Operations and Supply Chain Management

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Lecture – 38

Transportation and Distribution Models (Continued)

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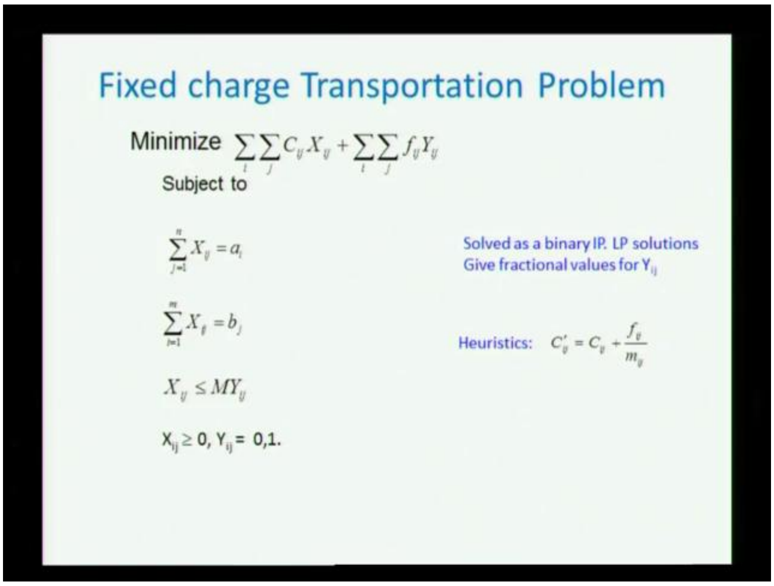

In this lecture, we see the fixed charge transportation problem. We have already seen this formulation of the fixed charge problem in the previous lecture, where I have explained that, this problem becomes significant when there is a fixed charge of transporting from a given source to a given destination. We have also seen that, the toll that is paid while moving items from i to j could represent the fixed charge between i and j. There could also be other fixed costs that are associated with travelling from a particular place to some other place. So, the fixed charge transportation problem has two components, C i j X i j plus f i j Y i j, the objective function is to minimize the two costs.

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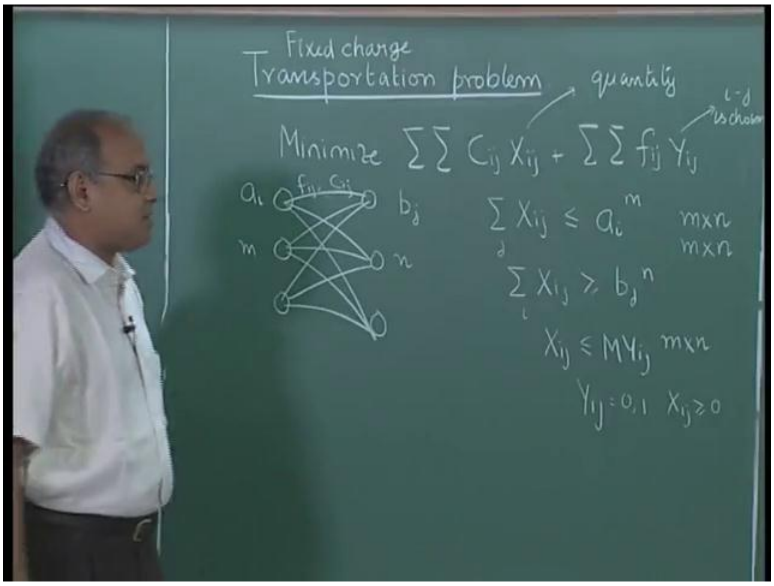

So, minimize C i j X i j plus f i j Y i j, so the decision variables X i j represents the quantity transported from i to j and Y i j represents, if arc i j is chosen. So, Y i j is a binary variable, it is equal to 0 or 1, it is equal to 1 if we choose to transport from i to j and it is 0 otherwise, X i j is the quantity that is transported from i to j. On the face of it, it appears that, this objective function is similar to that of the multistage transportation problem, but it is not so.

In the multistage transportation problem that we saw in the previous lecture, we had an i, j and k, so there were three entities. Now, we have only i and j, we have two entities, so it is very similar to the regular transportation problem, where there are only two entities a set of supply points, which may be warehouses and a set of destination points, which could be retailers or customers. So, there is only one stage and there are two entities and they the material flows from one set of entities to another set of entities.

Typically, material we could think in terms of three supply points and three demand points and items flow this way. Now, there is an a i, which is the supply available here and there is a b j, which is the requirement for this. Now, there is a f i j, which is a fixed cost and a C i j, which is the unit cost of transportation, if we move from i to j. So, we have typical constraints, which will be, whatever goes out of this should be less than or equal to a i.

So, sigma X i j summed over j is less than or equal to a i, whatever that reaches here should be greater than or equal to b j, so sigma X i j summed over i is greater than or equal to b j. So, once again the same caution that, the balanced problem would have total supply equal to total demand, in which case we can even change this with the equations. All the items will be transported and all the items will be received, we will not be receiving more than what is demanded, because the C i j's and the f i j's are not, none of them are negative.

Therefore, there will be a solution, where all these are consumed and all these are taken, now if we have a situation, where the total supply exceeds the total demand then all the demand will be met. More than the demand will not be given, because if we have to transport more than the demand, the total cost will only increase. And some of these supplies will not be utilised fully, so the same structure will hold. Only if we have a situation, where the total demand is more than the total supply then this formulation would give us infeasibility, because it would not be able to meet all the demand.

In such cases, what we do is, we balance it by creating another supply and then say at the end after solving, that whatever goes out from this supply actually does not go and therefore, correspondingly some of them will get less than what is demand. So, we need to observe that, particularly when we have a situation, where the total demand exceeds total supply, we have to be little careful in using this formulation. But, when we choose to use this, we have to create another dummy supply or a non existing supply.

Now, we also have to relate the Y i j variables to the X i j variables, so X i j is less than or equal to M into Y i j which means that, if I am transporting from a particular i to j then Ii have to choose that arc first and then I transport which means, when I choose that arc, I incur the fixed cost of transportation. So, I incur the f i j, so only when I choose, I will be able to set and this big M is a very large number. So, when I choose, I can send as much I can and if I do not choose this, I cannot send.

So, when Y i j is 0, X i j will automatically be 0, so we will now have if there are m supply points and n destination points, there are m into n X i j variables and there are m into n Y i j variables. So, it has 2 times m n variables, where this m n X i j variables are continuous variables, they need not be integers, the other m into n are binary variables, these Y i j’s are binaries. So, we have a problem that has binary variables as well as continuous variables.

