Operation and supply chain management
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Lecture - 37
Transportation and Distribution Models

In this lecture, we look at Transportation and Distribution Models in a supply chain. We spend some time on understanding the broad issues related to transportation and distribution. And then we address a few mathematical models and provide solutions to them through numerical examples and illustrations. The supply chain decisions we have seen can be categorized into location decisions, production decisions, inventory decisions, transportation or distribution decisions. Information related decisions also can be added to this, so transportation and distribution decisions are among the most important decisions in planning and executing a supply chain.

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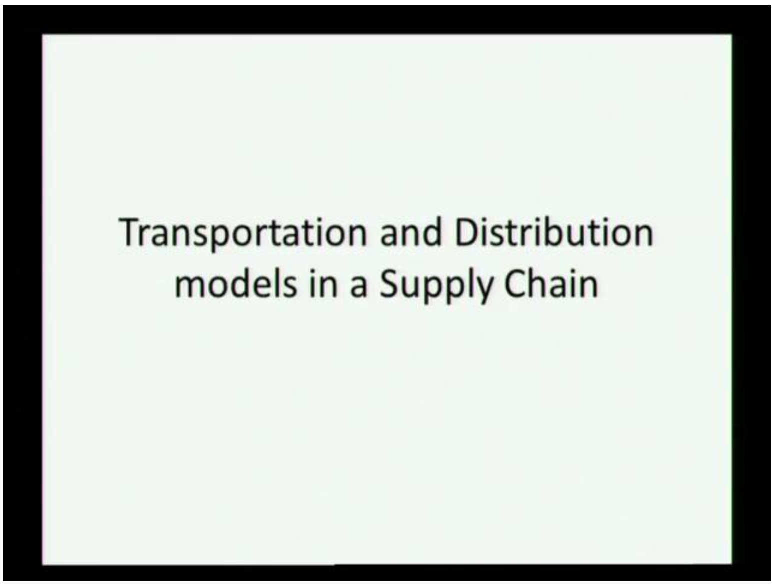

Distribution essentially talks about moving the manufactured items to the end customer, in an earlier lecture we defined supply chain as a network of facilities and distribution options that procure a set of parts or products, transform them into finished goods and distributes them to the customers. So, you can find the role and relevance of distribution in the supply chain by it is presence in the very basic definition of the supply chain.

Distribution is important, because for two reasons, one is the manufactured products should reach the customer in the right quantity at the right time.

While achieving customer service by distributing them in time, it is also important for the organization or the supply chain to ensure that, the cost of distribution is suitably managed and the cost of distribution is minimized or is as small as possible. So now, we will see some aspects of the costs related to distribution directly and indirectly, the most important cost is the vehicle related cost.

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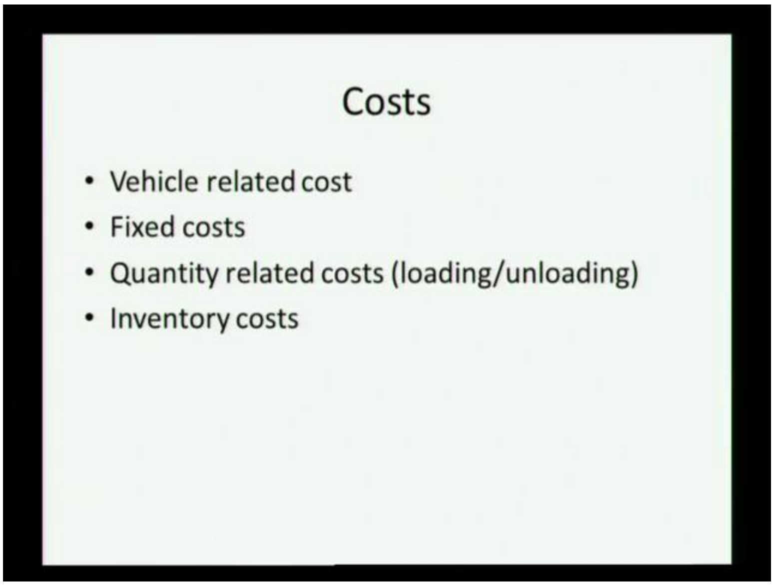

Because, it is manufactured, products are transported from one place to another, we definitely need a mode of transportation, which involves different kinds of vehicles. Most frequently used is the road transportation, though sea transportation and transportation by air and mixed modes of transportation are also prevalently used. Whatever be the mode of transportation, there is the vehicle that carries these items from one place to another and there is vehicle related cost.

Most of the times, this cost is proportional to the distance and there are times, there is also a fixed cost in addition to the vehicle related cost, which could be the cost of owning the fleet of vehicles or the cost of hiring the some of these vehicles or trip specific cost such as toll and so on. So, there are some fixed costs, which are also there in addition to the vehicle related cost. These costs are not proportional to the distance of movement of the products, there are also costs related to the quantity that is being transported.

The vehicles have fixed capacity and depending on the quantity that is being transported, we would be a requiring certain number of vehicles. So, the number of vehicles used will first depend on the quantity that is transported, it would also depend on the volume or the size of the product that is being transported. In addition, there are also loading and unloading costs, where the items have to be loaded into the vehicles or the trucks and they have to be unloaded, when these vehicles reach the customer destinations, so there are costs related to loading and unloading.

The fourth one that we see here is inventory costs and there is always the cost of holding the inventory at any point between the factory or the place where it is manufactured and the customer destination. At any point in between these two, the inventory is held and there is a cost of holding this inventory. We have already seen that, the cost of holding inventory can be treated as the interest that is to be paid on the money that is invested in making the product.

We have also seen some aspects of the tradeoff between inventory cost and distribution, if we centralize the distribution and keep it in the central warehouse, the safety stock or excess or extra inventory that is to be held will be less, but the transportation cost will be more. If we locate warehouses and distribution points nearer to the customers then the transportation cost will be less, but the cost of holding the extra inventory or buffer inventory will be more.

So, there is always this tradeoff between inventory cost and transportation cost, so these are some of the costs that are related to distribution. Now, depending on the nature of the product that is being transported, the distribution network has to be designed. Some of these products may require what is called point to point transportation which means, a truck or a vehicle will go from the warehouse to the customer and come back to the warehouse will not visit any other customer.

So, whatever is the customer demand, met by this way and most of these instances, either the product requires such a type of transportation or the demands would be closer to full truck loads, then the mode of distribution would be a point to point distribution. Other examples of modes of distribution that we will see, as we move along are called milk run and cross docking. When we see milk run, milk run is about transporting from a place, from a warehouse, the truck would visit multiple customers or more than one customer and deliver and then come back to the place where it start, so that is a different type of distribution system.

The third one which we will see is called cross docking, where we actually do not have a warehouse, items can come from the factory, come to a warehouse or come to a place, which does not physically store the items. When we say use the word warehouse, most of the times we mean that, items come, they stay physically in the warehouse and then they are moved from the warehouse to the customers or retailers. In cross docking, they do not stay in the warehouse, there is a place where trucks arrive from the factories and items are transported to other set of trucks in a central place.

And that place is where the cross docking takes place and the items that are transported will now move towards the retailers or the customers. So, the distribution mechanism does not store the inventory inside the warehouse, so like this we could have different types of distribution networks in practice. Some of the aspects are the ownership structure of the distribution network also has an impact; choice of the distribution network has an impact.

