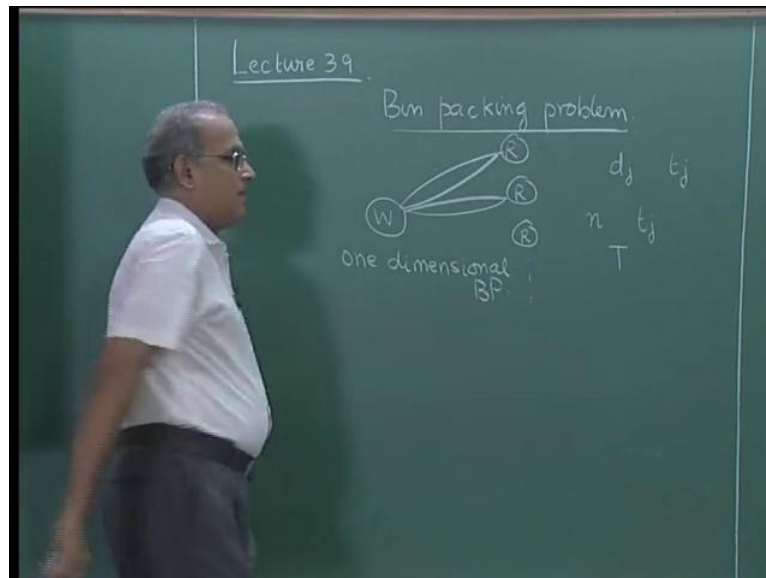


**Operations and Supply Chain Management**  
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**Lecture - 39**  
**Bin Packing and Travelling Salesman Problems**

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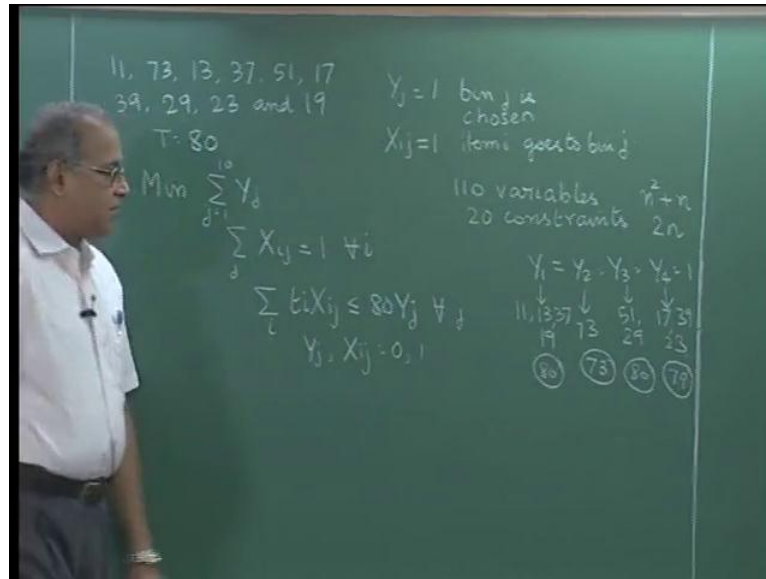


In this lecture, we continue the discussion on the bin packing problem. The application to transportation model comes from, assuming that we have a central warehouse and we have to transport to a large number retailers or customers and there is a point to point transportation, which means a truck will go from here to this and then it will come back to the starting point and then it will go to the next customer after picking up material from here and so, on.

So, the distance and time the truck takes from the warehouse, to customer  $j$  can be given as  $d_j$  or  $C_j$  or  $t_j$ , if it is the time taken, so this time would include the loading the unloading as well as the transportation time. Now given  $n$  customers, where customer  $j$  requires a time of  $t_j$  and if a vehicle is available for  $T$  time units, the problem is to find out, which of these customers are orders will be allotted to each of these vehicles. Such that, the number of vehicle is minimized and the sum of the  $t_j$ 's allotted to each vehicle will be less than or equal to  $T$ .

As mentioned in the previous lecture, this reduces to what is called, a one dimensional Bin packing problem, where the one dimension is time here, time required by each of the customers, to be compared with time available with each of the trucks. So, the problem is to find out the minimum number of trucks, now we explain it through a numerical illustration.

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So, we would, so we assume that, there are 10 requirements and the time, for each of these requirements are given in these 10 numbers and then the total time availability is 80. So, the problem is to find out the minimum number of trucks, such that we are able to meet all the demand, so now  $t_j$  is given here, now we have to find out, which of these items, now we can treat each one of these as items that goes into a bin, the bin's length is 80 items have lengths 11, 73, 13, 37 etcetera.

So, which of these items will go to which of these bins, such that we minimize the number of bin's, as well as ensure that the length of items that go into a bin does not exceed the bin length. So we have seen the formulation in the previous lecture. So we now assume that,  $Y_j$  equal to 1, if bin  $j$  is chosen and  $X_{ij}$  equal to 1, if item,  $i$  goes to bin  $j$ . Now, we know that with 10 bin's, we can definitely meet all the requirements, so we are going to assume that, there are smaller sub set of  $Y_1$  to  $Y_{10}$ , which we would like to minimize.

So, the objective is to minimize  $\sum Y_j$ , where  $Y_j$  equal to 1, if bin  $j$  is chosen. This depends on the notation, now we are using  $j$  for the bin and  $i$  for the item, therefore summation  $\sum_{j=1}^{10} Y_j$ . Now, each bin each item is to be associated to only one bin, so  $\sum_{j=1}^{10} X_{ij}$  summed over  $j$  is equal to 1, for every  $i$ , which means each item,  $i$  will go to exactly one bin.

And then for each bin  $t$   $\sum_i X_{ij}$  is less than or equal to 80  $Y_j$ . Now  $X_{ij}$  equal to 1, if item  $i$  goes to bin  $j$ . So for a particular bin, sum of the  $X_{ij}$ 's is the number of items that are attached to that bin.  $\sum_i X_{ij}$  is the length of sum of the lengths of the items attached to that bin. So, this is the sum of the lengths of the items attached to bin  $j$  and this should be less than equal to 80 and we can assign  $i$  to  $j$  only when it chosen, only when  $Y_j$  is 1.

Therefore, this constraint less than equal to 80  $Y_j$  or capital  $T Y_j$  will ensure that, the not only items will be assigned to bins that are chosen, it will also ensure that when items are assigned a sum of the length's does not exceed this 80. And then we have  $Y_j$  and  $X_{ij}$  as binary variables, now this is a binary integer programming formulation, this can be solved optimally using an IP solver.

