Operations and Supply Chain Management
Prof. G. Srinivasan
Department of Management Studies
Indian Institute of Technology Madras

Lecture - 5
Aggregate Planning, Tabular Method, Linear Programming

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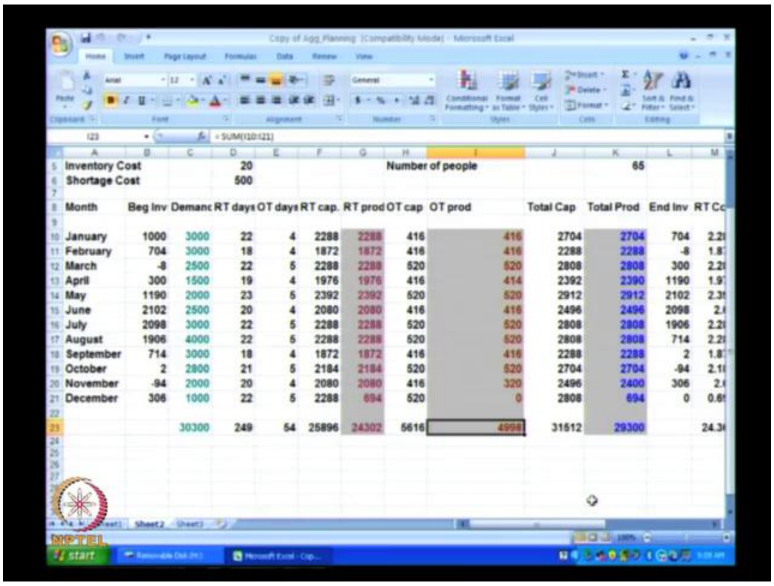

Inventory Cost			20				Number of people			65		
Shortage Cost			500									
Month	Beg Inv	Demanc	RT days	OT days	RT cap.	RT prod	OT cap	OT prod	Total Cap	Total Prod	End Inv	RT Co
January	1000	3000	22	4	2288	2288	416	416	2704	2704	704	2.2
February	704	3000	18	4	1872	1872	416	416	2288	2288	-8	1.8
March	-8	2500	22	5	2288	2288	520	520	2808	2808	300	2.2
April	300	1500	19	4	1976	1976	416	414	2392	2390	1190	1.9
May	1190	2000	23	5	2392	2392	520	520	2912	2912	2102	2.3
June	2102	2500	20	4	2080	2080	416	416	2496	2496	2098	2.
July	2098	3000	22	5	2288	2288	520	520	2808	2808	1906	2.2
August	1906	4000	22	5	2288	2288	520	520	2808	2808	714	2.2
September	714	3000	18	4	1872	1872	416	416	2288	2288	2	1.8
October	2	2800	21	5	2184	2184	520	520	2704	2704	-94	2.1
November	-94	2000	20	4	2080	2080	416	320	2496	2400	306	2.
December	306	1000	22	5	2288	694	520	0	2808	694	0	0.6
		30300	249	54	25896	24302	5616	4998	31512	29300		24.3

In the previous lecture, we introduced the aggregate planning problem. We started by saying that there is a demand, we said that we assume a single item that is being produced or a single product that is being made, the demand for 12 months is given as 3000, 3000, 2500, and so on. We also said that there are some regular time days of production available, which are 22, 18, 22, and so on, in the 12 months, we also said that the overtime days available are known which are 4, 4, 5, and so on.

Now, once we know the regular time days, we multiplied it by a constant and said that if there are 22 days total of 2288 units of regular time capacity is known. So, the regular time capacity column can be calculated by multiplying, these regular time days into a constant, which happens to be 104. In this case 22 into 104 is 2288, it means that in one day with the people that are available, we can produce 104 units of the item or the product.

Similarly, if 4 overtime days are available in the month, 416 is the maximum that can be produced using the overtime capacity, and this column that as total capacity is a sum of regular time capacity and the overtime capacity. Now, as a decision maker we have to now say, how many we are going to produce using regular time, how many we are going to produce using overtime? So, the total production will be the sum of the regular time production that is planned, and the overtime production that is planned.

So, if we give 2288 here and 416 here, the total automatically becomes 2704, we also have to give numbers here, which means the planned regular time production quantities should be less than or equal to the capacity that is available here. And similarly, the planned overtime production quantities should be less than or equal to the capacity that is available here, now instead of filling these two individually what the user does is the user will try and fill a number here, which will be less than or equal to 2704 in this case we have filled 2704.

Now, the moment we fill 2704, we can assume that the maximum possible 2288 will go to regular time production, and the balance will go to overtime production. This is because regular time production is assumed to be less costlier or cheaper than overtime production, and therefore, the moment we decide on 2704 we will produce the maximum possible through regular time, and the balance will come to the overtime. So, if we write 2704 here, which means we decide to produce 2704 the ending inventory will be beginning inventory of 1000, which is assumed to be there plus production which is 2704 less demand, which is 3000, so it becomes 704.

