Operations and Supply Chain Management
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Lecture - 40
Vehicle Routeing Problems

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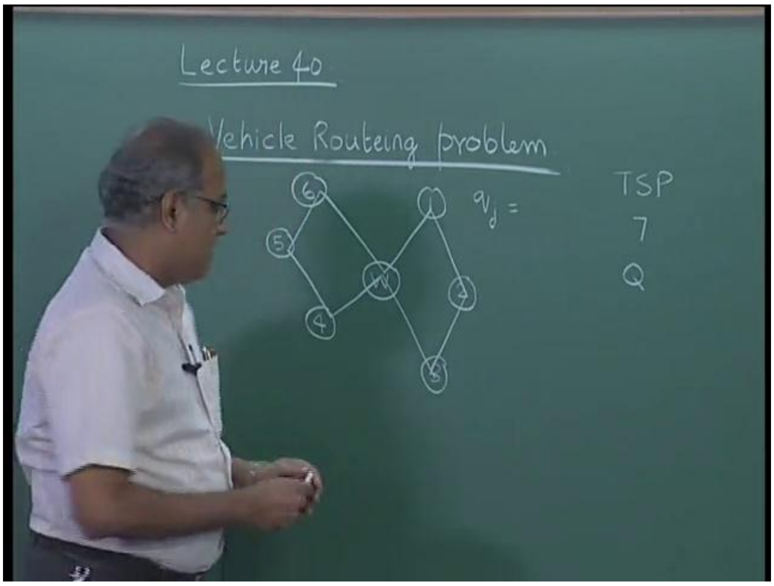

In this lecture, we address the Vehicle Routeing Problem. The vehicle routeing problem occurs in distribution, when we have to distribute from a central warehouse to multiple customers. Let us say there is a warehouse and the 6 retailers or 6 customers and we have to distribute. Now, each person has a certain requirement, which is given by q j, is the requirement of the material by customer j and we will assume that trucks take this material, from the warehouse, distribute to the customers and returns back to the warehouse.

This is an example of the milk run that, we spoke about in an earlier lecture, now if these q j's are small and such that all the 6 demands put together as less than one truck load, it is economical for the truck to start from the warehouse, visit each of these customers once and only once and come back to the warehouse, because the truck capacity will be is assumed to be greater than or equal to the sum of the requirements.

In such case, it becomes a traveling salesman problem with 7 nodes the warehouse plus 6 nodes. Many times that does not happen, the individual q j's will add up to a number, which is more than a single truck load, it is customary to use the notation Q, for the truck capacity. So, if sigma q j is greater than Q, then we need multiple trips, we either need multiple trips or we need multiple trucks or multiple vehicles, to do it, if time is a constraint.

If for example, we need 2 trips and we have only one truck, then we have the issue, of which of these customers or cities, it is customary in vehicle routeing problem, to say these customer nodes as cities and the warehouse as a depot. So, which are the cities that will go in the first trip and which are the cities that, we will go in the second trip or the next trip. And then in what order would the vehicle, visit the customers, who are assigned to a trip is a vehicle routeing problem.

We may have 1 vehicle and have multiple trips, where for example, in the first trip, it might go to 1 and then 2 and then 3 and back to the warehouse and in the second trip, it might do 6, 5, 4 and back to the warehouse. Now let us assume that, we start at time equal to 0 and let us it takes some time to finish the first trip and comeback, let us say it starts it takes about, 4 hours to finish the first trip. Then the second trip can start only after 4 hours and by the time, we reach the last customer on the second trip the time may be delayed; so that would necessitate multiple vehicles.

So, whether we talk of multiple vehicles or multiple trips, the problem is the same as long, as we are not looking at time windows, which means as long, as we are not looking at, times either before which or after which or in between, which we need to visit the customer. So, we use the terms trip and vehicle interchangeably, but wherever it is required, we talk about vehicles, it is also customary to say that in this case, even though there are 2 trips are possible, we would say there are 2 vehicles.

And the first vehicle will visit 1, 2, 3 and back and the second will visit 6, 5, 4 and back, they may do it sequentially, they may do it parallelly, but for the purpose of argument or discussion, we would say that there are 2 vehicles. So, a vehicle routeing problem essentially has more than one vehicle and if the vehicle routeing problem has a single vehicle, then it becomes a traveling salesman problem. So, T S P can always be seen, as a case of a vehicle routing problem, where there is only one vehicle.

Now, we have already seen some methods to solve the traveling salesman problem, now we will look at some ways to solve the vehicle routing problem. The vehicle routing problem is widely used in almost all kinds of distribution. It is used extensively in pickup problems, employee pickup, school children pickup, mail pickup, it is also used in a lot of delivery problems and so on. So, we will now see formulation for the vehicle routing problem and then we will explain it to an example and then we will also see some heuristic algorithms to solve the vehicle routing problem. So, several formulations exist for the vehicle routing problem.