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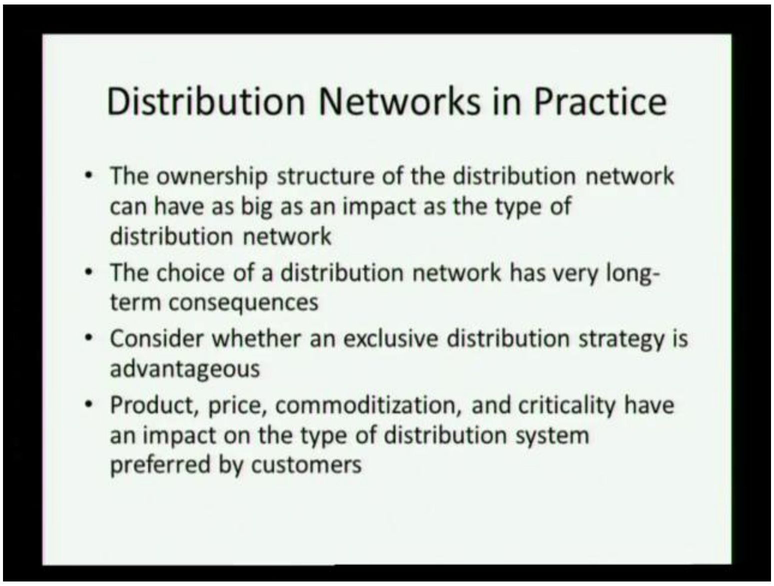

Also check whether distribution requires an exclusive strategy and last but not the least, product, price, commoditization and criticality also have an impact on the type of distribution system, that are preferred by the customers. So, this slide essentially tells us an adequate care has to be taken and a lot of thought has to be go in, when we decide the type of distribution network and that would depend on the customer, that would depend on the product and that would depend on the costs as well, here are some examples that I just now mentioned.

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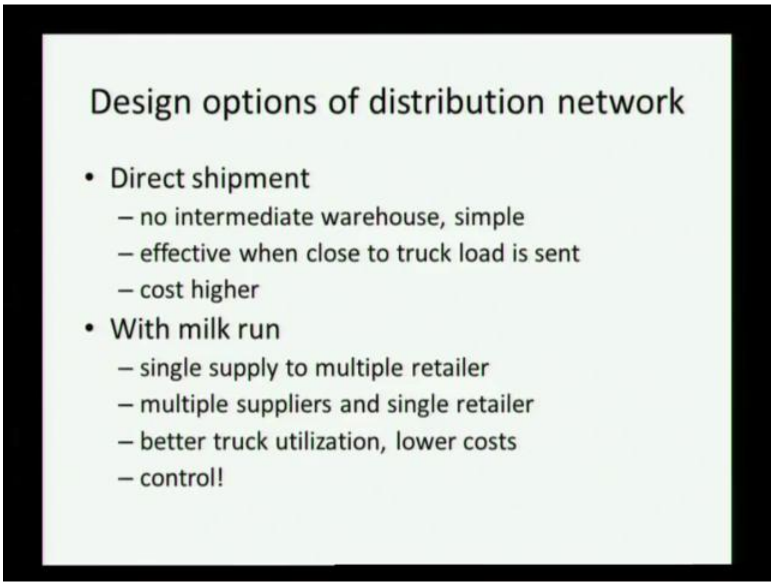

So, direct shipment it means, there is no intermediate warehouse, it goes directly from the factory to the customer. It is a very simple method, but it becomes effective when close to truck loads are set, if the demands are such that, we do not have close to truck load then direct shipment is not advantageous. Because, there is always a fixed cost of hiring or owning a truck and transporting items through the truck and that cost will increase.

And therefore, direct shipment will not be a cost saving proposition or it will not be economic, therefore the cost can be slightly higher. It becomes effective, only when we have close to truck load that is being sent from the factory directly to the retailer or the customer. In such cases, it is also advantageous to use milk run, therefore the truck will start from the factory, use a direct shipment mode and go to multiple customers and it will come back.

Here, we have better truck utilization, the cost become lower and there is a little more control. Next model is to ship via an intermediate distribution centre, so there is a distribution centre that stores the inventory and transfers the items and from there, it goes to the end customers. We could also have distribution centre across docking, where it is not exactly physically stored in the distribution centre, but cross docking takes place and this kind of a design can also be with or without milk run. So, if it has milk run then it goes to multiple retailers from a single supply or multiple supply to a single retailer and it will be designed that way.

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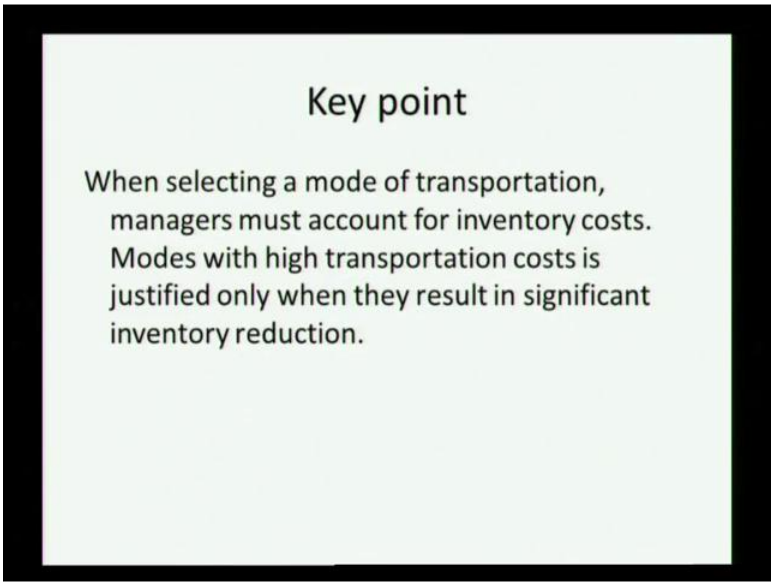

It is also important that, we should look at inventory costs. Distribution modes with higher transportation cost is justified only when they result in significant inventory reduction. We have already seen that, there is a strong relationship between the transportation cost and distribution cost. There will be situations and modes, where the transportation cost would appear lower, but that would result in inventory being distributed in several places, increasing the cost of holding the inventory.

And particularly when we consider the buffer or safety inventory that is required to take care of uncertainties, the cost of holding them will be very high. On the other hand, some modes will require a higher transportation cost and that would result in lesser inventory cost. So, a distribution mode or a channel that would result in minimum sum of transportation and inventory cost and is able to maintain the service level is the most desirable thing.

We have seen in an earlier lecture, that if we have N warehouses and if we centralized, we could save of the order of 1 minus 1 by root N if we aggregate. But, when we use those models, we should also look at the combined effect of the inventory cost as well as the distribution cost. It also leads to this inventory aggregation a very desirable thing. Aggregate means to add, so inventory aggregation is about adding the inventory or keeping the inventory together in a certain place.

So, inventory aggregation talks about storing in few warehouses which means, the transportation cost will be high and as I had mentioned, the cost of buffer inventory will be low when we use this model.

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Here again, we also have to look at tradeoff between inventory cost and facility cost, because when we setup few warehouses, the cost of setting up the warehouses has to be looked at and the capacity of these warehouses also has to be looked.

