Operations and Supply Chain Management
Prof. G. Srinivasan
Department of Management Studies
Indian Institute of Technology, Madras

Lecture - 36
Location Problems

In this lecture, we continue the discussion on supply chain management. In the previous lecture, we introduced two types of supply chain, which are called the efficient supply chain and the responsive supply chain. The efficient supply chain is to be designed for what are called functional products and the responsive supply chain is for the innovative products. We have also seen some ways, by which we can classify products into functional and innovative. And we also mentioned that the efficient supply chain for functional products concentrates more on cost minimization, while the responsive supply chain for innovative products would concentrate a lot more on service and delivery.

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Now, we will see some aspects of, how we design efficient supply chains or what are the things that go into creating an efficient supply chain. Some of these are continuous replenishment programs using EDI links for information sharing, capturing point of sale data for updating the forecast, invest in supply chain partnerships on the inbound and out bound side. Integrate the MRP or planning systems to benefit from improved data visibility and develop robust inventory control mechanisms to accurately fix reorder points and order levels.

We saw all these five points in the previous lecture and the common thread that goes along these five points is the reduction in inventory. Now, we will quickly see how that happens, now when we talk of continuous replenishment program through information sharing, we immediately talk about reducing the inventory in the partnering entities. Periodic replenishment systems involve higher inventory than continuous replenishment system.

And when we have a continuous replenishment system using information sharing, the buffer also will come down. Accurately updating the forecast would immediately help in reducing the variation in the forecast and therefore, reduce the extent of extra inventory that organizations would hold. Invest in partnerships on the inbound and outbound would also reduce both transportation cost as well as inventory cost. And integrating the various systems would automatically reduce the buffers in terms of inventory that various entities in the supply chain would have.

So, we can now see the thread in all these points, which is the reduction in the inventory and reduction in the transportation cost, which would reduce the total cost. Now, this fits into our description of efficient supply chains concentrating more on cost minimization. Now, we will see how responsive supply chains react and what are the things we need to do there - accept uncertainty in demand, and large forecasting errors as reality. We have already seen that, responsive chains are for innovative products and innovative products show higher variation or uncertainty in demand and also the forecast for these cannot be accurate leading to higher forecast errors.

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So, we need to accept this and devise systems, which will take care of these, which also implies that, there would be slightly higher inventory in these kind of systems to the extent possible. And because the profits margins are higher, the overall profits can be increased, even if the amount of inventory is a little high. But, what is also needed here is, improve responsiveness by cutting down lead times, use postponement strategies and delayed product differentiation strategies and deploy modular design and product platform strategies.

Now, in all these we will realize that, the focus is more on reducing the time taken to produce as well as to be able to produce a variety, both postponement and delayed product differentiation are ways, by which... If there are variety of products, the variety actually happens towards the end of production process and for most of the production process, the product would be the same upto the point, where the product differentiates. Now, the advantage of doing that is to reduce the inventory that is there, so delayed differentiation that would mean, the inventory upto the point, where the products looks similar.

The inventories are smaller and the inventories change when the product becomes different. But, more than the change of inventory, it is also easy to manage or bring down the time to produce through standardization, because the product would be the same upto a very significant point of the production cycle. So, the emphasis on the responsive supply chain design is more on cutting down the lead time or reducing the time taken to bring the product to the customer.

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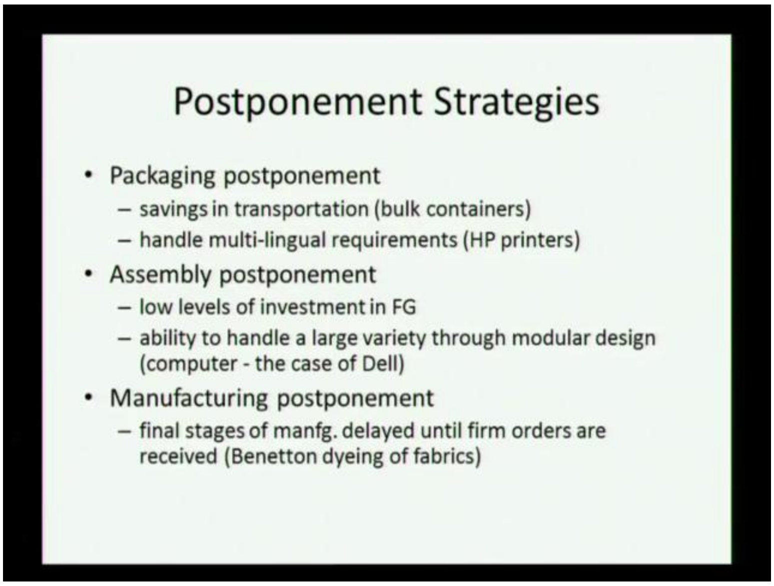

Postponement strategies, packaging postponement is saving in transportation and to be able to handle different requirements. Some examples are also given, postponing the assembly, postponing is trying to do it in the end and not to try and do it in the middle. And manufacturing postponement talks about, unless the firm orders are received, delay the manufacturing so that, the amount of inventory in the system is also reduced.

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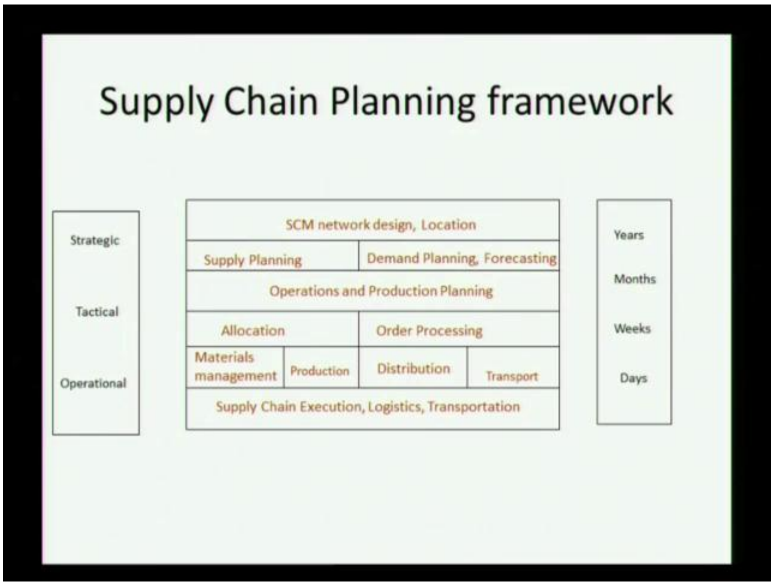

We come to the last slide in this presentation, which is called supply chain planning framework, where we show the strategic tactical and operational decisions in the supply chain through a planning framework. And we have already seen several aspects of this, supply chain location and network design come under strategic decisions, we have seen this in an earlier slide. Supply planning, demand planning, forecasting, operations planning are all tactical decisions, while materials management, distribution, execution, logistics, they all come under operational decisions.

We also know that, the strategic decisions are long term decisions, whose effect of strategic decisions are there for several years. Tactical decisions are medium term, they are there for several months or several weeks, while operational decisions are short term and they are there for several days of activity. So, when we start designing a supply chain and we start talking of organizations coming together, now these are the various aspects that have to be looked at.

The location decisions which are strategic in nature, the production decisions which are tactical in nature, the inventory and distribution decisions, some of the production decisions are also operational in nature, inventory decisions are both tactical and operational in nature and distribution decisions are more operational in nature. And the information decision or information sharing or information technology brings all these aspects together and binds them. So that, both inventories that exist in the system and time taken to do an activity come down.

If we realize that, the organization would make more profit by reducing these two dimensions, which is the extent of inventory in the system and the time it takes to make the final product and get it to the customer. So, these two aspects are crucial and these two have to be brought down so that, the overall cost in the supply chain comes down. There are differences that we have seen between the two types of the supply chain, one primarily concentrating on cost reduction and the other concentrating primarily on delivery and reduced time to market.

In spite of these differences, if you see that if we bring these two costs together, the cost of inventory and the loss of profit by delaying the time taken to produce or the gain by reducing the time taken to produce, organizations can benefit and make a lot of profit through proper relationship, which is the essence of supply chain management. We just go back to one of the slides that we have seen earlier, which talks about the major supply chain decisions, which I have indicated five of them.