Now, there will be m constraints that relates the supply, there will be n constraints that relate the demand and then there will be m into n constraints that link the supplies to the demands. So, this fixed charge transportation problem, which has an additional fixed charge on the arc, will now have 2 into m n variables, where m n are binary and the other m n are continuous.

And it has m plus n plus m n constrains, where these m and n are the supply demand and these are the linking constraints that link the X i j to the Y i j, so Y i j’s are binary and X i j are greater than or equal to 0. Now, we can solve this as a optimization problem like a mixed linear integer programming problem, where some of these variables are binary variables, some of these variables are continuous variables. Now, we can solve this to get the optimum solution to this problem.

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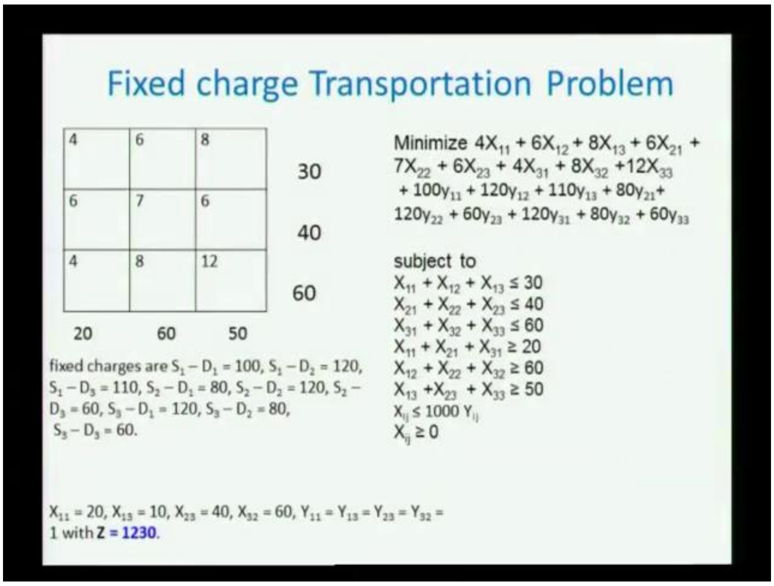

So, let me look at a 3 by 3 problem, that we have already seen and try and add a fixed charge to it.

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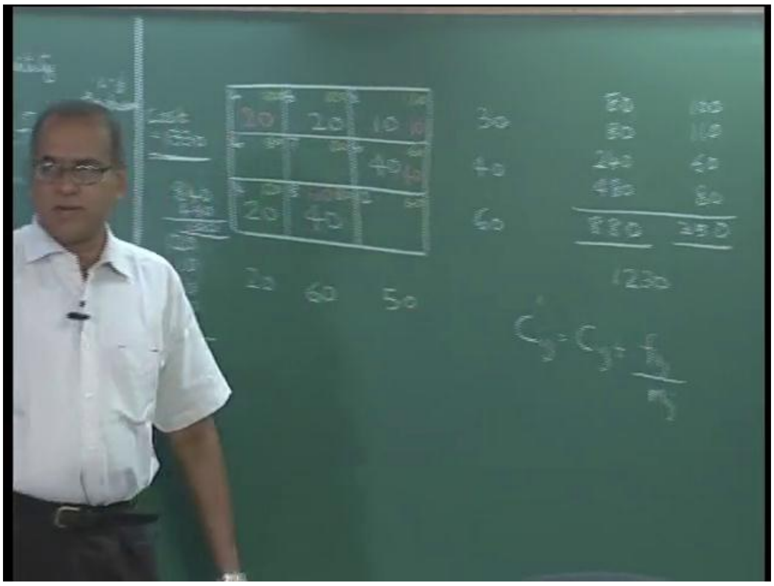

So, we look at this 3 by 3 problem, we look at the same problem that we saw in the previous lecture to explain the transportation, except we also add a fixed charge to it. So, the fixed charge is now shown here, that the fixed charge is 100 120 110, so 100 120 110 80 120 60 120 80 and 60. So, the values shown here are the f i j values and the values shown here are the C i j values. Now, if all the f i j values are 0 which means, there is no fixed charge, the problem will become, automatically it will become a transportation problem.

So, if all of them are 0 then there is no need to have this set of constraints, so it will become a transportation problem. Now, for the purpose of illustration, let us look at the optimum solution to the transportation problem without the fixed charge and then let us see, how the solution looks like. We set X 1 2 is 20, X 1 3 is 10, 40 20 40, 2 3 is 40, 3 1 is 20 and this will be 40. So, this is the optimum solution to the transportation problem without the fixed charge.

So, the C i j X i j component of this is 20 into 6, 120 plus 80, 200, 200 plus 240 is 440, 440 plus 80 is 520, 520 plus 320 is 840 that we saw. Now, if we have to look at this solution and implement the fixed cost associated with the transportation then we would be incurring a 120 here, because we have used this. We will incur 110 here, because we have used this, we would incur 60 we have used this, we would incur another 120 and we would incur another 80.

We have already seen that, as a transportation problem, there will be five allocations here, because the problem is balanced and non degenerate. So, this would give us a cost of 490, so the total cost will be 840 plus 490, which is 1330, now this is a feasible solution to the fixed cost transportation problem. So, what we have done is, we have not actually solved the fixed charge transportation problem, we have actually solved the transportation problem without the fixed charge.

And then we have added the fixed charge to it, to try and get a solution, whose cost is 1330, but if we solve this formulation of the fixed charge transportation problem, the formulation that we have written here. And for this example as I said, there will be 9 plus 9, 18 variables, there will be 3 plus 3 6 constraints plus another 18 constraints, so 24 constraints and 18 variables. If we solve this problem optimally, our solution would be X 1 1 is 20, so let me write it with a different colour, so X 1 1 will be 20, X 1 3 is 10, X 2 3 is 40, X 3 2 is 60, this is the solution.

So, 20 10 40 and 60 is the optimum solution to the fixed charge transportation problem, so let us try and evaluate the costs associated with it. The cost of transportation is 20 into 480, the fixed cost is 100, the cost of transportation is 10 into 8, 80, the fixed cost is 110, the cost of transportation is 40 into 6, 240, fixed cost is 60, cost of transportation is 60 into 8, 480 and this cost is 80. So, now this will be ((Refer Time: 14:00)) 880 and this will be 350, so the total cost will be 1230.