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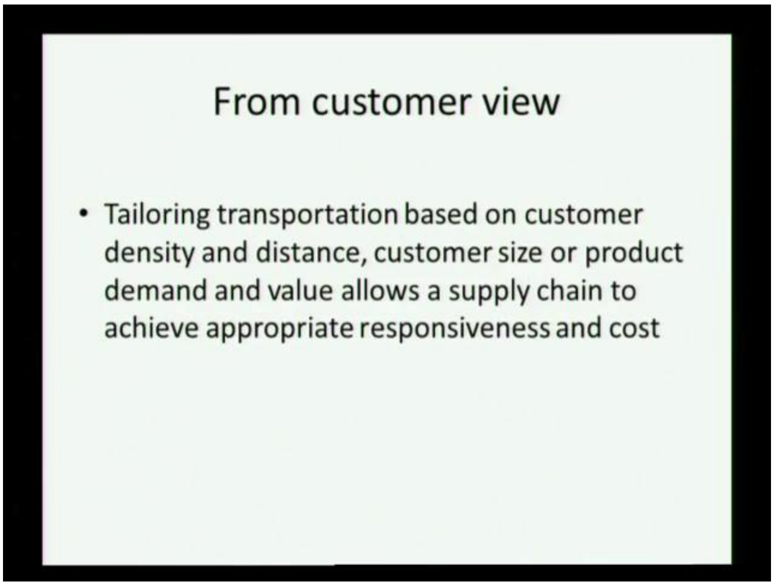

Now, from a customer's point of view, tailoring the transportation based on customer distance, size or product demand, allows a supply chain to achieve responsiveness and cost reduction. So, at the end of the day, we also have to make sure that, customer expectations are met, service levels are met and responsiveness which is a measure of customer expectation or ability to meet the customer expectation is achieved. So, all these factors are to be considered when distribution channels and transportation modes are being considered for distribution.

Finally, all these will have to be linked to the very basic definition of the supply chain, which talks about procurement, manufacturing or transforming them into finished products and distributing them. To be able to meet the customer demand and maintain service levels and to be able to do it at minimum cost is the very purpose of the supply chain. And the minimum cost is a cost that is aggregated or added considering all the players in the supply chain. So, with that in mind, we have to look at all aspects of decision making in the supply chain including distribution decisions.

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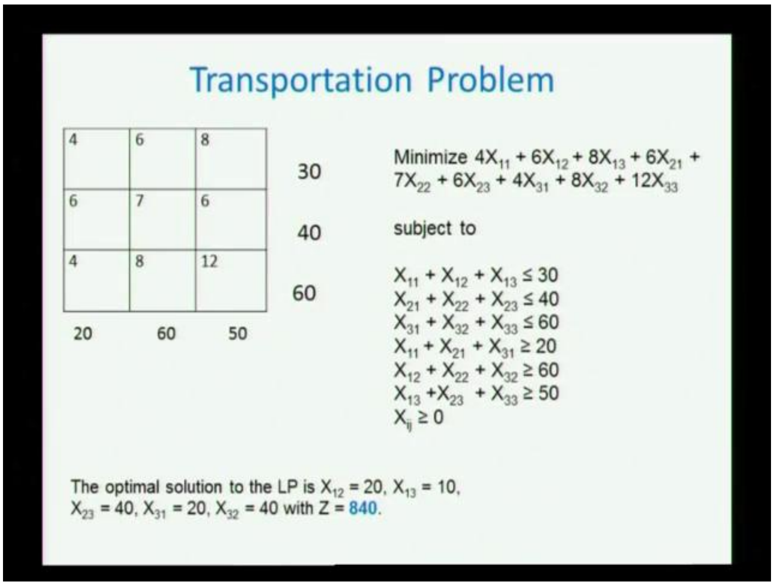

Now, we move to some mathematical models, which are available, which can be studied, which we are going look at, which help us in making distribution decisions in the context of the supply chain. Some of these are familiar and some of these we would have seen earlier in this course as well as in some other courses. But, I will address them for the sake of completion and for the importance that, they have in the context of distribution in the supply chain. The first problem that we will be looking at, is the transportation problem.

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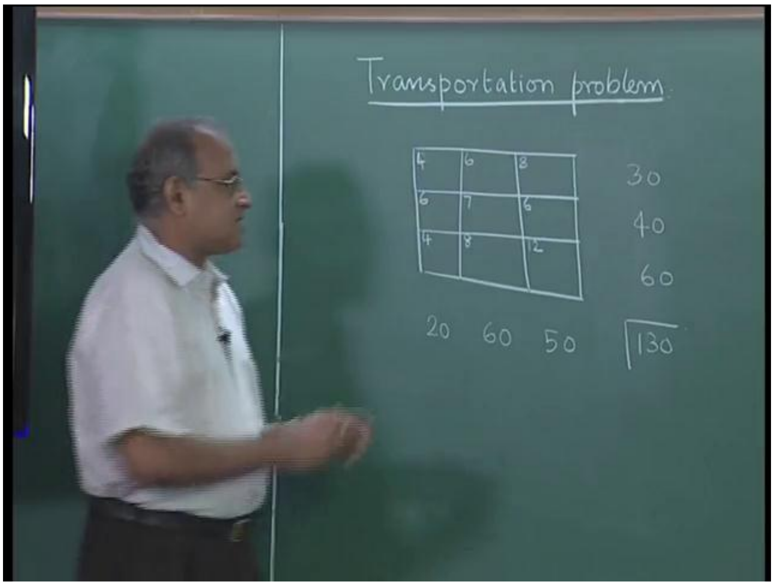

Now, we have seen this transportation problem earlier, we saw some aspects of transportation modeling in the previous lecture, where I had described the transportation problem as the first and the most basic problem, that we have to study in distribution. I have also explained transportation in the little more detail in the operations research course, where we have several lectures that address the transportation problem. I would still take an example now and explain some of these principles that are used in solving the transportation problem.

So, a numerical illustration to explain the transportation problem is like this, we explain the transportation problem using this example. Now, we can assume that, there are 3 supply points or we will assume that, these three are warehouses. There is a single item which is being transported, these are the requirement points or the destination points. In the context of supply chain, these can be seen as retailers or customers, now the single item has to be transported from these three warehouses to these three retailers or customers.

We have given an example, where the total supply is equal to the total demand and both are equal to 130. Now, this number represents what is called the unit cost of transportation which means, it represents the cost of transporting one item from this to this, is represented by this four. So, we assume that, the costs are proportional or linear, so if we transport 1 item, it is going cost 4, if we transport 20 items from this to this, it would cost as 80.

Therefore, we are trying to minimize the total cost of transportation such that, all these items from which are available here, are transported to the customers or the destinations.

