Operations and Supply Chain Management
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Module - 1
Lecture - 7
Aggregate Planning, Dynamic Programming, Backordering

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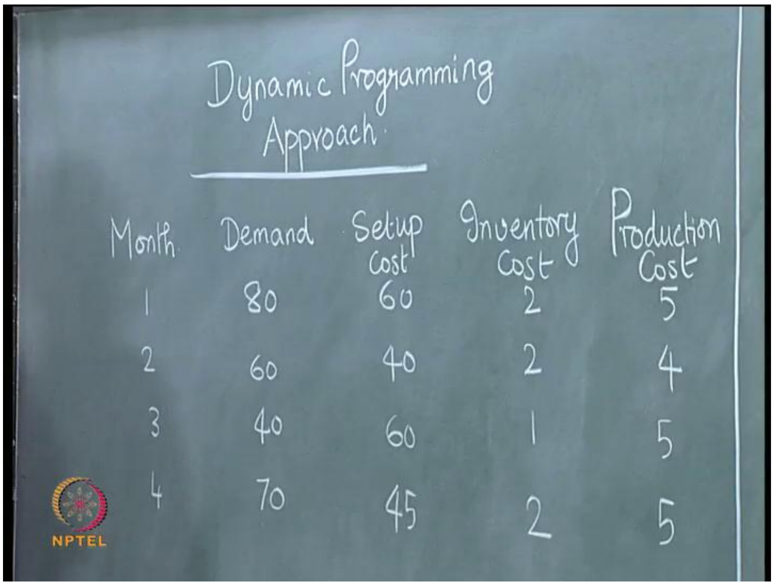

In today's lecture we will see another approach to solve the production planning problem and this approach is based on dynamic programming. Let us consider this problem that we have shown here. Again let us assume that we make a single product and we have the demand for four periods or four months given as 80, 60, 40 and 70. There is a setup cost that is incurred whenever there is production. For example, if we chose to produce in the first month then in addition to the cost of production there is a cost of setup which is say rupees 60.

So, the setup costs are given as 60, 40, 60 and 45. There is also a cost of production which is given as 5 4 5 and 5. In addition there is an inventory cost which means one can produce in say month one and use the production to meet the demand of month two. If we do such a thing then we incur a certain inventory cost or inventory holding cost. We are also going to assume that the inventory cost of 2 2 1 and 2 represent the cost of holding the ending inventory at the end of this period.

For example, if we chose to produce hundred in the first month and consume 80 the remaining 20 is assumed to be carried from the first month to the second month, and it attracts an inventory cost of 2, which is the inventory carrying cost at the end of month one. So, these costs represent the inventory carrying at the end of each month, there is also a production cost which is the cost required or cost that is incurred in producing these quantities. At the moment we do not have an upper limit on how much we produce. For example, if it is economical and profitable one can produce the total demand is 180 plus 70, 250.

So, if it is economical and profitable, one can produce all the 250 in the first month itself there is no limit on the production capacity at the moment. As we move along we will also realize that it is not very difficult to incorporate the production capacity if it is there. So, there are three costs that are involved the cost of setting up, the cost of production and the cost of holding inventory. In our problem we have also assumed that the setup costs are different in different periods, inventory costs are different in different periods and production costs are also different in different periods. So, the decision or the planning decision is to find out, how much is to be produced in these four months in each of the four months such that the total cost of setup total cost of inventory and production the three cost put together is minimized.

So, it is a minimization problem to minimize the cost of setup inventory and production. Right now we are not bringing time into consideration. We have not explicitly modeled the time taken to produce these items and the capacity that is available in terms of time. One should also note that the setup cost 60 also involves a setup time and since we are not explicitly modeling the time, we do not bring the effect of setup time into the analysis.

But if there is a production in a certain month a certain amount of setup cost is incurred if we decide to produce in this month. This model is slightly different from that the earlier models that we have seen - the tabular approach, the linear programming approach and the transportation approach. The difference comes from the fact that for the first time we are including a setup cost. Previous models that we discussed did not include setup cost explicitly. This model includes setup cost explicitly. We also said that whenever we decide to produce in a particular month, we have to incur the setup cost. For the purpose of modeling we are assuming a single product whose demands are so much, we also assume that if it is decided to produce in months one as well as in month two, we will incur a setup cost of 60 in month one and 40 in month two.

