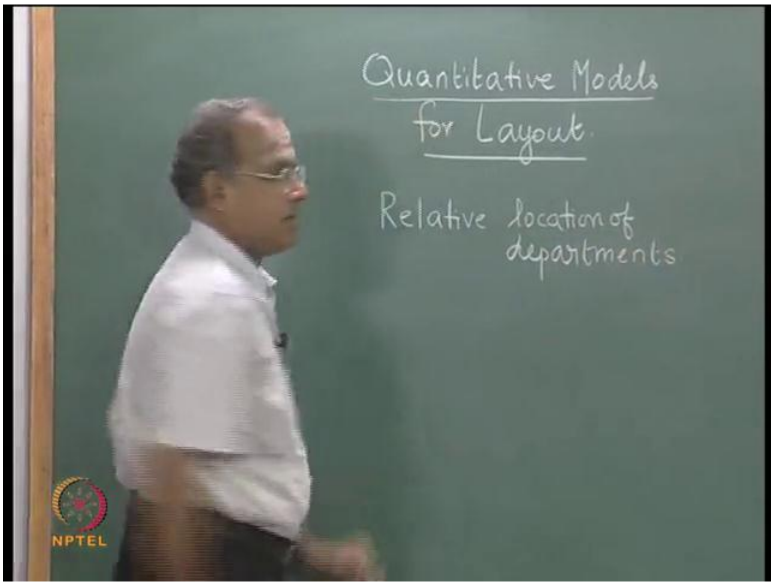
Operations and Supply Chain Management
Prof. G. Srinivasan
Department of Management Studies
Indian Institute of Technology, Madras

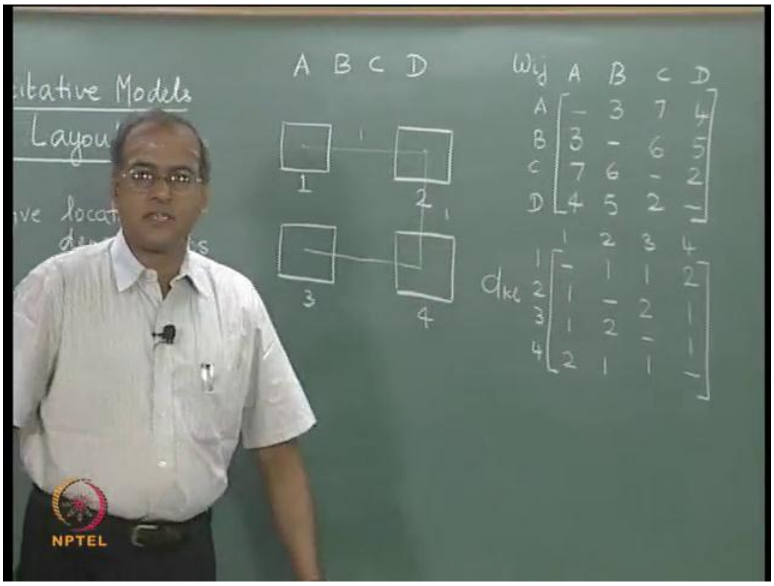
Lecture - 34
Quantitative Models for Layout, Summary

(Refer Slide Time: 00:11)



In this lecture, we look at Quantitative Models for Layout. In the previous lecture, we saw one qualitative model, where the layout was made based on a qualitative assessment of the proximity or nearness of the departments to be located. So, in this lecture, we will look at some quantitative models for layout, we have already seen that layout is relative placement or relative location of facilities or departments, now let us assume that four departments have to be placed.

(Refer Slide Time: 01:05)