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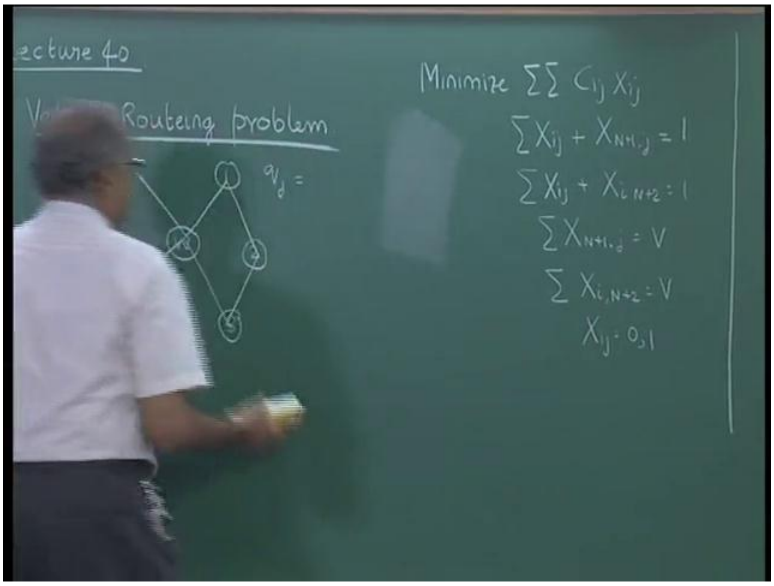

Now we will give one formulation which is this minimize C i j X i j.

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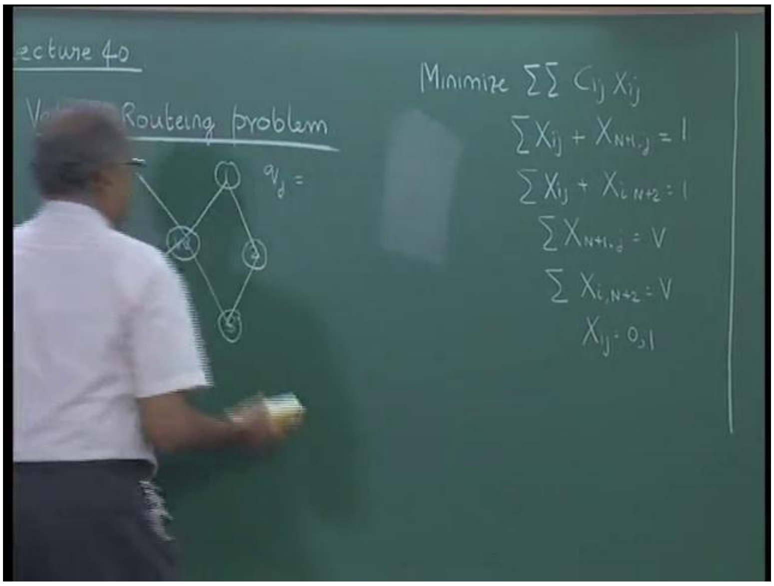

Now, we have this formulation for the V R P, we will try and explain this formulation in 2 stages, we will first explain this part of the formulation and then we will add this part of the formulation. Now, what we do in this formulation is this.

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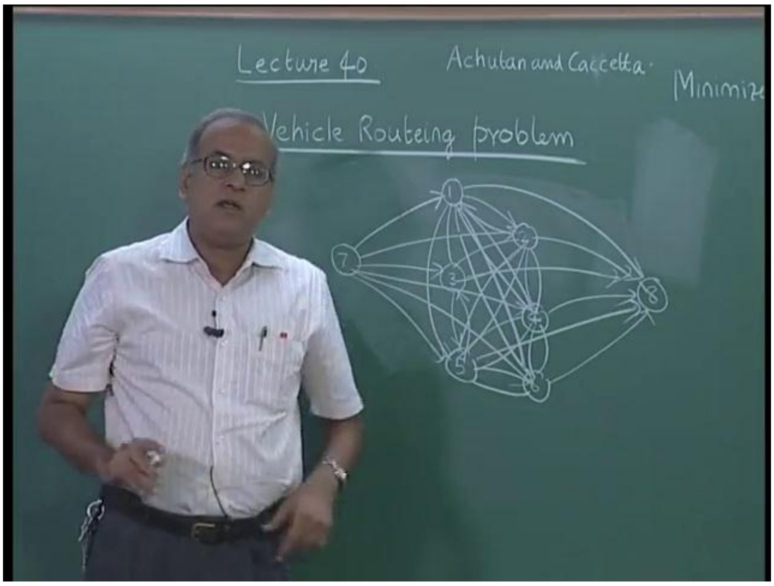

Now suppose, we have a 6 customers and we call them 1 2 3 4 5 6, now what we do is the matrix will now have then there is a depot, so we will have the depot and 6 customers, so the distance matrix is a 7 by 7 matrix. So, we will now call a 7, which is called the start node, then we will have an 8, which is called the destination node. Now what we do is each of these 6 customers are connected to itself. So, there will be a 1 to 2 a 2 to 1 and then 1 to 3, 1 to 4, 1 to 5, 1 to 6 and then you will have these arcs.

So, each one is essentially connected to each of the others, so this will be complete and so on, 2 and 5 will be connected like that, we connect all these 6 and then we complete the network by connecting 7 all these are directed arcs. So, there are connections and then 7 is connected to all the nodes and all nodes are also connected to 8, this formulation is by Achutan and Caccetta.

So, we compete the network by adding these 6 customers here having them in the middle, connecting each of them with the other and then this 7 and 8 act as some kind of source and sink assuming that, the warehouse is now split into 2 entities. One from where the vehicles go and the other, the where the vehicles at the end come into, so both actually represent the warehouse, but then they are like a source and sink on this network.

So, each of these distance between i to j on this is given from the distance matrix depot to each one is written here and each one of them to depot is also written here, so minimizing C i j X i j is on the entire network, so it is on all arcs in the network, so we would still have variables like X 7 1 X 7 2 X 7 3 and so on. X 7 we will not have variables like, X 1 7 2 7 from 7, it will only be 7 1 7 2 7 3 7 4 7 5 7 6, there will not be arcs that go into 7.