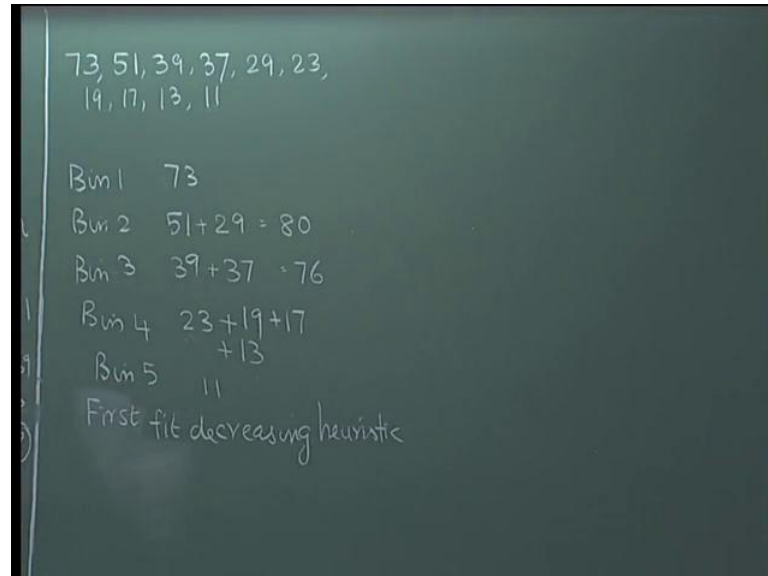
But, right now as I mentioned for 10 items, we will have 10 variables here and 100 variables here, so we will have 110 variables and then there will be 10 constraints here, there will be 10 constraints here, so there will be 20 constraints. Now, this constraint is for every item this constraint is for every bin, so there will be 110 variables and 20 constraints, so in general, if there are  $n$  items, we would have  $n^2$  plus  $n$  variables and  $2n$  constraints.

Now let us we can solve this optimally and when we solve this optimally, we would get a solution, which would be like this.  $Y_1$  equal to 1,  $Y_1$  equal to  $Y_2$  equal to  $Y_3$  equal to  $Y_4$  equal to 1, which means 4 bins are chosen. So, the item with 73 goes to this bin items with 11, 13, 37 and 19 go into this bin. The item with 51 and 29 go into this bin and the item 17, 39 and 23 go to this bin. So the solution will be this will be item number 1, so  $X_{11}$ ,  $X_{31}$ ,  $X_{41}$  and  $X_{101}$  and so on.

Now, the sum of these is eleven plus 19 is 30, 43, so 80, 73, 51 plus 29 is 80. 17 plus 39 is 56, 56 plus 23 is 79. So we find a solution with 280's and 79 and a 73. Now, let us assume, we want to solve this using heuristics, several heuristics are available, we will

see 1 or 2 of them, so most of these heuristics first sort these items in the decreasing order.

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So, when these items are sorted in decreasing order, we get 73 51 39 37 23 29 23 19 17 13 and 11, so the we first apply a very commonsensical heuristic, so we take the first item and then we observe, we create a bin, so we say bin 1 and put the first item 73 here. Now, we take the second item and see, if that would go into the bin 51 will not go into the bin, because the length of 80 will be exceeded, so create the second bin, so bin 2 has 51.

Now take the 3rd item, which is 39 and see whether, it will go into this bin first it would not go and then see, whether it can go into this bin 51 plus 39 does not go into this bin. So create a 3rd bin, bin 3 with 39. Now, take the 4th item with 37, so it would not go into this bin, because the length will become 110, it would not go into this bin, because the length will become 88, it will go into this bin, so 39 plus 37 with length equal to 76.

Now take the next item 29, see it will go to this bin it would not go, then try this bin, so it fits in 51 plus 29 is equal to 80. Now look at the 6 item with 23, it would not go into this bin, it would not go into this, it would not go into this, create a 4th bin with 23. Now take the next 1, 19, so 73 it would not go, again it would not go, again it would not go, so 23 plus 19. Now, take the item with 17, it is still would not go because this will be 90, it

would not go into this bin, it would not go into this bin, it will go into this bin 23 plus 19 is 42 42 plus 17 is 59, so it will go into this bin.

Now, look at the 13, it would not go into this bin, it would not go into this bin, it would not go here, now here it is 23 plus 19, 42, 42 plus 17 is 59, 59 plus 13 is 72, so it will go into this bin. And then take the last one, it will not go here, it would not go here, it would not go here, now let us see this is 72, so it would not go here, so create a 5th bin with 11. So, now, this method gives us 5 bins, whereas the optimum gives us 4 bins, it is a heuristic method, it does not guarantee optimum all the time, therefore it would do that.

Now this, this method gives us it is a minimization problem, so a heuristic algorithm would give, what is a feasible solution to the problem will be an upper bound to a minimization problem, so the number of bins 5 is an upper bound to the optimum value, which is 4. Now, this is a very popular heuristic, this is called the next best decreasing heuristic, next fit decreasing heuristic, it is called the next fit decreasing heuristic, important thing, it is a decreasing heuristic, because the items are already arranged in decreasing or non increasing order, therefore, it is a decreasing heuristic.

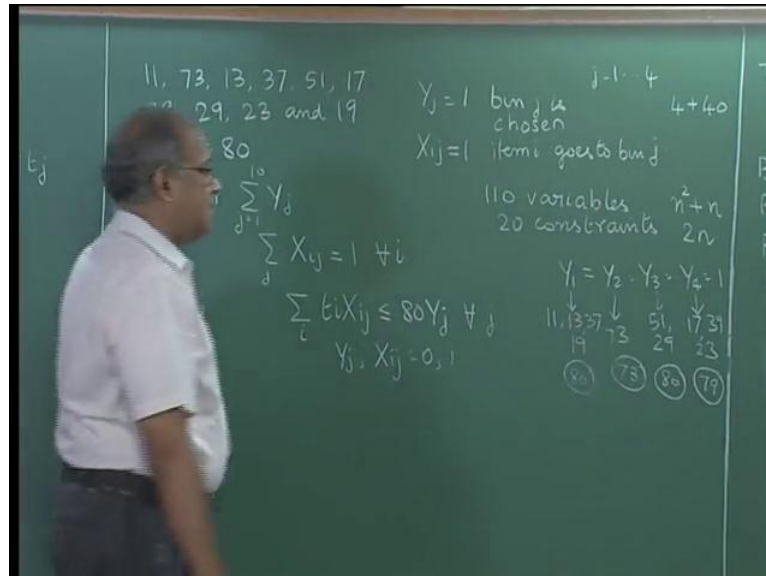
In fact, this is called the first fit decreasing heuristic, this is called the first fit decreasing heuristic, which means every item, we see, we take it, we see whether it will go to the first bin, if not we will see, whether it will go to the next bin and then it will go to the 3rd and then so on. So, whichever it fits first it will be put there, so it is called a first fit decreasing heuristic, now we have other heuristics some of them are called best fit decreasing heuristic and so on.

