Operations and Supply Chain Management

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Module - 01

Lecture - 04

Forecasting – Winter's Model, Casual Models, Goodness of forecast, Aggregate Planning, Tabular Method

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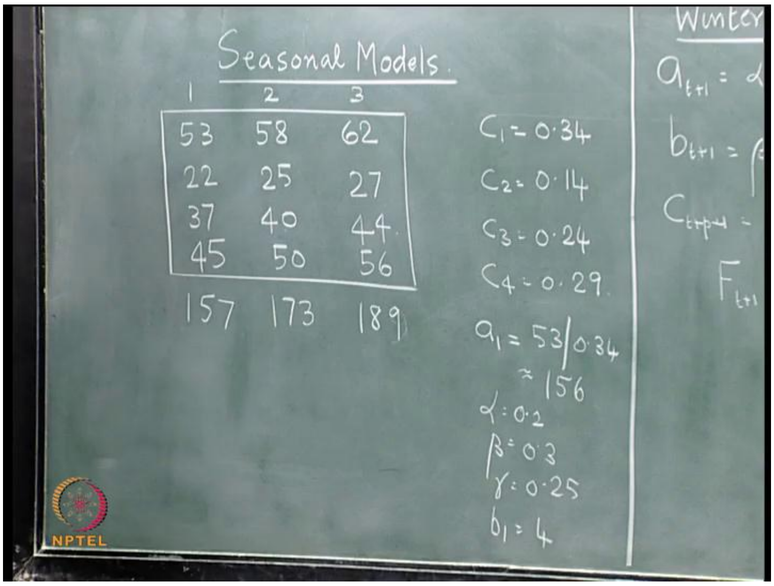

In today's lecture we continue our discussion on seasonal models; in the previous lecture we demonstrated a basic model considering seasonality. So, we looked at this data and this data would represent the demand over 3 years and each year is divided into 4 quarters. In the previous model, we computed a seasonality index, for example seasonality index for the first quarter data of year one would be 53 divided by 157.

The seasonality index for this number which represents second quarter of year 1 would be 22 by 157 and so on and we computed a seasonality matrix and then we averaged the seasonality indices for every quarter. We also try to forecast the next year's total demand by using the regression or the method of least square fit. Then, we said that we could multiply the forecasted value with each of these indices to get the forecast for the 4 seasons for the next year.

Now, what we will see is can we use exponential smoothing in some form to try and get a model which can in principle forecast better in the sense that more recent data will have more weightage. For example, when we used the method of least square to fit a line here we gave equal weightage to all these points. Now, the question is can we use exponential smoothing and try and build another model to do this. Now, that model is called the Winter's model and the Winter's model has the following equations.

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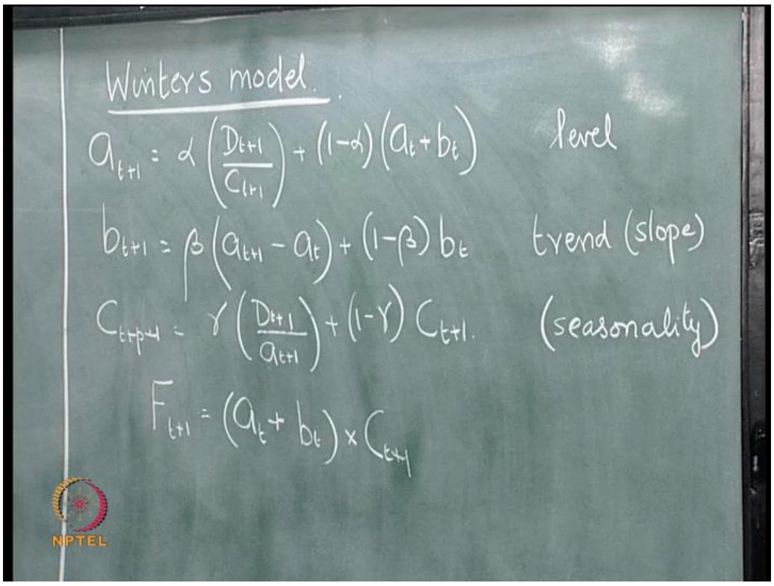

So, we use these equations for the Winters model at present, these equations may look a little complicated, but it is easy to explain these 3 equations. Now, a represents the level or which is the base data b represents the trend on the slope and c represents the seasonality. Now, there are three things in a seasonal model, a level, a trend and seasonality, for example to explain with this data the level values are like 157, 173, 189. It actually exhibits both level and trend one could think of 157 as a level and then there is an increasing trend. So, there is a slope which takes it to 173 and then to 189 and so on, there is seasonality, for example the seasonality index associated with this is 53 divided by 157.

So, now if we see that the level for a period t plus 1 has two parts exactly as in the exponential smoothing equation alpha is a smoothing constant. So, alpha times d t by c t d t plus 1 by c t plus 1 plus 1 minus alpha times a t plus b t, now if we are in period t and at that time the level value is 80, then the level value for the next period will be the earlier level plus slope. So, you have an a t plus b t which kind of represents the a t plus

1, similarly if there is a demand d t plus 1 in that period and there is an associated seasonality of t plus 1.

