, sometimes common operations are outsourced while the critical operations are done in house so that certain capacity is saved by outsourcing this.

So, outsourcing can happen in more than one way, most of the times the entire product can be outsourced only a certain part of it is outsourced, so to that extent that capacity can be saved and used under regular time capacity or under overtime capacity. So, organizations look at all of these variable workforce variable or flexible overtimes sometimes an extra shift, sometimes an extra day and a combination of these two, a bit of outsourcing. All these are integral part of aggregate planning to try and get as much capacity as possible to meet the demand, particularly if the demands are fluctuated. Now, another important thing is that being done is what is called demand planning.

Now, when we worked out the aggregate production plan, we assume that these demands are known. These demand cannot be changed, so we have to meet this 2500 in month one, 3,000 in month two and 4,000 in month three and let us try and workout some costs for this and see what we do. Now, we assume that the regular time capacity is 3,000 per month, let us assume the overtime capacity is 1,000 and any number can be outsourced. Now, the best way to try and meet this 2500, 3,000 and 4,000, we will also assume the regular time cost regular time cost is 100, we will say RT cost is hundred OT cost is 120 and outsourcing cost is higher than this.

So, what we will do is we will assume here that the overtime capacity is 500, so to meet this 3,000, we will use the 3,000 regular time and 500 over time to make this 3,000, we will use 3,000 regular time. Now, for this 2500, we will produce 3,000 regular time and have carry have an inventory of 500, let us say here also we carry an inventory of 500. So, the total cost associated with this plan will be 9,000 into regular time cost of 100 is 900, 1,000, this 500 overtime would cost us 500 into 120, which is 60,000. This inventory is 1,000 units, 500 is carried here, 500 is carried here, suppose we say that the inventory carrying cost is 10 rupees per period.

There is another 10,00, so we have a cost of 900 and 70,000 for this, now if the unit price is 150 rupees, then, the total money or profit or sale will be 150 into 9,500 which would give us 14,25,000 and the expenditure is 9,70,000. So, we will have 4,55,000 as the net profit in business, now the next question is can we kind of plan it better, if we have a way to work around with these demands and do it, very simple terms if instead of 2500, 3,000 and 4,000 which adds up to 9,500. If the demand itself where 3,200, 3,200 and 3,100, we know for sure that the cost will come down because the amount of the overtime would be distributed.

Here, it will be distributed a little bit here and then the inventory cost can be saved the 500 that we used for overtime will be distributed in evenly, but the cost will be the same. The inventory cost will come down, therefore the total cost will come down and the total profit will go up, but what organizations try and do is also to have different ways by which they can actually adjust this demand. Now, that is called demand planning and how do they do that they do that by giving some price discounts. So, when price discounts are given in certain periods say particularly during the lean period the off season discounts are given.

So, when off season discounts are given the demand which is going to happen later in a peak period is going to be spread out evenly so that the demand during the lean period also goes up. So, the peak demand will come down, but the demand gets spread out, but the total demand also increases a little bit because of the discount that is given. So, if we look at a situation, where now after say an off season discount the demands for the periods become 4,00, 4,500 and 4,000. If this is the demand that is given, now what we will do is we will produce 3,000 regular time.

In each of these we will produce 500 overtime and then we will have to do outsourcing. So, 3,500, 500 outsourced 1,000 outsourced and 500 outsourced. So, total cost will be a regular time cost will be 9,00,000 total overtime cost will be 1,500 into 128 and 80,000. If the outsourcing cost happens to be say 130, then we have 2000 into 130, which is 2,60,000 which gives us a total cost of 1,300 and 40,000 or 13,40,00 as the total cost as against 900 and 70,000, here the reason being the demand is now higher.

Now, we have to look at the profit, now when we worked out the profit here we said that the total cost is 150, let us say we give a discount and the total cost becomes rupees 145. Then, the total money will be 145 multiplied by total demand is 13,500 which give us 1957, 500 and 13,40,000 which gives us 500, 716, 1,700 as the net profit compared to 4,55,000. Now, a quick look at these one will be first tempted to say that this is excellent mainly because the profit has gone up, but then we also understand the profit went up not because we only spread the demand, but we also increased the demand. As a result of this discount of 5 that is given.