So, if there is more than 1, where it can fit, now at that point, we try and fit it into the 1 where it fills up to the maximum, that is called a best fit decreasing and then the worst fit decreasing would be, if there is a choice, put in the bin, where it would actually fill the minimum after filling. So, like that we can develop different kinds of heuristics for this particular problem, now one of the advantages of using this heuristic is now here, we have a solution with 5 bins.

So, we know that, there is a solution with 5 bins and once, we know that, there is a solution with 5 bins or 5 trucks in this example; we are only interested in solutions, which are 4 or lower, because we already have a solution with 5. So, using a very simple

heuristic like this, we get 5 bins and if we still want to solve it optimally and we want to check whether, 4 is possible then we have to come back.

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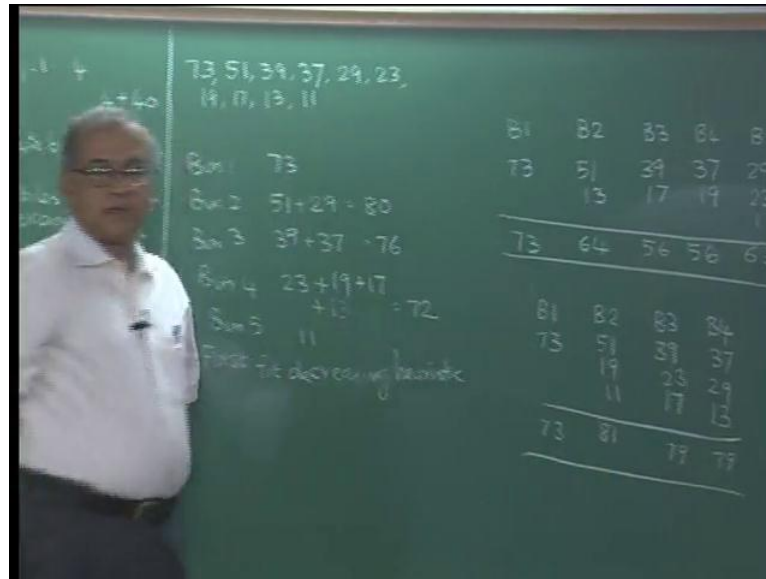


And solve this optimization problem, but the nice thing about, this is now you be do not have to define  $j$  equal to 1 to 10, it is enough to define  $j$  equal to 1 to 4, because we already have a solution with 5, so it is enough to define  $j$  equal to 1 to 4. And now see that simply define  $Y_j$  equal to 1, for  $j$  equal to 1 to 4 and  $X_{ij}$  equal to 1, if item  $i$  goes to bin  $j$ , so we will have 4 plus 40, which is 44 variables. And then we will have 14 constraints  $n^2 + n$  no here it will change now, this one be  $n^2 + n$ .

Now, here the number of variables  $Y_j$  will be 4,  $X_{ij}$  will be 40, so we will have 44 variables and in terms of constraints, this is for every  $i$ , so there are 10 constraints, this is for every  $j$ , there are 4 constraints, so there are 44 variables and 14 constraints, whereas in the old formulation, if we use 10 bins then we would have 110 variables and 20 constraints.

So, here are a set of problems, where it is actually advantageous to solve it, using a heuristic first and then based on the heuristic solution, if we approach the optimization, we observe that the number of variables and constraints in the optimization will come down.

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So, that is one advantage of using this heuristic. At the same time, if we are all right with the heuristic solution, then we could say that well, this solution would be acceptable to us. But, if we actually look at this solution and then we can tweak this solution, now that is not part of the best fit decreasing heuristic, the best fit decreasing heuristic would stop here, because it is interested in minimizing the number of bins and this heuristic gives 5 bins.

But, then we could do many things, if you look at the loads, it is 73, 80, 76, now this will become 72 and this is 11, now we can do a few more things by adjusting some of them by a little adjustment one can try and actually, reduce the number of bins from 5 to 4. But, if we were to implement this solution particularly for our truck example, then we observe that one truck is going to be used for 73 minutes another for 80, 76, 72 and 11. So when we start applying this heuristic to the situation that, we are talking about, we are also interested in some kind of a load balance among the trucks.

Because, it is also hard to say that, one truck will be utilized only for 11 minutes, while few trucks are utilized for 80 76 73 and so on, so we also need to have a secondary load balancing objective, if we were looking at solving this kind of a problem. We can do few more things, we can actually relate this problem to a traditional machine scheduling problem, in fact, we can try and do a few more things.

Now, if we want to solve with the better load balance, for 5 bins, we can try something else, we can now look at something like bin 1 bin 2 bin 3 bin 4 and bin 5, now we will go back and put this 73 here. Now the next will not go to this bin we will put a 51, we will put a 39, we will put a 37 and then we will put a 29 and then the next one, we can put a 23, so it will come here, so 29 plus 23 is 52, now 19 will come here, because if we see, where we assign for example here.

Now at this point, the first 5 go to the first 5 bins, the 6th one is 23, so 51 plus 23 is 74, 39 plus 23 is 62, 37 plus 23 is 60 and 29 plus 23 is 52, so the best place to put that 23 is here, because the loads will get balanced. Even though, we can put that 23 here, which will be closer to 80, but then we realize that the loads will be will be unbalanced and then there will be a load imbalance, therefore, the best thing to do is to put that 23 here, so that the loads are also balanced as they can be, so we put this 23 here to get 52.

Now the next one is 19, now if we look at the loads are 73, 51, 39, 37 and 52, so this is the smallest, so we get better load balance by putting this 19 here. Similarly, the 17 will come here, the 13 will come here, now this 11 cannot come here, because 73 plus 11 is 84. Now we check this is 64, so this can take that 11. This is 56, this is also 56, this is 52, so this will finally, take that 11 and now the loads for 5 bins will be 73 64 56 56 and 63.