We are not assuming that one set is setup is going to remain the same for all the four months. Every month if there is a production there is going to be a setup. One may also say that this facility can be used to produce more items and products and therefore, if this particular product is made in month one, as well as in month two, then we incur two setup costs one for each month. The problem can be formulated and solved as an integer programming problem with of course, a binary variable that would say that y i equal to one, if we decide to produce in month i and y i equal to 0, if we do not produce anything in month i. And we can have constraints that relate the decision to produce which is the y i and x i which are the quantities that are produced in this month.

So, integer programming - binary integer programming is definitely a methodology to solve this problem. We also observe the dynamic programming is also a very good methodology to solve this problem, where the stage will correspond to each of these four months. The state variable for dynamic programming would correspond to the inventory that is available at the beginning of each period. Also note that this problem does not have an initial inventory there is no product that is available, there is no desired ending inventory. So, we do not wish to have any ending inventory at the end of this period, there is no beginning inventory as well as there is no ending inventory, in this particular example that we are seeing.

So, if we use dynamic programming to solve this particular problem, then the state variable will be the amount of inventory that is available at the beginning of each period. The decision variable will be the quantity that is produced in each period and the criterion of effectiveness or the objective function will be to minimize the sum of the setup costs, inventory costs and production cost. Now, we all know that dynamic programming, though very useful to solve certain classes of problems has its own issues with respect to model. So, if we consider a problem of this type, the state variable can take very large numbers. For example, total demand for the four periods is 250.

So, if we use the tabular approach of dynamic programming to solve this, the state variable, which is the amount of inventory available can take very large number of values, and the tables can become lengthy and cumbersome, though dynamic programming can be used to solve this.

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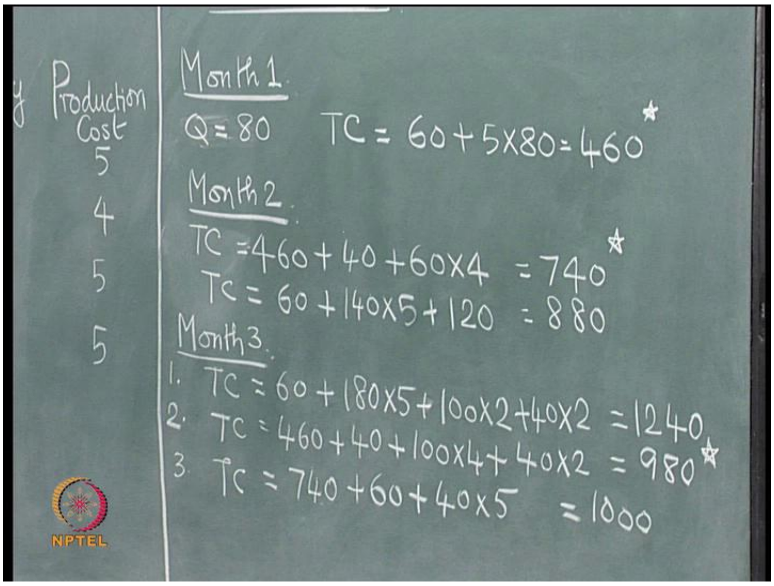

One of the most interesting simplifications to the dynamic programming solution came from Wagner and Whitin and it is called the Wagner and Whitin algorithm which uses dynamic programming to solve this particular problem. The Wagner Whitin idea or principle is as follows. Now there are four periods and we are going to assume that we are going to make decisions on production at the beginning of each of these four periods. Now, it is only logical that if we produce in a certain period, then we will have to meet the demand of that period at least. Otherwise we will have to incur one more setup and one more setup cost, therefore it is uneconomical to consider two setups in a particular period.

So, the if there is only going to be one setup in a particular period then it implies that what we produce with what we produce, we should be able to meet the demand of the entire period. One other assumption important assumption is that the demands have to be met at the beginning of each of these periods, shortages and backorders are not allowed in this particular model. So, under that assumption if we produce something in a month, then either the quantity that is produced should be at least the demand of that month or with the beginning inventory and production in that month, we should be able to meet the entire demand of that month.

So, if there is a beginning inventory in the beginning of a particular month, let us say we are looking at month number two and let us say if there is a beginning inventory of say 20 in the beginning of month two, then we should produce 40 or more, but then it is also logical to say that if there is a beginning inventory at the if there is inventory at the beginning of this month. Then if that inventory is sufficiently large to meet the demand of this month, then there is no need to setup and produce in this month.

So, Wagner and Whitin algorithm works around these ideas. So there are two important points there - one is there will be production in a particular month, when only when the beginning inventory is zero. And if there is production in a particular month then that production quantity will be the demand of that month or the demand of that month and the next month and the demand of three consecutive months or the production quantity is always equal to the demands of certain number of consecutive months. The second if there is beginning inventory in a month, then that inventory should be sufficient to meet the demand of that month and there will be no production in that month.