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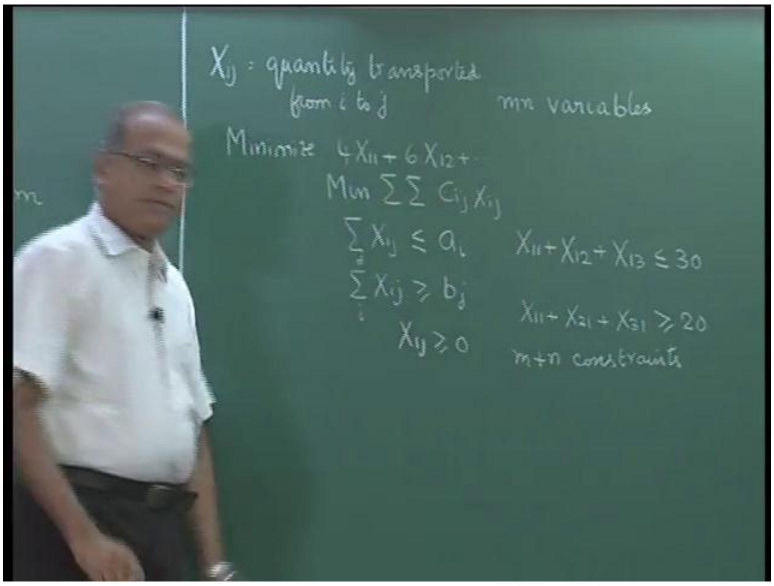

Now, the formulation of the transportation problem is as follows, X i j is the quantity transported from i to j. So, if X 1 1 is transported from 1 to 1, next 1 2 is transported from 1 to 2 and so on, the total cost of transportation will be 4 into X 1 1 plus 6 into X 1 2 plus 8 into X 1 3 and so on. So, we need to minimize this, so we minimize 4 into X 1 1 plus 6 into X 1 2 and so on, so we minimize 9 terms, which are there from X 1 1 to X 3 3.

And this is generally written as minimize sigma C i j X i j, i equal to 1 to 3, j equal to 1 to 3, there are 3 suppliers and 3 customers and it is not necessary that, we should have an equal number of suppliers and customers always. Now, we also have to make sure that, whatever goes out of this, does not exceed 30 40 and 60 respectively, and whatever that enter here, do not exceed 20 60 and 50 respectively. So, that gives us sigma X i j less than or equal to a i which means, X 1 1 plus X 2 1 or X 1 1 plus X 1 2 plus X 1 3 is less than or equal to 30.

So, X 1 1 plus X 1 2 plus X 1 3 less than or equal to 30, we will have two more constraints, one for this and one for this. The general expression is this, where a i is the availability in place i, similarly what is required, this is a minimum amount of items that are required. So, the amount that come into this is X 1 1 plus X 2 1 plus X 3 1, so we will have the other constraint of the type X 1 1 plus X 2 1 plus X 3 1 is greater than or equal to 20.

So, there are three constraints like this, one is for the first one, second one and the third one, this will be X 1 2 plus X 2 2 plus X 3 2 greater than or equal to 60 and so on. So, this in general is sigma X i j greater than or equal to b j summed over i here and summed over j here and X i j is greater than or equal to 0. So, this completes the mathematical formulation of the transportation problem, now this transportation problem with m supplied points and n demand or destination points, will have m into n variables.

And in this model, we will have m plus n constraints and this X i j also need not be integer, X i j can take continuous values. We have seen elsewhere in the other course in the operations research course that, even if we solve with X i j as continuous variables, we will get integer valued solutions as long as the a i and the b j's are integers, which is what we have here. In the earlier course we have also seen, how to solve a balanced transportation problem.

Now, this is a balanced transportation problem, because the total supply is 130 and that is equal to the total demand. So, if you have a solver, if you have an access to a solver then you can easily solve this problem as a linear programming problem, which has 9 variables as I mentioned, i equal to 1 to 3, j equal to 1 to 3 and 6 constraints. And an LP solution would straight away give us the quantities that can be or have to be transported from each of these i's to each of these j's.

Now, the transportation problem is a very important problem in operations research and it is a well known problem and there are better ways of actually solving it, which are computationally a little faster when we compare the LP way of solving it. So, if you have an access to a solver, you could easily solve it using the linear programming approach and the solver will give the solution very quickly for a problem of this size, which has 9 variables and 6 constraints.

But, if we want to study other ways through which we solve this, assuming that we do not have access to a solver. Now, there are very efficient ways of solving it, which would be in terms of computational time and effort, a little superior to solving the linear programming. A more detailed version of this is given elsewhere in the other course, but I will very quickly out line one method to solve this problem.

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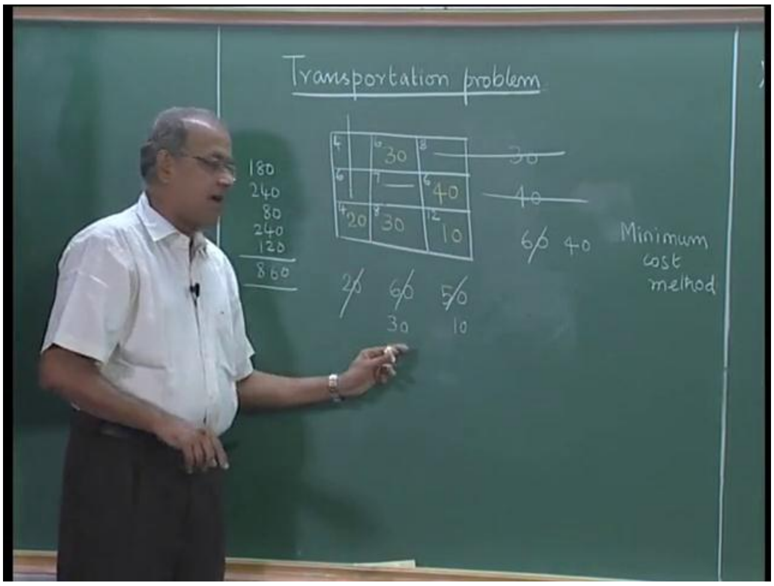

So, what we can do is, we would assume that, when we have to transport, after all we are interested in minimizing the total cost of transportation, so what we will do, is to try and transport according to the minimum cost. So, we first find out, what is the minimum cost of transportation, so that can happen here as well as here, so we could take any one of them. So, at this if we consider this, the demand 20 and the supply is 60, so we will transport only 20 here, which to meet the entire demand.

So, when we do that, this demand is completely met, this is met and this 60, the availability becomes 40, now we look at the next smallest cost, which happens to be here or here. So, let us take this one, we can take any one of them, the supply is 40 and the requirement is 50, so we can take all the 40 here to meet this requirement, this goes and this becomes 10. Now, we take the next minimum, which happens to be here, supply is 30, demand is 60, so we provide 30 here, this is completely exhausted and then this becomes 30.

And we realize that, we have to use this 30 and this 10 so that, the supplies and demands are met. So now, if we add this from row wise, we will get 30 40 and 60, and column wise we have met. Now, this is possible only if the problem is balanced which means, total supply is equal to the total demand. So, we now have a solution and at the moment we do not know, whether it is the optimum or the best solution, but it is expected to be a good solution, because we have made allocations progressively based on cost, by considering the least cost one first.

So, this method has a generic name, it is called minimum cost method, now the total cost of transportation would be 6 into 30 is 180 240 80 240 and 120 with the total cost of 860. Now, there are some interesting things about it, we also realize that, every time we make an allocation here, we are either completely using up what is available here or completely meeting what is required here. Because, every time we update the supply and the demand, therefore every time we make an allocation, we are either completely utilizing what is available or completely meeting what is required.