Similarly, there will not be arcs that, that go out of 8 you will have all arcs come into 8 1 8 2 8 3 8 4 8 5 8 6 8, there is no 7 8, so that is how the network is drawn, so summed over all arcs. Now, what happens is if I take a particular node 1, now few things can happen to that node, one is 1 the node 1 or any node say let us say node 3, I can reach node 3, either from the source, which means it is a first node that the vehicle comes or I can reach it from some other node.

So, X i j plus X n plus 1 j is equal to 1 for all with the j from 1 to 6, so if I take any of these nodes, I reach that node either directly from the source or through some other node, which is given by this set of constraints. Similarly, if I take a node from a typical node, I can go either directly to the destination or to some other node, so X i j plus i n plus 2 is equal to 1. So, either I go to the destination or I go to some other of these nodes, from here, I do not go back to 7 here, this is not defined for 8, this is defined this is not defined for 7.

Here, the i j is only up to this 6, 6, 7 comes here, 8 comes here, so n is equal to 6, so n plus 1 is 7 N plus 2 is 8, so this represents the n plus 2 node. Now, both these will represent that, if there are 2 vehicles that, I am considering, then capital V is equal to 2, which means exactly 2 of these arcs out of these 6, there are 6 arcs, exactly 2 of these arcs will take the value 1, which means this will force from 7, I will directly visit 1 2 out of this 6 directly.

Similarly, X i n plus 2 equal to V, again V is equal to 2, which is the number of vehicles, so this will force 2 out of these 6 arcs to be equal to 1, which means, from 2 out of these 6 arcs, we will reach directly to the destination. It may also happen that 1 vehicle may just go from 7 to 1 to 8, while the other vehicle could do 7 2 4 3 6 5 8, it can happen, but it will ensure that, 2 arcs go out of this, 2 arcs reach this and X i j equal to 0 1 minimizing the total distance would give us the distance that, we travel in the vehicle routeing problem.

So, this is a very nice formulation, when it comes to solving a V R P, more as a flow problem, solving it as a as a multiple path problem, from source to destination with some constraints and restrictions on the path that, we actually have here. So, this represents the formulation for the V R P. Now this would have for example, if we have the warehouse and then we have 6 customers or 6 destinations, now this X i j will have 36 variables.

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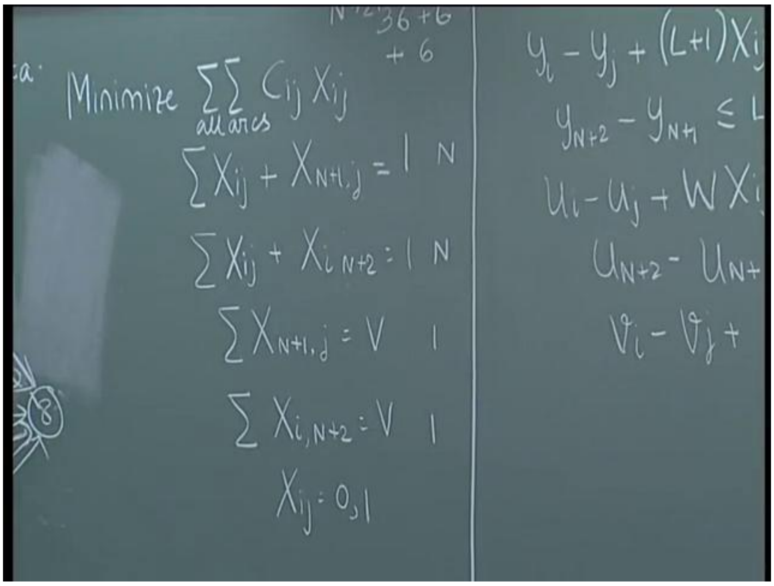

I equal to 1 to 6, j equal to 1 to 6, it will have 36 variables, then X 7 1 to 7 6 will be 6 variables and 1 8 to 6 8 will be another 6 variables, so 36 plus 6 plus 6, so if I have N customers, it will have N square plus 2, N variables, where N square X i j variables will be there and another 2 N variables formulas. The number of constraints, this will be N constraints this will be N constraints, this will be 1 constraint and this will be 1 constraint, so we will have 2 N plus 1 constraints.

So, this will have N square plus 2 N variables and then it will have 2 N plus 1, 2 N plus 2 constraints and you can solve it, as a binary I P to get the solution to this particular problem. And it is always possible to define only one path here and solve the T S P, so once again the T S P will also have N square plus 2 N variables and it will have 2 N plus 2 constraints. So, it is a more precise formulation to solve the T S P instead of having the other set of constraints, so essentially, we have N square plus 2 N variables and the 2 N plus 2 constraints to solve the T S P also, this can also be used to solve the V R P. Now we have to explain some of these, so a typical V R P is not only about, finding the vehicle to which each city or customer is assigned and the order of visit of the customers by the vehicles, there are few other constraints that, we may like to have.