So now, we have a solution, if we were to use this 5 bins, then this is a solution with better load balance than this, so depending on the problem and depending on more than one objective, we can use suitable heuristics. Now, the same problem, we can try something more and see that, if we have 4 trucks or bins, what happens to and if we start allocating this 10, what happens to the loads. So, 73 will come here 51 will come here, 39 will come here and 37 will come here, so automatically from a load balance point of view, 29 will go here, because this is the smallest load 37 plus 9, 46 plus 20, 66.

Now, 23 will come here, 39 plus this will give a 62. Now 19, this is 51, so 19 will come here, 51 plus 19 is 70. 17 cannot come here, because it will exceed, now to put this 17, 17 also will not come, 17 can come here, 51 plus 19 is 70. So, it will 7 plus 17 will exceed, 39 plus 23 is 62, so 62 can handle is this 17, this is 66, so this will handle get the 17 and this will get this 13. And then we do not have a place to put this 11, because this the load will exceed 80, if we add 11 here, also it will exceed 80, here the load will be 62 plus 17, 79, it will exceed 80, this will become 66 plus 13, 79, it will again exceed 80.

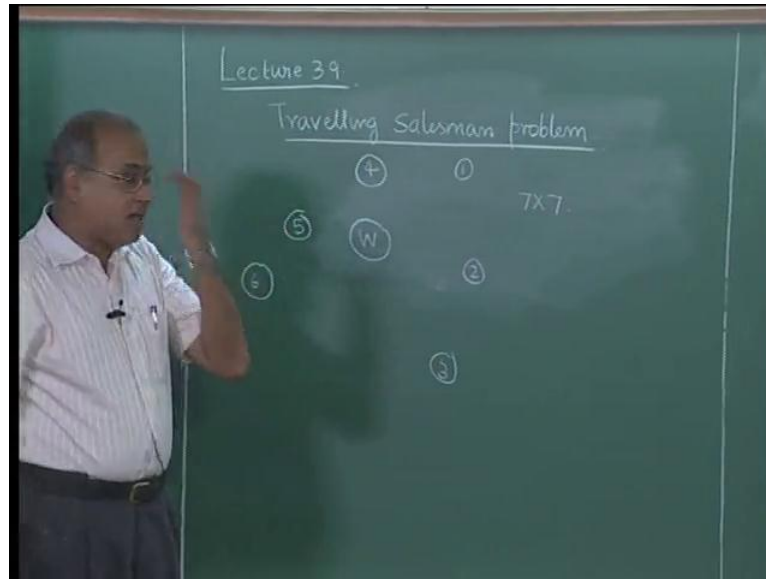


But, then we have to put this 11 somewhere, so we can put this 11 here and then say that, this is 73, this is 81, this is 12 plus 7, 19, 79, this is 12 plus 7, 19, 79. Now we realize that, the optimum solution was able to give us, 80 73 80 79, the heuristic solution gave us 73 81 79 79. So, using this heuristic solution, now we observe that, we would not be able to squeeze all the orders in 80 time units, if we have only 4 trucks or if we use this heuristic.

And if we want to minimize the number of trucks, such that, I can squeeze everything into 80, the answer is 5 whereas, the optimum is able to squeeze all the 10 items into 4 trucks or 4 bins, such that the length is 80, here it says, it requires an additional length of 81. Now, we can either solve this optimally, this is easily the best thing that, we would like to have, but in the absence of solving the integer programming optimally and if we rely on heuristic solution then we have to understand here, there are times, we would get even the optimum solution through some of the heuristics, we can try several heuristics.

Now, we will be able to minimize the number of bins for a given length and then within that, we may be able to get a load balance and we may also say given certain number of bin, what is the length of a bin that, I require. Now, it happens to be 81, what is the minimum length that, I require, now this problem is equivalent to the machine scheduling problem, it is called a parallel machine scheduling problem. Now, for this particular example, if I force 4 bins and then apply a load balancing algorithm, it says that, i may require 81 minutes on one of the trucks or one of the bins whereas, the optimum is able to do it, within 80 minutes. Right?

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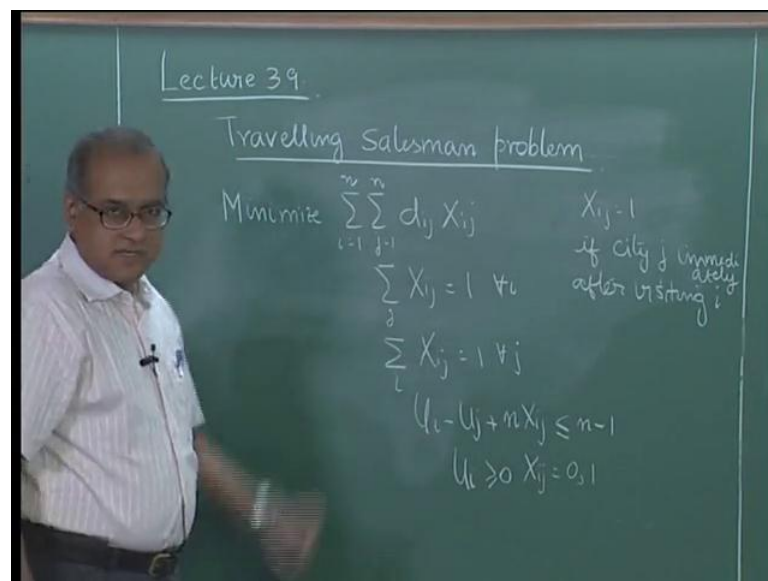
We next move to the travelling salesman problem. Now here what we do is we assume that, there is a central warehouse and then we have a set of customers. Now we - this is like the equivalence of where we look at milk run. So, we start a truck from the warehouse and then we meet the requirements of all these customers in one trip, which means that even, if each of these have a requirement.

The sum of all the requirements is less than or equal to the capacity of the truck or theoretically or mathematically, we can assume that the truck has a very large capacity, which is close to infinity and can handle the requirements of all these customers. Practically, we would say that, we know these requirements and the truck capacity is higher than the requirements of all of these, in such a case that truck will start from the warehouse, visit each customer once and only once deliver and it will come back to the warehouse, now that is called the travelling salesman problem.

Now, if you assume one warehouse and 6 customers, we have 7 points, we can assume that, we will have a 7 by 7 distance matrix and then starting from W, we want to visit each customer once and only once and then we come back to the starting point. Now, we have already seen several aspects of the TSP, in the advanced operations research course, where I have given the formulation for, the travelling salesman problem. And also I have explained many 2 or 3 different types of branch and bound algorithms to solve the travelling salesman problem.