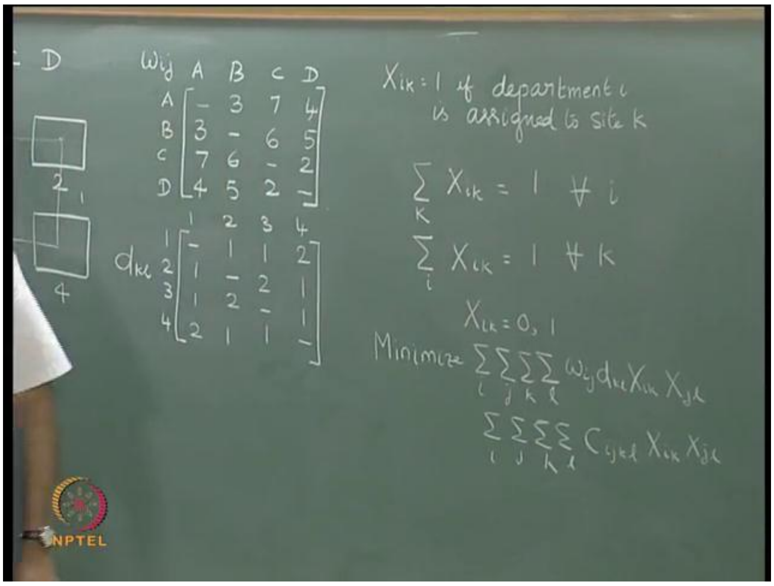
And let us call these four departments as A B C and D let us also assume that there are 4 sites which we call as 1 2 3 and 4. Now, these 4 departments there is going to be either some material movement or some people movement or some movement that happens among these departments. So, which we call as w i j, where w i j is the amount of movement from i to j, and let us describe a w i j matrix between A B C and D.

Now, we have assumed this matrix to be symmetric, this says that movement from A to B is 3 units, usually it is in tons or some kind of weight measurement, A to C is 7 units A to D is 4 units and so on. We have assumed symmetry though in many cases it need not be symmetric because the direction of flow of material inside a factory or a department can be different from i to j and j to i. So, in reality this need not be symmetric, but just to make the computations easy, we are assuming a symmetric matrix here which also means that from B to A the weight is 3 and so on.

We also create a distance matrix amongst these 4 sites, which we call as 1 2 3 4, so the distance matrix is given as. Now, initially we assume that all these 4 are of the same size, and all these 4 are placed equidistant from this point therefore, if this distance is treated as 1, this would also be 1. And distance between this and this, we do not take the Euclidean distance, we take the rectangular distance and therefore, the distance will be 1 plus 1 which is 2.

So, there are 4 sites and we create what is called a d k l matrix, where d k l is the distance between sites k and l. So, the 4 sites are written as 1 2 3 4, and the distance will be 1 to 1 the distance is 0 or it can be a dash 1 to 2 is 1, 1 to 3 is 1, 1 to 4 is 2, 1 to 4 is 2 because we take the rectangular distance. So, this plus this we do not take the crow flying or Euclidean distance 2 to 3 is 2, 2 to 4 is 1 and 3 to 4 is also 1, so; obviously, this 1 is also symmetric, so we have 1 1 2 2 1 and 1. Now, the problem is to locate the facilities to sites such that, the product of the weight and distance is minimized.

(Refer Slide Time: 04:59)



So, mathematical formulation to this problem will be like this. Let X i k equal to 1 if department i is assigned to site k. So, now, each department has to be assigned to only one site, we assume an equal number of departments and sites, so sigma X i k equal to 1 summed over all k for every department i. So, every department i goes to only one site, every site gets only one department X i k summed over i equal to 1 for all k, k is the site.

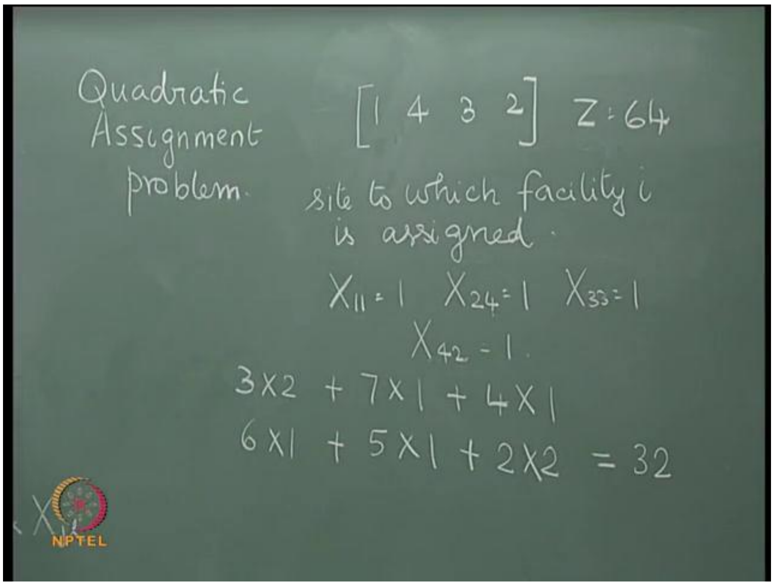
So, every site gets exactly only one department allocated to it and X i k is equal to 0 1, which is a binary variable. So, either department i if department i goes to site k, then the value is 1, if department i does not go to site k the value is 0, now these are the constraints the objective function is to minimize the product of the w i j's and d k l's. So, the objective function will be of the type, minimize w i j d k l X i k X j l this is summed over i j k l.