So, these are the two important results of Wagner and Whitin, which I once again repeat. If the beginning inventory is 0 or only when the beginning inventory is 0 we will decide to produce in that month. And if we produce, the production quantity is equal to the demand of that month or the demand of two months or demand of three months, or demand of a certain integral number of months. If there is beginning inventory then we will not produce and that beginning inventory should be sufficient to meet the demand of this particular month.

The moment we accept and use these two results that came from Wagner and Whitin, the dynamic programming solution to this problem becomes extremely simple. Now, we will look at how to solve this using dynamic programming, under the assumption that backordering is not allowed. So, let us go back and use the DP solution - dynamic programming solution. So, let us start with trying to meet the demand of month one. So, we first look at trying to meet the demand of month one, there is no beginning inventory so we have to produce in month one. So, demand of month one is met only in one way by deciding to produce.

So, we produce the cost equal to production quantity is equal to 80 because demand is 80 and the best way to meet this 80 is given by total cost is equal to setup cost is 60, which is to be incurred production cost is 5 into 80 equal to 460. There is no inventory that is carried because demand is 80, production is also 80, there is only one way to do it which is the optimal way. So, we just put a star indicating that this is the best way to produce and meet the demand of the first month.

Now, we go to demand of month two, now the month two demand can be met in two ways one is to try and meet the first month's demand in the most economical way and use the previous result, which is the essence of dynamic programming where the optimal decision up to the previous stage is computed and used for decision making in the next stage. So, one, is to try and use you meet the month one's demand in the best possible way, which is 460 which comes from here plus produce what is required for month two by incurring a setup cost of 40 plus production quantity is equal to, this is Q equal to 60 TC is equal to this.

We just write TC total cost is equal to - total cost of meeting this second month's demand as well as the first month's demand, two months demand is to meet the first month's demand in the best possible way, which is 460 that comes from here. And then meet the second month's demand alone by incurring a setup cost of 40 and a production cost of 60 into 4, 240. So, this gives a total cost of 240 plus 40 is 280, 340 plus 400, 740 is the cost that is associated with this solution. The other way to meet month one and two demand is to produce everything in the beginning of the first month. So, that would give us Q equal to, but it is produced in the first month.

So, all the 140 units are produced in the first month so there is a setup cost of 60 that is incurred, all the 140 units are produce in the first month. So 140 into 5, 700 plus these 60 units are carried from first month to second month. And therefore, there is an inventory carrying cost of this 60 into this two, this two represents the cost of carrying one unit of inventory at the end of the first month to the second month. So, 60 into this 2 is 120 so this becomes 700, 820 plus 60 is 880. Now, the demand of months one and two put together can be met in two ways one which gives us a cost of 740, the other that gives us a cost of 880. And since we are trying to minimize 740 is the best way to do this.

Now, we go to month three now. Month three can be met in three ways, one is to meet month one in the best possible way and then produce for two and three in month two. The other is to meet month two in the best possible way and produce for month three in month three. And the third is to produce everything in the first month and carry for months two and three. So, we have three ways of doing it. So, let us assume the first way is we produce everything in the first month and then we carry which means, there is a setup cost of 60 the total quantity produced is 60, 80 plus 60, 140 plus 40, 180, 180 into 5, which is 900 plus 100 units are carried at the end of the first month. So, 100 into 2 plus another 40 units are carried at the end of the second month so plus 40 into 2.

So, this gives us a total cost of 900 plus 200 is 1100, 1180 plus 60 is 1240. Second way to do it is to produce the first months demand in the best possible way, which is 460 and produce the demands of months two and three in the second month. So, in the second month we incur a setup cost of 40, we produce the demands of months two and three which is 60 plus 40 hundred in the second month so hundred into 4, 400 plus.

Now, this hundred is produced in the second month, but 40 is carried to the third month so there is an inventory cost of 40 into 2. Now, this gives us a total cost of hundred into 4, 400, 480, 520, 580 plus 400 is 980. The third way to do this is by producing the demand of months one and two, in the most optimal manner which is by incurring the 740 and producing the demand of month three in the third month.

So, this is to incur the 740 and then produce the demand of the third month, in the third month by incurring a setup cost of 60 quantity produced is 40, production cost is 5, 40 into 5 there is no inventory cost here because this 40 is produced in the third month and it is not carrying. So, total cost here is 40 into 5, 200, 260, 300 plus 700 is 1000. So, the best way to do it is here. Now we go to month four.