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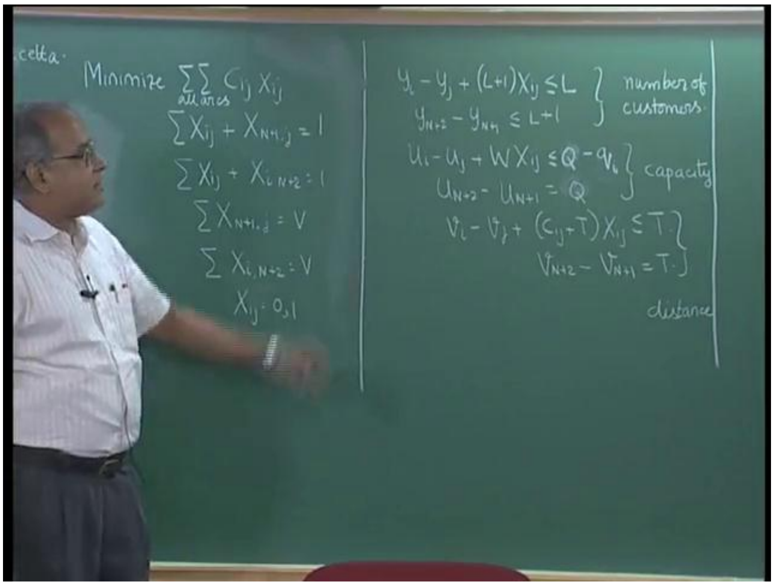

Now, this constraint is called this is a restriction on the number of customers that each vehicle visits, so this is a restriction on the number of customers that, we want the vehicle to visit, so we say each vehicle will visit less than or equal to l customers. Now, we could have restriction on capacity, in fact, if we remember right, the motivation to the vehicle routeing problem comes from the fact that, there is capacity restriction, so each customer would require a certain demand, which I said, which I used small q i as a notation.

So, we could either use capital q i or we could have small q i and this W is actually the capacity of the vehicle, we could use W or we could use Q, to represent the capacity of the vehicle. Because, if we do not have the capacity constraints, then it is always possible, if the vehicle has infinite capacity or if the vehicle has more capacity, than demanded, it is always possible to solve it as a T S P.

And then get minimum distance, because the minimum distance for a T S P with 7 nodes will always be less than or equal to the distance that, we travel when we use multiple vehicles or multiple trips as the case may be, if distances follow triangle inequality.

Generally Euclidean distance follow triangle inequality and in practice, we would know that d i j plus d j k is greater than or equal to d i k triangle, inequality is followed and therefore, a T S P will always give in terms of distance minimum distance compared to the V R P.

But, what can also happen is that, we might want to do parallel movement of this, we want to reach the customers within a certain time and so on, therefore we might say that the 2 vehicles individually may travel, if it is a T S P, then it travels a certain distance. Now, if we solve a 2 vehicle V R P the distance will be higher, but then from a time point of view both the vehicles could start at the same time and by the time, they will finish and comeback to the depot they can do it with sooner time.

So, that is another motivation to go for a V R P type of an implementation rather than a T S P, even if we do not have capacity constraints, but if we have capacity constraints, this will necessarily force us to use multiple vehicles and then we have to solve the V R P. Now, these set of constraints are called distance constraints, now we do not want a situation where, for example, if I have my total capacity of a vehicle, let us say is 50 and then and let us say the total capacity of the vehicle is 50 and the total demand is 55.

So, when we actually solve the problem as a 2, this would necessitate 2 vehicles and if we solve a 2 vehicle V R P, we might get into a situation, where one vehicle does about, 45 distance and the other vehicle does about 10 distance. So, we like to have a certain balance in the in terms of distance traveled, so we would like to say that each vehicle does not travel more than a capital T, which is the distance.

So, we could have either all 3 types of constraints, which would be number of customer constraint, capacity constraint and distance constraint or we could have a subset of these depending on our actual requirement. So, this would also all of these would require of the order of N square constraints, because now we realize that y i and y j are some other continuous variables at we have introduced. X i j is the same L is known. Similarly y N plus 2 and y N plus 1 are another set of continuous variables that have been defined.

Similarly, u i to these are very similar to our u i minus u j plus N X i j less than equal to N minus 1 that we have seen, so necessarily they take care of making sure that within a certain allocation, it does not exceed a certain number. So, again we have something like N square plus N square plus N square, so put together there will be about, 3 N square N square constraints, for each type. The only addition is a set of y i and y j, so that will be for in this example with 6 customers, there will be 8 y variables, there will be another 8 u variables and there is another 8 V variables.

The other catch of course, is that the y i's the u's and the V's are continuous, so when we add this the problem does, now it moves away from a 0 1 or a binary I P to a mixed I P, where some of these variables are continuous. The y i variables are continuous, the u i and the V i whereas, the X i j's are binary. But, we could still use this formulation very effectively to solve V R P's provided, we actually have a solver. If we do not have a solving facility then we use heuristics or thumb rule based algorithms. Now let us take a a small example and then show, how we solve this.

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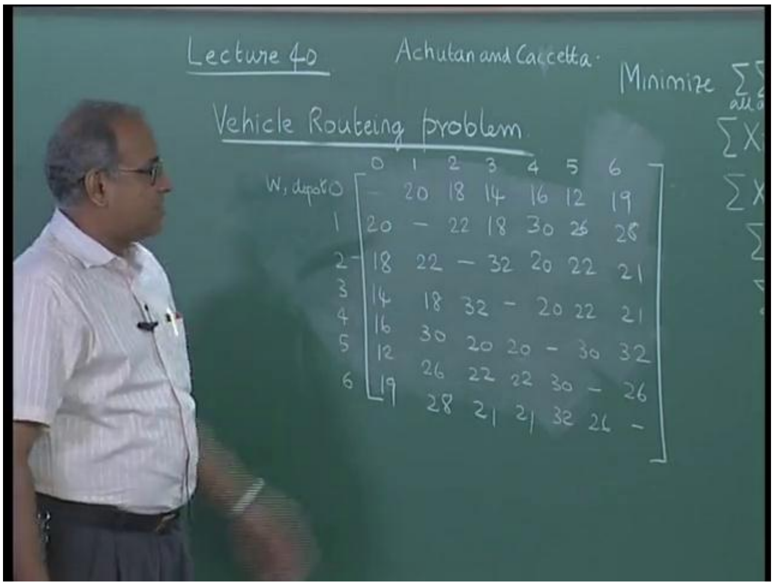

So, we have a problem with a warehouse and 6 cities, this is the distance matrix, 0 is the warehouse or depot 1 to 6 are the 6 cities or 6 customers, the matrix is right now assumed to be symmetric. So, distance between 0 to 1 is the is the same as the distance between 1 to 0, 0 represents as I said the depot or the warehouse, in a previous lecture, I had also explained that in many practical situations this matrix need not be symmetric, due to multiple reasons.