Now, let me explain this objective function. If facility i goes to site k, and facility j goes to site l, then the product X i k into X j l will be 1 otherwise it will be 0. So, if facility i goes to site k and facility j goes to site l, then there is a material movement of w i j between facilities i and j, which travels a distance d k l from sites k to l. Therefore, the material travel which is the product of w's and the d's will become w i j, which is the material movement from i to j, this much material travels a distance k to l if facility i goes to site k and facility j goes to site l.

So, we have this objective function which has four summation terms over i j k and l, the most important thing about this objective function is that, this objective function has a product of two variables X i k X j l summed over all possibilities. So, this objective is not linear because it has a product term that comes it, it is also customary to write this instead of w i j d k l as C i j k l X i k X j l, where C i j k l represents the product of w i j into d k l. So, the problem is to minimize w i j d k l into X i k into X j l subject to these two sets of constraints and a binary restriction.

So, for a 4 by 4 problem we have 4 constraints here 4 constraints here, which means we have 8 constraints and we have 4 into 4 16 variables. So, there are 16 binary variables and there are 8 constraints for a 4 department, 4 site problem, in general for a n department, n sites problem we have 2 n constraints and we will have n square variables. Now this problem which has this product term or a quadratic term in the objective function, and otherwise has assignment constraints, this is very similar to the assignment problem in operations research. So, it has a quadratic objective function, and it has the assignment constraints it is called the quadratic assignment problem.

(Refer Slide Time: 09:58)



And we can solve this formulation optimally to get the best layout for the 4 departments and the 4 sites, given the w i j's and given the d k l's, w i j represent the amount of material movement from the department i to j, d k l represents the distance travelled or distance between sites k and l. We solve this problem optimally to get the solution 1 4 3 2 with Z equal to 64, so let me explain first the notation and how we get this 1 4 3 2.

So, in this notation now this represents the site to which facility i is assigned, so this would mean that facility 1 is assigned to site 1. So, X 1 1 equal to 1, facility 2 is assigned to site 4, X 2 4 equal to 1, facility 3 is assigned to site 3, X 3 3 equal to 1, and facility 4 is assigned to site 2, X 4 2 equal to 1. So, this is what this notation explains, and let us show this computation of this 64, so now, there will be 6 interactions, interaction between facilities 1 and 2, 1 and 3, 1 and 4, 2 and 3, 2 and 4 and 3 and 4.

So, facilities 1 and 2 the w is 3 the material movement between 1 and 2 is 3, 1 and 2 are allocated to sites 1 and 4 respectively. So, distances between site 1 and 4 is 2, so we get 3 into 2 plus interactions between facilities between 1 and 3. So facility 1 is here facility 3 is here the weight is 7 into 1 is allocated to site 1, 3 is allocated to site 3. So, distance between sites 1 and 3 is 1, so 1 into 1 plus facilities 1 and 4 have weight equal to 4, facility 1 is allocated to site 1, facility 4 is allocated to site 2.