And I have also looked at a lot of heuristics extensively in that course, where we have studied the travelling salesman problem in detail. What we will do now is will take another look at the formulation of the T S P. And then we would also look at some heuristics and I would also try and give you, some of the optimum solutions to the travelling salesman problem and make it a little brief, because a more elaborate treatment is available already in the other course.

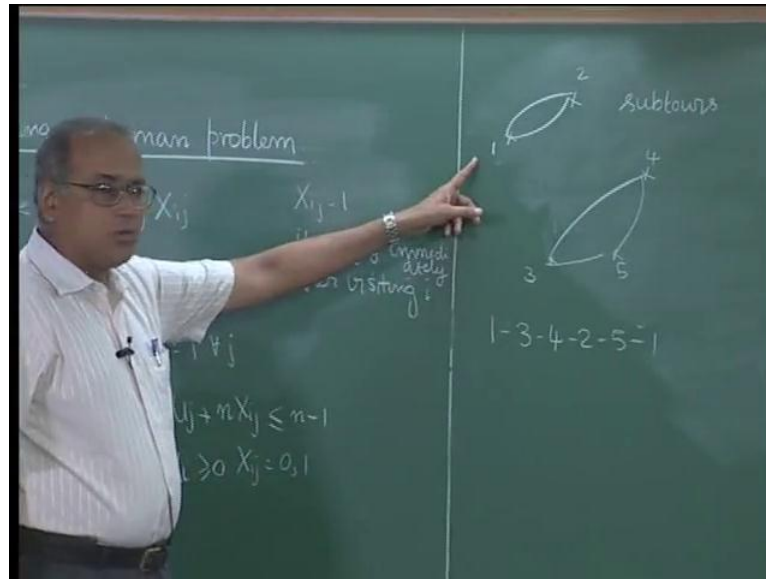
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Now, we have already seen this formulation of the travelling salesman problem. Now  $X_{ij}$  equal to 1, if the salesman visits city  $j$  immediately after visiting city  $i$ . Now  $d_{ij}$  is the distance matrix. Now, again in the context of the travelling salesman problem, we will assume  $d_{ii}$  that is distance between a point and itself to be infinity and not 0. So, a distance between a point and itself, so large that, we does not visit the point immediately after visiting it, for example, if put  $d_{11}$ ,  $d_{22}$  etcetera to be infinity or very large value, the variables  $X_{11}$ ,  $X_{22}$ ,  $X_{33}$  etcetera, will not be in the solution.

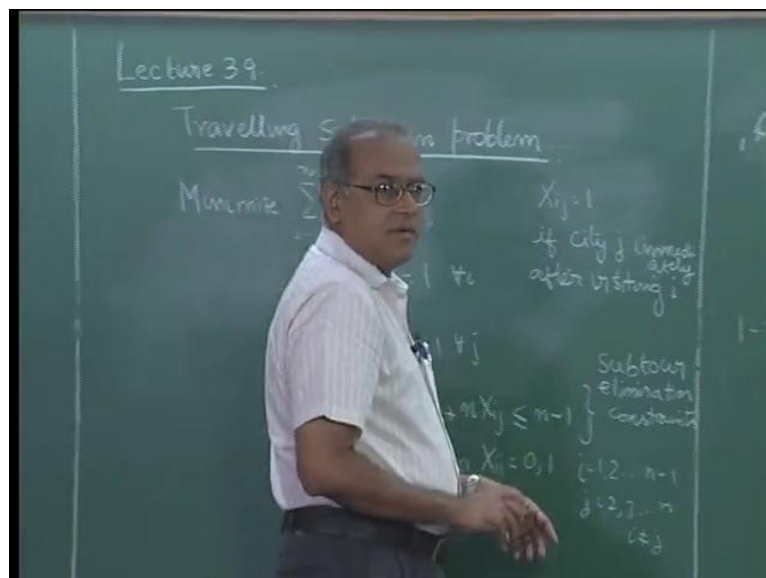
Now this will ensure that, if salesman is in a particular city  $X_{ij}$  equal to 1 summed over  $J$ , which means, if the person is in a particular city, the person has to visit only one city immediately. So, that will be taken care of by this and if this person is in a particular city he or she should have come from 1 city, so that is taken care of this. But, then we need the extra constraints otherwise the T S P will become an assignment problem or if we have 5 points, including the warehouse.

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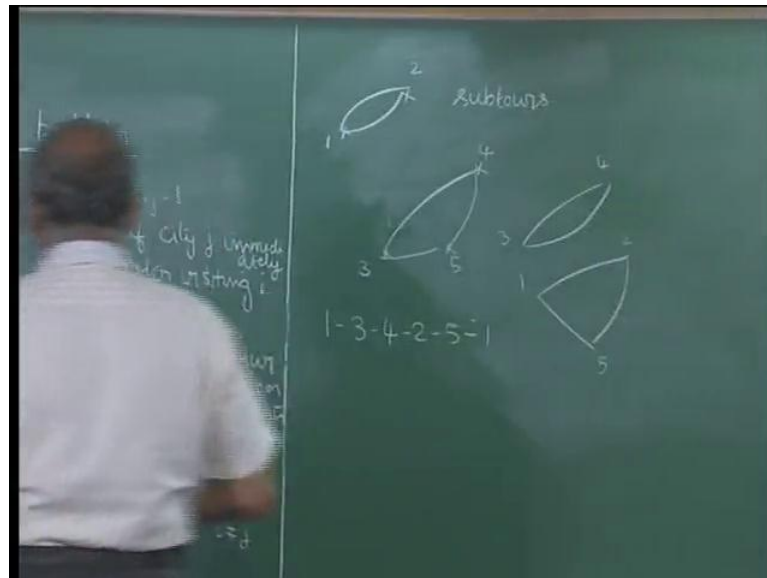
It may give solutions like this, which means it goes from say 1 to 2 and then 2 to 1 and then the salesperson suddenly goes 3 to 4, 4 to 5, 5 to 3. Now they will satisfy this, so this will be like solution to the assignment problem. Now, things like this are called subtours and these have to be eliminated. The solution to the T S P is always a full tour, which means you will have a solution like 1 3 4 2 5 1, this is called a full tour 1 to 2, 2 to 1, 3 to 4, 4 to 5, 5 to 3 is called a subtour.