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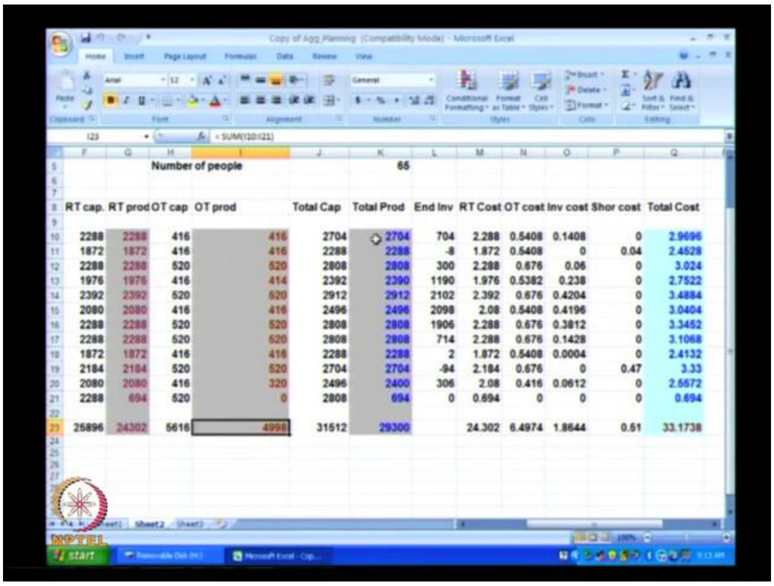

Now, we also calculate the costs associated with producing 2704, out of which 2288 are produced using regular time and 416 through overtime. So, we assume that the regular time production cost is rupees 100, so we incur 2.288 lakhs as regular time production cost. We assume that the overtime cost is 130 and to produce 416 units, we incur 0.5408 lakhs, we assume that the inventory is charged at rupees 20 per unit per month. So, to carry an ending inventory of 704, we multiply it by 20 and incur a cost of 0.1408 lakhs, there is no shortage cost, because the ending inventory is positive.

In this example, we are charging inventory based on inventory cost is computed based on the ending inventory value. So, there is no shortage cost and the total cost as a sum of these four costs which is regular time cost overtime cost inventory cost and shortage cost and that is 2.9696 lakhs. Now, the ending inventory of January which is 704 becomes the beginning inventory for February, which is shown here 704 is shown here, now the user has to make a decision here as to what is the total production.

So, if the user decides to produce all possible 2288 here, out of this 1872 which is the maximum possible will go to regular time production, and the balance will go to overtime production, now the ending inventory will be beginning inventory 704 plus 2288 minus 3000, which is minus 8. So, for the month of February regular time cost is 1.872 lakhs, which is 1872 units into 100 rupees, overtime cost is 0.5408 lakhs which is 416 units into 130 rupees.

No, inventory cost, because ending inventory is negative, but shortage cost 8 units are short ending inventory is minus 8 indicating that 8 units are short shortage cost is at rupees 500. So, 8 units into 500 will give us 0.04 lakhs and the total is 2.4528 lakhs, now we come to march with ending inventory of February is minus 8. So, beginning inventory of March is minus 8, this minus 8 means that 8 units are back ordered and are met in March, so what we defined as shortage cost of 500 is actually the back order cost, which is the cost of back ordering per unit per month.

So, like this we proceed the user makes these 12 decisions here for the total production, and the spreadsheet helps us in computing the cost, so the cost for this production plan is 33.1738 lakhs. So, the spreadsheet is an evaluative tool, where given the total production choices made by the user the spreadsheet calculates the cost as 33.1738. So, the important question or the next question is what are these quantities, such that the total cost is minimized, so the aggregate planning problem can be defined as, given that demand for the single product for 12 months.

Now, we are assuming one product, later we will tell you how we take care of multiple products, so right now considering one product given the demand for a certain number of periods, given the regular time capacity and the overtime capacity. The user has to decide on the total production quantity, which will be split into regular time production quantity, and overtime production quantity, using the balance equation that beginning inventory plus production less demand is equal to ending inventory.

And ending inventory in a month becomes beginning inventory of the next month, and the costs associated with regular time production, overtime production inventory and shortage, what should be the total production quantities such that this cost is minimized. Now, this is the aggregate production planning problem, now we try and answer a few other questions the first question that we need to answer is are we going to consider only one product or multiple products, because in practice every organization makes multiple products.

Now, we defer that question, we answer it a little later, next question that we have to answer is are only regular time production and overtime production are these the only two means of production or can we have other means of production. For example, can we consider outsourcing as a means of producing or as a means of meeting the demand? The other question is do we have only these fore costs, or are there other costs, the third question that we would like to answer is this in this spreadsheet, when we said that in 22 days we have a regular time production capacity of 2288.

Now, that came from a number 104 which is a number of units that we produce per day, and if we look at these calculations, these are made out of this we assume that 65 people are working it takes 10 hours to produce one unit. So, we have 650, 65 people are working, each person spends 16 hours in a day, so in a day we have 65 into 16, which is 1040, and then we assume that it takes 10 hours to produce a unit therefore, 104 units are produced in a day. Now, the next question is now the 104 comes out of this 65 which is the number of people who are working.

Now, can we increase the number of people working in different months, so that the regular time production capacity and the overtime production capacity can also vary, the question is it going to be 104 per day right through the planning period or can we increase or decrease that 104 by varying the number of people at our disposal. So, we try to answer all these questions, using another spreadsheet and by considering additional costs and also by considering varying manpower.