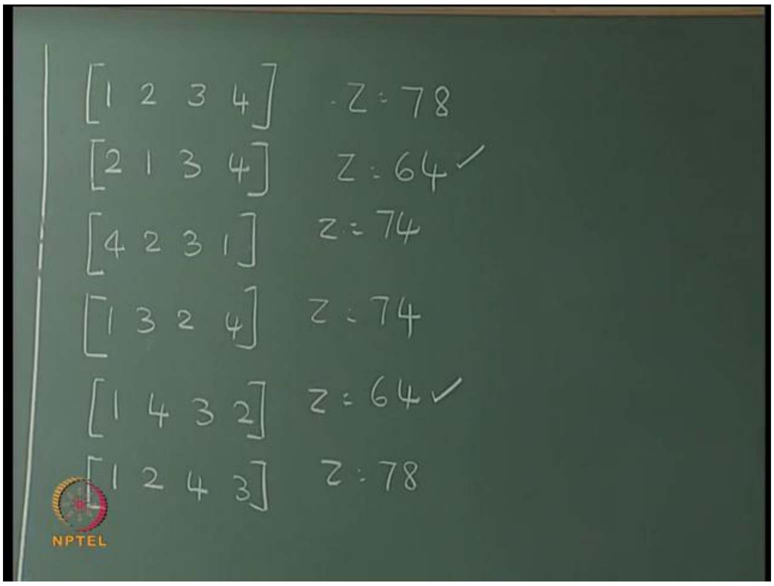
So, distance between 1 and 2 is 1 facilities 2 and 3 have weight equal to 6, 2 is allotted to 4, 3 is allotted to 3. So, distance between 3 and 4 is 1 plus facilities 2 and 4, so 2 and 4 have weight equal to 5, facility 2 is allocated to site 4, facility 4 is allocated to site 2 distance between 2 and 4 is 1. 3 and 4 the weight is 2, 3 is allocated to 3, 4 is allocated to 2. So, between 2 and 3 the distance is 2. So the sum of all these is 6 plus 7, 13 plus 4, 17 23 plus 5, 28 plus 4, 32 and due to symmetry, we have another two.

Actually we should be multiplying this interaction of when we take this pair, we should look at interaction between 1 and 2 and distance between 1 and 4. And interaction between 2 and 1 and distance between 4 and 1 by symmetry these 2 are equal, so what we compute here is 32 should be multiplied by 2 for the purpose of the objective function. Therefore, we get a solution with 64. So, the solution of course, can be found out using a solver or by using several known algorithms to solve the quadratic assignment problem.

And there are very efficient algorithms, which work on the principle of branch and bound to solve the quadratic assignment problem. Nevertheless the quadratic assignment problem is a difficult problem to solve, and as the number of as n increases, as the number of departments and the number of sites increases, the computational effort to solve this problem optimally is of exponential order. And it becomes difficult to solve the quadratic assignment problem optimally, because the computational time will be very high.

So, people have resorted to using heuristic algorithms which try to give the best solution, a near optimal solution or sometimes slightly more than the optimal in terms of objective function value. Because, this problem is a minimization problem, so we look at one simple heuristic which people have been using to solve the quadratic assignment problem, and that is shown here.

(Refer Slide Time: 16:20)



Following this notation we could start with any feasible solution and the easiest is to do 1 2 3 4. Now, please note this notation once again, this notation is it represents the site to which facility i is assigned, so here the site to which the facility 1 is assigned, facility 1 is assigned to site 1, facility 2 is assigned to site 4, facility 3 is assigned to site 3, facility 4 is assigned to site 2. So, we could start with the easiest 1 which is 1 2 3 4, we have already explained how to calculate the objective function value, given a vector like this is which is what we explain here.

So, we can follow the same computation to try and get the objective function value, if we start with 1 2 3 4 which is the most intuitive solution to begin with. So, this gives us Z equal to 78, now we could follow some kind of a pair wise exchange heuristic, where we exchange two at a time to get a series of solutions. So, we could start with 2 1 3 4 and on computation we get Z equal to 64. We could do 4 2 3 1 which has Z equal to 74, we could have 1 3 2 4 with z equal to 74, we could have 1 4 3 2 with Z equal to 64 and we could have 1 2 4 3 with Z equal to 78.

Now, these are all some of these solutions which we could get by exchanging some of the facilities for example, exchanging these two would give this. Now, from here we can exchange 4 and 2 to get this, we can use this and then exchange 2 and 3 to get this and so on. So, we could do a lot of possible exchanges to try and get better and better solutions, and after a series of exchanges we realize that we could either pick this solution or this solution, as the one which has minimum value of the objective function.