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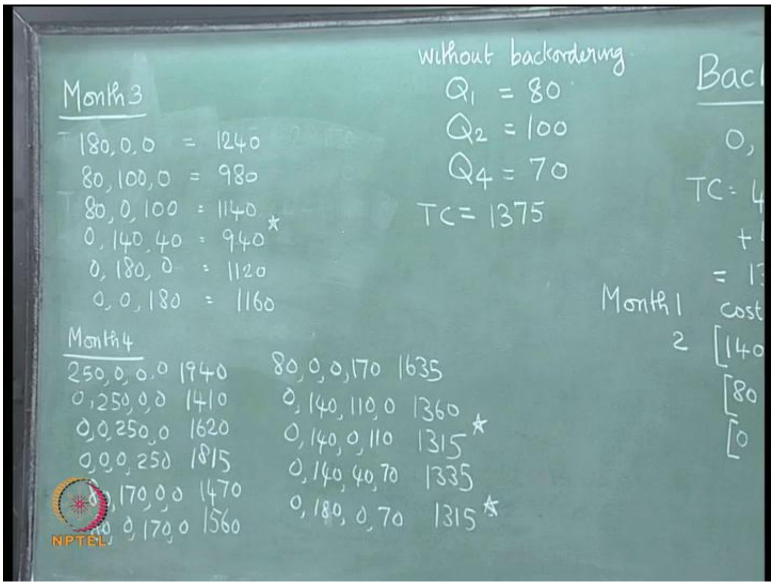

Now, we compute the total cost for month four in four ways, one is to produce all of these in the first month itself and carry. The second is to produce up to this optimally and then produce the rest here, the third is to produce upto this optimally and produce the rest here. And the fourth is to do upto this optimally and produce this.

So, we will compute the cost for all of them. So the first one will be to have all four. So, produce 80 plus 60, 140, 180, 250. So, TC1 first way to do it is to produce all the 250 in the first month itself by incurring a setup coast of 60, production cost of 250 into 5 unit production cost is 5 totally 250 are produced. Now, out of the 250, 170 is carried to the second month. So, 170 into 2 plus 110 are carried to the third month 110 into 2 plus another 70 is carried to the forth month, so out of the 110, 40 is consumed 70 is carried 70 into 1 plus 70 and this gives us a total cost of 1250 plus 340 is 1590, 1590 plus 220 is 1810, 1880, 1940.

Now, we look at the second one where we do up to month one optimally so you incur 460, and we produce this 2, 3 and 4 in month 2. So, production quantity is 170 into production cost is 4 so plus 170 into 4 plus out of the 170, 60 is consumed here so 110 is carried 110 into 2. Now, out of the 110 that is carried 40 is consumed 70 is goes to the fourth period so 70 into carrying cost of 170, and this gives us a total cost of this is 680, 740, 1140, 1160, 1260, 1330.

So, the total cost is 1330, let me repeat 170 into 4 is 680, 740, 1140, 1260, 1350, 1430.

Let me do it again 460 plus 680, 680, 740, 1140, 1000 plus 220 is 1360 plus 70 is 1430 setup. So, I have to include the setup time also, there is a setup cost of 40 which comes here so that 40 has to come here. So, let me do it again from the beginning so the 460 is carried from here there is a setup cost of 40 for this period. So, there is a setup cost of 40, the production quantity is 170 and the cost is 4 so plus 170 into 4 plus out of the 170, 60 is consumed 110 is carried, 110 into 2 plus out of the 110 that is brought here 40 is consumed 70 is carried, 70 into carrying cost of one is 70.

Now, the total is 460 plus 40 is 500, 500 plus 680 is 1180, 1180 plus 220 is 1400, 1470 that is the total cost associated with this. Now, in the third way of calculating it, we produce up to this optimally incurring a cost of 740 and then we chose to produce these two in period 3. So, it is 740 plus there is a setup cost there is a setup cost of 60. So, plus 60 there is a production quantity of 110 production cost is 5.

So, 110 into 5 plus out of these 110, 40 is consumed 70 is carried 70 into one is 70 so this is 740 plus 60 is 800, 800 plus 550 is 1350 plus 70 is 1420. Then the forth one we produce upto this optimally and produce the requirement of month 4 in 4. So, the optimal way is 980 from here plus there is a setup cost of 45, there is a production quantity of 70 and production cost is 5 plus 70 into 5 there is no inventory involved.

So, 980 plus 350 is 1330 plus 45 is 1375 which is the optimal one for this, so out of these four the cheapest is 1375. So, what are the production quantities the minimum cost is 1375, what are the production quantities? Now, the production quantity here in the forth one production quantity is 70, so we will now say Q 4 is equal to 70.