So, demand by seasonality would also give the level value at that period, therefore the actual level value of t plus 1 is a weighted average of this term and this term where is multiplied by alpha and this is multiplied by 1 minus alpha. So, if you are in period t and then we want to find out the level for period t plus 1, one can use this particular formula to get the level at period t plus 1. Now, similarly slope at that period also has two components now always the difference between two levels represents a slope. So, a t plus one minus a t is one measure of slope and the slope at period t which is b t is another measure of slope.

So, the slope at or for period t plus 1 is a weighted sum of 2 slopes, so beta times a t plus one minus a t which is again equivalent of the slope and 1 minus beta times b t where beta again is a exponential smoothing constant. Similarly, seasonality you can see the subscript is t plus p plus 1 which essentially comes because if I am somewhere here, then I am this is if this is called C 1, C 2, C 3, C 4, C 5, C 6, and C 7. So, if you are at two and the number of periods is four t plus p plus one is c seven is what we are computing which is essentially computed from this as well as this.

So, you will have d t plus 1 by a t plus 1 demand by the total the level which represents the total demand this is a measure of a seasonality index. This is also the measure of seasonality index from the previous one, so once again it is a weighted sum of two seasonality measures, where gamma is the exponential smoothing constant. Now, after we compute all these, we try and get the actual forecasted value which is given by this equation. So, what we will do now is we will try and show some computations and explain how we do the calculations for the Winters model.

So, let us take the same example and do that, so in order to do this we also need to do some initialization so we will assume C 1 to be equal to 0.34 which actually is 53 divided by 157, C 2 is 0.14, C 3 is 0.24 and C 4 is 0.29. So, let us initialize these three values, now we also a need to initialize a 1 where a 1 is the level at this period. So, a 1 is initialized to 53 divided by 0.34 which is 156, there is small rounding of error which can which is acceptable. So, 0.34 was obtained on the division of 53 divided by 157, so it is rounded of there, so the rounded value gives as a 156 as the level value.

So, we now know the level value here which is a 1, now we have to find out the level value for a 2 at the corresponding slope b 2 and so on. So, we also use alpha equal to 0.2, beta equal to 0.3 and gamma equal to 0.25 as the smoothing constants and we also need to use a value of b 1 which is taken as 4 which is roughly the slope here because 157, 173. The difference is 16 over 4 periods gives us a slope of 4 once again 173, 189 gives as a slope. So, we now have initialized all the value, so that we can do these computations, so let me show those computations again.

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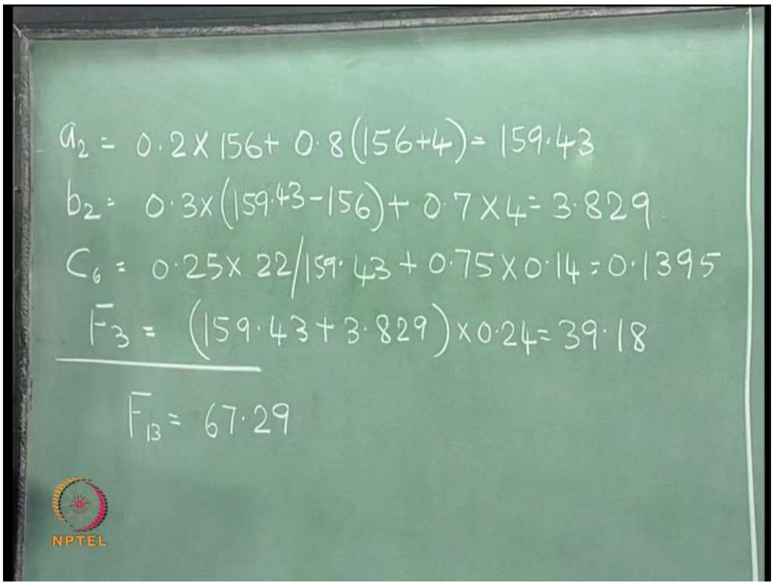

So, a 2 which is level for period 2 is 0.2 into 156 plus 0.8 into 156 plus 4 which is 159.43, now how do we get those numbers a t plus 1 level for period 2 is alpha times d t plus 1 by c t plus 1 alpha is 0.2 here. Now, d t plus 1 by c t plus 1 is 156 that we have plus 0.81 minus alpha into a t plus b t which is a 1 plus b 1 160, so the level moves up from 156 to 159.43 for the next period a 2. Now, b 2 is obtained from 0.3 into 159.43 minus 156 plus 0.7 into 4 which is 3.829, now this comes from this equation beta times a 2 minus a 1 159.43 which we just now calculated minus a 1 which is 156, 1 minus beta is 0.7 b t is b 1 which is the earlier slope 4.

So, you get 3.829, then we compute C 6 is equal to now essentially we are trying to look at seasonality for somewhere here using these seasonal values. So, C 6 will be 0.25 into 22 divided by 159.43 plus 0.75 into 0.14. Now, these things come from gamma into d t plus 1 by a t plus 1 gamma has 0.2 d t plus 1 is 22 a t plus 1 we just calculated as 159.43.

So, this is a measure of seasonality of this quarter which is represented by C 2, C 6, C 10, so 22 divided by 159.43, this is a measure of seasonality plus 0.75 1 minus gamma into 0.14 which is the initialized value for C 2.