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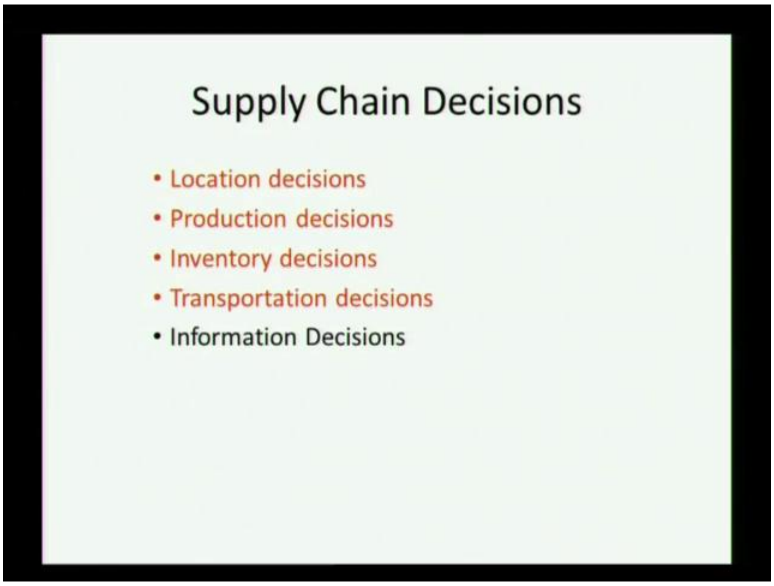

Location decisions, production decisions, inventory decisions, transportation decisions and information decisions. Now, we go back and see, how much of these we have covered in the course and what are the things that are remaining to be done in this course. When we started this course with a lot of emphasis on operations management, as I mentioned we have spent a lot of time on production decisions. Starting from forecasting, understanding forecasting completely to the extent possible, understanding the various models in forecasting.

And then we looked at aggregate planning, we saw several models for aggregate planning and then much later we did scheduling and sequencing, we did little bit of MRP, so all of these come under production decisions. We have seen several inventory models for what are called cycle inventory and safety inventory which means, models that relate the order quantities and models that relate the reorder levels and safety stocks. So, we have spent some time covering the inventory decisions and we have spent some time addressing the production decisions in this course.

Very briefly, we have looked at some aspects of location decisions and we are yet to look at some aspects of transportation decisions and the role of information technology on the supply chain. So, what we will do now is revisit some of the location decisions that we have seen and continue to discuss some location models that talk about locating facilities and warehouses. And then we will move on to transportation decisions and models for transportation and then we will look at information systems and the role of information system in effectively making the supply chain performance better.

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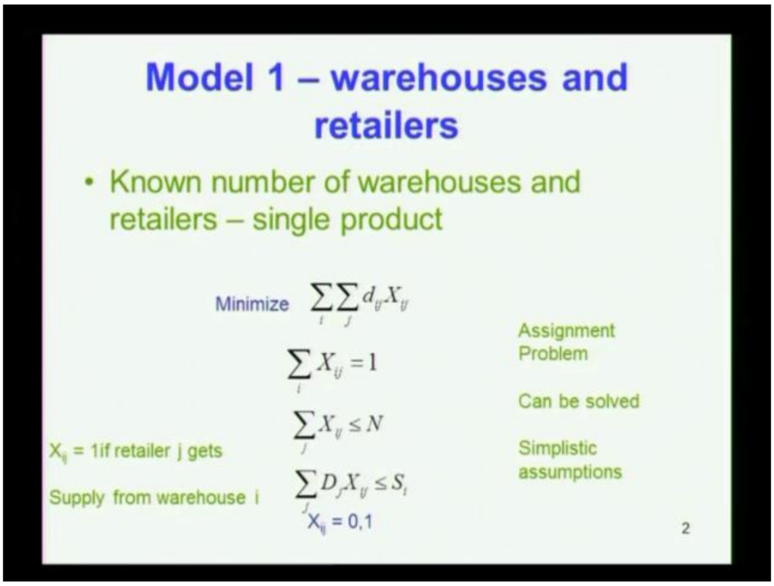

The models that we have actually have seen, related to location earlier in this course are three models that we have seen.

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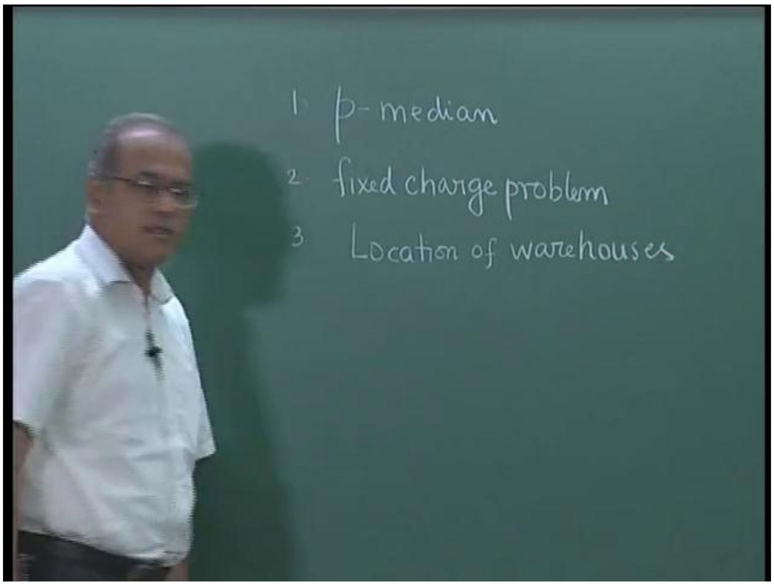

So, we have seen the p median model, we have seen the fixed charge problem and we have seen location of multiple facilities or location of warehouses, earlier in this course through the location models. Now, we will see few more location models and try to link them with this.

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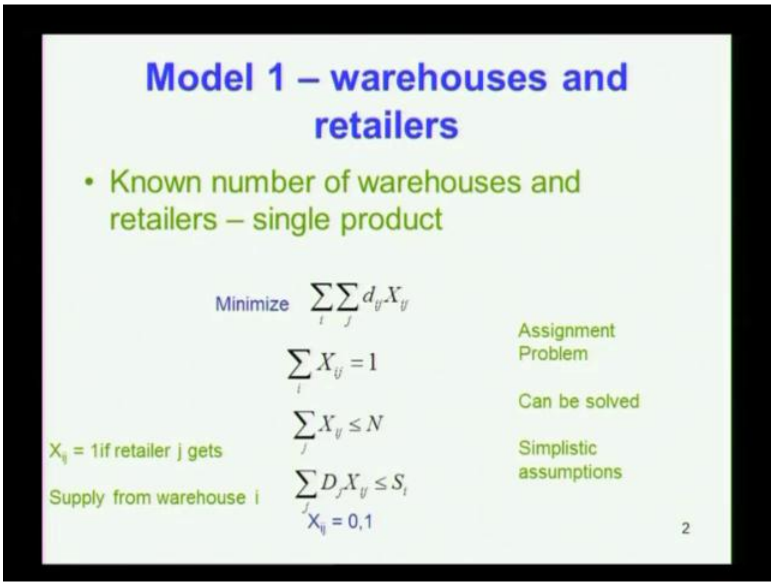

So, the most basic model when it comes to locating warehouses and retailers for a single product is an assignment problem, where we minimize d i j X i j, where X i j equal to 1 if the retailer j gets supply from warehouse i and gets all the supply from warehouse i. The model that we show here is to minimize d i j X i j.

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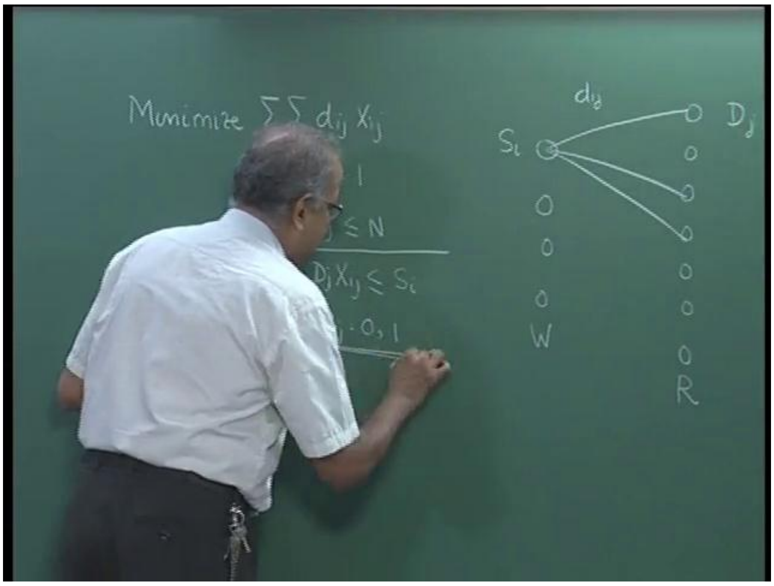

Here we assume that, we would have several warehouses and a large number of customers or retailers. So, we call this as warehouse and we call this as retailers, now we are going to assume, when we look at this formulation, that there is a demand D j from each of these. Now, there is a supply from each one of these S i, we are going to assign each of these retailers to this. For example, we may have a solution, where if we have something like this then we would say that, retailers 1, 2 and 4 get their supplies from supply S i.

Now, at present we need not consider the D j's and S i's explicitly, we would simply say, that there are certain numbers of retailers and certain number of warehouses. And when we actually do up to this, the problem will be to minimize the distance, so there is a distance d i j that we would like to minimize. We also would like to use all these supplies and then we have to say that, each of these retailers will get their supplies only from one particular warehouse.

There is a demand for D j for that item from each of these, there is also a restriction that a particular warehouse will not supply to more than a certain number of retailers. So, this would still be related to a typical assignment problem, even though a typical assignment problem in the OR literature would talk about sigma X i j is equal to 1, sigma X i j is equal to 1. Now, this can be expanded to a simple assignment problem, even when we put a restriction that this it will supply to a maximum of certain numbers.