So, there is some kind of a loss because the 150 has become 145, but unless the organization is able to increase the demand number 1 as well as spread the demand if the

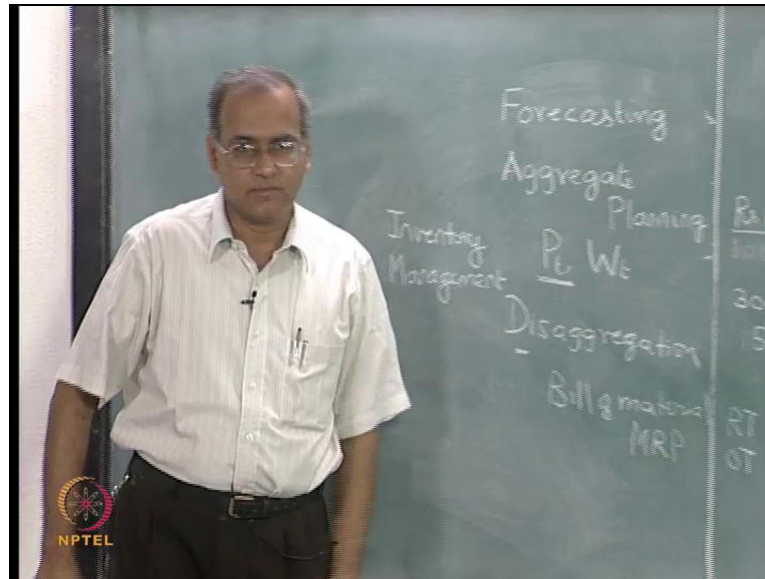
organization is able to do these two things by offering this discount. Then, it is profitable for the organization to do this, but on the other hand, if the demand total demand is near about 9,500, say 10,000 is the total demand instead of 12,500. So, the total cost will be 12,500 into 145, let me do that which is one eight, 1,25,000. So, this is 18,12,500 4,71,500 is the net profit which is still more than the 4,45,000 that we had here.

Now, the organization should be able to it has brought down the cost or price by 145 by giving a 5 rupees discount on this. Now, if the 5 rupee discount is adequately compensated by 2 things one is a spread and balance in the demand and the other is the increase in demand. So, if you are not able to get the increase of 3,000, then it will not be profitable to give this discount. So, two things have to happen what is the extent of discount that is given when is the discount when is the discount given. So, these two things are important, so if the discount is given particularly during the lean period so that the demand can be brought forward from the peak to the lean the peak would always mean that we have to increase the capacity.

This means the cost will go up in these calculations we have assumed that this three thousand is intact sometimes a regular time capacity also can be increased by increasing the workforce. So, the cost will go up so if we are able to bring the demand the peak demand and evenly distribute it particularly through the lean periods. It becomes advantageous for the organization to do this and even gain a little bit of profit if the net demand goes up because of the discount. Now, this is an important aspect which is called demand planning and this is something that organizations now have started looking at.

So, it is actually profitable for an organization to give an off season discount provided the demand for the period increases because of the discount and the peak demand gets averaged over certain periods. So, these are some of the aspects that one looks at particularly when one does aggregate planning. So, if we go back to one of the earlier lectures where we looked at the various topics in production planning and control.

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We said we would start with forecasting where we would look at forecast of the demand where we answered the question how much is the demand that we have to produce and meet. Then, we answered the next question in aggregate planning where, now if this is the demand that we have to we have to produce and meet how we are going to do it. In terms of regular time capacity should it be fixed can it vary with the workforce overtime capacity what are the other strategies is there an outsourcing strategy.

What are the other strategies are we going to hold inventory or are we going to incur some back ordering, so we looked at all those questions. So, the decision that comes out of the aggregate planning takes input from forecasting. The output from the aggregate planning are essentially the P_t and sometimes W_t w_t is the workforce incurred in terms of man hours is that I am going to employ more importantly P_t has the number of man hours of production that we are going to employ. Now, we need to actually going to do two things one is how this P_t is the number of hours of production going to be distributed into three things regular time overtime outsourcing which the aggregate planning has already done.

Let us say we are going to use this many hours of regular time this many hours of overtime. Now, how are we going to use these hours to produce the various products which product is going to be produced first what is the order in which we produce these products. Now, that is answered in what is called disaggregation sometimes called

master production scheduling. Now, after we answer this disaggregation, then we need here at this point we are going to say that well this product is going to be made in next two periods. So, we also try and find out when these products are going to be made after we answer that question or sometimes even parallels what are all the things that we make individual quantities of each of these final products.

Then, these each of these final products has its own set of assemblies and sub assemblies and bought out components. So, if the certain number of final products are to be made what is the corresponding number of the brought out items that we should have. Now, that is answered by what is called the bill of material as well as by what is called MRP materials requirement planning. This tells us if we are going to produce and make the demand for this many quantity of each of the products, what are the corresponding bought out items and in what quantities they are demanded or required.

Then, how are we going to manage our material or inventory buying policies for these items so that leads us to another topic called inventory management, which we will see in the next lecture. After we study inventory management, we will go back and look at disaggregation and then we will look at MRP and then after MRP is studied and the inventory management is covered. We will then move into production control which is sequencing and scheduling.