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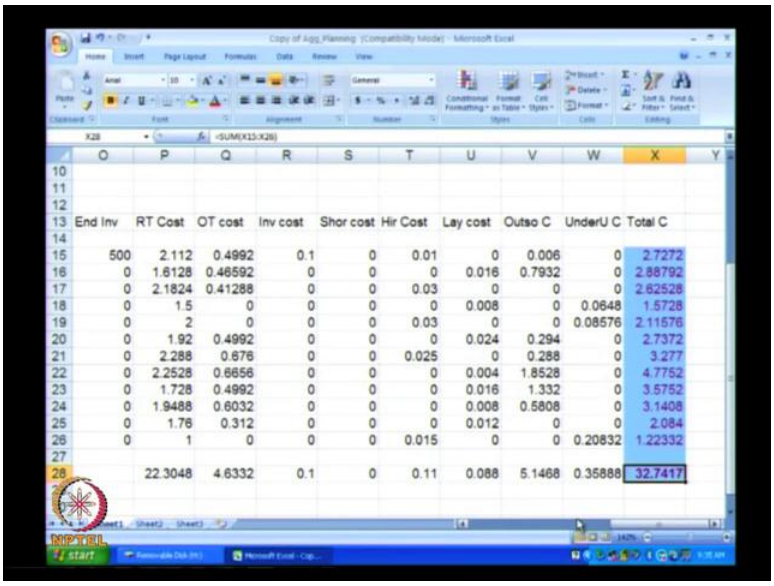

	O	P	Q	R	S	T	U	V	W	X
13	End Inv	RT Cost	OT cost	Inv cost	Shor cost	Hir Cost	Lay cost	Outso C	UnderU C	Total C
15	500	2.112	0.4992	0.1	0	0.01	0	0.006	0	2.7272
16	0	1.6128	0.46592	0	0	0	0.016	0.7932	0	2.88792
17	0	2.1824	0.41288	0	0	0.03	0	0	0	2.62528
18	0	1.5	0	0	0	0	0.008	0	0.0648	1.5728
19	0	2	0	0	0	0.03	0	0	0.08576	2.11576
20	0	1.92	0.4992	0	0	0	0.024	0.294	0	2.7372
21	0	2.288	0.676	0	0	0.025	0	0.288	0	3.277
22	0	2.2528	0.6656	0	0	0	0.004	1.8528	0	4.7752
23	0	1.728	0.4992	0	0	0	0.016	1.332	0	3.5752
24	0	1.9488	0.6032	0	0	0	0.008	0.5808	0	3.1408
25	0	1.76	0.312	0	0	0	0.012	0	0	2.084
26	0	1	0	0	0	0.015	0	0	0.20832	1.22332
28		22.3048	4.6332	0.1	0	0.11	0.088	5.1468	0.35888	32.7417

So, that we try and show in the other spreadsheet, which is like this same problem, we consider here the same demands of 3000, 3000, 2500, 1500 etcetera. You can compare this 3000 same demands, we consider 30300 is the total demand same 30300, regular time days are the same overtime days are the same. Now, in this model we are trying to include the workforce that is ((Refer Time: 13:00)), we explained this using another spreadsheet, where we consider additional costs as well as varying manpower.

Now in this spreadsheet, we use the same values of the demand and these demand adds up to 30300, we use the same values of regular time production days as in the previous spreadsheet as well as the same values for overtime production days, as used in the earlier spreadsheet. The change that we bring in this spreadsheet is that, we are now going to vary the workforce. So, we are assuming that say, this represents January of a particular year where the demand is 3000, we assume that in the previous December we had 58 people who were working.

Now, in this spreadsheet calculation, we are going to show that this 58 is now increased to 60, so we introduce another column which talks about hiring people. So, from 58 to 60 we have hired 2 people for this period, so the number of people hired is 2 we have not laid off any one because the number has increased. So, if there is an increase in workforce it is hiring, if there is a decrease in workforce it is a layoff, so we do not have layoff here layoff is 0.

Now, with 60 people working for 22 days, the regular time production is 2100 regular time capacity is 2112, now this comes from we have 60 people each works for 16 hours in a day, so that is 960 divided by 10 is 96. So, each day we can make 96 units, so in 22 days we can make 2112 units, similarly in 4 overtime days multiplied by 96 we have overtime capacity of 384. So, now, the regular time capacity and the overtime capacity not only depend on the number of days, it also depends on the number of people who are working in that month.

We also assume that once we define 60, these 60 people are going to work on all the 22 days of this month. So, we are now defining the regular time capacities and the overtime capacities, now the user has to give a decision on the total production. So, once the user decides say 2500 here, the first 2112 or 2112 out of 2500 will be made using regular time production, because that is the maximum capacity, and the balance is made from overtime production which is your 384 from overtime production.

Now, we have decided to produce 2500 in this month, our regular time capacity is 2112 our overtime capacity is 384 together the capacity is 2496, but we have decided to produce 2500. Now, out of the RT and OT capacities we can get only 2496. So, the balance 4 will be treated as being out sourced, so if the user enters 2500, first we will check what is this RT capacity? So, that much is given to RT production, and then the balance is taken then the OT capacity is seen, and then the maximum possible is given to the OT production.