Now, upto this is either directly an assignment problem or can be modelled as an assignment problem. Now, this constraint also becomes important, when we bring a certain supply to each one of these which means, there is a capacity here and then we need to do this. Now, this is not exactly a transportation problem, the difference comes, because in the transportation problem, there is a supply quantity and there is a demand quantity.

Transportation problem does not put an explicit restriction that all the demands have to come through only one supply. Transportation problem would allow that the demand of this can be met from here as well as here. So, the moment we put an additional restriction to the transportation problem that, the demand of this should be met only from one of them, it becomes what is called a generalized assignment problem. So, generalized assignment problem, which can be solved through binary programming, X i j takes values 0 or 1 and this can be solved as a 0 1 or binary optimization problem.

So, the one of the models is, which given a set of warehouses or given a set of supply points, given a set of retailers or given a set of demand points with known demand D j. Now, which warehouse will supply to which retailer such that, the capacity is considered and all the demand of retailers has to be met from the warehouse. And there is an additional restriction that, for reasons of control, that this does not supply to more than a certain number of retailers.

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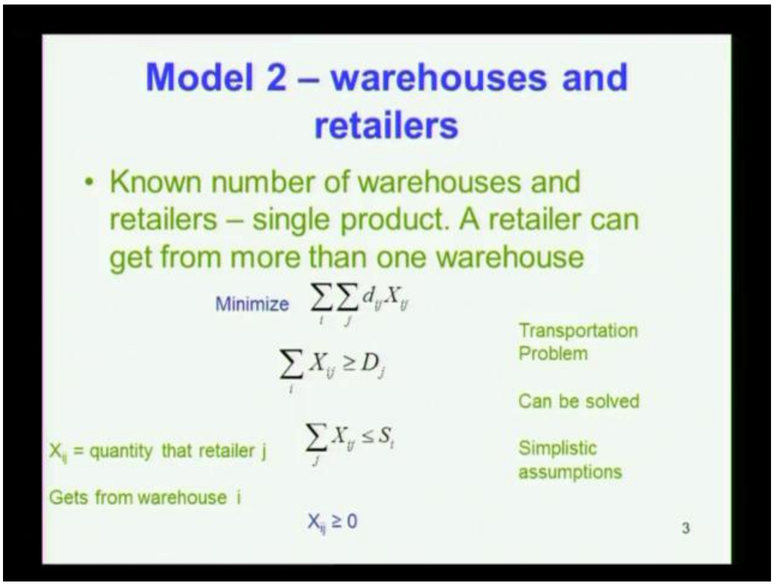

So, this is a standard problem where we model, how the demand of each thing can be met. The second model would be known number warehouses and retailers, single product and a retailer can get from more than one source. So, the second problem is a simpler version of this problem, so we would still have this, we would still say that, this constraint is modified.

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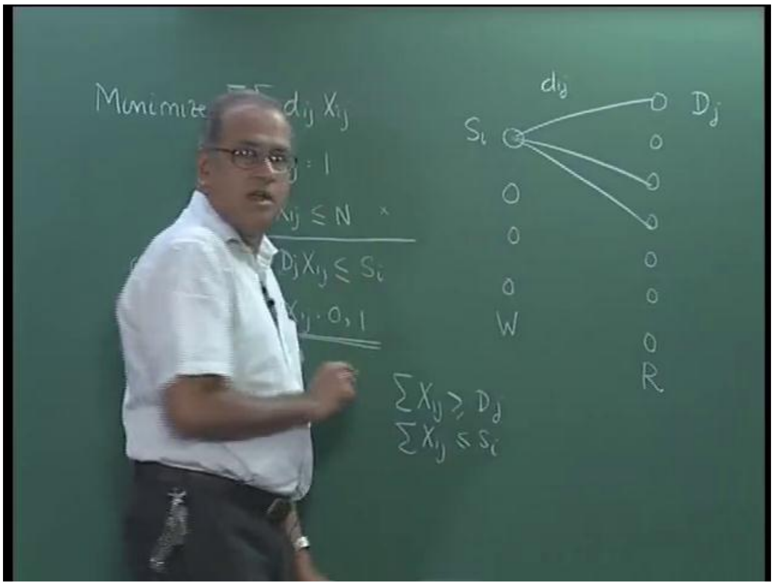

Now, this constraint will be modified to sigma X i j is greater than or equal to D j for the retailer j and sigma X i j is less than or equal to S i and this does not exist, this does not exist. X i j is not binary any more, X i j is the amount of demand or quantity that is met from transporting from i to j and it becomes a straight forward transportation problem, this does not involve dedication. For example, here in model 2, we do not assume that, all the demand is going to be met from only one, you may have a situation, where the demand of this can be met from both these.

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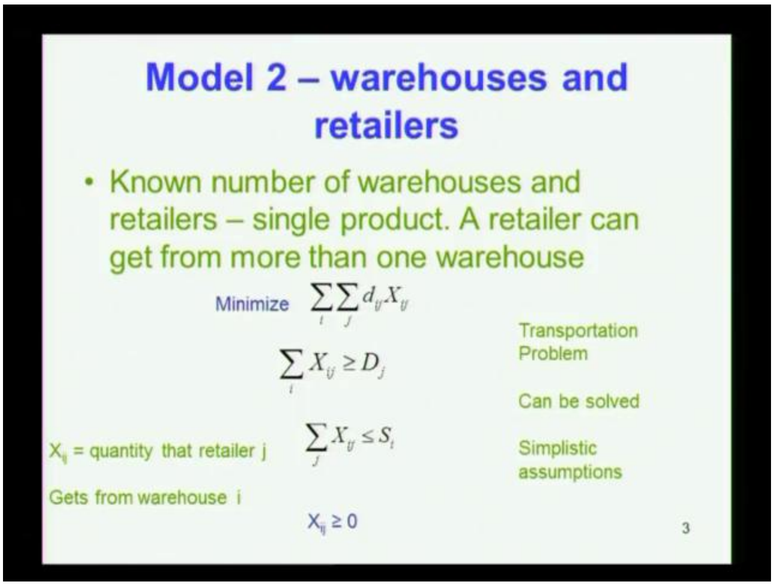

So, the model 2 is actually a simpler version than model 1 in terms of assumptions, where a retailer can also can get from more than one supply and in terms of solution, because it becomes a straight forward transportation problem, which is simpler to solve than the generalized assignment problem. Now, having seen those two models, we will now come back to first the p median

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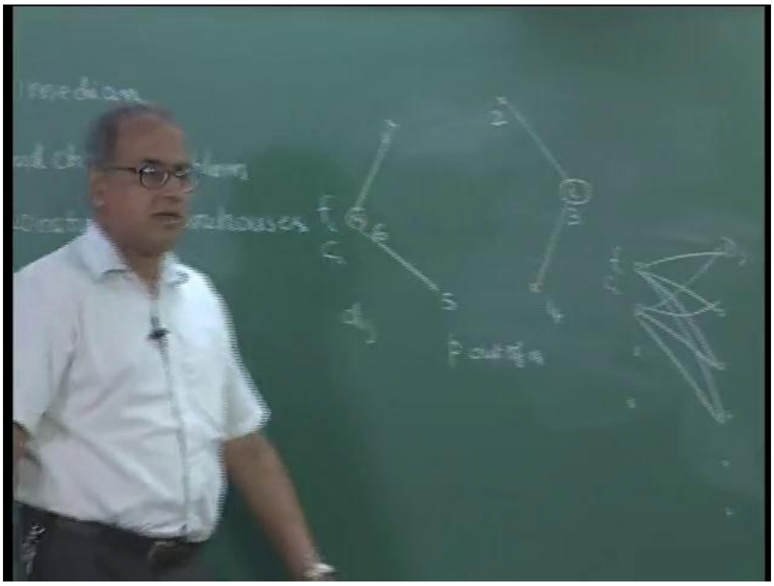

and then the fixed charge and then we will go to the model where we talk about, which is called the fixed charge transportation problem and I will explain the difference between them. So, though I have shown this as model 3 in this PPT, after the model 2, we need to look at the p median as well as the fixed charge problem. We have seen p median and fixed charge in an earlier lecture, so I will just provide a very quick recap of both of these before we go to the next one, which is called the fixed charge transportation problem.

P median, one of the ways of introducing the p median problem is to say that, there are points here, let us say there are 6 points or there are 6 customers. And now, let us say, we want to locate say 2 factories or known number of factories or p factories or p medians. Now, which of these six will be medians such that if for example these two are the medians. Now, first let me describe this problem mathematically then I will describe this problem with respect to the context, in which we are looking at it.

Now, there are 6 points and there are n points, there is a distance matrix d i j that is known which means, the distance between each of them is known. We call these as 1 2 3 4 5 6, distance between a point and itself is 0, so the d i j matrix is known, a p median is to choose p out of these n points. There are n points, to choose p out of these n points, which act as medians and the remaining n minus p, each of these point is attached to a median.