So, when we do this C 2, C 6 becomes 0.1395, now from this we can initialize f 3 forecast period 3 is 159.43 plus 3.829 into 0.24 which is 39.18. Now, let us go back to F 3 this is our F 3, this is the actual d 3 is known, so the forecast for this is 139.18 that comes from the level at the end of 2 is 159.43. So, level plus slope would take it to 159.43 plus 3.829 which is here and that is multiplied by the seasonality which is 39.18. Now, we have shown one full round of calculations, now once this is known we can continue the calculations by now starting to compute a 3. So, a 3 will now become alpha times d 3 by C 3 plus 1 minus alpha times a 2 plus b 2.

So, now we know all the values d 3 can be got from here which is 37, C 3 is 0.24, alpha is known, 1 minus alpha, therefore is known a 2 and b 2 are calculated are available there. So, we can calculate this a 3, similarly we can calculate b 3 as beta times a 3 minus a 2 a 2 is known a 3 has been just calculated 1 minus beta times b 2, b 2 is known from the previous one. So, we can calculate this one, similarly we can calculate C 7 because gamma is known d 7 by a, this is 7. So, d 3 by a 3 is known plus 1 minus gamma C 3 is also known, so we can calculate c seven from which we can calculate F 4.

So, we can continue with the Winters model and finally, we have a value f 13 forecast for period 13 a 67.29 with the updated values seasonality values are also calculated accordingly and kept updated. So, finally the forecast for period 13 is 67.29, so if we want to forecast the value here from using Winters model we would get 67.29 here and then we use this model to compute this and so on. Now, what are the advantages of the Winters model as we have use exponential smoothing. The very idea that we consider all the data points we give more importance to or higher weightage to the more recent data points and so on.

Now, all the three important things like level slope and seasonality index are updated using a simple exponential model where in some sense two things are captured the value at period t and then the value at period t plus 1. So, they are captured in this each has 2 components connected by the weights and the alpha beta and gamma are the smoothing constants. Then, we have a relationship that the forecast is level plus trend multiplied by seasonality. Now, this also answers a very interesting question that we posed in the previous lecture, now if we have a data like this, now can we simply take 53, 58, 62 and forecast the next value can we take 22, 25, 27 and forecast.

The next value the answer to this is no from a forecasting point of view the reason being the assumption that even though there is an increasing trend here and the increase in trend is visible when we actually take the demand total demand for the year. It becomes visible when we do that, if we see here we see an increasing trend, but we do not actually see an increase in this direction because the values are very different. They are multiplied by the corresponding seasonality index by using the Winters model what we have trying to say here is that in some sense the 16 increase here is actually spread as 4 in each season.

That is the reason we also got something like 67.29 which is which is sixty 2 plus some four plus something that plus came because of the increased weights to more recent data. So, to that extent Winters model is able to capture is able to put into each period the fact that there is an increase and that increase is captured by deseasonalizing it. Second advantage perhaps is if we were given the demand here let us say if the actual demand here turned out to be 67 and then we want to make a forecast. Here, we could directly use this formulae and get the forecast here. Where as if we are delink them and said I am going to do this and then some something here, without doing any of these we are not capturing the effect of these data points on a subsequent forecast.

So Winters method also helps us and trying to do the forecast with data which is not fully complete with respect to a year. You could have only 2 or 3 pieces of data and then you could do the four and then you could proceed. So, with this we have kind of seen forecasting models essentially time series forecasting models for level data which i called as a constant model then for level plus trend. Then, level plus trend plus seasonality, so we have essentially covered three aspects of it which is level trend and seasonality. Now, there are other things which also need to be covered in forecasting models it is also a customary to do what are call cyclic models.

Usually cyclic models have a larger time period than the models that we have seen here and cyclic models are usually represented using sinusoidal curves. So, we will have models of the form y is equal to a cos 2 pi t plus b sin 2 pi t where t is the period and a and b are the constants that we try and find out. Once again, we can use methods of least square to try and get the values of a and b which are for the cyclic models.

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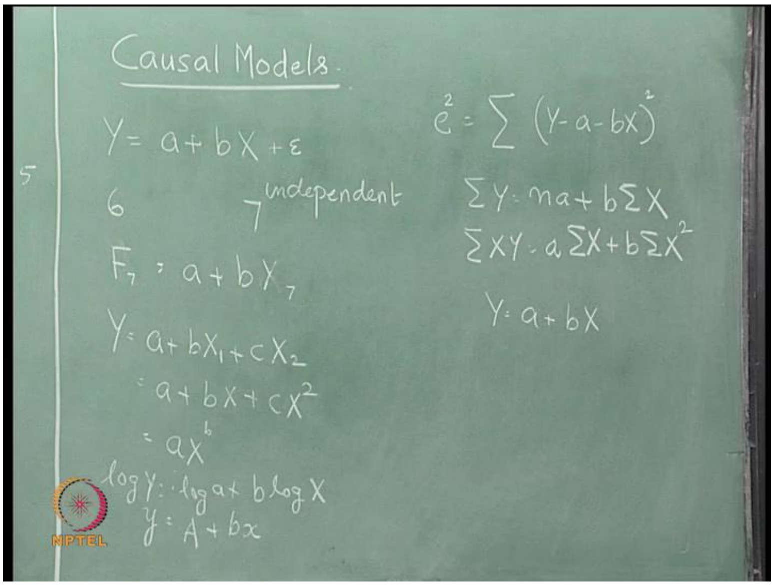

The next thing that we will see right now is what are called causal models. In the causal models, we try and fit something of the form y is equal to a plus b x where y is the variable that we are actually trying to forecast. Now, it depends on another variable x which is the independent variable where y is the dependent variable that we are trying to forecast. The normal example given is for the grain output would depend on the amount of rainfall, so x is the independent variable which is your rainfall for which we have the data y is the dependent variable and then we want to forecast y for a given value of x.