So, the cost will be 1230, now the solution that is shown here, this is the optimum solution to the fixed charge transportation problem by solving this formulation. Now, we compare this solution to the other solution, where we evaluated the total cost by considering the fixed cost. Now, 1330 was the cost here, while the cost is 1230 here, there is a saving of 100.

We also realize that, there are only four variables in the solution here, this cost has come down, this cost has marginally gone up, but this cost has come down, the very fact that, there are only four variables in the solution implies that, we are going to have four arcs in the solution and only four fixed costs. Whereas, this would force us to have five and then there were five fixed costs and then we realized, there are lot of saving there, which was resulted roughly in the saving of 100. Though this has gone up a little bit, this has gone down significantly to give us a saving of 100.

So, fixed charge transportation problem, if we have the fixed charge assumption and there is a cost associated with it then we need to look at this formulation and then solve it. So, the next question that comes is, can we have another method like our minimum cost method or the stepping stone method, to try and actually solve the fixed charge transportation problem without the formulation. A very simple thumb rule would be, now to define C dash i j is equal to C i j plus f i j by m i j. So, let me explain this thumb rule, now let us look at the same 3 by 3 problem.

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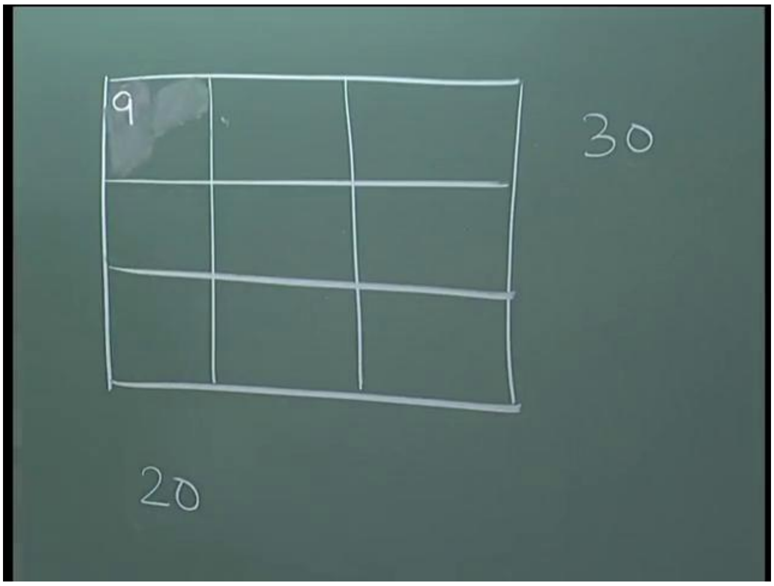

Now, the unit cost of transportation from 1 to 1 is given by 4 and the fixed charge is 100, now the supply that is available here is 30 and the demand is 20. So, if we consider this position, so C dash i j will be C i j, which is 4, plus f i j is 100 divided by m i j is the minimum of this and this, minimum of the supply and the demand, so minimum of 30 and 20, which is 20. So, this would give us a number equal to 9, so I am going to write this number here, this number is 9.

So, like this we can calculate this number, which captures both C i j as well as f i j into to get a new number, which is called C dash i j, which could represent some kind of an equivalent cost, considering both the C i j's and the f i j's. The motivation comes from the fact that, this fixed charge, if we are going to use this to transport, we are incurring an additional charge of 100. Now, this additional charge of 100 is used in transporting from this to this.

The maximum that can be transported from this to this, is the minimum of the two numbers 30 and 20, which is 20. Therefore, this 100 should be used in transporting a maximum of 20 from this to this, which is the minimum of this 30 and this 20. So, per unit, the apportionment of the fixed cost can be 100 by the minimum of these two, which is 100 by 20, which is 5 and therefore, C dash i j becomes C i j plus f i j by m i j. So, this way, we can complete the calculations for C dash i j and then the values now become 9.

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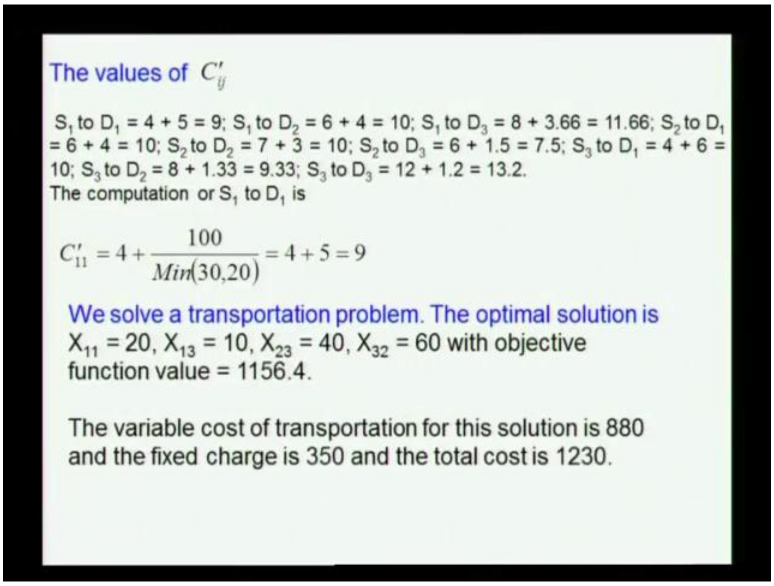

So, the values become 9 10 11.66 10 10 and 7.5 10 9.33 13.2.

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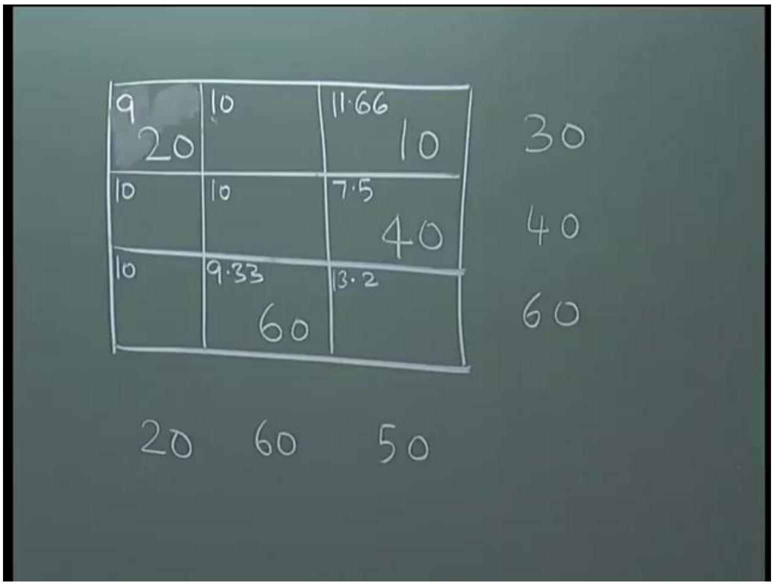

So, let me again explain this 13.2, 13.2 would come as 12 plus 60 divided by the minimum of these two, so 12 plus 60 by 50, 12 plus 1.2, which is 13.2. Now, we can solve a transportation problem with 30 40 and 60 and 20 60 and 50. So, this becomes the normal transportation problem and when we solve that transportation problem, we would get a solution, which is 20 10 40 and 60. So, we will get a solution like this and the cost associated with this will be different, it will not be 1230, it will be different.