For example, if we choose these two as median then these four are the non median points and each of the non median gets attached to a median point. For example, this might get attached to this and this might get attached to this, this would be attached to this, this would be attached to this such that, sum of these distances is minimized. There will be 6 distances, now these two points attach to themselves, so their distance is 0, these four points are going to get attached to a median, so there will be 4 distances.

Now, which are the two medians, the best two medians and to which medians, the non medians get attached such that, sum of the distance, the four distances is minimized is called the p median problem. Now, what is the relevance of the p median problem in the context of, what we are looking at? Now, we could say that, here are 6 customers, now we want to establish say 2 factories, p equal to 2; we want to establish two factories such that, we are able to meet the customer demands based on distance.

Now, we want to find out the best location out of these six, the best two locations out of these six such that, the remaining four are allocated into the two locations and the distance is minimized. So, that is the relevance of the p median problem in the context of, what we are looking at. So, there are 6 customers and we want to or who are located in six different places, the d i j matrix is known and if p equal to 2, which out of these two six locations, we would locate a factory.

And these two located factories will cater to the requirement of the four remaining points. Now, which of these four remaining points are attached to the factory is given by the p median problem. So, we have seen ways to solve the p median problem, which is our next location model. Now, the fixed charge problem, automatically it can be seen as logical extent of the p median problem, it can also be seen in a slightly different context.

Now, when we extend this p median problem and then we start saying, in the p median problem we are going to assume, that whether I made this one as a factory or this one as a factory, the cost of making this as a factory is the same across all the points. Now, in the fixed charge, I am going give an f i, which is the fixed cost of making a factory from this location, so the problem changes to f i Y i plus d i j X i j. So, there is going to be a fixed cost, now the other way is, there can also be a capacity of this, which I may call as C i or capacity of i.

Now, in the p median, we have not included cost and capacities, now suppose I solve a p median and for example, let us say these are the two best locations and these two are the things then at the end of p median I would say, locate factories here based on distance. The individual cost is going to be same, now if the demands are D 1 plus D 6 plus D 5 then it is necessary to have a capacity of D 1 plus D 6 plus D 5 here and a capacity of D 2 plus D 4 here.

Now, in the fixed charge we do it slightly differently, we start defining what the capacities are going to be and then decide, which are the two best locations and how the demands of rest of them are going to be handled by this. Fixed charge does not, again there are two ways of fixed charge, where one of the ways is to say that, if there is a factory located here and this one does not get a factory then all the demand of this should come only from one of these, either this or this, which is p median kind of a assumption.

But, in practice we could say, part of the demand can come here, part of the demand can come here, depending on the capacities of these two places. So, fixed charge can be seen as a logical extension of p median, where there is a fixed cost of creating, which is different for different points. As well as, the capacities are predetermined in the fixed charge, whereas in some ways, they are post determined in the p median, they are already given to us.

And the difference is, fixed charge does not necessarily restrict, that all demand should only come from one of the chosen factories. Other way to look at the fixed charge problem is to say, here are some potential sites for the factories and here are the customers. So, in this way of introducing the fixed charge problem, we are not going to locate factories in any customer locations. Now, these specific sites where factories are located are different from customer sites, therefore this as a graph is a bipartite graph.

So, there is no connection between or amongst these or amongst these, now again the d i j's are now like this and so on. So, here the d i j will be a square matrix, there are 6 points, we will get a 6 into 6 d i j. Now here, if there are 4 sites and 6 customers, the d i j matrix will be a 4 into 6 matrix then we want to find out, which two of these we select, there is an f i, there is an C i and there is a D j. So, we have seen some aspects of p median and also we have seen some aspect of fixed charge problem in an earlier lecture. Now, with this background we move to model 3, which talks about warehouses and retailers, we will visit this location of warehouses later.

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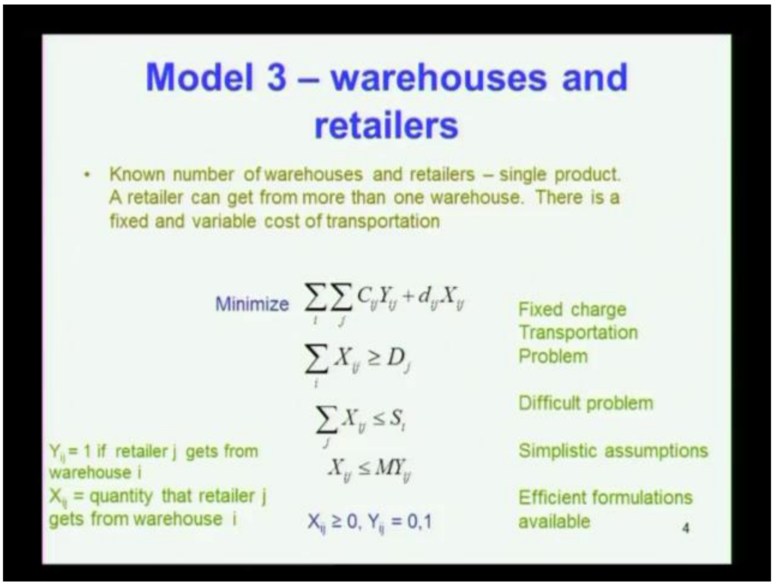

So, this model talks about warehouses and retailers, where we have a known number of warehouses and retailers. We are talking of a single product, we also say that a retailer can get items from more than one warehouse and there is a fixed and variable cost of transportation. So, the important change is that, there is a fixed and variable cost of transportation. So, let me go back and explain this through this network, now here we have a set of warehouses and then we have a set of retailers.

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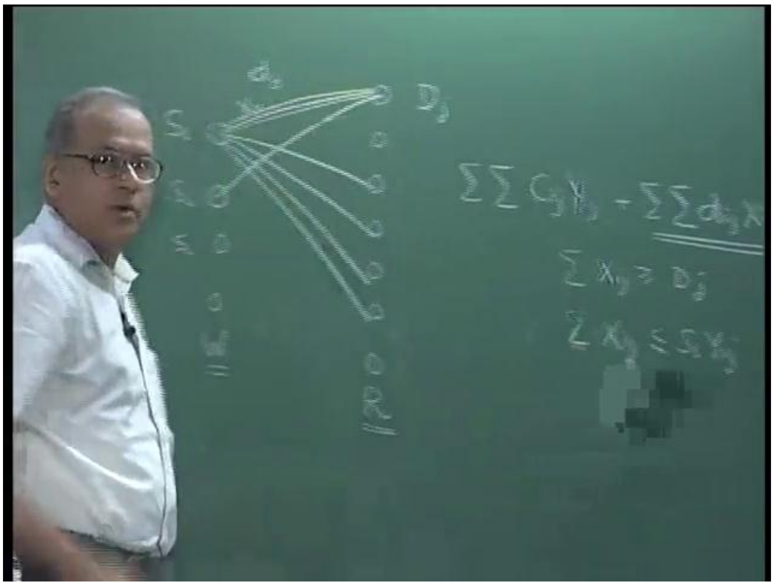

Now, the question is, there is a supply S i in each one of these, this is not a location problem per say, the decision here is not where to locate. In a p median the decision is where to locate, in a fixed charge the decision is where to locate, now here we assume that, there are a set of warehouses and there are set of retailers. For the sake of illustration, there are 4 warehouses and there are 7 retailers, now each one is connected to every other, so it means, every warehouse can supply to every retailer.

The normal transportation problem, which was our model 2, talks about there is an S i, there is a D j, how much do I send. Assumption is, this can be met even partly from here and partly from here, the additional dimension in this problem if you look at the objective function, the old transportation problem will have C i j X i j. And now, we also have another d i j, here we have C i j Y i j and d i j X i j, so there are two terms in the fixed charge transportation problem, whereas in the old transportation problem, there was only one term, which is d i j X i j, X i j is the quantity transported from i to j.

Now, we introduce a Y i j and say that, if for example, I transport a quantity X 1 1 from the first supplier to the first demand. Now, the cost of transportation will be d 1 1 and X

1 1, but there is an additional cost, which is called a fixed charge of doing this. Now, this fixed charge would be like an additional toll that I may have to pay if I choose to transport from this to this. So, there are two decisions, first of all whether I choose to transport from i to j and then if I choose to transport from i to j, what is the quantity that I transport from i to j.

Whether I choose to transport is given by the Y i j and the quantity that I transport is given by the X i j. So, the constraints will be that, each sigma X i j is greater than or equal to D j, whatever comes into this will meet this demand. And as far as the other one, sigma X i j is less than or equal to S i and Y i j, where we have shown it as two different constraints.