So, if we have for example, six periods of data for the food grain production and we will require 7 periods of data for the rain. So, for example, if we want to fit y equal to a pus b x if we want to do y 7 which is the forecast for period 7 for the variable y or we can call it F 7 which is the forecast for period 7. Then, that will be a plus 7, now we need a plus b into x 7 where x 7 is the data for the independent variable for period 7 unlike in time series in time series the forecast for period 7. The data we would use only up to period 6 in a causal model we also require data for the independent variable for period seven methodology wise the fitting a plus b x and a plus b t are one and the same except that x will replace t in the causal model.

So, methodology wise it is the same you can once again take y minus a minus b x the whole square which represents the error sum of squares, we call it e square is equal to sigma y minus a minus b x the whole squared. Now, one should also note that actually while this is the model, we assume that the given data has a small noise or randomness which we call as epsilon. Then, that creates a small error therefore, y minus a minus b x is actually not 0, but some value otherwise if we start looking at y equal to a plus b x here. When I write y minus a minus b x then one would think that it is 0, it is actually not there is a small error component whose square we actually try to minimize.

Now, we can partially differentiate with respect to a and b which are the unknowns to try and get the two equations which is sigma y is equal to n a plus b sigma x and sigma x y is equal to a sigma a sigma x plus b sigma x square. Now, with these two equations we can try and solve for a and b, now the solution is very similar to the time series. Therefore, I am not taking a numerical example to explain the same solution once again you will have columns for x x square x y x square and x y. Then, we can solve for this the how to solve this has been covered in an earlier lecture in this course the only difference as i said is having obtained a and b.

Now, if we have data for six periods for x and y, we could use data for six periods of x and y and finally, we will get something like y equal to a plus b x. Now, for the seventh period if we know the value of x seven if we know the value of the independent variable, then we will be able to substitute because using these 2 we can get a and b. So, we can substitute to get the value of y which will be the forecast for the seventh period or a subsequent period. So, this is how we address causal models now when it is always possible to build causal models of different types and need not be a plus b x. For example, we can build causal models of the form a plus b x 1 plus c x 2 or we can build a plus b x plus c x square.

We can build y equal to a x power b and so on, now when we build y equal to a plus b x one plus c x 2 it means there are two independent variables which influence the behavior of the dependent variable y. So, the methodology will now shift to minimizing sigma y minus a minus b x 1 minus c x 2 the whole square. So, there are three unknowns a b and c so partially differentiate with the respect to all the three of them to get three equations.

Here, instead of two and then we can solve those three simultaneous equations to get a b and c and for the period to which we want to forecast we need the values of the x 1 x 2, they can substitute and try and get the actual value. If we fit a model like this there is only one independent variable, but then there is a quadratic term here. So, we will try and minimize y minus a minus b x minus c x square once again there are three things that we have to calculate a b and c. We will partially differentiate to get three equations, so we will get a third equation which will says sigma x square y is equal to a sigma x square plus b sigma x cubed.

So, there will be another term also, there will be a third term also, you can partially differentiate it to get three equations involving three terms a b and c. So, we can fit a quadratic and do it if we fit a causal model of the type y equal to a x power b, then what we need to do is to take logarithm on both sides. So, get log y is equal to log a plus b log x so this will become log y is equal to log a plus b log x. Now, log y will be replaced by a variable y this will be another constant a plus log x become another small variable x now we fit small y is equal to capital a plus b small x. Now, this model is very similar to this model.

So, we need to now instead of log a we will actually end up getting this a and then from this a we need to go back to get this small a because just its representative of log a. So, at some point these things become a little bit of substitution in mathematics, but the basic methodology is to try and fit something of the you know a plus b x or a plus b x plus c x square or a plus b x 1 plus c x 2. Then, we try and minimize the error sum of squares and get the line of best fit assuming linear.

If it is quadratic the quadratic curve of the best fit, so in almost all the causal models the underlying methodology is to try and convert it to linear or quadratic as the case may be and try and determine the line of best fit. Then, we proceed as I said the error sum of squares thing is very similar to what we have seen in the basic time series forecasting model. Now, we come to the last part of the forecasting which is to try and find out the goodness of a forecast.