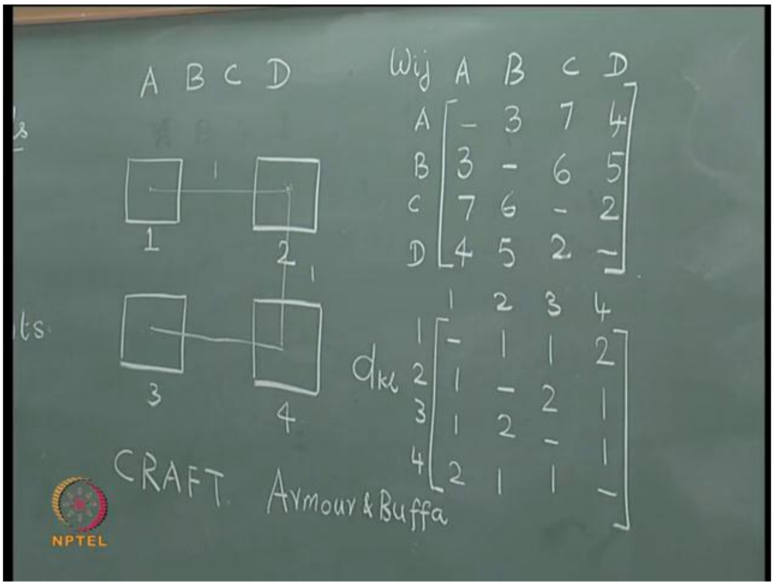
And the algorithm can terminate with the best minimum possible value that it can give, at the moment here if we are done only this we know that we have a solution with 64, but we do not know that it is the optimum solution. Right now, since we have already seen that Z equal to 64 is optimum, we could say that this exchange algorithm is able to give the optimum solution for this problem instance.

But, many times particularly with as n increases the exchange algorithm is unable to give the optimum solution in many instances, never the less it is a good heuristic to solve the quadratic assignment problem. Now, let us come back to one of the assumptions of the quadratic assignment problem which is this, now first of all we have an equal number of facilities and sites that is the first assumption that we have. The second assumption that we have is also that, all the departments which are here require the same area they do not require different areas.

And therefore, and all the sites are of the same area and therefore, any department can go to any site. Now, if we want to apply this basic idea to practice where in a manufacturing shop floor, we have different departments and they have to be placed relatively closed to each other. So, that the w into d is minimized w into d represents the amount of material movement that goes within the shop floor.

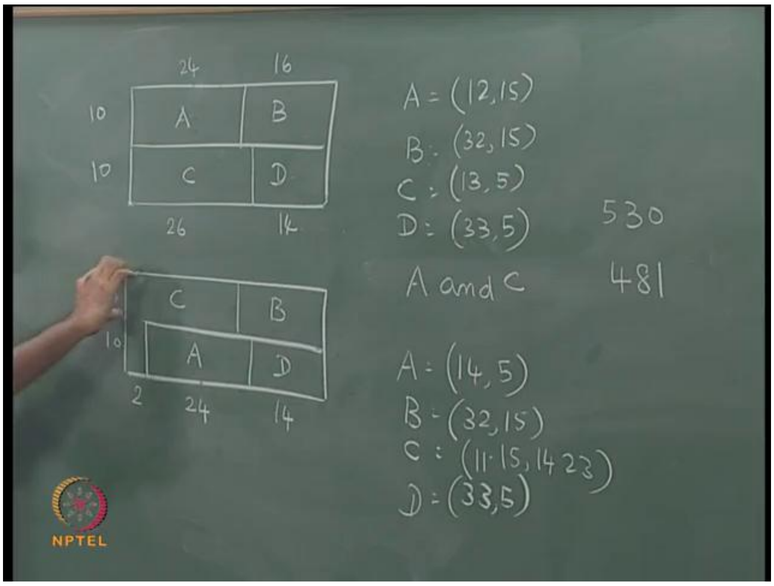
Now, in such cases the first thing is that the areas required by the various departments can be different. And since, if the areas are different how do we adopt or how do we modify this idea of a quadratic assignment problem suitably or how do we modify the idea of the exchange heuristic suitably to get a good solution, which can be implemented on the shop floor.