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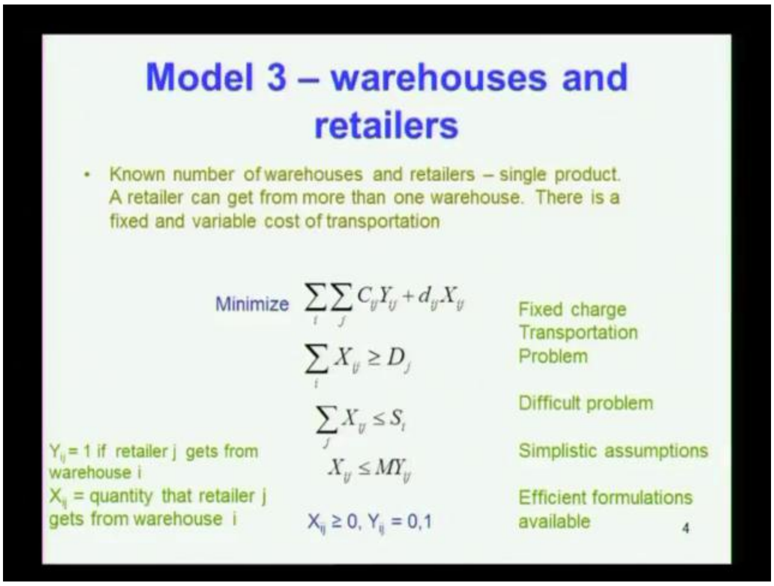

We have shown two constraints, which is X i j is less than equal to S i and then said X i j is less than equal to M Y i j. They can be brought together into a single constraint, which will say that, this supply or capacity of S i is available, only when to meet the requirement of a j partly, only when we choose the root i j, so only if Y i j is 1, this in whole or part is available for this. So, this is the constraint, Y i j will be binary and X i j will be a quantity, so this is called fixed charge transportation problem.

See, it is different from the fixed charge problem, the difference comes from the fact that, in a fixed charge transportation problem, there is a charge fixed charge f i j between i and j, whereas in a fixed charge problem, there is an f i associated with this i. Now, in the fixed charge problem, the fixed charge is for a node and in a fixed charge transportation problem, the fixed charge is for an arc or an edge that connects i and j, here it is for a node or vertex, which is i.

So, there is a cost associated here and there is a cost associated here, this cost should be seen as the cost of setting up a facility. And this cost should be seen as a fixed cost of transporting, which is like a toll that is paid when you move items from i to j. So, that is our model 3, which talks about warehouses and retailers.

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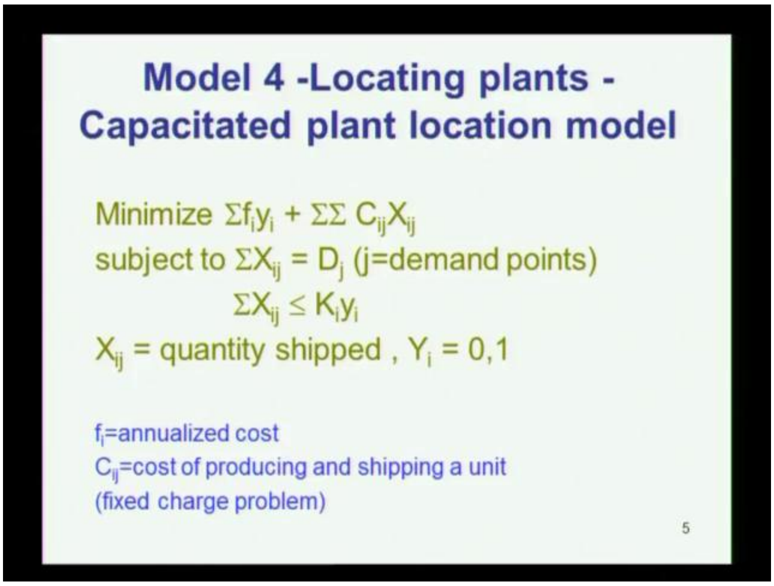

Then, we move to the 4 th model, which is the capacitated plant location model, which is actually the fixed charge problem. So, f i y i, so this is what is shown there as model 4, which is the fixed charge problem.

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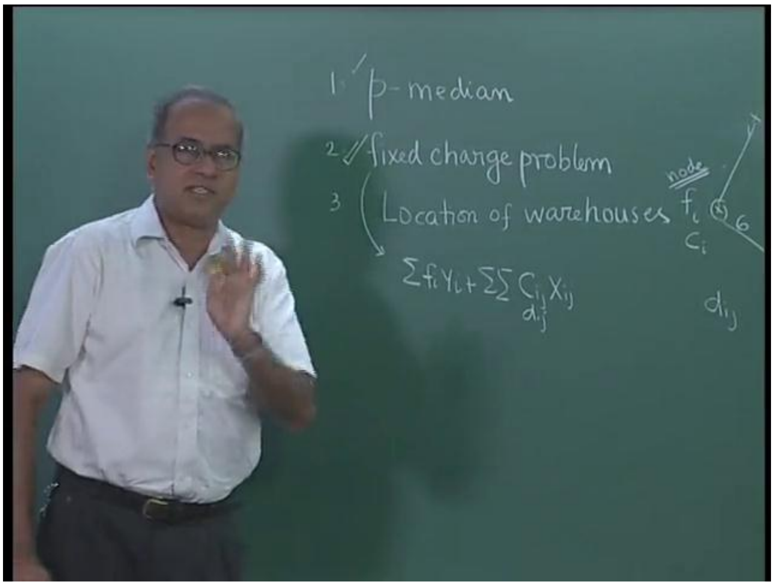

Now, you see the objective function is sigma f i y i plus sigma C i j X i j or d i j X i j, if this is a generic expression, which talks about the unit cost of transporting from i to j, we could use d i j X i j if the unit cost is proportional to the distance and the proportionality is 1. For example, it costs 1 rupee to transport the unit distance then you can use d i j X i j or C i j X i j. So, we have already seen some aspects of the plant location or a fixed charge problem earlier in this course.

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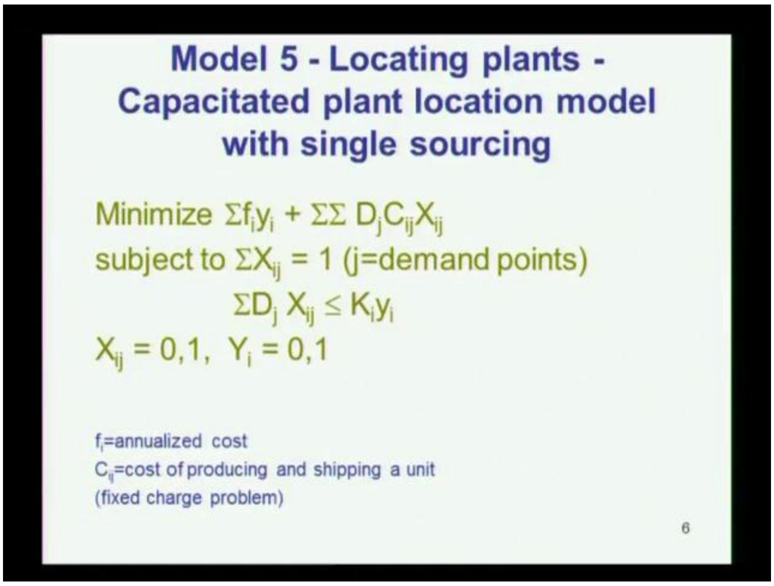

Now, we look at locating plant, capacitated plant location model with a single sourcing, so as model 5, what we see is something like this, the same fixed charge problem here.

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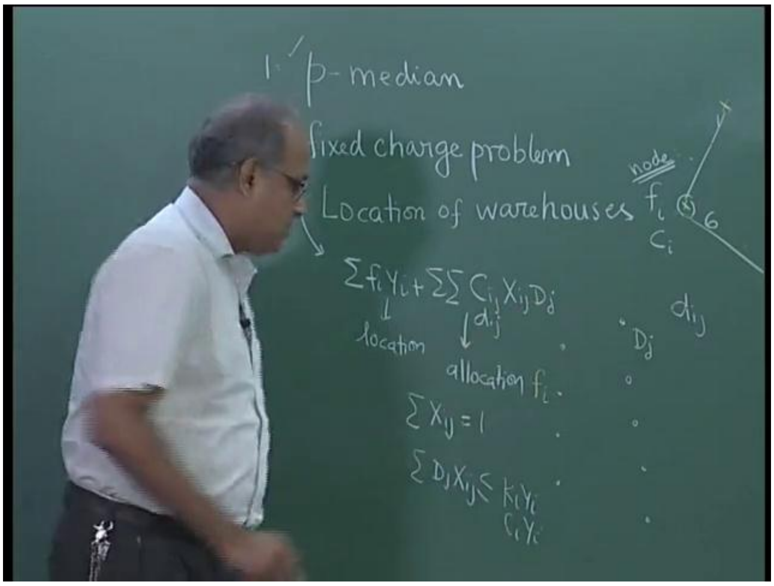

So, when we go back to this network which I draw again, these are the possible sites, these are the customers. So, there is an f i associated with this side, there is a D j associated with this customer, now all that now we have to f i, so y i equal to 1. If this site is chosen, so the objective function will minimize, this is the cost of location, this is the cost of allocation, so it is also called location allocation problem. Because, we talk of locating facilities as well as we talk of allocating from the capacity of these facilities allocating to meet the demand, it is called allocation problem.