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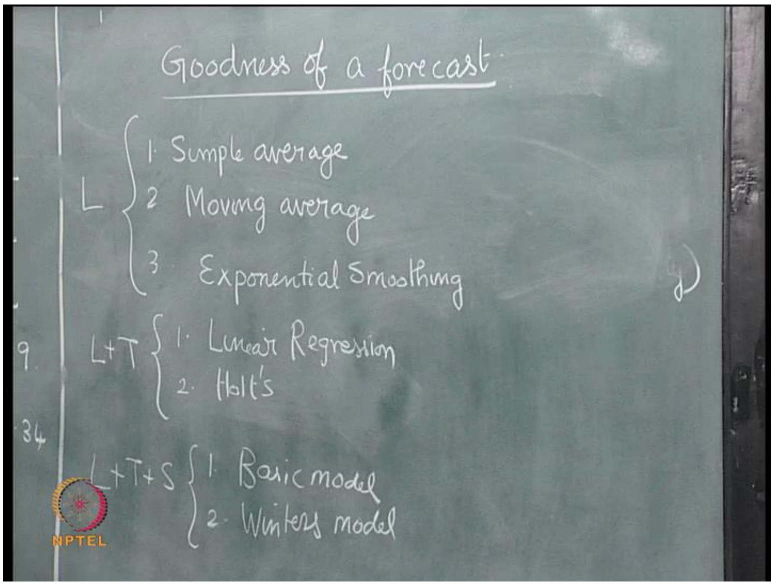

Now, how good is the forecast is the question that we have to ask, the reason being depending on our assumption we actually have more than one model that we can use to forecast like if our assumption is that the data represents a constant model. The data represents only level then we have seen several models we have seen simple average. We have seen moving averages, we have seen exponential smoothing all these if we have only level if we had l plus t that is level plus trend we have seen two model. One is linear regression line of best fit and then we saw Holt's model and if we have l plus t plus s which means seasonality is also included.

We saw a basic model and we saw Winters model so depending on our assumption that the data represents only level or l plus t or l plus t plus s we could use more than one model to try and get the forecast. So, how do we compare solutions of various models under the same assumption if we believe that it represents only level, then we could use any one of the three or even a weighted moving average and so on. So, we could test with 4 or 5 models and then finally, we have to choose one.

Similarly, if the assumption is l plus t, then we could use both these models and other models that are available in the literature and then chose one the forecast from one of these and so on. How do we do that, now that is called measuring the goodness of a forecast, now let me take a simple example to explain the goodness of the forecast, now let us explain it using a very simple example.

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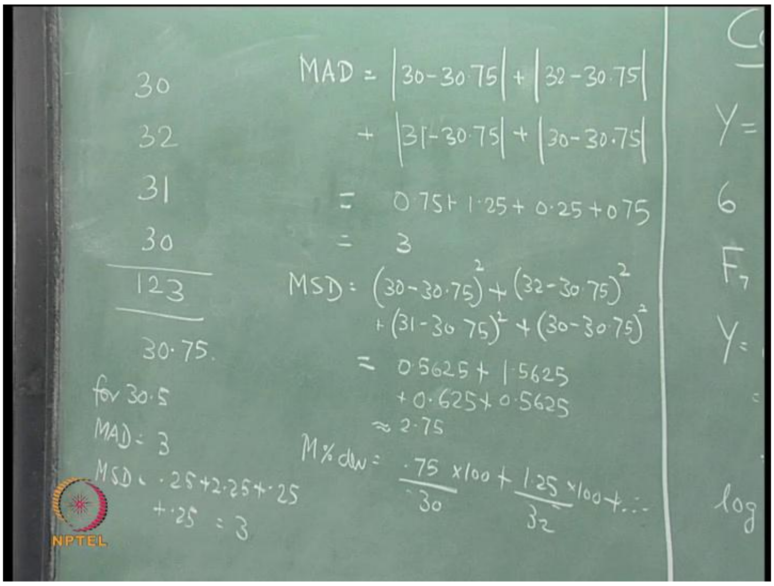

So, let us take some data which is 30, 32, 33 and 35, let say this is let us assume we take data which is 30, 32, 31 and 30. Now, we can assume that this represents only level because at the moment we do not see a visible trend here. So, let us say we take only level data and let us say we are going to use the total comes to 123, so simple average is 30.75 is the simple average. Now, there are several measures that we can define for the goodness of the forecast. So, we are going to do one is called mean absolute deviation mean squared deviation, mean percentage deviation and so on, so let me show the calculation for some of these.

Now, let us assume we are using simple average and therefore the forecast for all the four periods is 30.75 and forecast for a fifth period is also 30.75. So, mean absolute deviation in this case is let us call it m a d which is mean absolute deviation mean absolute deviation would be sum of this. So, 30 minus 30.75 plus 32 minus 30.75 plus 31 minus 30.75 plus 30 minus 30.75, now please note that we are finding out the absolute value and not the actual. So, this is 0.75 plus 1.25 plus 0.25 plus 0.75 which is 1.5 plus 1.5 is 3. For example, we did not do 30 minus 30.75 plus 32 minus 30.75 and so on.

If we had done that value will be 0 because it is a simple average, so we take the mean absolute deviation, now mean squared deviation is 30 minus 30.75 square plus 32 minus 30.75 square plus 31 minus square plus 30 minus square. If you are coming back to this this is not mean because this is the total absolute deviation which is 3. So, mean absolute deviation will be one by n number of point y, so 3 by 4 will give us a 0.75 as mean absolute deviation. Now, mean squared deviation will be this is 0.75 square which is 0.5625. This is 1.25 square, so 1.5625, this is again 31 minus 30.75 is 0.25, so 0.625 plus 0.5625.