But, once we get this optimum solution, we now have to go back and map this optimum solution here and then try to find out the fixed cost as well as the transportation cost. Transportation cost will become 880, fixed cost will become 350 and the total will be 1230. If we do 9 into 20 plus 10 into 11.66 and so on, that will be different, so it will not be 1230, we have to do that separately and try and get the cost to the original problem. But, this thumb rule need not give us the optimum solution all the time, in this example it has given us the optimum solution.

There could be instances, where this thumb rule may give a cost that is actually higher than the cost given by the optimum solution, which is solved using the integer programming. That is because from an operation research point of view, the fixed charge transportation problem is a hard problem. And therefore, we do not have effective thumb rule based methods, which can give the optimum solution in all the instances, in some instances it may give the optimum solution, as it did in our example.

So, if we really want the optimum solution to the fixed charge transportation problem then we can solve this or use this and use a solver to solve this, which again necessitates the availability of a solver. And if we do not have a solver, we could think in terms of other methods such as a Branch and Bound algorithm and so on, such methods exist in the literature, they can be used to get the optimum solutions. At the same time, if we are not very keen on the exact optimum solution.

But, if we are content with the good solution, which can be slightly higher than the optimum in some instances, but a good performance on an average, one could look at this thumb rule. And then create an equivalent transportation problem from the fixed charge transportation problem then solve it and get the solution, so we can do that as well.

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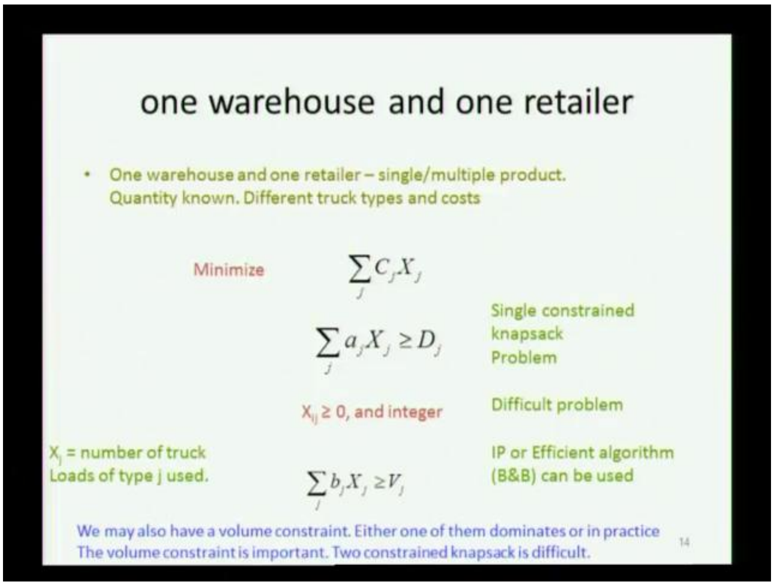

The next problem that we will look at, is what is called a point to point one warehouse one retailer problem.

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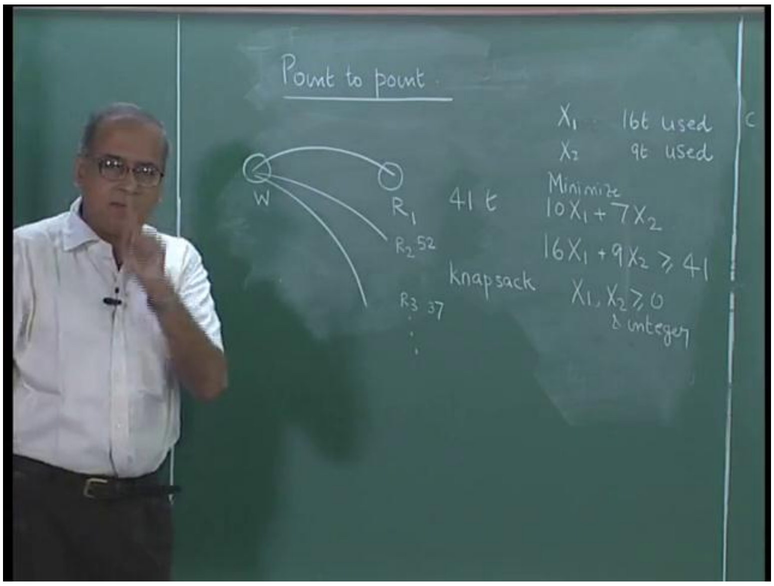

Simple point to point one warehouse one retailer problem, now let us assume, we explain it through a numerical example. So, we will assume now that, we want to transport 41 tons of items from this warehouse to this retailer. Now, in order to transport these 41 tons, we can do that using different types of trucks. So, let us assume, we are looking at two types of trucks, one truck can carry 16 tons per truck and the other can carry 9 tons per truck.

Now, the cost of hiring and using this is 10 and the cost of hiring and using this per truck is 7. So, the question boils down to, how many of these trucks do we take, how many of these trucks do we take such that, we are able to transport the 41 tons and we are able to do it with minimum cost. So, the problem would be, let X 1 be the number of 16 ton trucks used, let X 2 be the number of 9 ton truck used. So, the total cost will now be 10 X 1 plus 9 X 2 or 10 X 1 plus 7 X 2 plus is the total cost, which has to be minimized.

Subject to, if I am using X 1 trucks of this type I can carry 16 X 1 items, if I use X 2 type trucks of this type I can carry 9 X 2, is greater than or equal to 41. So, the problem is to find out X 1 and X 2, that minimizes 10 X 1 plus 7 X 2, subject to 16 X 1 plus 9 X 2 greater than or equal to 41, X 1 X 2 greater than or equal to 0 and integer. Now, the integer is extremely important, because we can only hire an integer number of trucks. So, it becomes a very simple linear integer programming problem.