Both distance and time to travel can change depending on the time of the day depending on the route that is taken and so on. But, just to illustrate the V R P, we would include a a symmetric matrix, so I will just give you some solutions here. So, if we apply the T S P formulation.

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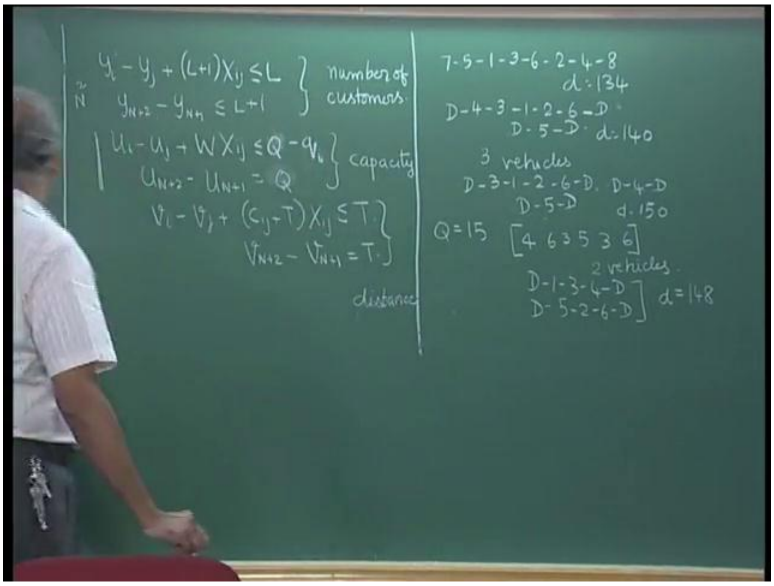

We would get the solution, 7 5 1 3 6 2 4 8 representing the solution, but distance equal to 134, which is the optimum solution to the T S P, which means we use this formulation and then we put V equal to 1, we do not use any of these, when it is a T S P, we simply use this formulation and put 1 V equal to 1, which means one vehicle is good. So, we get that solution with 134 on this, now if we ignore the load constraint all these, we ignore, we simply solve for 2 vehicles and then if you solve for 2 vehicles, we have a solution, which is D 4 3 1 2 6 D and D 5 D, this gives us a distance equal to 140.

Now, when I use D 5 D, what also happens is we would have something like this, we would, if the solution to this, will give something like 7 4 3 1 2 6 7 and 7 4 3 1 2 6 8 and 7 5 8. So, it will be like in the network that, we had here, 1 path will start from the source just go to 5 and then go to the destination, so if we look at it from this formulation point of view the solution will be 7 4 4 3 3 1 1 2 2 6 6 8 7 5 5 8.

So, this will be the solution with 140, it is also obvious that, I have to increase the number of vehicles, force, it to certain number of vehicles, we realize that the distance actually increases, which is expected. If we have 3 vehicles, we would have D 3 1 2 6 D D 4 D and D 5 D with 150, so when I use a capital D, I am using this D as a depot, so as I said, if we use that formulation on this, we would have 7 to 3, 3 to 1, 1 to 2, 2 to 6, 6 to 8, 7 to 4, 4 to 8 and 7 to 5, 5 to 8.

Now, the total distance will be 150 in this you can see that D increases, now if we add the constraints vehicle capacity Q equal to 15 and the 6 individual capacities are 4 6 3 5 3 6 and if we solve this formulation, for 2 vehicles. Now, we realize that the total is 4 plus 6 10 13 18 21 27, so we need minimum of 2 vehicles, so if we solve for 2 vehicles, we will get the solution D 1 3 4 D and D 5 2 6 D with total distance total distance D equal to 148.

So, similarly, we can solve this, for various combinations, we could this, we have right now looked at we have looked at this right now. For example, we could go back and then we could add some other constraints also. We could now here, we have a situation, where it visits 3 customers, now we want to restrict it to visiting 2 customers, then we could bring this and put l is equal to 2 and add this, now we will realize with 2 visits, it may not be possible with 2 vehicles.

Then, we have to increase the number of vehicles to 3 and we can solve it, for 3 vehicles, we can look at this as well as this, we will get a different solution, the number of constraints will get added and so on. So, depending on our requirements, we can actually do this, but then as I said this is how we solve the vehicle routeing problem optimally using this formulation. One of the important requirements, when we look at distribution is that in addition to all these.