Therefore, we would be able to do it in, there are 3 supplies and 3 demands, so using 5 allocations, generally we will be able to do it, unless one allocation does both of utilizing the entire supply as well as meeting the entire demand. Such problems are called degenerate problems and how to handle degenerate problems, I have explained in the operations research course. So, we will now look at a solution, which will have 5 allocations if there are 3 plus 3 items.

We will be able to do it in 5 allocations, because the problem is balanced and the last allocation will automatically exhaust the supply as well as meet the demand. So, generally if we have m supplies and n destinations, we talk of m plus n minus 1 allocations, with which we will be able to complete this. So, the first stage of this solution is a minimum cost method that I have spoken about, now this gives us a solution with the cost of 860.

Many times when we solve the transportation problem by hand as I have illustrated here, we would be using another method called Vogel's approximation method, which on an average, can give slightly better solutions than the minimum cost method. I have also explained the Vogel's approximation method in much more detail in the course on operations research. But, right now for the purpose of this discussion, we would use the minimum cost solution and then try and see, whether this solution can be made better or can be improved, now we do that in a in a different way.

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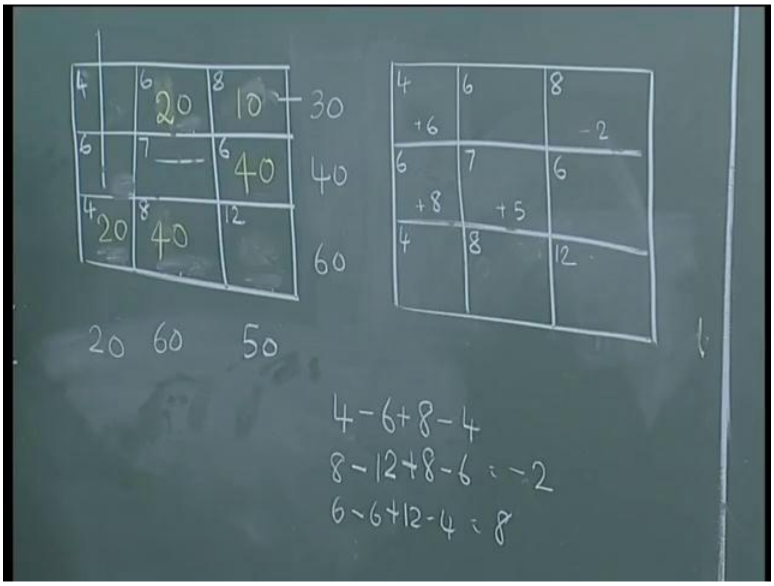

Now, if this solution is not the one that has the minimum cost then there is some other solution that has a cost lower than this and such a solution is expected to have at least one different allocation. So, right now there are 4 places, where there is no allocation, so we check, if any one of the presently unallocated position can actually get an allocation. So, if we try this, so we have 30 here, so if we try to put 1 to make an allocation here then we have to reduce 1 here to meet this requirement of 30 40 60 20 60 and 50.

So, if we put a plus 1 here then I have to put a minus 1 to meet this then I have to put a plus 1 to meet this and I have to put a minus 1 to meet this as well as to meet this, this is called a loop. So, when I put a plus 1 here then my cost will go up by 4, it will reduce by 6, because I have taken away 1, it will go up by 1 by 8, because of this plus 1 and it will go with the minus 4, because I have removed 1. So, the net cost of putting 1 here is, 4 minus 6, which is minus 2 plus 8, 10 minus 4, 6, so it is plus 6 and by putting 1 here, my cost is only going to increase and it is not going to decrease.

In a similar manner, if I try and put 1 here in this position, so I will get a plus 1 minus 1 plus 1 and minus 1. So, the cost will be a plus 8 minus 12 plus 8 minus 6 and this is minus 2, so there is a gain by putting a plus 1 here, so 8 minus 12 plus 8 minus 12, so there is a gain in this, so I get a minus 2. So now, when I try to put a plus 1 in the other unallocated position, which is this position, so I will get a plus 1 minus 1 plus 1 minus 1, so I will get a plus 6 minus 6 plus 12 minus 4, which is equal to 8.

So, the cost will increase and if I put 1 here, it will be plus 7 minus 6 plus 12 minus 8, which will again be positive. So, 7 minus 6 1, 1 plus 12, 13 minus 8 is plus 5. So now, we observe that, putting something here, adding 1 here can reduce the cost, so we try to add as much as we can so that, the cost will come down. So, the maximum that we can do here is 10, so that this will become 0, this will become 40 and this will become 20.

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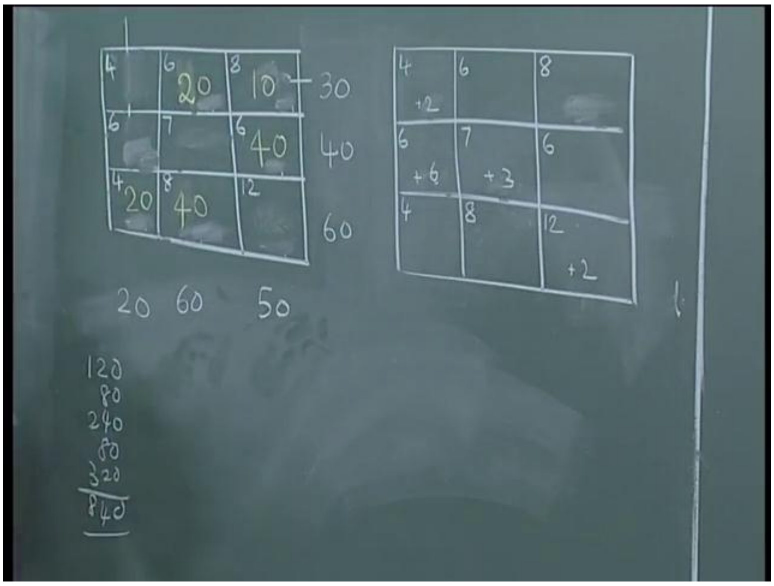

Now we realize that, the total cost of this solution is 120, 6 into 20 plus 80 plus 240 plus 80 plus 320, the cost is 840 and that 840 comes from the fact that, we found out that the gain is 2 per unit and then we have added 10 units here, so the gain is 20 and therefore, 860 becomes 840. Now, this is the next solution with the cost of 840 and now we need to check, whether this solution can be improved further, so we try and add 1 to all unallocated positions.

So, when I do this here, I get plus 4 minus 6 plus 8 minus 4, so 4 minus 6 is minus 2, plus 8 10, 10 minus 4 is 6, 4 minus 6 is minus 2, minus 2 plus 8 is 6, 6 minus 4 is 2 which is positive. So, I will get a plus 2 here, now when I put something here, this will become a plus 1, this will become minus 1 plus 1 minus 1 plus 1 minus 1 and the loop is closed, so 6 minus 6 0, 0 plus 8 is 8, 8 minus 6 2, 2 plus 8 10, 10 minus 4 is 6. So, I get plus 6, when I put a plus 1 here, this is the loop, 7 minus 6 is 1, 1 plus 8, 9, 9 minus 6 is 3 and when I put here, this is the loop, 12 minus 8 4, 4 plus 6 10, 10 minus 8, 2.