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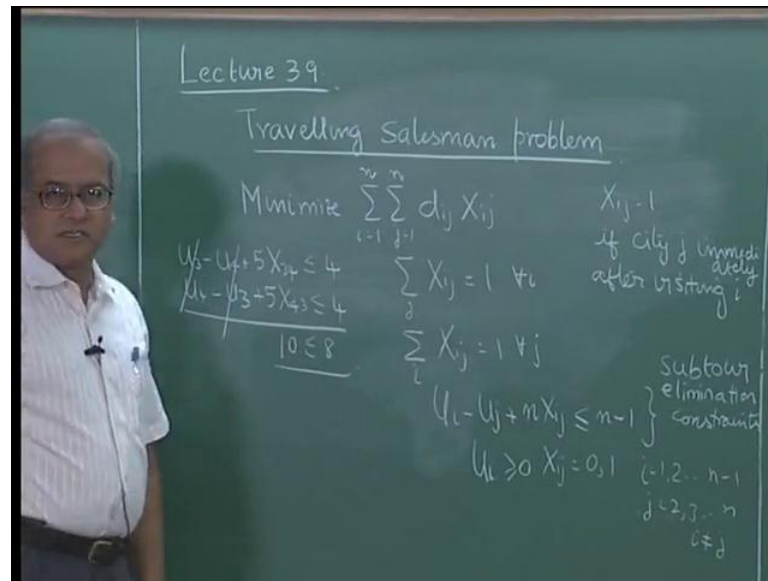
So, we need this plus we have, what are called subtour elimination constraints. Now, if there is a subtour let us say, there is a first, we define  $u_i - u_j + n \times x_{ij} \leq n - 1$  and then we define it for  $i$  equal to 1 to  $n - 1$ ,  $j$  equal to 2, 3 up to  $n$  and  $i$  not equal to  $j$ .

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Now, if there is a subtour, let us say let us say, there is a subtour 1 to 2 and 2 to 1 or first let us assume that, there is a subtour, which is there is a subtour, which is 3 to 4 and then let us say for a 5 city problem, there is another subtour that is 1 to 5. So, a subtour 3 to 4 will be eliminated here.

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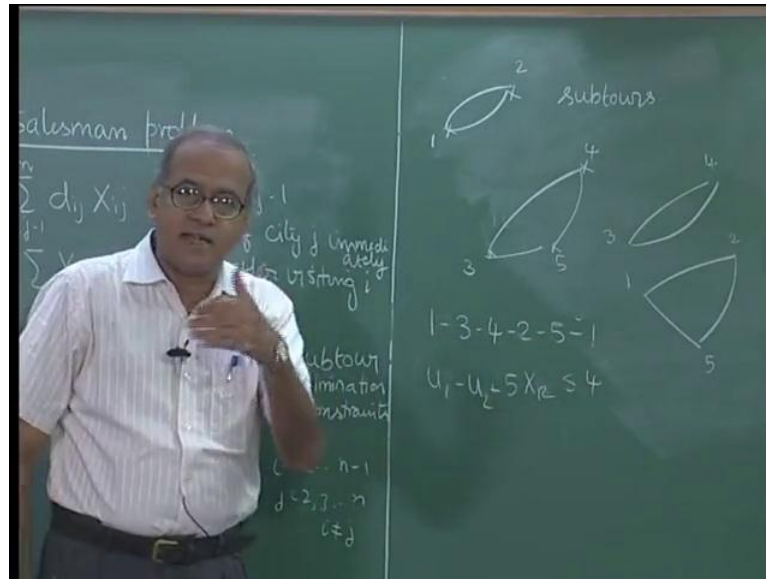


Because, there will be a constraint which will be say  $u_3 - u_4 + 5X_{34} \leq 4$  and  $u_4 - u_3 + 5X_{43} \leq 4$ , there will be a constraint like this. Where I have just written  $u_i - u_j + nX_{ij} \leq n - 1$ , now if this, if such a subtour exist then these 2 constraints have to be satisfied.

Now, we realize that adding vertically, you are you will get, if both are equal to 1, you will get  $10 \leq 8$ , which is not true, therefore this constraint will ensure that subtours involving say a 2 city subtour or whatever  $3 \ 4 \ 4 \ 3$  are automatically eliminated. The same thing will hold, for any for example, if we have  $3 \ 4 \ 5$  then there will be  $u_3 - u_4 + \text{something}$  and  $u_4 - u_5 + \text{something}$  and  $u_5 - u_3 + \text{something}$ , so vertically adding it will give us an inconsistent set.

Therefore, all these subtours involving this and this are eliminated, now if there are subtours that are involve city 1, the reason one is important is because of this  $i = 1$   $2$  to  $n - 1$   $j = 2$  to  $3$  to something. So, if there is a subtour involving city 1, then that subtour apparently 1 will get a feeling that, it will not be eliminated by this, for example, if we start writing for this, then you will get a thing like.

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$u_2 - u_1 + 5x_{21}$  is less than or equal to 4, if I add  $u_1 - u_1$ , so  $u_1 - u_2 + 5x_{21}$  less than equal to 4, but then you do not have a  $2-1$  plus  $j$  to  $i$ ,  $j$  is not defined from for 1, so you would not get a second constraint. But, the way we understand this is if there is a subtour that involves 1, then there has to be another subtour at least one more subtour, which does not involve 1 and since all subtours not involving 1 are eliminated, automatically subtours involving 1 will also be eliminated.

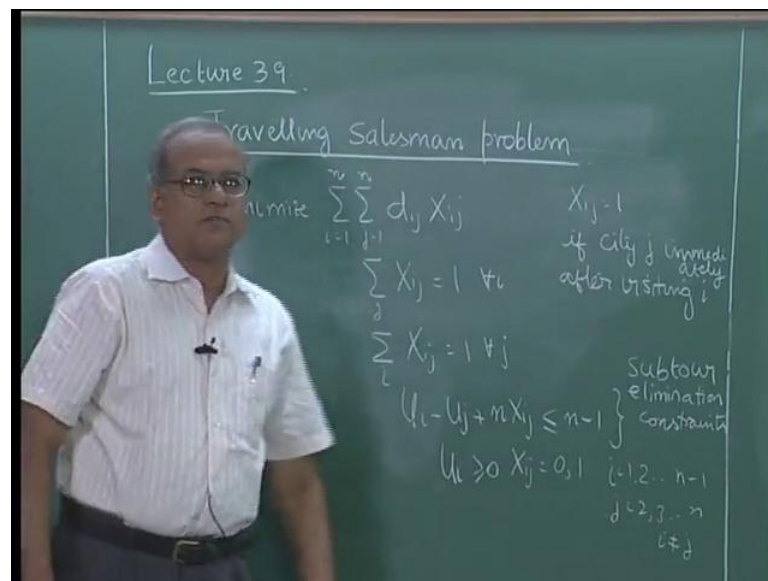
Now, this will not violate a full tour, because a full tour will have something like this 1 to 3, 3 to 4, 4 to 2, 2 to 5, 5 to 1, now that 5 to 1 constraint does not exist in the formulation, therefore you will have only 4 constraints here, you would not have 5. Now, the  $u_i$ 's and  $u_j$ 's can be, so defined, such that these set of constraints will be satisfied, so these set of constraints are called subtour elimination constraints and they are used extensively, if we have to solve a T S P optimal.