And if we are going to consider only point to point from one supply to one destination or one warehouse to one retailer, there is going to be only one constraint. So, this is a single constraint integer programming problem, all integer problem, because both X 1 and X 2 will have to be integers. So, there is an objective function that is to be minimized, there is a single constraint and there is a all integer. Now, this problem is called a single constraint knapsack problem.

The standard single constraint knapsack problem is a maximization problem with a less than or equal to constraint and variables greater than or equal to 0. Now, this version of the knapsack problem is a minimization problem with the greater than or equal to constraint, nevertheless it is also an knapsack problem, so one need not look at it as a different problem. The original knapsack problem talks about putting items into a knapsack such that, the value that we put in is maximized, X j is the number of units of different items that can be put in.

Now, this coefficient could represent, there is a single constraint, which could either be a volume constraint or a weight constraint, not both. If we have both then it becomes a two constraint problem, this is a single constraint and if it is a volume constraint, this would represent the volume of each item and the volume that we put in should be less than or equal to the volume available. So, it is a maximization problem with a single constraint of a less than or equal to type.

This version of the knapsack is a minimization problem with the single constraint, which is greater than or equal to type. Now, the same problem can become a little different if we have different warehouses and different retailers or if we have one warehouse and multiple retailers R 1 R 2 R 3 and so on. This person may require 41, this person may require 52, this person may require 37 and so on. So, again we may have to now define X 1 1 and X 1 2 as the number of trucks of 16 ton and 9 ton, X 2 1 and X 2 2, X 3 1 and X 3 2 and so on.

As long as there is no availability restriction on the number of 16 ton trucks and the 9 ton trucks, the problem can be separated into several single constrained knapsack problem, one for each arc. But, if there is a capacity restriction or the availability restriction on the number of 16 ton trucks and 9 ton trucks available then we would get then it would simply become 10 X 1 1 plus 7 X 1 2, there will be another 2 1, 2 2 and all that.

And then there will be a constraint, which will say X 1 1 plus X 1 2 plus X 1 3 plus X 1 n will be less than or equal to the available number of 16 ton trucks. So, we could have another set of availability constraints and the problem becomes very different, it is not a single constraint knapsack problem anymore. Now, let us restrict our discussion only with the single retailer or a single customer and therefore, we will have the single constraint knapsack problem.

Now, if we again we have a solver, one can easily solve this using the solver, there is only a single constraint. If we have only two variables and a single constraint, one can even do an enumeration and try and solve it. But then there are better ways of actually solving this using some kind of a Branch and Bound algorithm.

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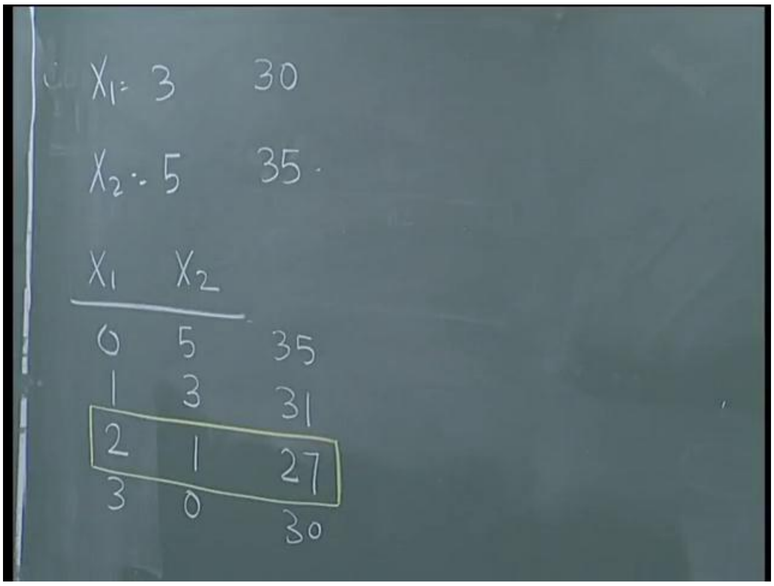

Now, if we use only 16 ton trucks then we will require X 1 equal to 3, which is upper integer of 41 by 16, so we will need 3 trucks here and the cost will become 30. If we use only 9 ton trucks then we would need 5 such trucks, so I would have 5 and then the cost will become 7 into 5, which is 35. So, we could simply think in terms of an enumerative algorithm, so we can simply enumerate and say X 1 X 2, so X 1 is 0, X 2 is 5, cost is 35. X 1 is 1 so then we have 1 here, so this would give us a capacity of 16, so from here, we need a capacity of 25, so we will have 3 here.

So, 1 into 10 is 10, plus 21 is 31, if I have 2 of this, so I have 32, out of this I require only 9, so 2 comma 1, so I will have 27 and I will 3 comma 0, I will have 30. So, only four enumerations are possible in this case and the best solution is here, which is use two 16 ton truck and one 9 ton truck to get this 41 and with the total cost of 27. But then we can also understand that, the enumerative method that we have shown here, is not computationally efficient and can become intractable when some of these numbers become higher.

Therefore, the single constraint knapsack problem also becomes a hard problem and some efficient method such as Branch and Bound algorithms are also available, which essentially do the complete enumeration. But, they do what is called implicit enumeration, they do not do a complete enumeration explicitly. Now, by creating some bounds, it is possible to reduce the number of computations here to a meaningful number.

And then to try and get the best solution using Branch and Bound algorithms or implicit enumeration algorithms, which reduce the number of explicit enumerations and tries to give us the optimum solution. I have actually outline the Branch and Bound algorithm to solve a single constraint knapsack problem with a maximization objective and less than equal to constraint in the advanced operations research course, where we look at the cutting stock problem.