So, this would roughly be 0.56 is 1.12, so 2.68, 2.2 say 0.74, so this is roughly totals to about 2.75 that is the mean squared deviation. So, a mean squared deviation will be this divided by 4, now mean percentage deviation can be now the demand is 30, the forecast is 30.75. So, the deviation is 0.75 fact we could even qualify this as mean absolute percentage deviation. So, 0.75 is the absolute thing here, so that divided by the forecast which is 0.75 divided by 30.75 into 100 plus 1.25 divided by 30.75 into 100 and so on. Now, the only difference here is these two will be related because the denominator is the forecast which is 30.75 which is common.

So, we can even do mean absolute percentage deviation with respect to demand and not with respect to forecast so that these values can change. Now, these denominators are become by 30 by 32 plus 2 more terms and so on, so one can define several terms like these which basically represent the goodness of a forecast. Now, this value mean absolute deviation has 3 that three is total absolute deviation as I said and mean absolute deviation is 3 by 4 mean square deviation is 2.75 by 4 and so on. Now. if instead of simple average if we had used a 2 period moving average which means the forecast would have become 30.5 instead of 30.75, then we can calculate the mean absolute deviation.

Now, for 30.5 forecast mean absolute deviation will become 0.5 plus 1.5, 2.5 plus 0.5 which happens to be 3 again. So, 0.5 plus 1.5 is 2, 2.5 plus 0.5 is 3. Now, mean squared deviation will become for 30.5, it will become 0.25 mean squared deviation will become 0.5 square is 0.25, 1.5 square is 2.25, 31. So, another 0.25 plus 30, another 0.25 say this will be about 3, so if is a 2.75 divided by 4, say this is 3 divided by 4. Therefore, this will show a smaller value of mean squared deviation than this forecast, this way if we use more than one model here we can choose one out of these goodness measures as the measure that we are going to use and say for the sake of discussion we have chosen mean squared deviation.

Now, we can evaluate the mean squared deviation for the forecast obtained by each of these and then find out that model which has given as the least value of mean squared deviation. The example is if we had used simple average the mean squared deviation is about 2.75 by 4, if we had used a 2 period moving average, then the forecast is 30.5 and the means square deviation is 3. Therefore, we could say that if we had used mean squared deviation as the criteria to measure the goodness of a forecast, then the simple average is a better forecast than the moving average.

There, one has to be a little guarded when we consistently choose mean squared deviation as the measure of goodness. If we look it at very carefully statistically simple average is always the measure that will minimize the error sum of squares or mean squared deviation. The very definition of simple average comes from there, therefore if we take this simple average, it will always be better than any other forecasting method if we are using mean squared deviation as the criteria. So, usually mean squared deviation is not taken as the measure for measuring the goodness of a forecast.

Normally, mean absolute deviation taken as measure for the goodness of a forecast simply because mean squared deviation is kind of little bit biased towards a arithmetic measurement, but another way of overcoming that is also this. Here, what we did was we took all the arithmetic mean and then we said 30.75, we could have done something else if we have only this data the arithmetic mean is 30, if we have this data, the arithmetic mean is 31, we have this data the arithmetic mean is 31.

If we have all four data arithmetic mean is 30.75, now we can find the error sum of squares as 30 minus 30 square, 32 minus 31 square, 31 minus 31 square, 30 minus 30.75 square. If we start doing that, then our basic idea that simple average will always be the best. So, there are several issues like this that we can look at, but the very basic idea is to look at one of the measures and use it consistently often the suggested measure is mean absolute deviation. Now, use that measure and then if you have forecast using more than one model evaluate it based on this measure and chose that one which gives the minimum value.

Then, we can get into another question for example; if we have a data which is like this if we have a data which is like 30, 34, 36 and 39. So, clearly this data shows trend, so we could use this or this for example, if we had 30, 32, 31, 30 at the moment, we can say that it does not show trend, but one could say that let me try and fit y equal to a plus b t b is going to be small. If it does not show any trend or b is going to be 0, if it does not show any trend can I do that the answer is not - in principle we have to check what are the assumptions and after the assumptions,

We try and get the model you do not want to fit a model and depending on the result of the model go back and correct our assumption oh well this may exhibit trend. So, that is not usually the practice the other question is if we have a set of data, now can I apply a model from here, can I apply a model from here and then chose the better one based on the goodness the answer is actually no being the reason being the model that we chose should come after the assumptions that we make regarding the behavior of the data. So, it is not a good idea to try and see whether moving average is better of whether linear regression is better after testing both the models and trying to find out the goodness that is usually not practiced.

What is actually good is to try and make the assumption that based on the assumption this is this exhibits level. Therefore, I am going to use one of these this exhibits level plus trend there for I am going to use one of these and so on. So, with this we actually come to the end of the forecasting part of this course or this lecture series. So, we kind of started the forecasting topic by saying that there is a need to forecast there is a need to forecast because manufacturing organizations needs to know how much they have to produce in the subsequent periods or in the subsequent months.

So, forecast is defined as the estimate of the future demand and then we also said that the forecast can depend on only time which lead us to time series models it can also depend on some other independent variables which lead us to causal model. Within the time series, we saw level plus trend level plus trend plus seasonality and we saw some causal models and we also saw how to measure the goodness of a forecast. The next question that comes is well after the forecasting, what do we do? Forecast has given us the estimate of the demand for a future period. Now, what is a next step that we have to do in order to use this data that has comes from forecast?