So, when Q 4 is 70 it means we are producing 70 here, which means we have used up to the third period, we have used the optimal solution. So, we have used this 980 which we see has also come here. Now, go back to this 980 and find out the production quantity which is hundred into 4, hundred is the production quantity so that hundred comes from 60 plus 40 so Q 2 is hundred. Let me explain how I got that hundred now the moment we are here we are going to see the production quantity for at this point, this is the production quantity hundred into 4 is the production cost. So, hundred is the production quantity. So, I am somewhere here up to this I have solved optimally and i realize that my production quantity is hundred which means, I am producing here 60 plus 40 so Q 2 is hundred.

So, we have written Q 2 equal to hundred and you also realize that 460 comes here indirectly you also realize that here I have produced hundred. So, I have to look the optimization here which happens at this point production quantity is 80 so Q 1 is equal to 80. Now, total cost is equal to 1375. So, we have now solved this optimally using dynamic programming now there are four periods the optimal solution does not ask you to produce in all of the four periods, it is enough to produce here it is enough to produce here and it is enough to produce here. In some sense it is it is understandable because the production quantity comes down here, so there will be a tendency to push the next month's production into this month, pushing this might involve a higher inventory holding cost.

So, it is a tradeoff between inventory holding cost and additional change over or setup cost, but if the production cost themselves change, then this will also have a bearing on the solution. Now, in order to bring out the features of the algorithm we have considered a problem where the inventory holding costs are different in different periods, production costs are different in different periods, and setup costs are also different in different periods.

Though one cannot exclude this from happening which means in organizations, we will realize at some point when these costs are measured, one would realize that to setup for the same product the setup cost can vary in different months. The reason could be many sometimes even the labour associated with the setup the cost can change depending on pay revision and additional payments made to the people. And therefore, setup costs can change.

Inventory cost can also change due to many reasons for example, if inventory space is added in a certain period or space is removed from the storage area, then there can be a situation where the inventory cost can change. Production cost can also change due to cost of labor and multiple other cause of consumable and so on. So, these things can actually happen, but this model is also applicable when the production costs are the same, the inventory costs are the same and the setup costs are the same across all periods. There it becomes a tradeoff between cost of holding and additional inventory for a particular period versus the cost of incurring one setup or one change over.

The point I am making is if all these costs are the same like 60, if all these costs are the same 2, all these costs are the same say 5, then it is a tradeoff between incurring a setup of 60 and carrying a certain quantity of the demand for the next period multiplying it by two. Wherever it is cheaper than we would chose to setup or we would chose to produce early, and carry the inventory. So, the Wagner Whitin algorithm actually has become, once we apply a Wagner Whitin idea into the dynamic programming, we realize that it has become extremely simple. All we have done is for four periods we have just evaluated 4 plus 3 plus 2 plus 1, which is 10 possible scenarios.

Whereas, if we had not applied the Wagner Whitin idea and done using dynamic programming then for example, the state variable can take any value from 0 to 250 and therefore, we would have evaluated many, many more scenarios than we actually have evaluated. So, the Wagner Whitin idea has helped in the dynamic programming solution to this problem.

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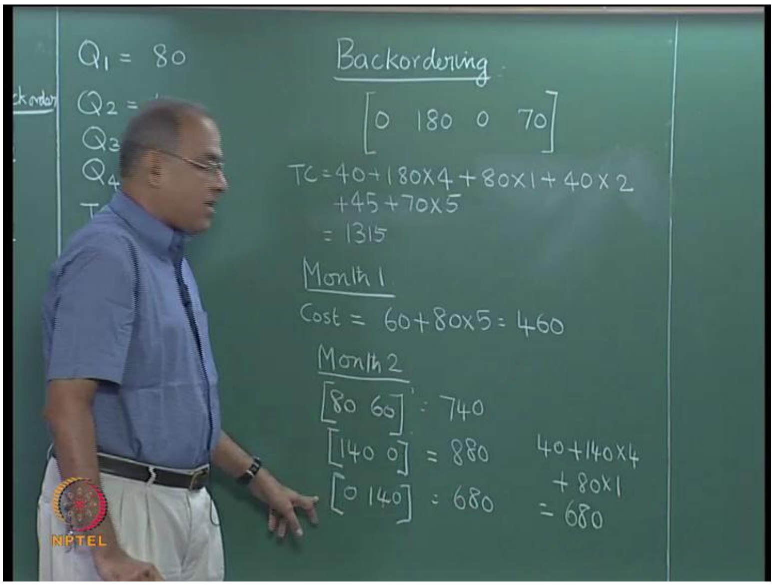

Then comes the next question do we allow backordering into the Wagner Whitin solutions. So, we also address the backorder problem, the problem gets a little more involved when we consider backorder. So, for example, how can we consider a backorder, see right here we said that see here when we did not allow backordering, there was no beginning inventory. So, the demand of the first month has to be met in the first month itself. So, we did not consider a situation where the first month's demand was actually met out of producing 140 here, if backordering is allowed then such a possibility exists, I do not produce anything here.