So, all of them will only increase the cost, the cost does not decrease, so this solution is the best solution or the optimum solution. The method which we have just now seen, which took the previous solution from the minimum cost method to the optimum is called stepping stone method and the stepping stone method will provide the optimum solution from a starting solution. The minimum cost method solution is called a starting solution, it is also a basic feasible solution and the stepping stone method can help us to get the optimum solution.

Many times when transportation problems are solved by hand or even when efficient programs are written, another method called the modified distribution method or the u v method is usually used, instead of the stepping stone method. I have explained the u v method in detail in the operations research course. Now, for the sake of illustrating this from a point of view of distribution, I have explained it using a combination of minimum cost method and the stepping stone method.

Both the minimum cost method and the stepping stone method are very intuitive methods, that give us good starting solutions as well as finds the best solution from the starting solution. Other methods such as Vogel’s approximation and u v method, which are widely used, are also intuitive. But, they are also computationally a little intensive and a little different and might also require some principles of operations research and duality, etcetera to explain how they work.

Now, for the purpose of making it simpler and for illustrating it, I have used a combination of the minimum cost method as well as the stepping stone method. So, this is how we solve the transportation problem optimally, one of the ways of solving it optimally. And this is the final solution, where this 30 that is present here, 20 will go to this customer, 10 will go to this customer, this 40 will entirely go to this customer and out of this 60, 20 will go here and 40 will go here.

So, both these people will get their items from two suppliers, this person will get it from these two, the other one will get it from these two, this person will get only from one person. Similarly, this person will be supplying to two people, this person supplies to one person, this person supplies to two. So, this is how the basic transportation problem works, some of the important aspects in the transportation problem is, the assumptions that there is a unit cost of transportation and that the total cost of transportation is actually a product of the unit cost and the quantity.

For example, when we computed this 840, we said 6 into 20 plus 8 into 10 and so on, so if 20 units are transported, the cost is 120. Now, this linearity, proportionality, additivity, assumption makes the problem a linear programming problem here, but in practice we also observe that, the actual cost of transportation need not be the per unit cost multiplied by the quantity.

When we actually look at truck loads that move, now the truck has a certain capacity and irrespective of what quantity that goes into the truck, as long as it is within the capacity of the truck then there is the cost of transportation, is actually the distance between the source and destination multiplied by the cost of traveling a unit distance and need not be dependent on the quantity that is being transported. In addition, the truck would have a fixed cost that would be there for a hiring the truck and utilizing it or the cost associated with owning the truck and if there are some tolls then additional cost is being paid.

So, when we looked at the cost structure, which is very different from the cost structure that we are looking at here, that would lead us to different problems, which can be solved using techniques and principles of operational research. We would be seeing some of them, at the same time we would also be extending this to multiple stages.

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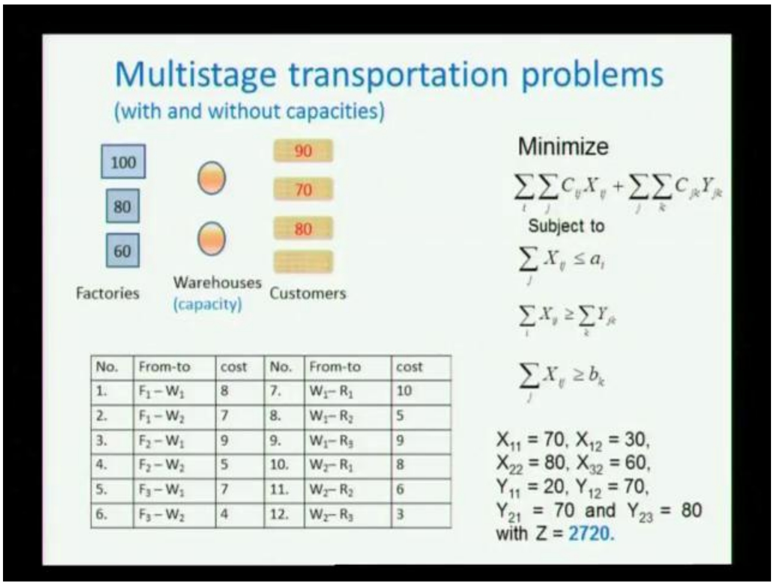

So, we will next see the multistage transportation problem which is here. So now, we explain a multistage transportation problem, which is quite common in the context of supply chain.

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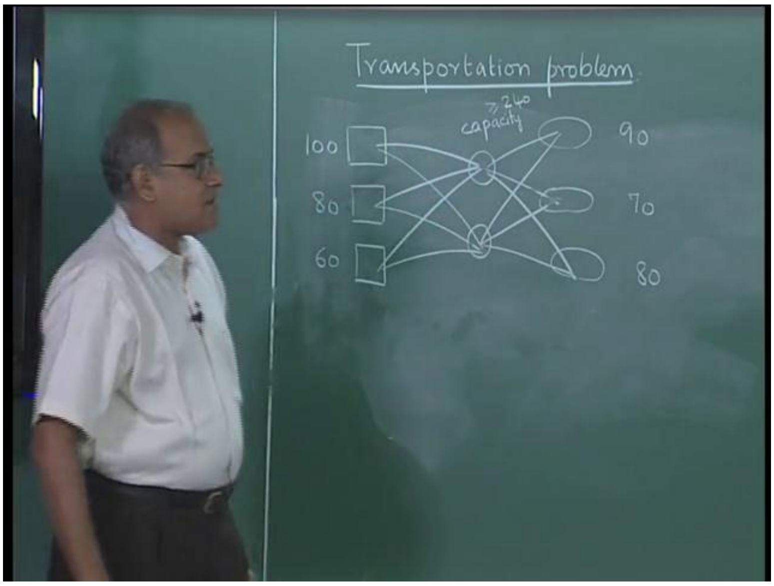

So, we could think in terms of 3 supplies or 3 factories, for the purpose of illustration, we have 2 warehouses or 2 intermediate points, where they are stored. And then for the sake of illustration, we have 3 customers, the supplies are 100 80 and 60, and the requirements are 90 70 and 80. Now, we are going to assume that, the items from here move like this, so if something goes from here to here, it would either go through this or it would go through this.

So, it would go through one of the warehouses and then it would reach the ultimate customer. The first model is, we are not going to restrict the capacity of the warehouse, we are going to assume, that the capacity of the warehouse is infinity. Or as far as this problem is concerned, the capacity of the warehouse is greater than or equal to 240 which means, we are allowing the possibility of all the items to go through this and then go there or the other possibility of all the items going through the second warehouse and then reaching the customer.

Now, when the capacity of these warehouses is less than 240 then it mandates that, some of the items will have to go through the second warehouse. So, the first model that we will see, will not consider capacity, it will just leave it like this. Now, we could again thing in terms of a similar formulation to the one that we have seen here. So, then we need to look at defining two sets of variables.