So, all the demand should be from only a single place, that is why we call capacitated plant location with single sourcing which means, the entire demand of this will be met from only one of these and that one has to be chosen. So, you will have a constraint which will say, sigma X i j equal to 1 which means, in this model, X i j is binary, y i is also binary, y i is 0 or 1 if we choose this, if it is chosen is 1, if it is not chosen it is 0.

So, whatever is chosen will incur a fixed cost, which is f i y i, now the demand of all of these have to be met. So, X i j equal to 1 if the demand of this j is going to be met from a chosen i. Since X i j equal to 1 will ensure that, the demand of each of these is met from only one of these. And the second thing that we need to look at is, sigma D j X i j is less than or equal to K i y i or C i y i.

So, not only this gets the entire demand from one of them, this should get it is demand from a chosen site. So, if this site is chosen then y i will be 1, only then it will get from this, if this is not chosen this cannot supply to this and if this is chosen, this has a capacity of C i y i. So, whatever you get from this capacity, should also be within what this supplies, so this C i y i is the capacity of the chosen one and this is given to several D j X i j.

So, the capacity is greater than the demand, so the single constraint takes care of all of them. The objective function changes, because X i j is binary, so D j X i j is the demand, so C i j into this, so cost into demand. So, this is called locating plants capacitated model, because K i or C i talks about the capacity of this when it is chosen, so it is capacitated. Single sourcing, because all the demand from here is met from only one supply, so that is our model 5, which is capacitated plant location with single sourcing.

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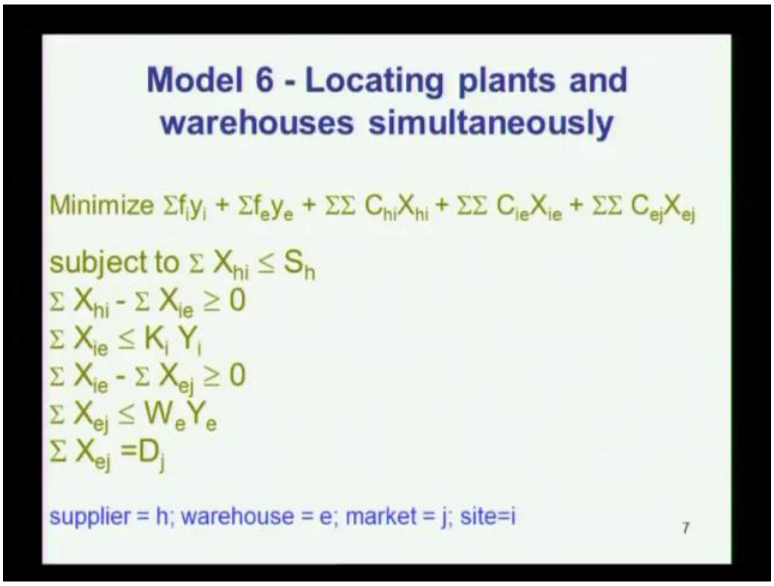

Sixth model is locating plants and warehouses simultaneously, now this model is exactly the same model that we have seen here earlier, where we have spoken about locating plants and warehouses simultaneously which means, we are looking at multiple stages.

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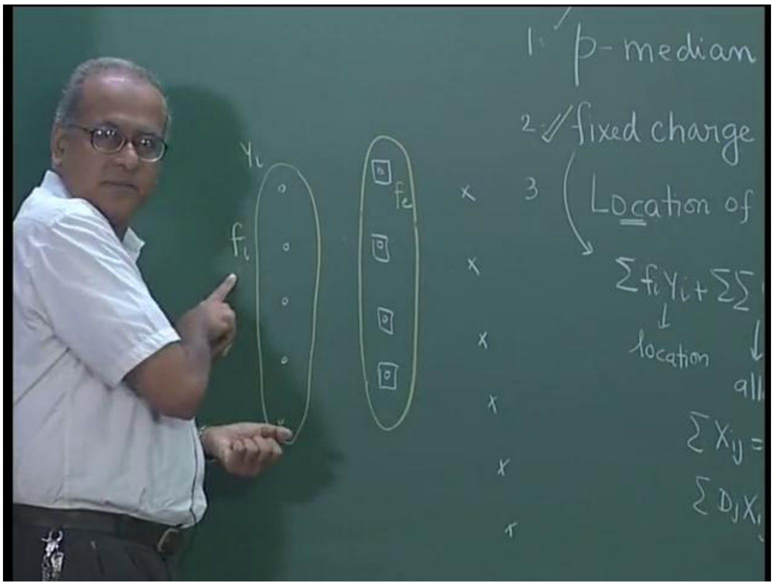

So, there can be a stage where we have plants or factories, there will be a stage where we will have warehouses and there will be a set of customers. So, the question here is, we can put a restriction on the number of factories that we wish to locate, we can put a restriction on the number of warehouses that we wish to locate. So, we will have multiple decisions f i y i, so f i would be a fixed cost here, y i will be the decision here and then we also have another one f e y e.

So, this will be a fixed cost f e and then we would possibly have another y e or z, whatever variable we can use, so if we choose some of these. So, there is a fixed cost of location here, there is a fixed cost of location here. Now, the moment there is a fixed cost of location here, there is a capacity, so things have to be produced and transported only to chosen facilities, because there is cost of creating this. Then these are transported to the chosen facilities and from there, they are transported to meet the requirements of these customers.

So, here again, we can have a dedicated or a single sourcing model or a multiple sourcing, where a chosen warehouse can supply to either or the other way, a demand is met only completely from one warehouse or it can be met from multiple warehouses. Now, there is a distribution cost here, there is a distribution cost here, there is a fixed location cost here, there is a fixed location cost here.

Therefore, you we see 4 terms or 5 terms and this depending on the number of layers and if there is one more layer here, there is a warehouse distribution as well as the customer then you have one more transportation cost. The location decisions usually concern only this and this, how many plants to locate, how many warehouses to locate.

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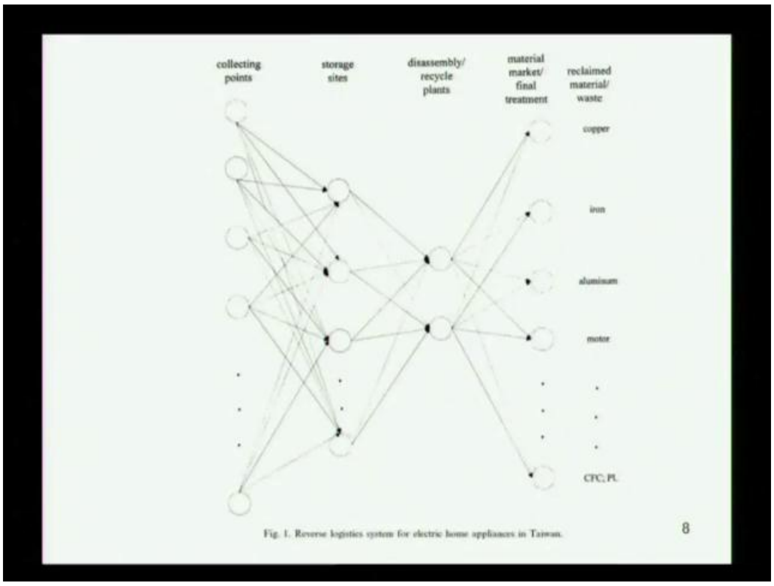

Rest of them are transportation cost and linking constraints, which will link the capacity as well as the demand, which is shown here 4 layers, so there are 3 types of distribution costs.

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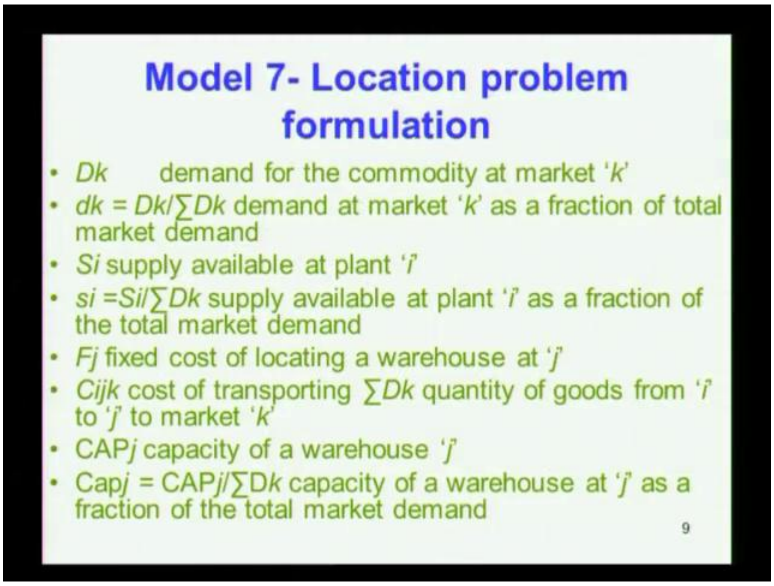

We could also look at more advanced location problems, particularly for multiple products, so far the models that we have seen, are largely for single products. And now, these can also be expanded to looking at multiple products and fixed toll.