Right now when we did not consider backordering, this model will definitely have Q 1 equal to something because the first month's demand has to be met at the end of the first month, and there is no beginning inventory. But in the backorder case we could actually have considered the scenario where we do not produce anything here we make 140 here to meet these two and maybe we could have considered another scenario, where we make 110 here to consider these. Or we could even consider a scenario of producing 180 here meet a backlog of 80 meet this month's 60 and meet the next months 40 here and the 70 here.

So, considering backordering throws open lot more scenarios then we can think of, but then we also have to include a backorder cost. So, let us not spend too much time evaluating all possible scenarios, but let us try understand one or two scenarios. So, let us include a backorder cost which comes here. So, let us consider the backorder cost as 1, 2, 1 and 2. So, I am going to write the backorder cost as 1, 2, 1 and 2 and this quantity is backorder cost is shown here. For example, if I chose to produce 140 here which means this period is backordered.

So, the backorder cost will be 60 into this 1 which is 60 now as I said it throws open several scenarios. So, let us just look at one scenario that involves backordering and let us try and find out the cost. So, let us look at a scenario of 0 180, 0 70 it means I am going to produce 80 plus 60 plus 40, 180 here which means I am backordering this demand I am meeting this demand in the month itself and I am carrying to meet this demand and then the fourth one that comes here.

So, if we do such a thing now what is our cost now? I am incurring a setup cost of 40. So, total cost will be setup cost of 40, I produce 180 here the unit production cost is 4, the unit production cost is 4. So, 180 into 4 now out of this production this 80 I am actually backordering first period 80 I am backordering. So, let us say my backorder cost is 80 into 1, this 60 I am meeting in the period itself, this 40 I am carrying to the next period. So, the inventory cost is 40 into 2, so inventory cost is 40 into 2 plus the fourth period I am going to produce 70 here, I incur a setup cost of 45, I incur a production cost of 70 into 5, there is neither inventory nor backordering.

So, it is 45 plus 70 into 5 so plus 45 plus 70 into 5 this is 720 plus 350 is 1070, 1070 plus

80 is 1150, 1150 plus 80 is 1230, 1270 and 1315. So, this is the total cost when we consider backordering in this case. Please note so far we have not solved the problem optimally, we have just illustrated a possible solution that includes backorder cost and we are able to show there, if we include backordering then the cost comes down from the optimum 1375 to at least 1315 because 1315 is a feasible solution considering backordering.

So, backordering is advantageous from a cost optimization sense, which is also understandable because when we solved this problem optimally, saying that there is going to be no backordering, which was one of the assumptions, which means we have restricted the normal problem that could include backordering and we had solved a restricted version of a minimization problem. So, a restricted version of the minimization problem will only have an optimum solution higher. Now, this was a relaxed version, so this optimum solution will be 1315 or lower.

So, we can actually solve the problem optimally using the Wagner Whitin ideas by modifying it by saying that if my inventory is 0, I may even skip this period and backorder it to the next period that is the only modification. But if I have an inventory I will not skip and backorder because if I am incurring both inventory cost and backorder cost I would not do that. So, if there is 0 inventory, I need not produce in this period I may still skip production and add it to backorder. Now, for the same problem if we consider backordering, we have to evaluate more scenarios than we actually have evaluated.

In this case there are only ten scenarios, but the case with backordering has more scenarios than we actually have. Month one we incur cost equal to setup cost of 60 plus we produce the demand of 80 of month one incurring a production cost of 5. So, plus 80 into 5 would give us 460 as the cost for month one. Now, we move to month two now we can meet the demands of month two in multiple ways. So, we could do 80, 60 which means we produce 80 in the first month and 60 in the second, we could do 140 0 and we could also do 0 140.

So, this gives us a cost equal to 740 140 0 would give us a cost equal to 880 and 0 140 would give us a cost equal to 680. So, let me explain the computation of this cost because this involves backordering so we produce all the 140 in the second month and we meet the demand of 80 of the first month and 60 of the second. So, cost to produce this 140 there is a setup cost of 40 plus there is a production cost of 140 into 4 plus - this 80 of the first month is backordered. So, the backorder cost for the demand of the first period is 1. So, plus 80 into 1 so this would give us 40 plus 560 which is 600 plus 80 which is 680. So, out of these possibilities this seems to be the best one and this is the one that involves backordering.