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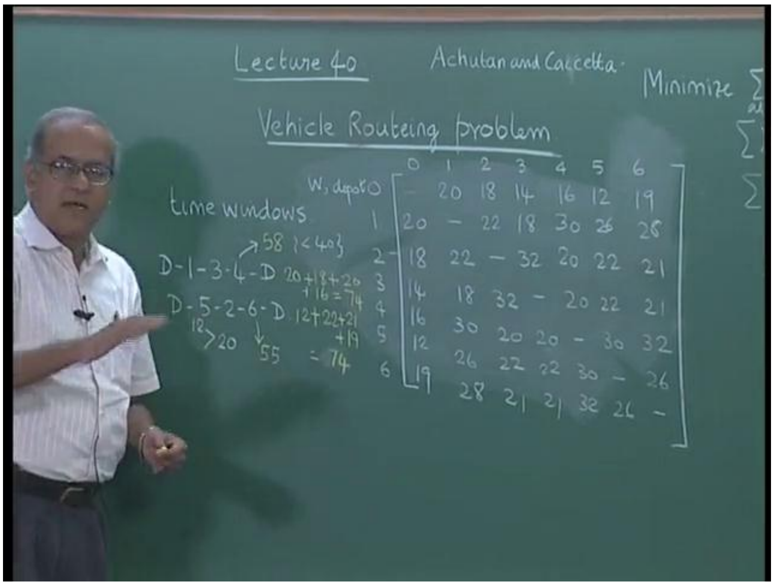

we have situation, which is called time windows, for example if we look at the solution

D 1 3 4 D and D 5 2 6 D, now let us assume that the solution D 1 3 4 D and D 5 2 6 D, now the distance traveled will be D to 1 is 20 use a different color. So, D to 1 is 20 plus 1 to 3 is 18 plus 3 to 4 is 20 plus 4 to D is 16 and here D to 5 is 12, 5 to 2 is 22, plus 2 to 6 is 21, plus 6 to D is 19, so this takes distance equal to 38, 58 plus 16 is 74. This has 23, 24, 44 plus 19 is 63, so D to 1 is 21, to 3 is 18, 3 to 4 is 20 plus 16, so 38, 58 plus 16, 74.

So, we get 137 here. So D 5 2 6 D, D to 5 is 12 plus 5 to 2 is 26, 5 to 2 is 22 plus 2 to 6 is 21 plus 6 to D 40, 40 plus 22, 62 plus 12, 74. So we get the total of 148. Now, for a moment instead of assuming these as distances, let us assume, these also as time taken and we have trying to solve the problem, such that the total time taken is minimized. Now, we have if both these vehicle start at time equal to 0 and both these vehicles will come back to the depot at time equal to 74, because it, so happens that both of them are equal.

Now, if for, now it will reach the customer number 4 at time equal to 38 plus 20, 58. Here it will reach customer number 6 at 34, 35 plus 35, now if we have a requirement that, we need to reach customer 4, before 40 say, then it becomes a problem with time windows. If we have a situation, where we should visit customer 5 after 20, right now it will visit customer 5 at time equal to 12, of course for this instance, we could simply go back and switch the sequence, we can do D 4 3 1 etcetera.

But, the point is if we have a larger pool of customers and we have larger number of vehicles, we and more constraints, we may not be able to do that. Then we get into problem with time windows, where there is an explicit restriction of visiting a customer, either before a certain time or after a certain time or in between a certain time. Now, time windows problems are not easy to solve, using this kind of formulation, because this kind of formulation is more a path type formulation, which talks about, which node to visit after another note.

Now, there in time windows, we need formulations, where we have the time of arrival as an explicit variable in the formulation, so other formulations exists to solve time windows related V R P’s. This is not, this is widely used, when we actually do not have time, windows and we solve the problem more as the problem to find out, which way, which customer goes to which vehicle and in which order and the subject to all of these. This does not explicitly consider time of arrival at a customer as a variable and therefore, would not solve the time windows based problems.

Several formulations exist, in fact, V R P has multiple formulation, depending on, how we wish to solve them, we could use different types of formula, there are other types of formulations exists, where we could use them for certain bounds and we could also use them to use some other relaxation based procedure. But, right now for the purpose of discussion with respect to distribution in a supply chain, we would look at this formulation, which helps us also model constraints on capacity, constraints on distance, as well as constraints on the number of customers.

Now, as I said all this is possible, if we have solvers, which can solve this problem for us, if we have a situation, where we have about 50 or 60 customers, then we are talking of, here we have 50 square, here we have here, we have if we have 50 customers. We are talking about the order of 50 square, which is 2500 variables and then we also have another 50 plus 50 and so on. The problem also gets unwieldy and becomes difficult to solve with large number of customers.

So, we then require heuristics or thumb rule based methods to solve and there are the most popular heuristic is called the savings based approach. Once again I have addressed heuristics, the Clarke and Wright savings based approach, as well as the improvement over the Clarke and Wright, which is called the Holmes and Parker method, I have addressed them in the advanced operations research course. But, one again for the purpose of completion of this, I will do only the Clarke and Wright solution to this problem and we will leave the Holmes and Parker for the viewer to view it in other lectures. Now the Clarke and Wright algorithm is based on a very simple idea

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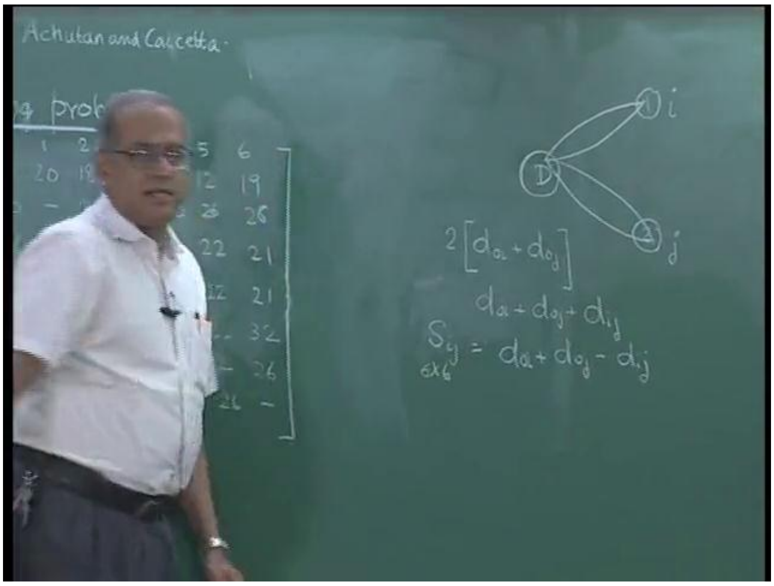

that I have a warehouse W or a depot as I might call it. We call it a depot and there are 2 customers 1 and 2 or in general i and j. Now if a vehicle goes from the depot, it goes to the customer and then it meets the customer requirement and it comes back. It once again goes to this customer and comes back, the distance traveled will be 2 times d naught i plus d naught j, d 0 is the origin or the depot i is the customer i, so 2 times d 0 i plus 2 times d 0 j.