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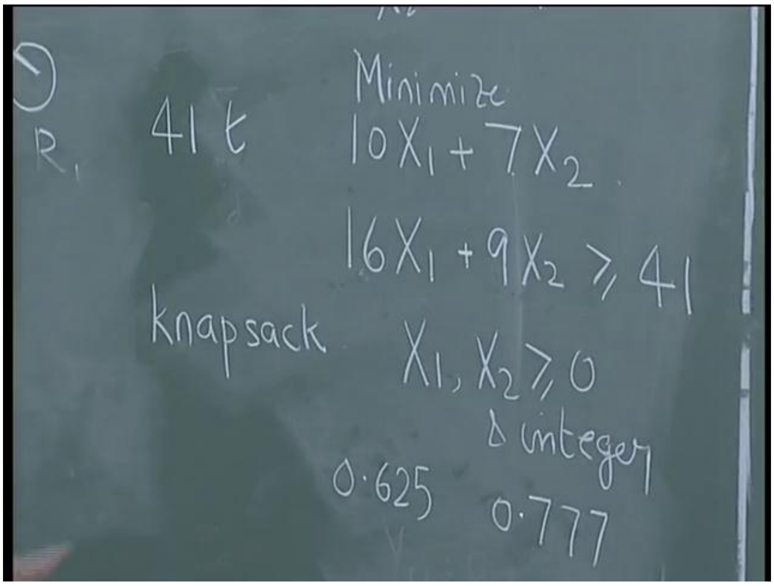

A very quick note on this, we can also do this, now what we can do is, this is a minimization problem. So, this is a minimization problem, if we can solve it as a linear programming problem, an LP solution to it which means, we are ignoring the integer restriction. Since it is a minimization problem, we can actually sort the variables in terms of the ratio 10 by 16 and 7 by 9, 10 by 16 is like 0.625, 7 by 9 is 70, so 9 7's are 63, 70, 0.77.

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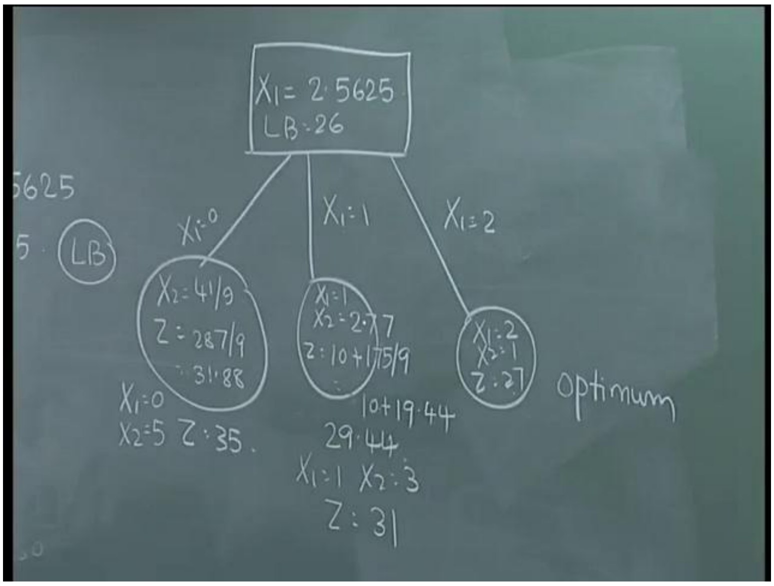

So, LP solution would give us X 1 equal to 41 by 16, which is 2.90, so 16 5’s are 80, 100 6 are 96, 40, 2.5625 and cost equal to 25.625. So, this will be the LP optimum, it would choose, because there is only one constraint, the LP will choose only one variable and it will choose that variable, which has the smaller coefficient. So, it will choose X 1 and give it a value 41 by 16, so 25.625 will act as a lower bound to this. So, we can start the problem by saying X 1 is equal to 2.5625 with lower bound equal to 25.625, which makes it 26.

Now, from this, since X 1 equal to 2.5625, we can branch of with X 1 equal to 0, X 1 equal to 1 and X 1 equal to 2 and then when X 1 equal to 0, we will solve another LP. So, when X 1 is equal to 0, this would we are solving an LP that minimizes 7 X 2, subject to 9 X 2 equal to 41. So, X 2 will be 41 by 9 and objective function value Z will be 41 by 9 into 7, so 41 into 7 by 9, 287 by 9 which is 9 3’s are 27, 180 9 8's are 72, 31.88.

Now, we look at this one, now the feasible solution for this will be X 1 equal to 0, X 2 equal to 5 with Z equal to 35, now this gives us an upper bound for this. Now, when X 1 equal to 1, we will now have a solution X 1 equal to 1, X 2 equal to 25 by 9, 25 by 9 is 9 2’s are 18, 70, 9, 7’s are 63, 2.77. Now, Z value will be 10 plus 25 into 7 by 9, 175 by 9, this is 10 plus 9 1’s are 9, 85, 9 9’s are 81, 40, 9 4’s are 36, 19.44, 29.44 is the lower bound and the corresponding feasible solution is X 1 equal to 1, X 2 equal to 3 with Z equal to 21 plus 10, 31.

Now, X 1 equal to 2 would give us X 1 equal to 2, X 2 equal to 1 with Z equal to 27, so this is integer feasible and this integer feasible is lower than both the lower bounds and therefore, it is optimum. Now, the Branch and Bound algorithm that we have seen, borrows ideas from lower bounds and upper bounds to between LP's and IP's. So, it involve certain operations research ideas and right now, I am just explaining it to show that, it is actually optimum.

A little more detail and depth and a detailed way of representation of this as I already mentioned, is considering a maximization problem with less than or equal to, which I have explained in further detail in the advanced operation and research course, where we address the cutting stock problem. Now, this Branch and Bound method is another method, which is slightly different, similar and different to the enumeration method that we saw here and gives the optimum solution of 27.

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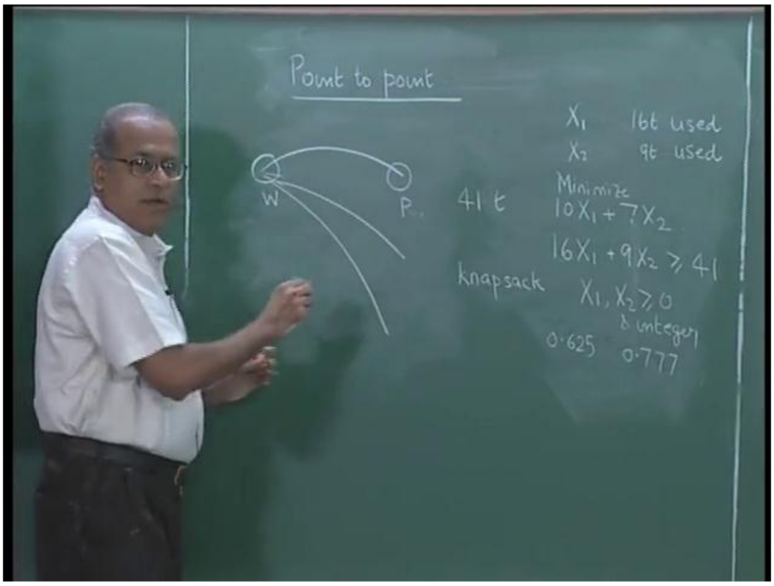

But, as I mentioned, if we expand this problem to multiple retailers and more importantly, keep a capacity restriction on the number of trucks available of different types. In practice, it need not be only two types, it can be multiple types, the problem becomes much more complex and it has to be solved using integer program.