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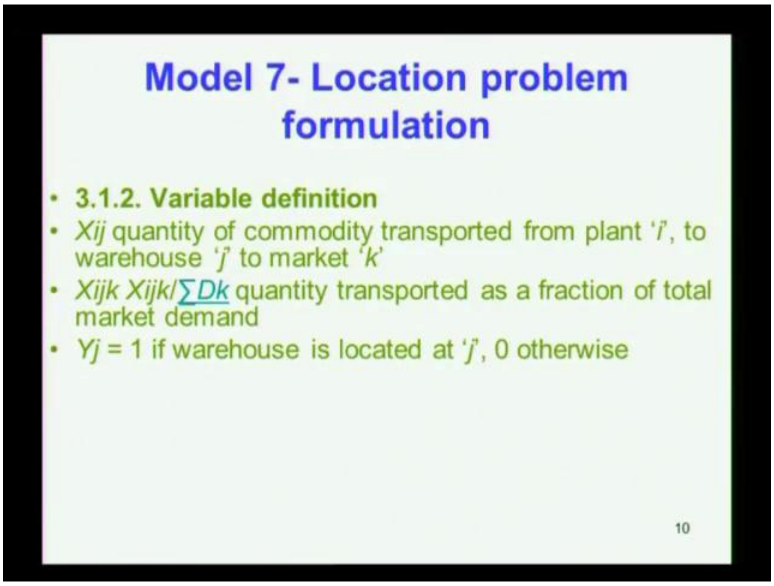

So, we get much more numeric formulation.

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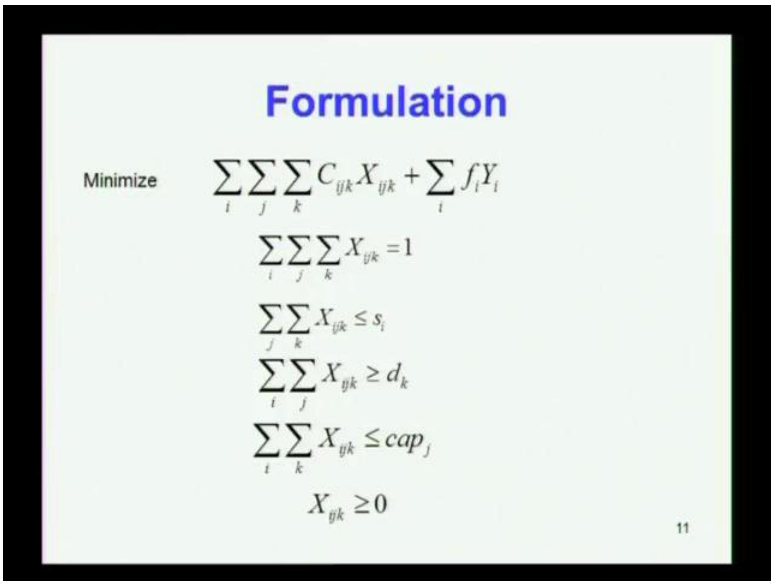

Which is shown here with C i j k X i j k, where k represents a product or a market i to j which simply means, from the plant to the customer. So here, we are not looking at multiple layers in the supply chain, we are talking about a set of factories and a set of customers and multiple products or multiple markets.

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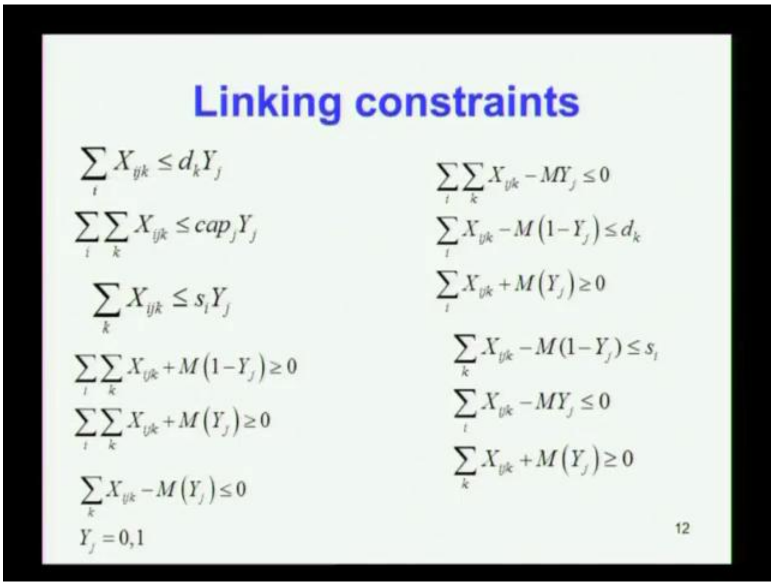

There is a location and there is an allocation and there are some linking constraints for each one of them.

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Now, we will look at a couple of more aspects, which are called warehouse aggregation and safety stock and service levels in this. Now, what exactly do we mean by warehouse aggregation. Now, when we look at a model like this, when we look at all over location and allocation models.

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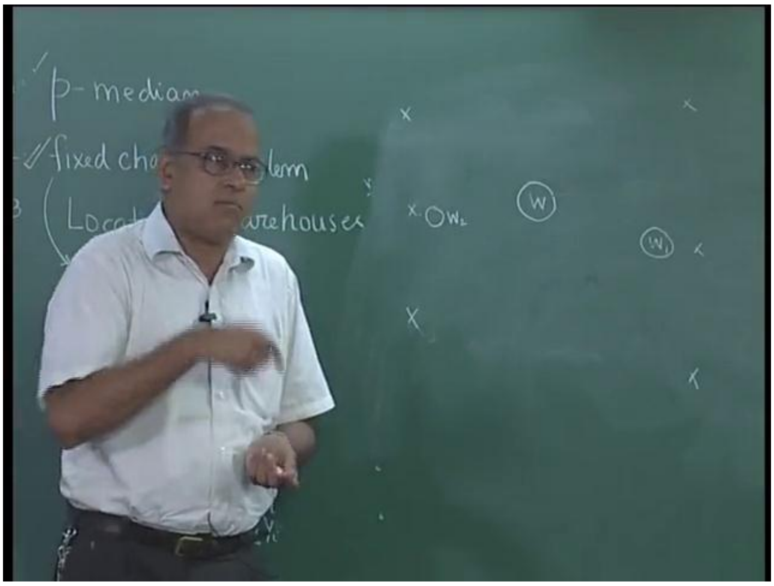

Now, let us see, there are sometimes when we have, let us assume there is a warehouse here and there are several customers here. Now, there are times we wish to make another decision which says, now can we actually have two warehouses. For example, can we have one warehouse here and we have one warehouse here. Now, if we replace this central warehouse with these two warehouses we realize that, the distance travelled will be less.

Because, here we are looking at these six distances, whereas here you are looking essentially three distances, if this merges with this, we are looking at 3 plus 3, we will look at 4 distances and it will be much smaller than the sum of the distance here. But now, what are the advantages and the disadvantages, now the advantages and disadvantages, advantages in reduced transportation cost, the disadvantage could be more inventory or buffer needs to be kept here as well as here.

Whereas, when we centralize them and bring them together, the inventory that we will have there, will be much less. So, there is this traditional trade off between a centralized warehouse, where the inventory is kept, the safety stock is less, transportation costs are more. Decentralized warehouses, where the transportation costs are less and the inventories would be more. Now, we look at some of these, we look at a single warehouse and multiple retailers.

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Now, one could simply easily calculate the economic order cost for each retailer for n retailers.

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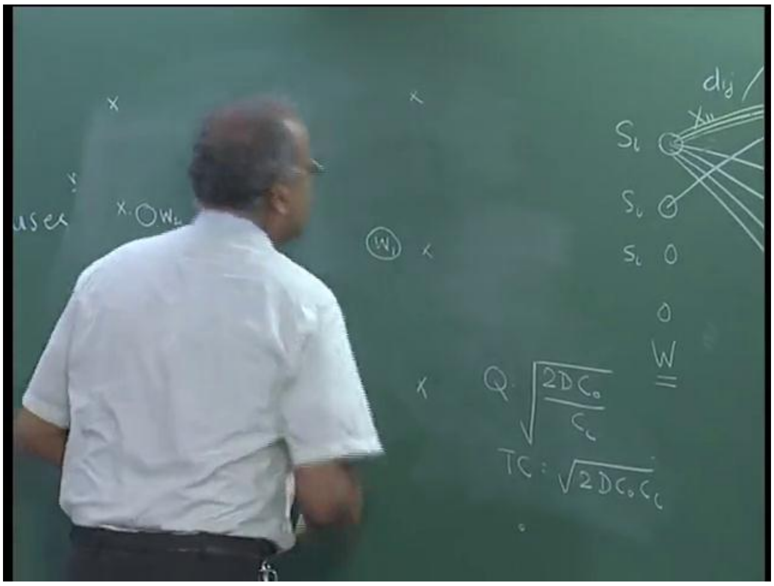

So, if we have n retailers, using our traditional formulae that we have used, the economic order quantity Q will be equal to root over 2 D C naught by C c, where this is the demand for the item, this is the order cost, this is the inventory carrying cost. So, for each of them for a single item, this will be the economic order quantity and the total cost at the economic order quantity will be root over 2 D C naught C c.