But, if there is enough capacity and we move from the depot to customer i and then to customer j and then we come back to the depot, the distance traveled would be d naught i plus d naught j plus d i j. Now, if distances satisfy triangle inequality, we know that this is smaller than this, therefore, there is a saving, if we move instead of going to customer i and back and then customer j and back, if we combine, there is a saving, which is called S i j, which is the difference between this and this is bigger this is smaller.

So, this on subtraction would give us d naught i plus d naught j minus d i j, so given a d i j matrix, such as this given a d i j matrix, such as this, it is possible for us to find out an s i j. Now, please note that this matrix is a 7 by 7 matrix with 0 representing the depot and 1 to 6 representing the 6 customers the savings matrix will be a 6 by 6 matrix and that will be only for customer pairs i and j, because the 0 is hidden inside the saving. So, now, what we do is we can compute the savings matrix, for the given distance matrix, let me write this savings.

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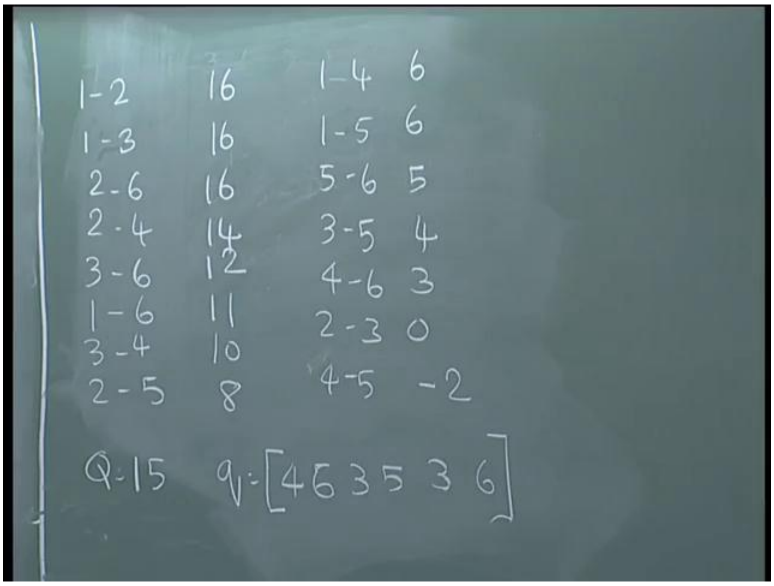

So, the savings would be 1 to 2, the saving is 16, 1 to 3, the saving 16, 2 to 6, the saving is 16, 2 to 4 it is 14, three to 6 is 12, 1 to 6 is 11, 3 to 4 is 11, 2 to 5 is 8 1 to 4 is 6, 1 to 5 is 6, 5 to 6 is 5, 3 to 5 is 4, 4 to 6 is 3, 2 to 3 is 0 and 4 to 5 is minus 2. So, this is our savings matrix, then let us assume Q equal to 15 and small q is equal to 4 6 3 5 3 6 and vehicle capacity is 15. Now, when we compute the savings matrix, now I have not shown the savings matrix as a square matrix, I have shown it pair wise, so there are 6 customers, so these there are 6 C 2, which is 15 pairs, it is also symmetric.

So, saving between 1 to 2 is the same as 2 to 1, so I have just shown the 15 values here. We just observe 2 things, that when we do this, particularly here, we now realize that 2 to 3, there is a saving of 0, which means the depot 2 and 3 lie on a straight line. Now, here the saving is minus 2 would indicate the numbers actually do not satisfy triangle inequality and there is a little violation in these numbers, these numbers are not realistic and practical from a triangle inequality point of view. But, it actually does not matter, when it comes to solving this particular illustration. Now, the other thing, I have done is I have already written these pairs or the savings in this sorted order in the decreasing or non increasing order, they have been written. So, the Clarke and Wright algorithm will begin by looking at the highest saving and then it would save 1 and 2.

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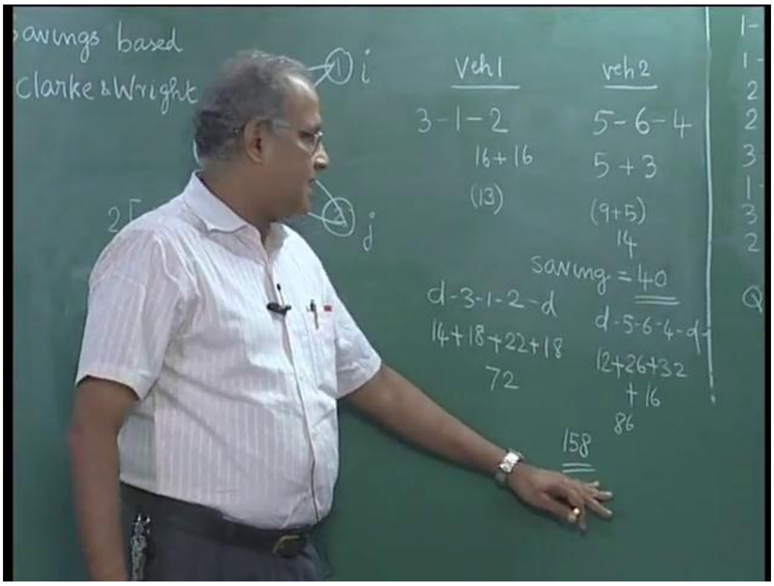