And if there is still something to be given that is given to the outsource, the reason and the motivation is that regular time production is less costlier than overtime production, which is less costlier than outsourcing. Therefore, maximum is given to RT production then to OT production and then to outsourcing, now the ending inventory is calculated in the same manner as beginning inventory. In this case beginning inventory is 1000, production is 2500, so 3500 less demand of 3000, which comes from here and the ending inventory is 500.

Now, we have various costs, we have actually 8 costs in this spreadsheet regular time cost, overtime cost, inventory cost, shortage cost, hiring cost, layoff cost, outsourcing cost, and under utilization cost. So, let us see all these 8 costs, now regular time cost is kept at the same value of 100, so 2112 units will give us a regular time cost of 2.112 lakhs, overtime cost is kept at 130, so 384 units will give us an overtime cost of 0.4992 lakhs.

There is an ending inventory of 500 the inventory cost is 20, so we get 0.1 lakh, the ending inventory is positive, therefore there is no shortage. Shortage cost is 0. In this period we increase the workforce from 58 to 60, therefore we hired workforce, cost of hiring is given as cost of hiring is 500 we hired 2 people, so the hiring cost is 500 into 2 people, which is given by 0.01. There is no layoff means the workforce reduces, so there is no layoff, there is an outsourcing cost. We have out sourced 4 units the cost outsourcing is taken as 150. So, 150 into 4 is 600, so 600 gives us 0.006. The 8th cost which we have to describe is cost of under utilization.

Now, in this particular case the regular time capacity is 2112, and we have used up all the 2112 of regular time capacity, if we do not use the entire regular time capacity, and if some regular time capacity is unused then that becomes an under utilization cost, in this case the under utilization cost is 0. Now, if you go back to few other months, where I can show some of these costs, and if we go to month 2, February we begin with 500 ending inventory of January becomes beginning inventory of February with 500.

Now, we define workforce to be 56 for the month of February, so there is a layoff there is a no hiring, so February has a layoff cost that you can see here, there is a layoff cost. Now, you also see if you look at the ending inventory column, you do not have negatives, so at no point we have shortage all shortage costs are 0. If we look at the month of say we look at this month, we look at the month of April, we now realize the regular time capacity is 1824, for this month overtime capacity is 384, but we have decided to produce only 1500.

Now, 1500 is smaller than 1824, so all the 1500 goes to regular time production, no overtime production, no outsourcing we have not used up the entire 1824, so 324 is available as unutilized or under utilization. So, the cost of under utilization is 324 into 2 which is 648, so the cost of under utilization is 0.0648, so there are 8 costs that we calculate and then the total cost of the production plan is 32.7417 using this production plan which has 8 costs. And more importantly, which has more options for the user, so in this case what should be the quantities, such that this total cost is minimized.

Now, in this spreadsheet the user actually makes 2 decisions, there is a decision on the workforce, and there is a decision on the total production, in the earlier spreadsheet the user made only one decision, which is on the total production. So, the optimization problem relevant to aggregate planning is given a certain decisions that the user makes with respect to workforce and production, whichever is relevant production. Of course, is very necessary there are situations, where we work with constant workforce, there are situations where we work with variable workforce.

So, depending on whether it is constant or variable the workforce decisions are made. So, the user makes production and workforce decisions, a maximum of 8 costs are considered here, we may consider all of them we may consider a sub set of them. Then the aggregate planning problem is what are the production and workforce decisions, such that the total cost is minimized. We now try and address the optimization problem by using an approach based on linear programming, so let me explain that next.

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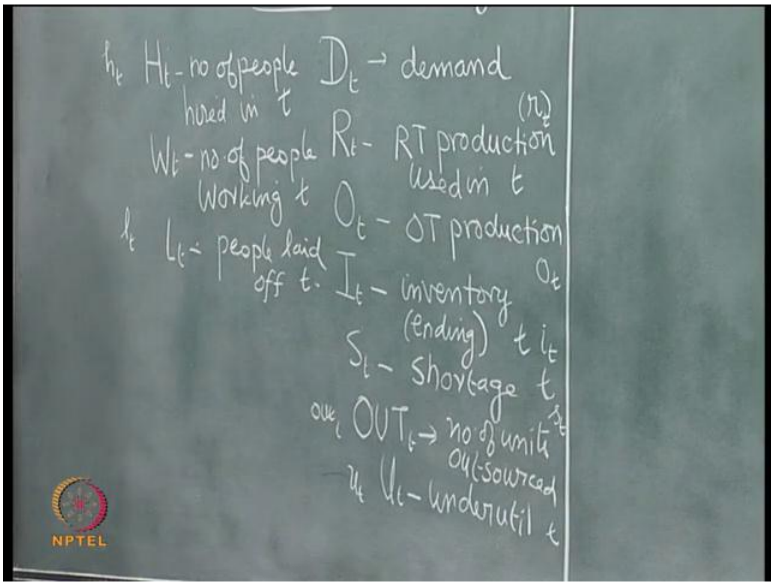

So, we try and look at what is called an LP approach to aggregate planning, so we first assume that demand for period t is known and given, so there is a demand for period t. The various costs are given, and the various decisions are given let us say RT is the regular time production used in period t, let us call OT as the overtime production used in period t. Let us define I t as inventory ending inventory at the end of period t, let S t be the shortage at the end of period t, let us call some outsource of t as the number of units out sourced.