Now, that comes from what is called an aggregate production planning, we will now see the basic aggregate production planning in this lecture. Then, continue a detailed discussion in the next lecture. Now having seen forecasting model, let us now look at the aggregate planning or the production planning problem where we use the results of forecast.

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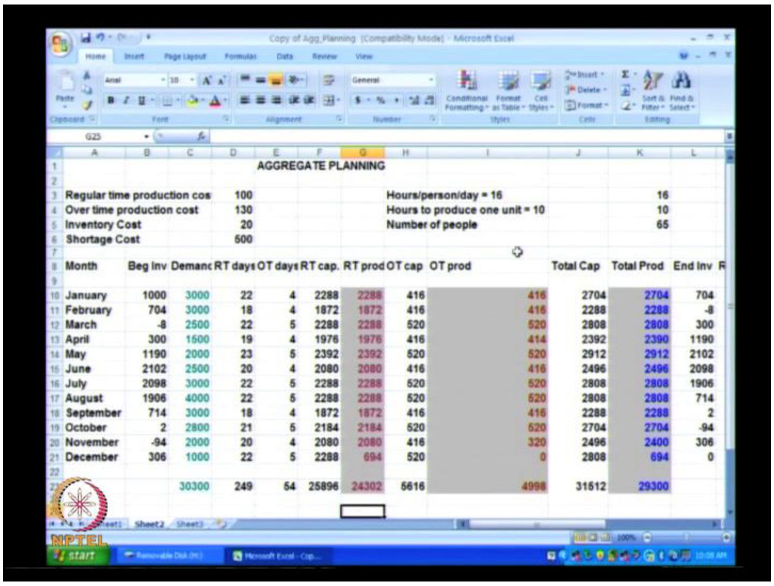

AGGREGATE PLANNING

Regular time production cos	100		Hours/person/day = 16	16
Over time production cost	130		Hours to produce one unit = 10	10
Inventory Cost	20		Number of people	65
Shortage Cost	500			

Month	Beg Inv	Demanc	RT days	OT days	RT cap.	RT prod	OT cap	OT prod	Total Cap	Total Prod	End Inv
January	1000	3000	22	4	2288	2288	416	416	2704	2704	704
February	704	3000	18	4	1872	1872	416	416	2288	2288	-8
March	-8	2500	22	5	2288	2288	520	520	2808	2808	300
April	300	1500	19	4	1976	1976	416	414	2392	2390	1190
May	1190	2000	23	5	2392	2392	520	520	2912	2912	2102
June	2102	2500	20	4	2080	2080	416	416	2496	2496	2098
July	2098	3000	22	5	2288	2288	520	520	2808	2808	1906
August	1906	4000	22	5	2288	2288	520	520	2808	2808	714
September	714	3000	18	4	1872	1872	416	416	2288	2288	2
October	2	2800	21	5	2184	2184	520	520	2704	2704	-94
November	-94	2000	20	4	2080	2080	416	320	2496	2400	306
December	306	1000	22	5	2288	694	520	0	2808	694	0
		30300	249	54	25896	24302	5616	4998	31512	29300	

Now, let me explain this problem using the spread sheet, now let us assume that there is a single product, now there is a only one product that the company makes and that we have made the forecast of that product which is shown here. So, in this sheet what is shown as demand is actually the forecast of the demand for the next 12 periods which represent the 12 months. We also right now assume by looking at this data that this forecast is for a product which exhibits certain seasonality.

So, there is a demand which is the forecast of the demand which is shown here, now let us assume that there is a beginning inventory of thousand units at the beginning of January that does available at the beginning for use. Now, we are also going to assume here that it actually takes about 10 hours to produce one unit and that there are 65 people working. Therefore, we also assume that the organization works for 16 hours a day and so on. So, all these assumptions finally, give us that it is possible to have 104 units which are produced in a day using what is called a regular time production.

Now, let us spend some time on this spread sheet, now this column shows the 12 months of the year that we have and right now. Let us look only at the beginning inventory of January which talks about 1000 units, now demand or forecast of demand is known here and these ads up to 30,300. Now, we also have what is a called r t day which is the number of days available in each month for regular time production and these are 22, 18, 22, 19 and so on so this data is also known.

We also have what is a called o t day which means the number of days available for overtime in each month which is given by 4, 4, 5, 4, 5 and so on. Now, if we have 22 days available in January which is shown here, now we can produce 2,288 units in January, a maximum of 2,288 units which comes from 22 multiplied by 104. This also is known and can be calculated once these numbers are known, once the r t days column is known r t capacity can be calculated. We can also calculate the o t capacity column because in 4 in a maximum of 4 o t days availed for overtime 100 and 4 into 4 which is 400 and 16 units can be produced using overtime. So, this column can also be calculated once the numbers in this column are known.

Now, we can also calculate the total capacity in each month which is the sum of the regular time capacity and the overtime capacity as shown here. Also the first term represents regular time capacity second term represents the overtime capacity. So, this is also known, now we are going to ask ourselves this question what is going to be the total production in each month so that we try and meet the demand or forecast of the demand. Now, as a user let us say that we start filling these values which are the total production in each month. So, if we fill this number 2704, now the total capacity there is available to the left is going to guide us in filling these values as total production, so total production cannot exceed the total capacity in each month.