So, it starts with vehicle 1 puts 1 and 2 together to have a saving of 16 and if we put 1 and 2 together, we use 10 units of the demand, the vehicle has a capacity of 15, now the requirement for 1 and 2 is 10, so we use of 10 out of the 15. We now, take the next saving, which is 1 and 3 now, one is already here one is at the end of it, so 1 and 3 can give us an additional saving of 16, now we are adding this 3 inside, so 4 plus 6 10 plus 3 13 is so we put this 13.

We have another saving of 16 here and then we have to add 3 here, the reason is the saving comes out of 1 and 3, therefore you we should not put here, if we put here, then it becomes 2 and 3, which would be a 0. So, you put 1 and 3 here, so we have now have a saving of 32 and we have used 13 out of the 15 capacity, then we look at 2 and 6, so we can try and add 6 here, but we cannot do it, because the capacity will be exceeded.

Similarly we cannot do 2 4, we cannot do 3 6, we cannot do 1 6, we cannot do 1 3, we cannot do 2 5, we cannot do 1 4, we cannot do 1 5 and the next thing, we can do is 5 and 6, but we have to put them in a separate vehicle. So, we start with vehicle V 2 or vehicle 2, we put 5 and 6 together, to have a saving of 5 and we used up the capacity of 9, 5 and 6 is three plus 6, which is 9. Then we look at 3 and 5, we cannot do that, because both are assigned look at 4 and 6, which has 3, so 6 to 4 will come here, so saving is plus 3 and 4 has another 5, so 9 plus 5, 14, 13, so total saving is 40.

And we can also show, how this saving comes, we can calculate this d 3 1 2 d and d 5 6 4 d, now d 3 1 2 d will give me, d to 3 is 14, 3 to 1 is 18, 1 to 2 is 22 plus 2 to d is another 18, so 14 plus 18 32 54 plus 18 is 72 and the other 1 d to 5 is 12, 5 to 6 is 26, 6 to 4 is 32 and 4 to d is 16. So, 12 plus 26, 38, 40, 70, 86, so this gives us 158, is the total distance that, we travel, the saving is 40 and if we realize that, if we are actually gone from the depot to 1 and back depot to 2 and back 3 and back and so on, the distance we would have traveled is 20 38 52 68 60 99, 99 into 2 is 198. So 198 minus saving 40 gives us 158.

Now if we do this through the Clarke and Wright this is called the savings based algorithm or it is called Clarke and Wright algorithm. For this particular example Clarke and Wright gives us a solution with distance of 158 and a saving if 40. So, one of the limitations of Clarke and Wright is that, once we start this 1 and 2 and 1 and 3 by fixing these 2 and by creating the first vehicle, we have got a saving of 32 here.

But, this would prevent us from looking at all of these and the next thing, we can look at is somewhere here, now the question is if we follow a different order of taking can, we make it better. Now, we can, so the improvement to Clarke and Wright is called Holmes and Parker method and as I have mentioned Holmes and parker method is computationally, more intensive than Clarke and Wright method. It is an extension of Clarke and Wright method with more computations, it is like a branching algorithm, it will which will eventually terminate by evaluating all possible paths.

But, we can also terminate it sooner or quicker depending on our requirements. I have explained the Holmes and Parker solution, in detail in the advance operations research course under the topic vehicle routeing, where for the same illustration using the Holmes and parker method, we obtained a saving of 50 instead of saving of 40. So, a saving of 50 would actually give us a solution with distance of 198. Now, this is 99, so 99 into 2 is 198 minus saving of 50 would give us a solution with distance equal to 148, which actually happens to be the optimum that, we have shown here 74 plus 74, which is 148 Holmes and parker is capable of giving us this solution.

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So, let me go back here and check, we our savings are 1 to 3 and 3 to 4 and then 5 to 2 and 2 to 6, so 2 to 6 and 2 to 5. So 16 plus 16, 32 plus 8, 40 plus 10, 50 is possible. Holmes and Parker method is capable of getting this solution. Clarke and Wright is unable to because Clarke and Wright is like a greedy heuristic, it simply sorts it in this order and takes them only in this order and does not try or does not make does not take it to consideration other ways of choosing them.

So, better methods than Clarke and Wright, such as Holmes and Parker exist, but these methods require more computation, than the Clarke and Wright method. So with this we come to the end of our discussion on vehicle routeing problems, for this course. We also end our discussion on transportation decisions or transportation modeling. A very quick recap, we started with the basic transportation problem, where we transport a unit a single commodity between a set of source nodes and a set of destination nodes.

And then we extended the basic transportation problem to multistage transportation and we also extended the transportation to do, what is called fixed charge transportation. Then we looked at from a given point to another destination, we looked at a model, where we could have different types of trucks. And then we try to find out the number of trucks that are needed of each time to do this transportation, so that gave us a single constraint knapsack problem and we saw, ways to solve that.

Then we looked at point to point transportation and then modeled it more like a bin packing problem and then we looked at milk runs and first looked at the traveling salesman problem formulation. And then we extended it to the vehicle routeing problem formulation. So, with this we wind up our discussion on distribution models and a supply chain and then we look at some aspects of information, in the next lecture.