And let U t be the under utilization in period t, let H t number of people hired in period t, let W t be the number of people working in period t, let L t be the number of people laid off in period t. Now, these are all where this is known D t is known, all these are variables that we have to find out, so there are 1 2 3 4 5 6 7 8 9 variables effectively, but there is a relationship among these variables.

Now, we also have costs associated with each one of them, which we denote by using the lower case numbers, so let small r be the regular time production cost, if this regular time production cost is going to be different in different periods, then we can add a small subscript RT, which is a regular time production cost in period t. If it is going to be the same for all the periods, then it will be r, so to generalize it we begin by saying that let small RT be the cost of regular time production in period t.

Let small OT is the cost of overtime production in period t, let i t be the inventory cost for period t, let s t be the shortage cost for period t. Let small OUT be the cost of outsourcing in period t, and let small u t be the cost of under utilization in period t. Let small h t be the cost of hiring people in period t, let small l t be the cost of laying off people in period t. Now, if we define these costs right now i am not defining small w, but I am defining the rest of the costs the objective function or the minimization function will be to minimize.

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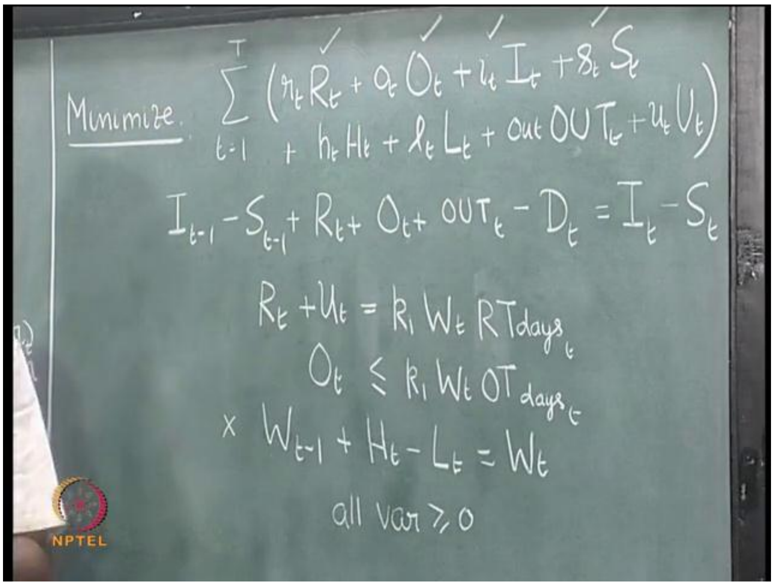

Summed over t for t periods, if r is the r t into RT, this is the variable, this is the constant, and this constant as assumed to be different for different periods, otherwise we can simply put an r if it is the same for all the periods. So, this represents the regular time production cost plus o T into O T, this represents the overtime production cost, this is the variable this is the unit cost, plus i t into I t represents the cost of holding the inventory.

This is the variable this is the cost plus s t into S t, which is the cost of shortage, if it is there plus h t into H t, h t is the number of people, who are hired this is the unit cost of hiring. So, h t into H t plus l t into L t cost of layoff plus small out into outsource of t, this is the variable, this is the cost plus cost of under utilization u t into U t. So, there are 8 costs that I have put in the linear programming formulation, these are the 8 costs.

Now, we need the constraints for any typical period I t minus 1 is the inventory at the end of that period, which is the beginning inventory for a particular period plus the productions, the productions come out of three ways regular time production plus RT plus OT plus OUT of t. So, production comes in 3 forms regular time production, overtime production, and outsourced production, so these three represent the production beginning inventory plus production less demand D t should be equal to the ending inventory, so should be equal to I t.

So, this is the typical balance equation, but the beginning inventory can also be negative, if there is a shortage at the end of the earlier period, so one way is to say that now these quantities are all greater than or equal to 0, these cannot be negative, but if we start defining I t as the inventory at the end of the previous period. And if there are shortage then I t and I t minus 1 can take negative values, which means in the linear programming formulation I t has to be defined, as a unrestricted variable which can take positive value or a negative value.

Now, we also know from linear programming, that every unrestricted variable can be represented as a difference of two variables both are greater than or equal to 0, plus we also have different costs for inventory and shortage. Therefore, it is to our advantage if we, now start representing this as I t minus 1 minus S t minus 1 plus RT plus OT plus OUT of t minus D t is equal to I t minus S t. So, if the beginning inventory is positive, we will have a positive value of our I t minus 1 S t minus 1 will be 0.

If the beginning inventory were a shortage, then it has a negative value, so this will be 0, this will be a positive value, so that minus S t minus 1 is negative. Similarly, if the ending inventory is positive or negative I t and S t will take the corresponding values, so this is our first equation which balances the inventories demand and production, so this is one equation. So, we will have if there are 12 periods we will have 12 such equations, and more importantly we will have thirteen variables for I t and S t, because we need to initialize the I t and S t.