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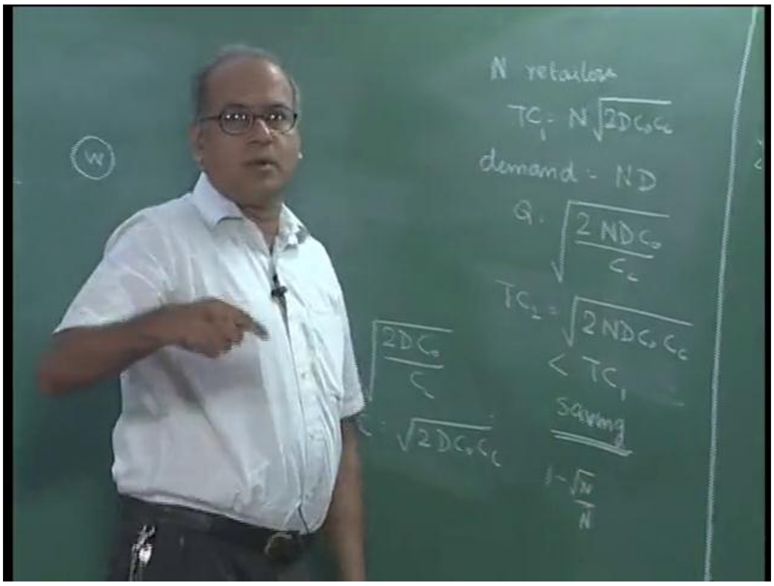

Then, if we have it for N different retailers, the total cost there will be, for n different retailers N into root of 2 D C naught C c. Now, if bring all of these together into a central place then the total demand will become N into D and therefore, the economic order quantity will be root over 2 N D C naught by C c. And the total cost T C 2 if we may call this, is equal to root over 2 N D C naught C c, now this is smaller less than T C 1.

Because, N into root of 2 D C naught C c will be bigger than root over 2 times N D C naught c c and now there is a saving in the total cost, if we bring it down together. And the same thing can be seen as either warehouse aggregation or can be seen as inventory aggregation for a set of N retailers and so on. So, the saving will come down and this saving can easily be shown as 1 over root N by N or 1 over 1 by root N or 1 over root N by N will, now be the saving when we try and bring the warehouses together.

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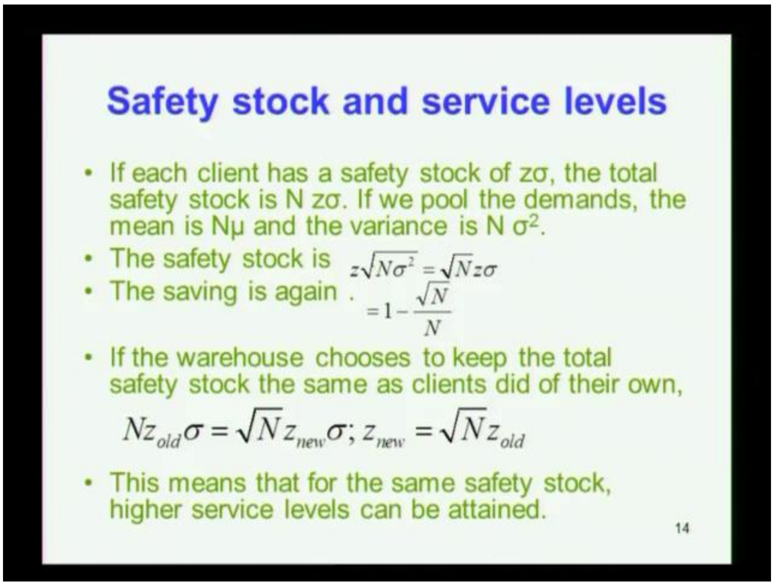

The other aspect that we can see, is also the saving in safety stock and in service levels, which we have actually seen in an earlier lecture on inventory. Now, we also know that, if each one of these has each one of these retailers, there is a lead time demand distribution.

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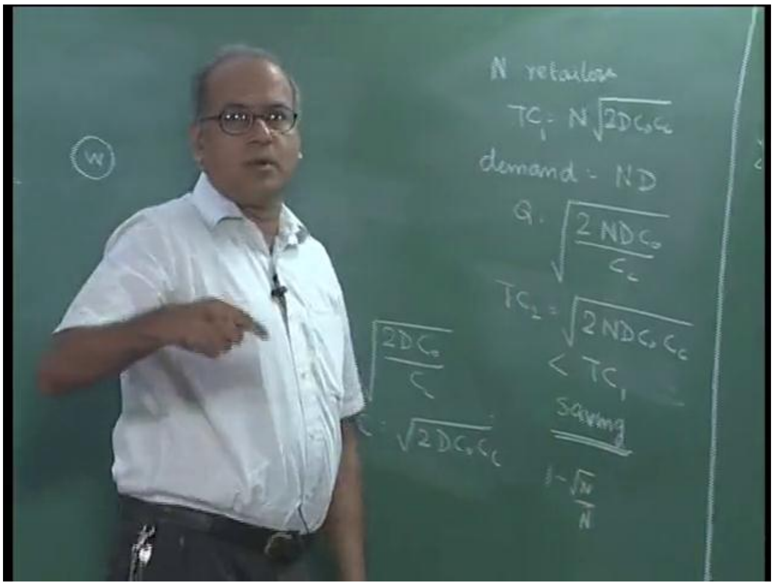

And if sigma is the standard deviation of the lead time demand and then we have a service level SL and based on this service level, we go back to the normal distribution and then find out a z from the normal distribution. Now, z sigma is the safety stock that will be maintained, now for N people, we will have N into z sigma in each of these places. Whereas, when we pool them together, we have already shown that, the standard deviation when combined together is actually root N into sigma.

And the same safety stock, the total amount of safety stock maintained if we bring them together will be z into root N into sigma, whereas if we maintain them in different places, it will be z into N into sigma. Very similar to N into root over D C naught C c and root N over root of 2 D C naught C c, similarly N into z sigma versus root N into z sigma. Once again it is possible to show that, the saving is of the order of 1 minus root N by N or 1 minus root 1 by root N.

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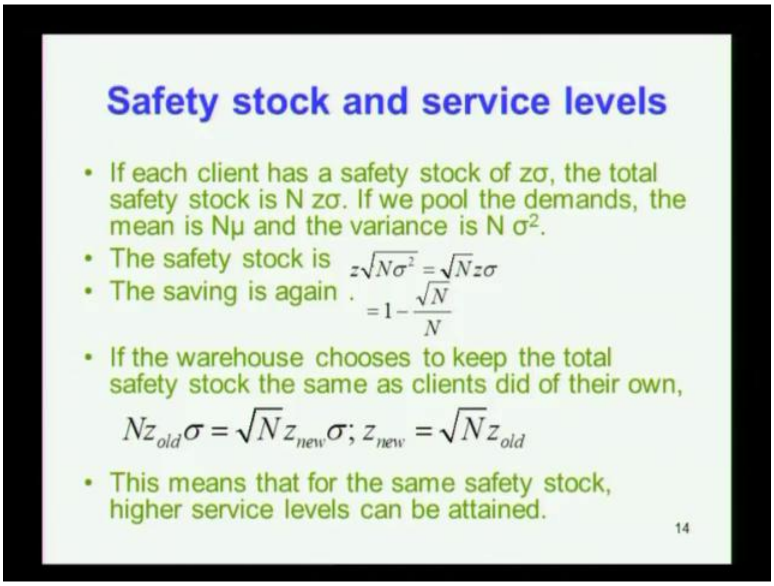

So, the advantage of doing this from an inventory point of view is to try and aggregate it or try and make decisions, which are for everybody together, which is an essential dimension of supply chain. Because, if the individual players make their own decisions or if the individuals have their own warehouses, the amount of safety stock will be higher and the total cost of inventory will be higher. So, by centralizing and by bringing them together, the total cost of inventory can be brought down.

But, we have already seen that, in some ways the cost of transportation will increase when we try, the cost of transportation will be lower if we have different warehouses, rather than a centralized warehousing. The important aspect is, for these organizations to come together and to be able to do centralized ordering or to be able to take from a centralized warehouse. So, the inventory cost may go up a little bit when you centralize it and take it from a centralized warehouse, but the total cost can come down.

Much later we will see that, by proper information sharing, we can actually bring down even the cost of transportation by aggregating or when we aggregate the demand. So, with this, we also look at some models for warehouse and models for warehouse aggregation. And what we have seen now along with, what we have seen earlier in terms of the p median problem, the fixed charge problem and location of factories and warehouses, would give us a good picture of the location decisions with respect to a supply chain and the ability to bring down the total cost through aggregation. Rest of the aspects such as transportation decisions and modes for transportation and distribution, we will see in the next lecture.