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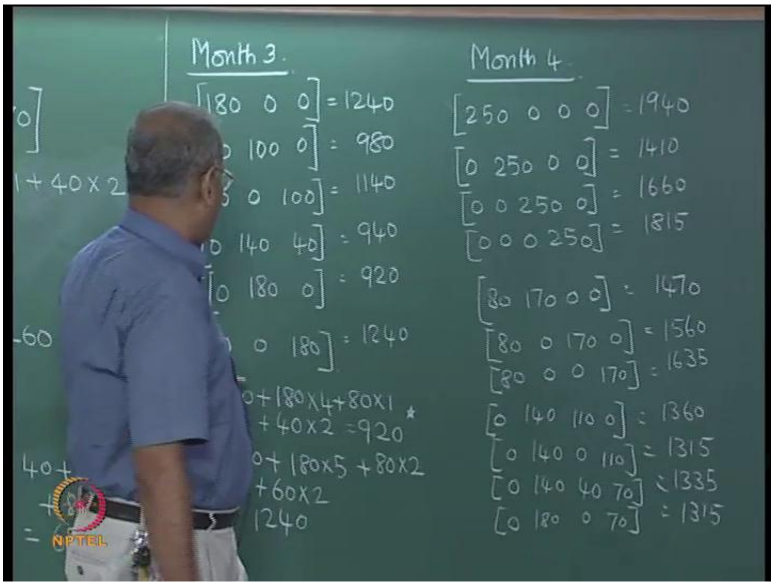

So, with this we move to month three, where we can look at several combinations for month three so we could use 180 0 0 we could use 80, 100 0, 80 0 100, 0 140 40, 0 180 0 and 0 0 180. Let us look at finding out the cost in two such instances so we will show the computation for 180 0 0 the cost is 1240 for 80 100 0 it is 980. So, let us explain the computation in detail for a couple of instances. Now, when we say it is 180 0 0 it means in this case, we produce 180 in the first month consume the demand of 80 and then carry over hundred to the next month. And then consume the demand of 60 and carry over the remaining 40 to the third month.

Now, let us illustrate the computation for two instances. Let us do it for this instance as well as this instance. Now, here is an instance where we produce 180 in the second month to meet the demands of the three months, which means there is a backorder here as well as there is an inventory carrying here. In this scenario we are assuming we produce the total demand of 180 in the third month. So, there is no inventory carrying, but there is backorder, but backorder for two periods. So let us first show the computation for this so TC for this.

So, let me illustrate this with some kind of a plus sign here so TC for the plus sign will be - I produce 180 here in the second month. So, setup cost is 40 plus production cost is 180 into 4 plus the first period demand of 80 is backordered - the cost of backordering the demand of the first period is 1. So, plus 80 into 1 now here the backorder 80 we consume 60 for this period. So, the balance 40 is carried over from second month to third month so inventory at the end of period 2 is 2. So, 40 is the quantity carried into to cost which is equal to 2 so this gives us a cost of 40 plus 720, 760 plus 80 840 plus 80 equal to 920 which is what we find here.

Now, let us show the computation for this one. Let me illustrate this with TC+ just to show that we are looking at this computation. Here, as I mentioned earlier there is no inventory, but there is backordering. So, the production is done in the third period by incurring a setup cost of 60 plus production cost is 180 into 5 plus the demand for both the periods are backordered. So, for this period the demand is 80 per period backorder cost is one, it is backordered for two periods. Therefore, it is 80 into 2 plus another 60 demand for this period is also backordered.

Now, backorder cost per period for month two is 2 therefore, plus 60 into 2, we should note the difference in the way we compute the inventory cost as well as the way we compute backorder cost. In this particular example, when we did this 180 0 0, if I were to calculate the total cost here now out of this 180 that is produced in the first month 80 is consumed. So, hundred is carried at the end of the first month so hundred into inventory cost of 2 plus another 40 is carried at the end of the second month so 40 into 2. So, when we do the inventory calculation we take the inventory at the end of that month and then multiply by the inventory cost at the given for that period. When we do the backorder calculation we go back here and say that the first period demand of 80 is backordered for two periods.