The first value that we assumed as thousand in the spreadsheet is an initial value given to I naught, and S 0 was 0 in that case, so we need some initial conditions, where I t minus 1 and S t minus 1, second one is our capacity. So, regular time capacity is available, so whatever is given as regular time production quantity, should be less than or equal to the regular time capacity, so this is clearly written as what is the regular time capacity. From the spreadsheet we said that if there are W t people working, then each person is capable of producing a certain quantity per day.

So, we let us call that as some k 1 into W t if w t people are working then each person can make k 1 per day, so this will be W t into k 1 into the number of days, which is a constant, if k 1 is the quantity that each person produces per day, which was our 1.6 in the previous spreadsheet example, now that has to be multiplied by the number of days. So, we will start defining RT is the regular time production in period t, is less than or equal to the number of people working into units produced per day, into another constant which is some k 2, where k 2 is known it is RT days.

So, I can either use it as k 2 or I can use it as RT days of t,, number of regular time days available in period t, or I can multiply this is a constant, this is a constant, so I can simply say RT is less than or equal to some constant times W t. Now, if we get into a situation, where RT is actually less than what is the capacity, then we have under utilization, which we saw in the second spreadsheet. And there is an under utilization costs which is also there, so this is rewritten as RT plus U t is equal to constant times W t, so this represents the regular time capacity constraint.

Similarly, there is a overtime capacity constraint, so OT is less than or equal to the same k 1 into W t into OT days of t. This W t is the number of people, who are available k 1 is the number of units this person is producing per day, this represents the number of OT days that are available. So, OT is less than or equal, here we do not have a under utilization cost for OT, and this formulation we are using under utilization cost only for RT, and we are not using it for OT.

Then we have another constraint, which is W t minus 1 plus H t minus L t is equal to W t, w t minus 1 is the number of people who have worked in period t minus 1, who are available at the beginning of period t it is like your inventory, you either hire or you layoff, so plus H t minus L t is equal to W t. Now, all variables greater than or equal to 0, now this is the linear programming formulation of this problem, now there is a hiring cost associated with this there is a layoff cost associated with this.

Since, both this h t and l t are greater than or equal to 0, both H t, and L t will not appear in the solution, only one of them will appear in the solution. For example, if net hiring people is 2, you will not say that I have hired 3 and I have laid off 1, and I have a net of 2 people hired that would increase, these 3 costs you would rather say hire two and do not layoff anyone, so this is the formulation. Now, if there are t periods, then there are as many constraints as the number of periods, as many constraints as the number of periods, once again number of periods, number of periods.

We also need some initial condition on W 0, also observe that in this formulation, we are not explicitly adding the payroll cost into the objective function. We are not doing that we assume somewhere that the payroll cost is getting reflected in the regular time cost, which is includes payroll perhaps even in the under utilization cost, which includes part of the payroll costs. That is the reason we do not have a small W t which is the cost of actually engaging these people in the month

We also have to understand that H t and L t are not the payroll costs, but these are the costs, these are also not the payroll cost of the additional people hired, H t and L t, which are called this H t and L t, which are the cost of hiring and cost of layoff. H t essentially concerns the cost of conducting interviews to hire people, the cost to train the people, it does not include the payroll of the additionally hired people, that is taken care in the W t, and in this case in RT and U t.

Similarly, L t is the cost of laying off, where some kind of an additional compensation is given to the person, who is laid off, so that is your L t. So, this formulation is the equivalent optimization or linear programming formulation, for the problem the second spreadsheet problem that we saw earlier. The spreadsheet was an evaluative solution, where the user made those decisions on in the spreadsheet the user made decisions on total production and workforce.

Here what happens the decision variables are many of them, but we also know that in an evaluative model when you make a decision for total production? You are actually making decisions for RT OT as well as outsource of t, when we made one production decision in the second spreadsheet, when we made a decision for W t there, we automatically made decisions for H t L t and W t. And the moment we made this decision for RT, we also made the decision for U t which is the under utilization, so this is exactly the formulation for the problem, that we had there.

Now, let us go back to that spreadsheet, and see what is the optimal solution for that particular example, when we solve that using this particular linear programming formulation. It is a linear formulation because all variables are greater than or equal to 0, we have taken care of that by this I t minus 1 minus S t minus 1, as well as taking care by the H t minus L t. So, if W t is bigger than W t minus 1 H t will take a value, if W t is smaller than W t minus 1 L t will take a value, both attract costs which are greater than or equal to 0, so let us go back and check in the spreadsheet as to what these solutions are.

Now, in this linear programming formulation, we have modeled all that we saw in the second spreadsheet, where we considered 8 different costs, which included cost of inventory regular time, overtime, inventory, shortage, hiring, laying off, outsourcing and under utilization. Now, if we where to model the first spreadsheet, where we considered only 4 costs, where we considered regular time cost, overtime cost, inventory cost, and shortage cost, which means we did not have this particular equation, where we varied the workforce.