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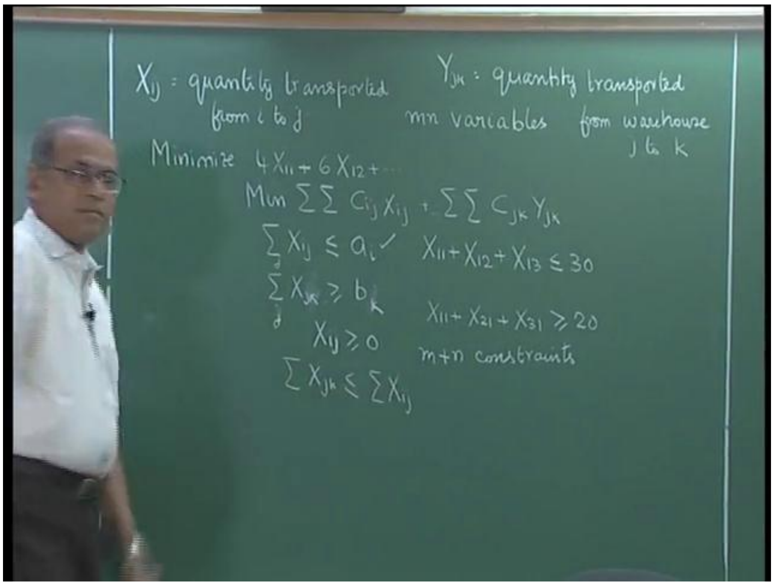

So, we could think in terms of C i j X i j plus double sigma C j k y j k, now here Y j k is the quantity transported, j to k. So, here j is the intermediate storage which we call as the warehouse, i is the production facility which could be a factory and k are the customers who are here. So, we solve a two stage transportation problem, transportation in the first stage involves the X i j variables, transportation in the second stage would involve the Y j k variables.

Now, the constraints also become a little different from this, so whatever goes out of this should be less than or equal to the capacity. So, X i j less than or equal to a i which is fine, whatever that reaches here should be greater than or equal to the requirements. So, X j k summed over j should be greater than or equal to b k, now k stands for the customer, b k is the demand of the customer, so whatever reaches the customer, is greater than or equal to b k.

Now, in the event of a balanced problem that is, total supply is 240, total demand is 240, exactly 90 70 and 80 will reach and all the 100s 80 and 60 will be exhausted. If it is not a balanced problem then depending on, if this is higher than this then all the demand will be met. Even in that case, more than the demand will not be supplied, because the cost will go up. It is a cost minimization problem, as long as these costs are greater than or equal to 0, there will always be a solution, where only the exact requirement is met and some of them will not move from this.

If we have a situation where the total supply is less than the total demand then all the total supplies will be exhausted, some of them will not get their demands. So, in such cases, we have to carefully define the problem, particularly if there is a situation, where the total supply is less than the total demand, if we put an explicit condition that it is greater than or equal to b k then it will show infeasibility, because it will not be able to meet all the demands.

So, this formulation has to be written accordingly, particularly for unbalanced problems, so it is actually customary to make it balanced by adding a dummy. If the total supply is less than the total demand then it is customary to add a dummy here and create a notional one so that, we balance it and then at the end say, that whatever goes out of it, actually does not go out of it. So, when we formulate it as a linear programming in this case, one needs to be a little careful to give due consideration to unbalanced problems.

Because, there can be situations, where total supply can be less than or equal to the total demand. So, there is also another set of another constrains which links this, because as far as this is concerned, whatever that goes out, should be less than or equal to whatever that comes in. So, you have a set of intermediate constraints, which will tell you that, for all these intermediate facilities, sigma X j k whatever that goes out, is less than or equal to whatever that comes in, sigma X i j (Refer Slide Time: 45:37).

So, y j k is what goes out, so in this definition y j k is what goes out, so both X i j and y j k are greater than or equal to 0. So, this two stage problem the way we have defined here, so there will be 6 X i j variables, there will be 6 y j k variables and then there will be 3 constraints here, there will be 3 constraints here, there will be 2 constraints here, so there will be 8 constraints and 12 variables. One can solve the linear programming formulation this way and then try to get the optimal solution.

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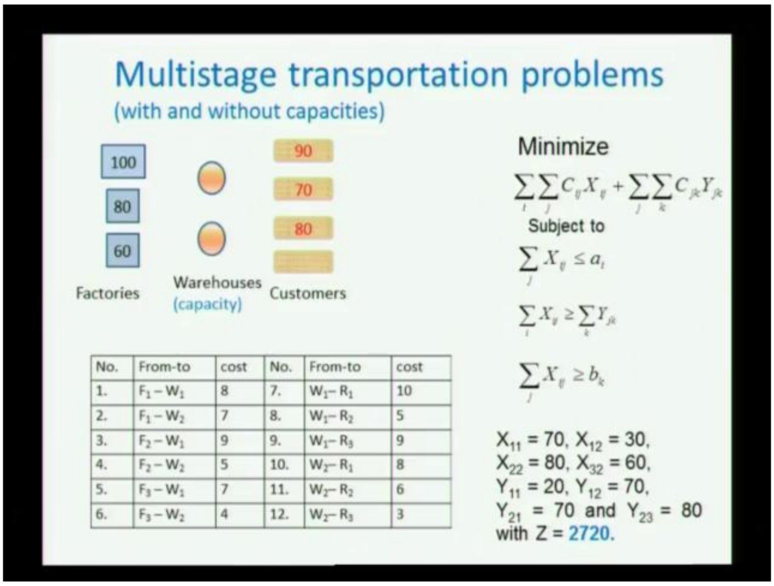

The optimal solution is shown here with X 1 1 equal to 70 and X 1 2 equal to 30, so X 1 1 will be 70, X 1 2 will be 30 from here, 2 2 is 80, 3 2 is 60.

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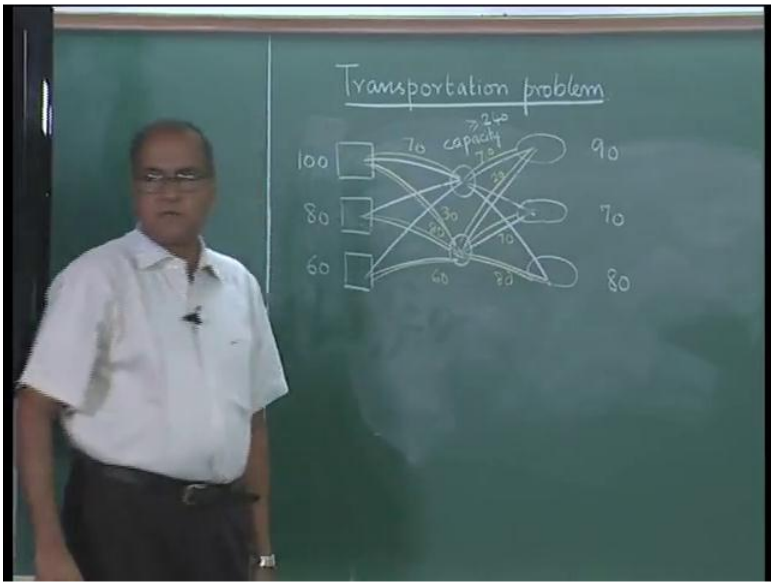

So, 2 to 2 is 80, 3 to 2 is 60, (Refer Slide Time: 46:44) y 1 1 is 20, y 2 1 is 70, and 2 3 is 80. So, this is 150, so 80 plus 30, 110 170, so this is 20, so this should be 70 and this should be 20. So, this is the final solution the 70 that comes here goes here, 30 comes here, 80 comes here, 60 would come here and they get distributed as 20 70 and 80 respectively. So, if we have capacity constraints then we put another capacity restriction on this.