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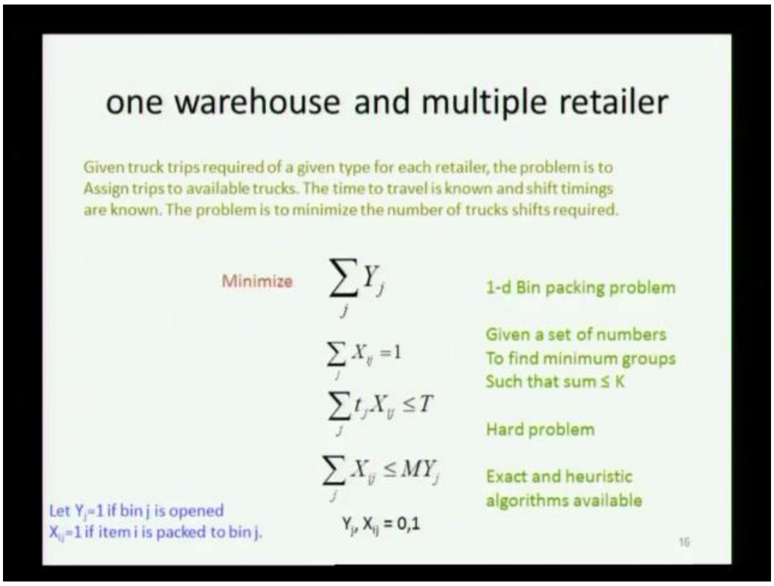

The next model that we can see is one warehouse and multiple retailer, which is a logical expansion of this.

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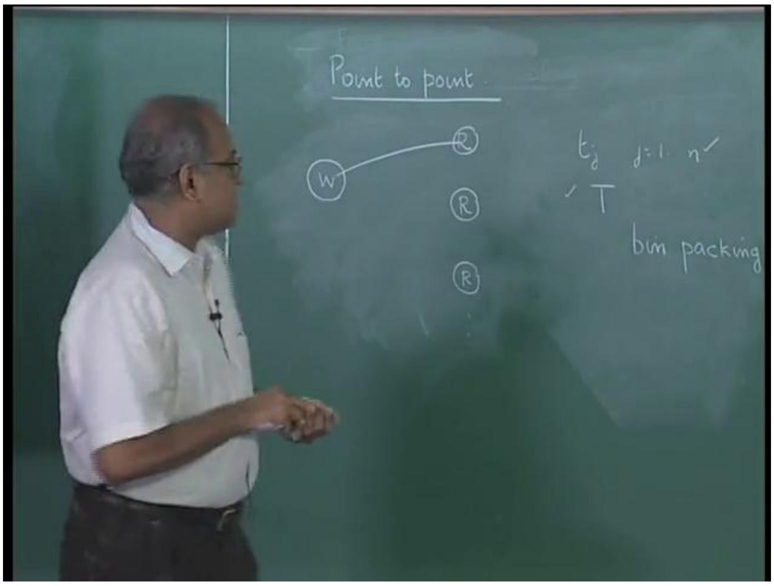

But, the problem is slightly different here, there is a single warehouse, there are multiple retailers and so on. Now, we will assume that, there is a truck, which goes from here point to point, it goes to the retailer and comes back and then it can go from here to another retailer and come back and the third retailer and come back and so on. So, let us assume that, there is a retailer j, there is a distance of d j, which the truck covers. Let us say, both ways put together, there is a distance d j, which is this into (Refer Slide Time: 40:23) this plus this into this is d j.

Now, we will assume that, there is a single truck which is going to transport, so whatever quantity that is transported from here to here, is less than or equal to a single truck load and or, but we also do point to point which means, from here it goes it will come back and then it will go to here or some other place and come back. Now, if there are j equal to 1 to n retailers, so we will have d j values d 1 to d n, now the total time required by the truck by a single, if a single truck has to do it, the total time required is the sum of d 1 to d n.

But, if this truck is available only, now we will call this time as using this notation, we would call this time as t 1 to t n. Instead of distance, we call them as time t j, j equal to 1 to n and the truck is available only for a time of capital T then the question is, how many trips should this truck take or how many trucks do I need such that, I meet the demand of all of these t to n. So now, that problem is called a bin packing problem, it is called a one dimensional bin packing problem or it is called a bin packing problem.

And the problem is very similar to saying that, I have a bins of height T and then I need to put items into this bin, I have n items, each item has a height, the j th item has a height t j. So, i have to put these items one above the other, it is a one dimensional bin, so I need to put them one above the other such that, I do not exhaust the height of the bin. How many bins do I need so that, I can pack all these n items, where the j th item has t j, is a one dimensional bin packing problem.

Problem can also be explained in different ways, I have several sticks of length capital T, now I need to make smaller sticks n number of sticks with each length equal to t j. How do I break these longer sticks to get these lengths and I have to use the smallest number of such long sticks with length T, how. What is the minimum number of the longer stick with length T that I require, through which I can break them and get these t j n lengths, where the j th smaller stick has a length t j, that is another way of looking at this.

The third way to look at this is, given n numbers t 1 to t n, form minimum number of groups such that, the sum of the numbers in each group is less than or equal to capital T, all these is the description of what is called one dimensional bin packing problem. So, the formulation of the one dimensional bin packing problem will be like this.

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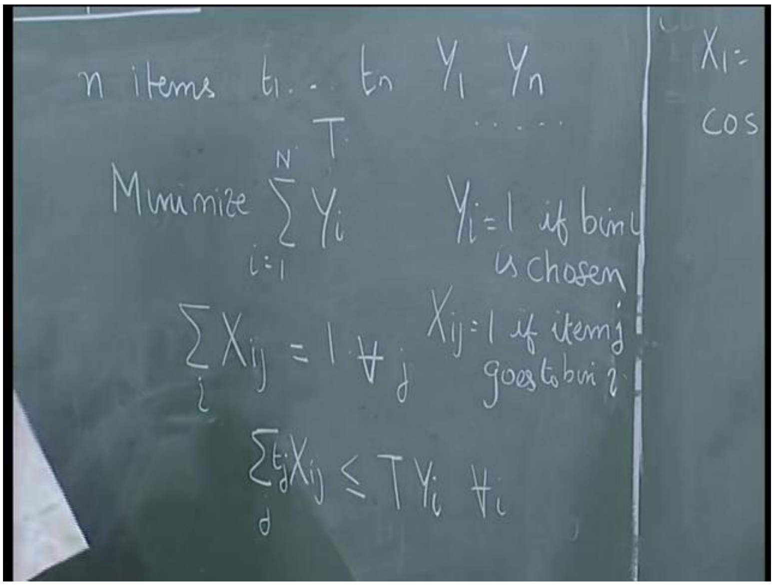

Now, we are going to assume that, there are n items with values t 1 to t n and there is a bin with length T, where we need to put it. Now, we want to minimize the number of bins, so we will assume now that, Y 1 to Y n represents the number of bins that we have. It is fairly obvious, that if each of these is smaller than T, which is not a bad assumption then we can take, there are since there are n items, we can take n bins and put each item into a bin.