Here there is only one W, which is a constant which was kept at 65 for the other in this spreadsheet, so we do not have these 4 costs and we do not have this OUT of t, there is no outsourcing. So, this linear programming formulation can also be modified to take care of the assumptions in the first spreadsheet, what I also wish to convey here is even though there are 8 costs that are there, it is not absolutely necessary all the time to consider all the 8 costs.

Sometimes, if we work with the constant workforce model, then we are going to consider only 4 out of the 8 cost, if we leave out the outscoring and under utilization, so we are going to look at only these 4 of the 8 costs. So, what we do now is we go back to the first spreadsheet, we try and see the solution given in the first spreadsheet, and then we also provide the solution in the optimization problem, where we are modifying this L P formulation. To meet the assumptions of the first spreadsheet, where we are considering only 4 costs, we consider a constant workforce, and we do not consider the outsourcing.

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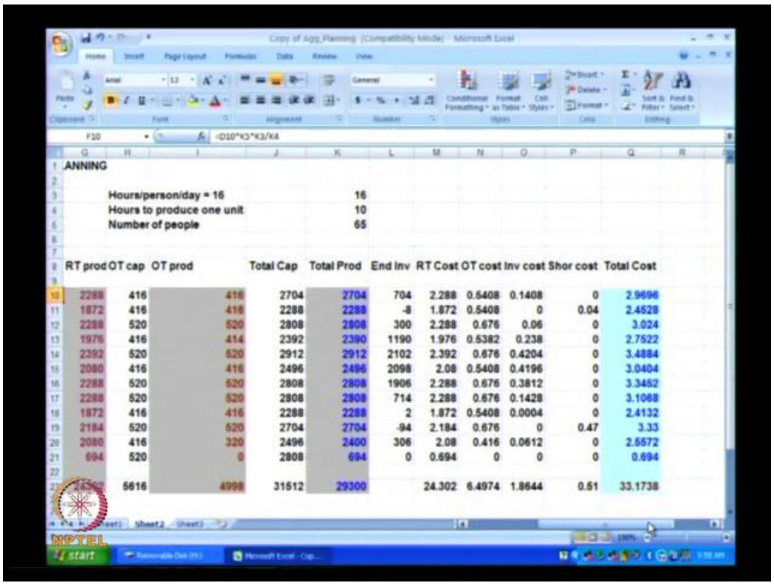

So, we are going to consider only regular time, overtime, inventory, and shortage, now in this spreadsheet the total cost that we see is 33.17 lakhs.

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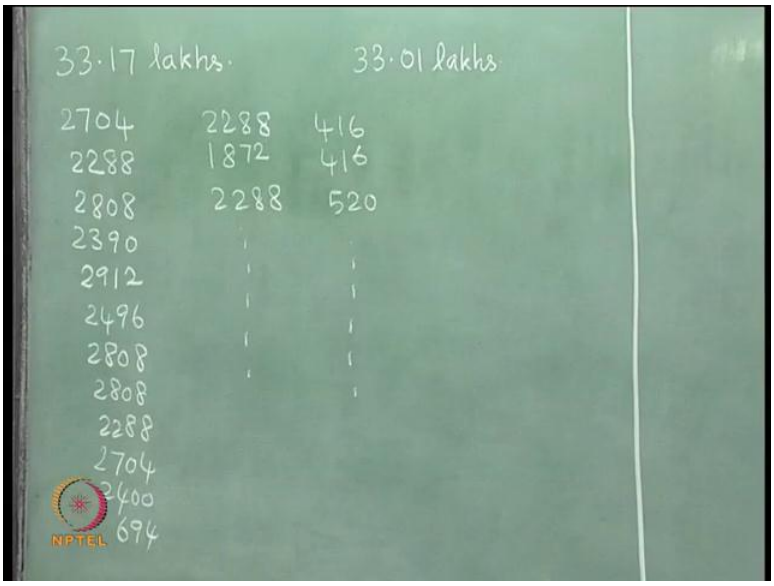

33.17 lakhs and the total production quantities that are given there are 2704, 2288, 2808, 2390, 2912, 2496, 2808, 2808, 2288, 2704, 2400, 694, these were the 12 production quantities, which you can see here, which are shown in this spreadsheet. Now, the cost is 33.1783 lakhs, now if we solve the optimization problem for this, the optimum solution is R 1 is 2288, R 2 is 1872, R 3 is 2288 and so on.

Now, the overtime values are 416 416 520 and so on, and the total cost is 33.01 lakhs, now the evaluative model using the spreadsheet gave us a solution with 33.17, the optimization gave us a solution with 33.01 lakhs, total production here was 2704, here total production is 2704. This is 2288, this is 2288, this is 2808, this is also 2808, but there is a change somewhere else in the optimal solution, so 33.17 lakhs from the evaluative model came down to 33.01 lakhs.

Now, this is the advantage of optimization, if we try to and get this solution from the spreadsheet, the user has to play around with these values, and for every change you can see that this value keeps changing. For example if we had used instead of 2704, if I had used 2600, then you see that 33.17 becomes 34.94, so the user can now play around with these total production values and such that the value actually comes down.