But I having said that this T S P is usually not solved as a mixed I P. because this is a mixed I P, you realize now that  $u_i$ 's are greater than or equal to 0, now if there are  $n$  cities then we have  $n$  square variables for  $X_{ij}$ . So, there will be  $n$  square variables for  $X_{ij}$  and  $n$  variables for  $u_i$ , so we will have  $n$  square plus  $n$  variables, where these are binary and these are continuous.

And in terms of number of constraints, now we will have  $n$  constraints here, we will have  $n$  constraints here, we will roughly have about  $n^2$  constraints slightly less than that

may be  $n C 2$  minus  $n$  constraints and so on. So, we will have of the order of  $n C 2$  constraints here, so this will still become a large problem, when the number of cities increases and this would roughly increase of the order of  $n$  square, so we will have a large number of constraints. And this is a binary formulation plus a mixed formulation, because this these set of variables are continuous, so we normally do not use this formulation to solve the travelling salesman problem, instead we use branch and bound algorithms.

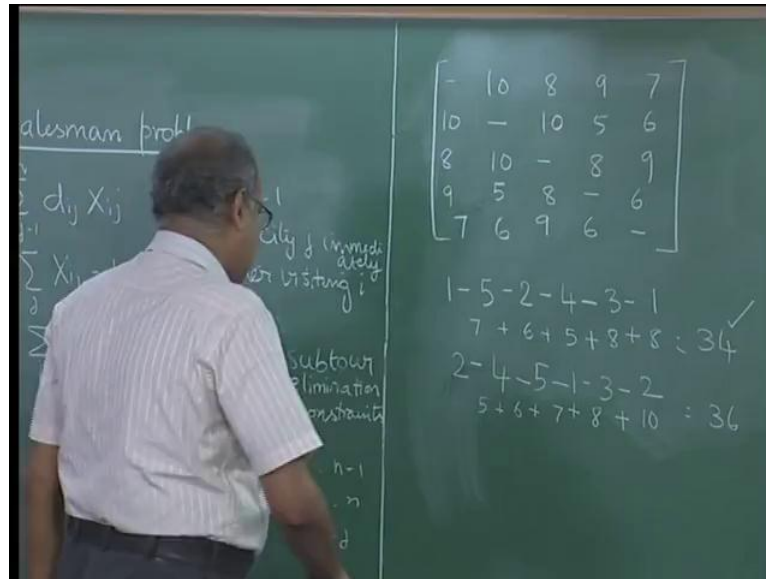
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We use branch and bound algorithms, there are several types of branch and bound algorithms that are available; the more popular 1 is the 1 by Little and 3 other authors. Now, this is based on the penalty approach, I have given a detail description of the Little's branch and bound algorithm in the advanced operations course and hence I am not repeating the Little's algorithm once again here. So, the reader the viewer could go to the other lecture and take a look at the Little's algorithm, through which we can solve this. Now, several heuristic algorithms are very popular for the branch and bound, for the travelling salesman problem and for the purpose of completion, let me just explain 1 or 2 heuristic algorithms here. So, we take a 5 city T S P and explain through this.



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So, a 5 city T S P with distance matrix is dash 10 8 9 7 10 dash 10 5 6 8 10 dash 8 9 9 5 8 6, now here we have to assume that, there is a warehouse and there are 4 other places that, we have to visit. So, one could be the warehouse 2 3 4 and 5 could be the customers. In fact, in a T S P does not matter, because the solution 1 2 3 4 5 1 is the same as a solution 2 3 4 5 1 2, therefore, it does not matter, where the truck actually starts.

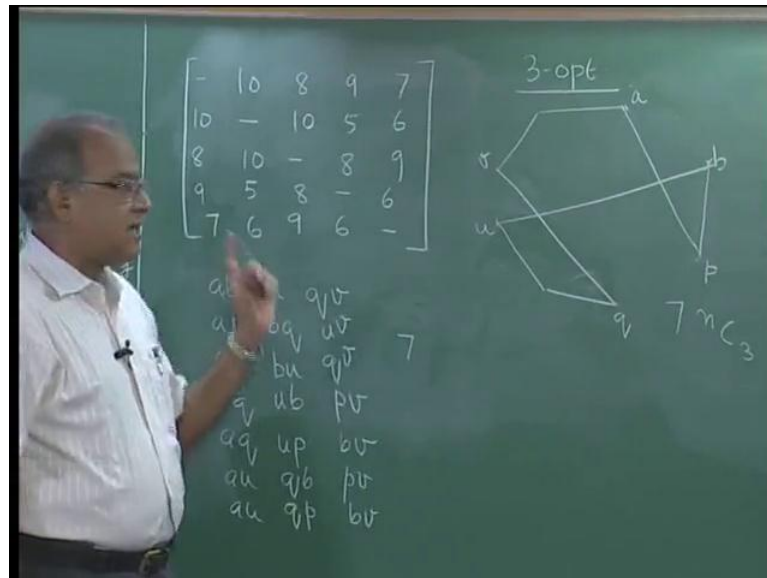
But, then it is customary to say then it starts from the warehouse and it completes one round by visiting each and every customer once and only once and then it come back to the starting point. Now, the simplest heuristic is called the nearest neighbor, if I assume that, I am in city 1, now I go to the nearest neighbor, whose is 5 from here, I go to 5, now from 5, I go back and realize, I can go to 2 or I can go to 4, so I go to 2. Now, I come to 2 and the nearest person is 4, now I have only 3 remaining, so 3 and 1.

Now, the length of this will be 1 to 5 is 7 plus 5 to 2 is 6 plus 2 to 4 is 5 plus 4 to 3 is 8 plus 3 to 1 is also 8, so this is 13 18 plus 8, 26 plus 8, 34. We do not know whether, this is the best solution or we could make this solution better. So, we simply expand this by trying that, we start with 2 first, so I start with 2, so I go to 4, this is the minimum from 4, I realize this is the minimum, but I have already visited 2, so I go to 5 and from 5, I go to 1 and then 3 and then 2.