So, that would give us a solution with n bins, but that is not what we want, we want to have the number of bins to be as small as possible. So, what we do is, we would assume right now that, we have n bins and then try to minimize the number of bins. So, we would say, minimize sigma Y i, i equal to 1 to n, Y i equal to 1 if bin i is chosen. So, by minimizing the number of sigma Y i, I essentially try and minimize the number of bins that I choose, so the objective function would be this.

Now, what are the constraints, each item should go to only one bin and before that, I need to introduce another variable X i j equal to 1, if item i goes to bin j. So, I will now say, sigma X i j equal to 1 summed over the bins j or let me look at it slightly differently, I have used subscript i for bin, so I would use subscript j for the item, so item j goes to bin i. So, summed over all the bins, X i j equal to 1 for every item j, so every item j will go to exactly one bin, which is given by this constraint.

Now, as far as the bin is concerned, sigma X i j summed over j, there are the items go to the particular bin and t j now is the length of the j th item, so t j X i j is the length associated with this bin. Now, this should be less than or equal to T that we have, which is the overall bin length and then this should be less than or equal to T into Y i for every i. So, Y i is equal to 1 when the i th bin is chosen and when the bin is chosen, it gives us a capacity of T and then whatever that goes into the bin, should be less than or equal to this, so this is one way of doing it.

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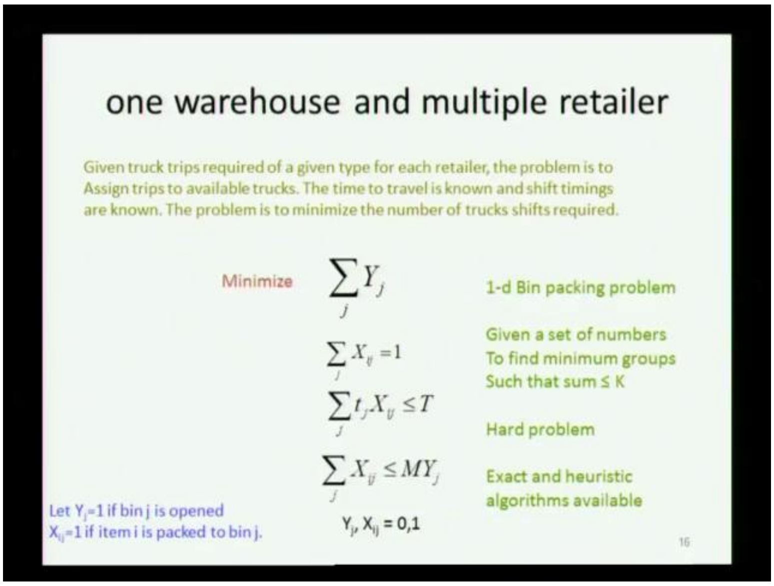

The other way of doing it is, what is shown in this slide and these are explicitly written as two different constraints. So, we could simply say, t j X j less than or equal to T which means that, we need to have a linking constraint between the X i j's and the Y i's.

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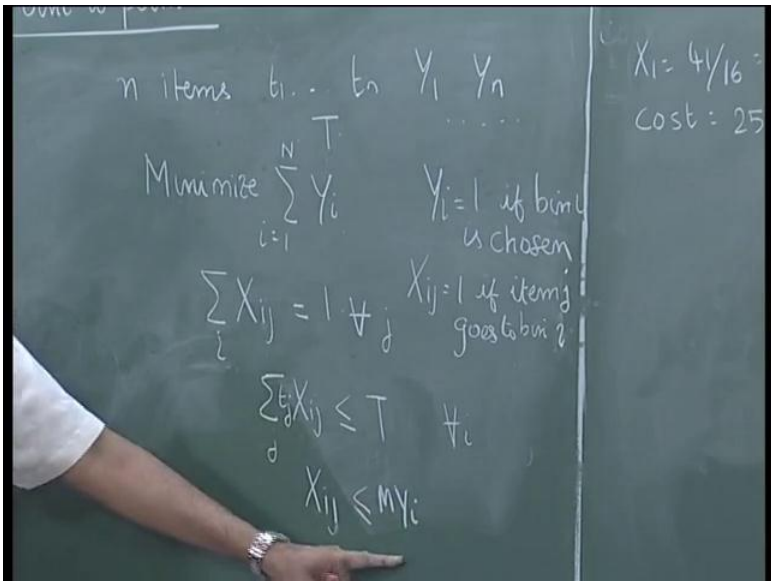

Now, we could also write X i j is less than or equal to M into Y i which means, only when the bin is chosen, I can put something into the bin. Now, this has more constraints, so this constraint can be eliminated by simply putting (Refer Slide Time: 48:00) T into Y i. Now, this is the formulation of the bin packing problem, so both Y i and X i j's are binaries. Now, one can solve this bin packing problem to try and get the optimum solution, but the way we have formulated for example, if we have 10 items then we are talking of...

We are now also defining Y i, so Y will take 10 values and X i j will take 100 values, so we will have 110 variables. And this formulation will be, this will give us 10 constraints, this would give us 10 constraints, so there will be 20 constraints. So, if we actually solve a problem with 10 items using this formulation, we would have 110 variables and 20 constraints, we can do that. Alternately, we could think in terms of some other heuristic or thumb rule based algorithms, which would say that, we may be able to pack this instead of a maximum of 10, we will be able to pack it in say, 4 or 5 bins.

Then, if we restrict the number of bins from 1 to 10, we can bring it down to 1 to 5 then the number of variables and constraints will reduce. So, it is customary to look at a heuristic solution to the problem then find out a certain number of bins and then solve the optimization by taking advantage of the fact that, a heuristic solution would actually result in lesser number of variables than constraints. So, we will look at some of the heuristic solutions to the bin packing problem in the next lecture.