Now, that is going to take enormous amount of time, if the user has to do it in a spreadsheet the optimization gives us the best value, and when you use the spreadsheet model or what is called as a tabular approach. You would not know, which is the optimum value, whereas here you know what is the optimum value.

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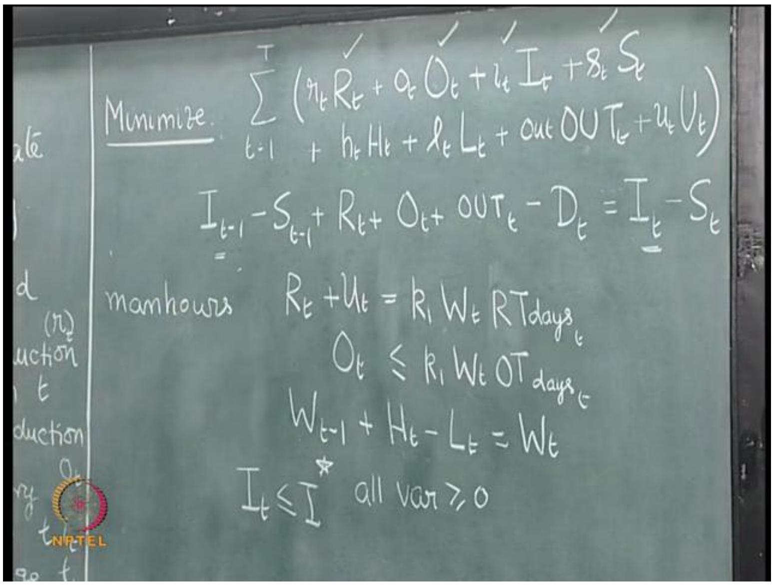

The other important thing is that, as I said this model is a very generic model, which captures 8 different costs, sometimes you use all 8 of them, sometimes you do not use all 8 of them, you may use fewer as we did we used only 4 out of the 8. Sometimes, the organization will say that I will use variable workforce, but I do not want to use outsourcing, which means you will not have the outsourcing cost here, but you will have the variable workforce, which means you will have the H t as well as L t.

So, depending on the specific requirements in the various organizations, we can modify this linear programming model suitably, and then solve to get to the optimum solution. We also made an assumption here, we did not explicitly use W t into this, I said the W t people are paid and that gets absorbed in this RT cost, as well as in some of the U t cost, when they are not going to be utilized. It could also get absorbed in the OT cost, if they are used for the overtime, another way to do it is to separate it and then bring this W t into the objective function, by saying that there is a payroll cost for W t people W into W t.

And, if we do that then this RT, this OT and this U t will not consider the payroll component, so this RT will only be the cost of material, the cost of consumable, the cost of other power and few other things, which will go into R t. Normally, RT and OT particularly the difference in these 2 costs comes from the additional money that is paid to work overtime, so that the additional amount also has to be absorbed here in this O t.

So, the formulation can also be modified by bring the payroll cost explicitly into the objective function, but linear programming provides just with a very generic framework to solve this. Now, in addition organizations may have some other kind of constraints, see here we do not have any restriction on the values, that I t and I t minus 1 can take. If it is cheaper and economical we do not mind building a lot of inventory through this model, because we are not restricting the value of I t though indirectly we are restricting it because inventory has to be built only through production.

So, if I t has to go up then it means that this RT and OT and OUT of t have to go up, but if we look at the costs very carefully, the costs of holding inventory is actually little less compared to every other costs that we have marked. So, there can be a tendency to increase I t, which times organizations would like to avoid by putting a restriction, that I t should be less than or equal to some I star, I do not want to hold more than a certain amount of inventory in this.

So, that is another way of handling, sometimes shortages should not be there, so you will leave out the s t variable completely and say that I will have, I will meet every months demand then and there, I will not meet the shortage. Now, we try and address the next question, what we do if we have multiple products the other thing at we also see is this, now some of these are in units, now we have to define these variables carefully.

Now, we defined some of them as units, number of units produced in regular time, number of units produced in overtime, and so on, if we start defining them, as number of units produced in regular time, overtime, number of units of inventory, number of units of shortage, number of units outsourced. But, H t and L t are number of people, who are hired and laid off, U t should ideally be the number of hours that are not utilized. So, we need to bring these three things the number of units produced, the number of hours that are unutilized, and number of people that are there plus in addition, we if we make multiple products then each product would require different times to produce.

Therefore, the capacity also has to be adjusted accordingly, so in aggregate planning, we do not and try and define these as number of units produced or number of people hired, but all the variables are now represented in a single unit, which is called man hours. So, this formulation will be let RT be the number of man hours of regular time production, number of man hours of overtime production, number of man hours of inventory, number of man hours of shortage, number of man hours outsourced, number of man hours underutilized, so it has a common unit, which is called man hours.

Therefore, some of these k 1 etcetera, will have to be redefined, which is not very difficult to do and the moment, we start using man hours we also know that we can handle multiple products. The word aggregate comes from the fact that we are aggregating all these products, looking at an equivalent product. And then we try and formulate a problem such that we compute the man hours of production, and many hours of workforce, that are going to be implied, and these are to be found at minimum cost now we look at further models of aggregating planning in the next lecture.