So, this will be 5 plus 6 plus 7 plus 8 plus 10, so this will be 11, 18, 26 plus 10, 36. Like this, we can do a nearest neighbor with each of the cities as the starting city and then we

can proceed and then choose the best out of these solutions, that we have. In this example, we can show that, this is actually the optimum solution and the nearest neighbor is able to give the optimum solution right away. Then we start from 1, this is because of the nature of the numbers that, we have in this particular example.

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Another very popular heuristic is called the 3 opt heuristic, the 3 opt heuristic essentially does the 3 way exchange, so what we do is if you have a T S P let us say a large T S P. We have a large T S P. Now, let us assume this is a solution, so 1 2 3 4 5 6 7 8 1 this need not be the optimum or the best solution, this is a T S P with 8 nodes or 8 cities, now I take a solution like this and then I take one of these as a b, another pair is p q and the third pair is u v.

Now, we realize, there are eight nodes, so a b p q u v can be chosen in many ways. We would also realize this is a b this is p q, but then I left a node and then I chosen u v here. Now, using this, I can actually create some feasible solutions, for example I can remove this, I can remove this and I can remove this, now I can do this, I can add these 2, I can add this and I can retain this, this will be another feasible solution. For example, I can remove this, I can do this, this could be another solution. So like this we have several possible solutions.

So, if we have a a b p q u v, then we can actually get 7 more solutions to this, we can have a b p u q v a, p b q u v a, p b u q v a, q u b p v a, q u p b v a, u q b p v a, u q p b v a,

so where every a b p q u v chosen, we can have 7 solutions. Now, if there are n, we take a starting solution, so there are n vertices, there are n edges in the solution, so a b, p q, u v represent 3 out of the n, so they can be chosen in  $n C 3$  ways and then each of these a b, p q, u v, we can write 7 more solutions from this.

So, if we are given a T S P with say 5 vertices, then  $5 C 3$  is  $5 X 2$ , which is 10, 10 into 7, we can get 70 solutions out of this. The only thing is because we have 5, then we realize that these 70 units are not unique, there will be overlap, because when we have a 5 by 5 to chose a b p q u v, then some of them have to overlap. So, a lot of those 70 will overlap, but then we can still chose the best of them and we can proceed. Whereas, if we have a problem with 10 vertices, (Refer Slide Time: 47:29)

The image shows a chalkboard with the following handwritten calculation:

$$\begin{aligned}
 & 10C_3 \\
 & \frac{10 \times 9 \times 8}{3 \times 2 \times 1} \\
 & = 840
 \end{aligned}$$

And now, we realize that we a b p q u v itself can be chosen in  $10 C 3$  ways, 10 into nine into 8 by 6 another 7, so for every solution will be able to this, so we have 800 and 12 7s are 84, we have 840 solutions, which we can get, from a single solution and then we can keep improving this. So, 3 opt is a very popular method, one can write a simple computer program, which would do the 3 opt heuristic and then we could get the best values of the solution.

But then when we apply these ideas particularly to trucks and distribution and so on, we also need to look at a little beyond this kind of a T S P and then add a few more aspects to the T S P. Now, this T S P is symmetric, which means distance between i to j is the same as j to i, but then when we actually apply these on practical problems, we

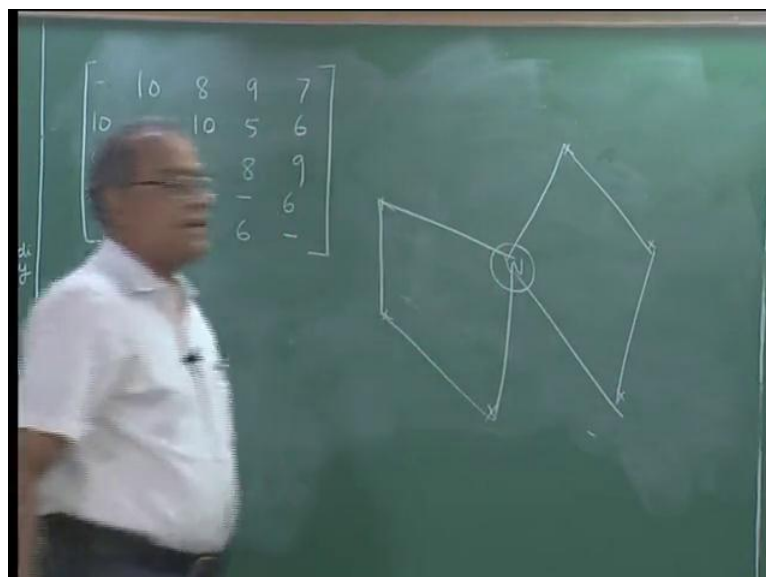
understand that actually the distances are not the same, because there could be one way and there could be different highways, you need to take when, you go from one place to another, so we have T S P, which are not symmetric.

We also observe that particularly in critical distributions like say mail motor and other such distributions, one would observe that actually, even if the routes are the same, the time taken to go from one place to another will depend on the time of the day. When the travel is made the travel is made early in the day one can reach a little faster and when we move into peak hours, then we have, we go much slower.

So, there is a speed dimension that come in, there is a distance the speed from, which the time is to be computed and the speed will depend on the time of the day, because most of these also at the end of the day result, in problems were certain delivery times have to be met and so on. So, we have to take into accounts speed of the vehicle also, we also could have windows, time windows, where delivery to a customer has to be made either, before a certain time or after a certain time or within a time interval.

So, all these result in handling the travelling salesman problem very effectively, in terms of the nature of the objective function, in terms of additional constraints, that we may have, also in terms of getting the data depending on dynamically updating and getting the data so that the correct data is used and the corresponding T S P are formulated and solved. The next thing that, we will have to look at is called the vehicle routing problem,

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where we would have a central warehouse, we would have a set of customers, now we realize that the demand of these customers is such that, we may require more than one trip, the vehicle capacity cannot handle the demand of all of them. So, we get into a situation, of which customer goes into, which trip and then how the trips are made; now that problem is called the vehicle routing problem.

A solution to vehicle routing problem could be, one trip will be like this and the other trip would be like this, but in general, which cities or customers will go to which vehicle, such that we minimize the total distance. We use minimum number of vehicles and then we are able to meet the demand is called a vehicle routing problem, we will see various aspects of the vehicle routing problem, in the next lecture.