Now, per period backordering cost for the first period demand is one. Therefore, 80 into 2 into 1, 1 is the cost two is the number of periods here. Similarly, 60 is the quantity backordered its backordered for one period with the cost of two so 60 into 2. So, this gives us 60 plus 900, 960, 960 plus 160 is 1120, 1120 plus 120 is 1240. Now, out of these we realize that this is the one with minimum cost then we move to month four.

Now, with month four we work out several combinations and these are shown here so we could do 250 0 0 0 which is 1940, 0 250 0 0 1410 0 0 250 0 1660, and 0 0 0 250 which is 1815, 80 170 0 0 which is 1470 80 0 170 0 which is 1560 80 0 0 170 1635, 0 140 110 0 1360, 0 140 0 110 1315, 0 140 40 1335, 0 180 0 70 1315. So, these are the various solutions which we evaluate here. We also realize that these four are the situations, where all the 250 is produced in a single month, and these three cases involve backordering because from here. We backorder this demand, from here we do both from here all the three are being backordered.

Now, when we do this backordering, the cost computation will be - this 80 demand is backordered for three time periods, the cost of backorder is 1. So, this will be 80 into 1 here it will be 60 into 2 into 2 because the 60 demand is backordered for two periods. The cost of backorder in that period is 2 so this will become 60 into 2 into 2 and in this one it will become 40 into 1 because the other 40 is backordered for one period, the cost is also one. So, that when added with the setup cost of 45 and production cost of 350 would give us 1815.

So, these four look at the cases, where all the demands are produced in a single period. Now, here what we do is we have this 80 and then the three cases, so if we look at this for month one we have 80, which is the first one the remaining three are now produced in three possible ways including backordering, there is backordering here, there is backordering here. Now, we move to the second month and we realize that the best value is 0 140. So, we take the 0 140 and look at the other cases that are here with 1335.

And the third one the best case is 0 180 so we have 0 180 0 70, 0 180 0 gives us this 920 plus 70. Now, we realize that the best value happens in two instances, which is 1315 here which also happens to be the solution here that we shown compare to 1375 of the earlier one this is the solution, without backordering. And these are solutions with backordering.

So, the 1315 that we calculated here and showed as a feasible solution is actually optimal with the case with backorder. So, this is how we work out and solve the production planning problem using dynamic programming. Now, there are three important things which we have to look at before we wind up this discussion on the dynamic programming model. The first and foremost is this model is different from the earlier models because here we have considered the effect of setup. In the tabular method, in the linear programming base method and in the transportation based method, we did not consider setup costs.

Now, dynamic programming gives us the opportunity to consider setup costs. The Wagner and Whitin assumption has made the solution much simpler. When we did not consider backordering we looked only at ten cases, when we considered backordering we did look at more cases I have not gone to the detail of calculation of each of these numbers, but these are not difficult considering that I have shown how each of these numbers are calculated. And also considering how I have shown how the one particular thing is calculated.

So, one can calculate all these costs and then find out the number of calculations has increased it is a little more than even double the thing that we have here, which means if you are looking at n periods then this is roughly 1 plus 2 plus 3 up to N and that would go about 2 to 3 times that number. Still the number is actually small and manageable because the Wagner and Whitin method allows us certain conditions, where we can use to solve this problem optimally. Then there are two other issues that we have to look at one is should we really consider backordering, what is the impact of backordering, what is the effect of backordering from a practical point of view.

One is for the purpose of illustration I have used that backordering cost here 1 2 1 and 2 are kind of comparable to the inventory cost of 2 2 1 and 2. In fact in this particular numerical illustration I have even used values that are smaller than this, but in reality cost of backorder is much higher than cost of holding inventory. Cost of backordering has an important component, which is the loss of customer goodwill because backordering is employed.

So, which is not a cost that can be easily measured so cost of backordering is much higher, many times backordering also results in for example, air lifting a certain product from one place to another place because there is a delay and already there is one period backlog that is accumulated. So, people would like to transport it as quickly as possible with increased cost. So, cost of backordering is much higher than cost of holding the inventory. So, should backordering be planned many times the answer is no because there is not very good to plan backordering, but unless there is a requirement and there are certain demands which can actually be backordered and not met, which simply means the customer is ready to postpone it because the customer has inventory already in thus in the system.

Then in such circumstances backordering may be permitted or considered. The third part is we assume that the demand all the two hundred and fifty could be produced in a certain period, many times there are capacity restrictions and we would not be able to produce all the 250. So, there can be restrictions on the maximum production that can be made in a particular period. Now, in the next lecture we will look at some more aspects of aggregate production planning, we will look at a couple of more models and one more model and a couple of other issues after which we will move from aggregate planning to materials management or inventory control.