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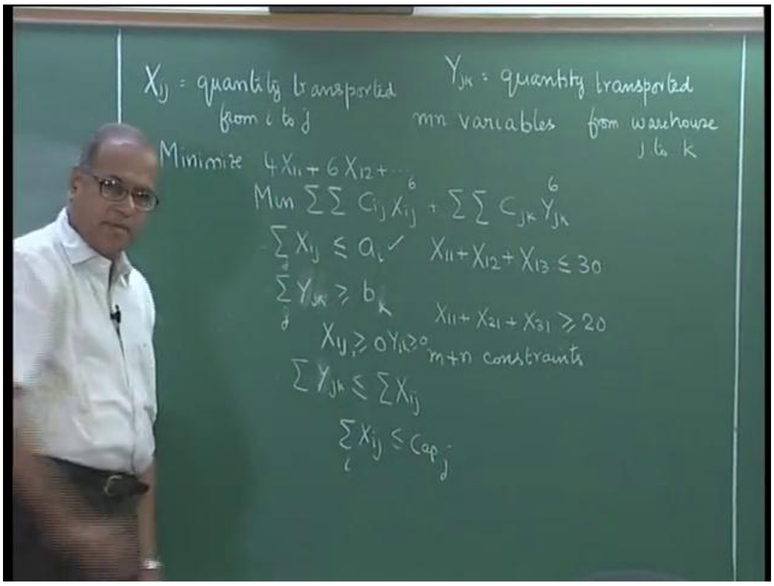

And that will tell us that, sigma X i j is less than or equal to capacity of j summed over i and the solution can change depending on the capacity constraint that we have introduced. So, one could solve the linear programming formulation here, it would still be an LP, one could do something else, if there is no capacity here. Then it is only true that, if I have to transport this from here to this or this or this, I will route it through an intermediate warehouse. It could be any one of them, I could route it through the intermediate warehouse such that, the distance travelled is minimized.

So, what we can actually do in this case, is to try and find the shortest path from each of these three to each of these three, passing through these nodes. So, try to find the shortest path and the shortest distance, so if we find the shortest path and the shortest distance, we will now have the 9 distances.

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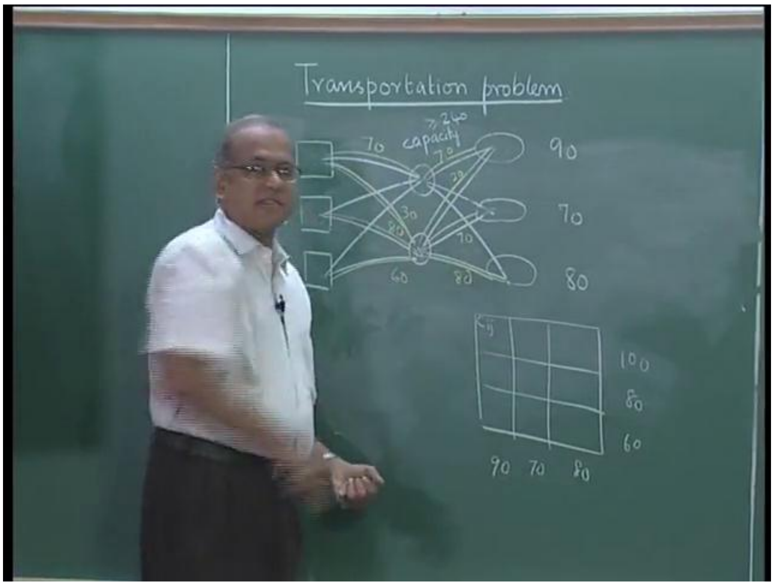

And we can now directly formulate a 3 by 3 transportation problem with 100 80 60 and 90 70 80 with the value of C i j representing the shortage distance from this. So, 1 to 1 what is the shortage distance, either this distance or this distance, so the minimum of them we can write here. So, if we actually compute this and then a multistage transportation problem can be looked upon as a single stage transportation problem, provided there are no capacities in the intermediate stages.

So, that is how we solve a typical multistage transportation problem, here we have shown the linear programming formulation to the multistage transportation problem.

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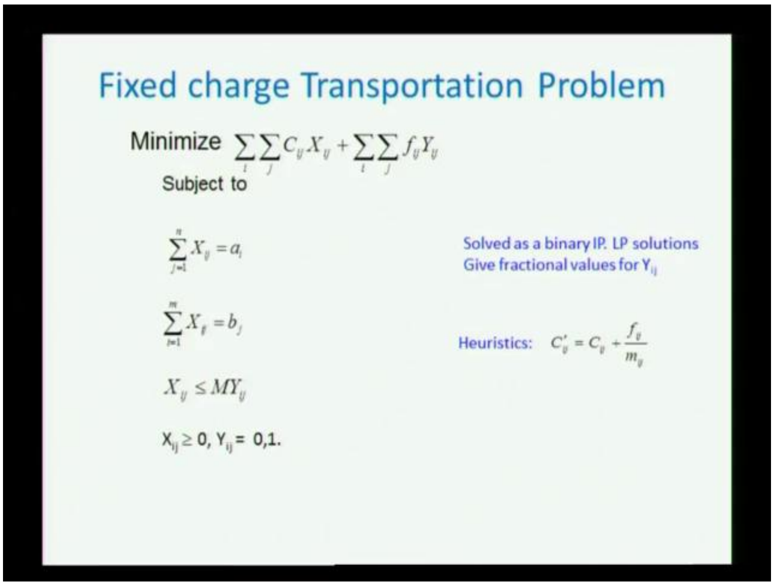

The next problem that we will look at is called the fixed charge transportation problem. We have already seen the formulation of the fix charge transportation problem in the previous lecture, now we will explain the fixed charge transportation problem.

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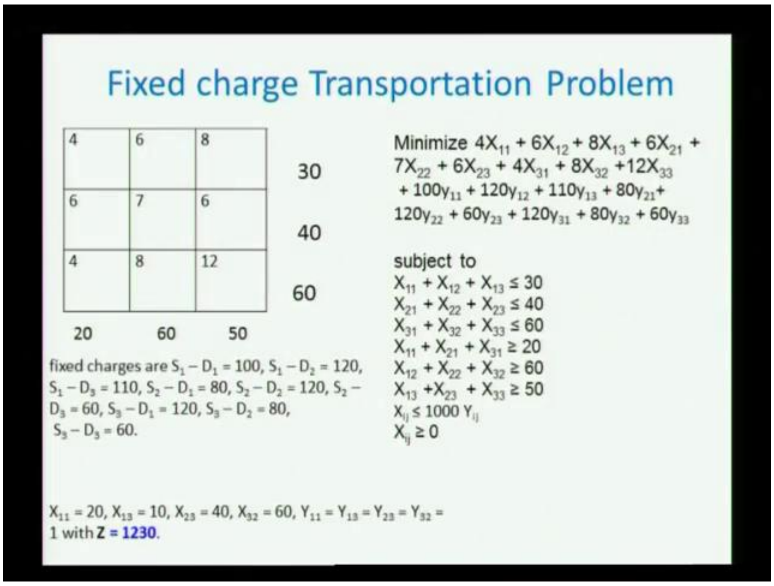

And then use a numerical illustration to explain the fix charge transportation problem and we will see this in the next lecture.