So, if we fill a 2,704 as the total production for the month of January now this 2,704 has to be distributed to regular time production and overtime production. There is a regular time production cost of hundred there is a overtime production cost of 130, since it is cheaper to produce in regular time out of the 2,704 that we have decided 2,288 will be produce using regular time. The balance will be produced using overtime, so the moment the user types 2,704, here it will automatically give 2,288 to regular time which is the maximum r t capacity and the balance will go to overtime. So, the user actually fills the production quantities for each of these 12 months.

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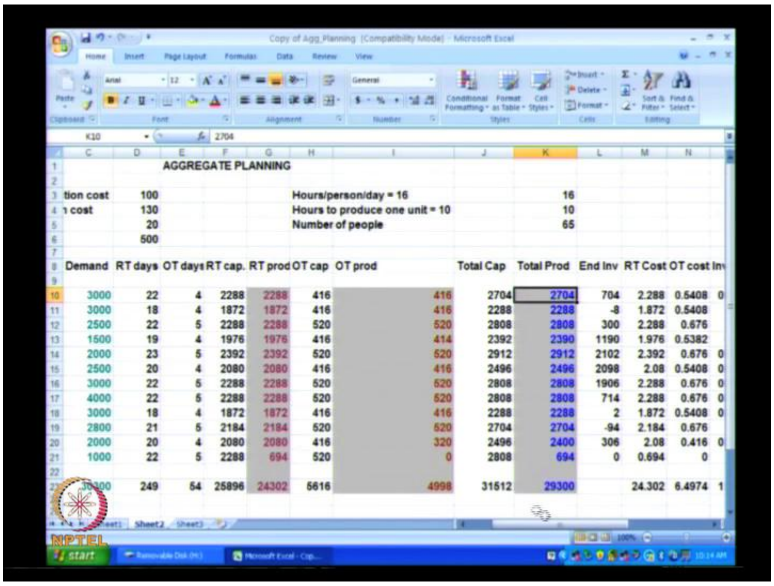

Now, the moment the 2,704 is typed by the user now there is a beginning inventory of thousand which is available here, there is a total production of 2,704. So, we get 3,704 less 3,000 which is the demand gives us the final inventory of 704, so the ending inventory for January is calculated this way.

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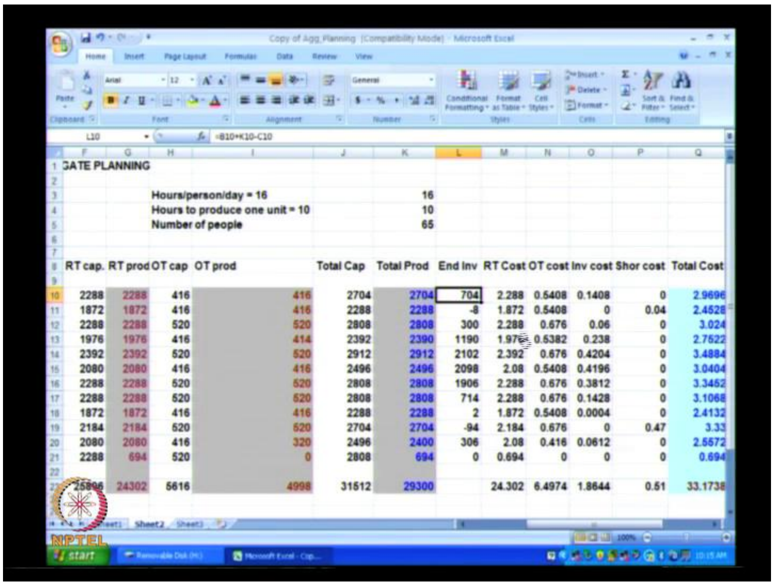

Now, what are the costs there is a regular time production cost there is a overtime production cost there is an inventory carrying cast there is a shortage cost. Now, we have already seen that the regular time production cost is 100, so have to produce 2,288 units the cost is going to be 2.288 lakhs, overtime cost is 130 to produce 416 units. It is 416 into 130, which is 0.5408 lakhs ending inventory is 704 units, there is a 20 rupees for the inventory cost. So, 704 into 20 would give us 0.1408, since ending inventory is positive there is no shortage, so shortage cost is 0 and the total cost is a sum of these four costs 3 costs in this case because shortage is 0.

It is 2.96 lakhs, now the ending inventory of January which is 704 will become the beginning inventory of February which is 704 and if we decide to produce 2,288 which is the actual capacity we have a negative ending inventory. We can calculate the r t cost the o t cost the inventory cost is 0 because there is negative ending inventory and a shortage cost is 0.04 because 8 units are short shortage cost is 500 into 8 is 4,000 which is 0.04 lakhs. So, total cost for February is 2.4528, so like this the user will type the values for the total production for the 12 months.

We can write this spread sheet in a manner that the total cost is 33.1378 lakhs, so if the user chooses to produce 2704 in January 2,288 in February and so on, the total cost is 33.17 lakhs. The question is what should be the production quantities in these 12 months such that the total cost is minimized. Now, this problem is called the aggregate planning problem where given capacities and given the forecast of demands and given the various costs in this particular spread sheet.

We have used regular time cost overtime cost, inventory cost and shortage cost, what should be the production capacities such that the total cost is minimized. Now, this problem is called the aggregate planning problem and we will see more about this problem in the next lecture.