(Refer Slide Time: 21:27)



Now, that is answered by a very popular algorithm called the craft algorithm, now which was created by Armour and Buffa. Somewhere in the early 60's, and CRAFT algorithm stands for Computerized Relative Allocation of Facilities Technique. So CRAFT stands for computerized relative allocation of facilities technique. And CRAFT is a method by which, we also bring the areas of the various departments and then try to optimize the ton kilometer or material distance in a layout. So, let us explain the CRAFT algorithm through an example.

(Refer Slide Time: 22:28)



Now, let us assume that we have an initial layout which is like this, now let us begin with an initial layout, where there are 4 departments A B C D, the material movement among these departments is given by the same w i j that we have. And this is the initial layout, where A requires 24 into 10, 240 square units of area, B requires 16 into 10 square units of area, C requires 10 into 26 and D requires 10 into 14 square units of area. So, now, what we can do is now this is an initial layout that we create.

Now, we want to see whether this is the best layout or can we have other layouts better than this. So, to begin with the first thing we do, is we keep this point as some kind of reference point or origin, and then we try to find out the centroids of A B C and D. So centroid for A will now become, so, this will become 12 comma 15 because this is the midpoint, so this distance is 12 this is 10 plus 5. So, for A it is 12 comma 15 for B it will be 24 plus 8, 32 comma 15 for C it will be 13 comma 5, and for D it will be 33 comma 5.

So, now based on these centroids we can now create a distance, so we can now call these centroids as 1 2 3 4, and then we can compute a distance matrix here based on the centroids. So, the w into d for this given layout will be 3 times this distance plus 7 times the distance between this centroid and this centroid plus 4 times the distance between this and this plus 6 times the distance between this and this plus 5 times distance between this and this and 2 times distance between this and this.

Now, distance between for example, this and this will be absolute value of 32 minus 33 which is 1 plus absolute value of 15 minus 5, 10. So, it is taken as 11, so right through rectangular distances are use Euclidean distances are not used, so distance between B and D these centroids of B and centroids of D will be 1 plus 10 which will be 11. So, w into d for a given layout can be calculated, and this comes to about 530 for the given layout.

Now, the next thing that we have to do is in order to see whether we can have a better layout than this, what we will do now is, we will create a pair wise exchange by initially assuming that A and B will exchange positions, which means A and B’s centroids are exchanged, we could think of A and C exchanging positions, so A and C’s centroid get exchanged, we could think of B and C exchanging. So, there centroids will exchange and so on.

So, since there are 4 there are actually 6 possible exchanges, but out of the 6 possible exchanges, we will consider only 5 of them, we will consider those exchanges where there is a common boundary between the 2 facilities. For example, we will consider A and B, A and C we will not consider A and D because they do not have any common boundary or area. We will consider B and C there is a common area C and D C and D.

So, 5 changes can be considered by simply changing the centroids from A to B then this will if we are looking at changing A and B, then this will become A's centroid, this will become B's centroid. So, like this 5 possibilities exist, and then the best out of these 5 are actually chosen, and then we realize that the best exchange happens between A and C with value equal to 481. So, we have considered 5 exchanges that is between A and B A and C B and C B and D C and D, by exchanging the centroids.

We have not considered A and D, because they do not have a common border, and out of the 5 we realize that exchanging A and C gives us a good value of 4 81. But, then there is a small approximation that we have assumed, for the purpose of this interchange when we interchanged A and C we said A's centroid will become 13 comma 5, and C's centroid would become 12 comma 15. But we actually cannot exchange the centroids like that because the areas are different.

So, when the areas are different the centroids will change, so what we do now is we try to incorporate that, and then we actually exchange A and C to see what happens. Now, that is shown here, so when we actually exchange A and C. B will remain as it is, so B will be here, D will also be here. Now, we have to exchange A and C in a way the advantage of choosing departments that have a common border is by saying that when I exchange A and C I have to put A here and I have to put C here.

Now, the area that is available for C can accommodate A, whereas, the area that is available for A cannot accommodate C. So, what we do now is we try to put the smaller area into the bigger area, and keep the rest of the area for the bigger one, so what we will do now is, we will try and put A here now this is 41 A is 24 by 10. So, I will take another 24 for A this is 40, so 24 plus 4 38 plus 240, and C will now come here, now C loses it is rectangular area.

Now, it is actually made up of two rectangles which are like this, now I go back and update the centroids. Now, you see that A's centroid is 2 plus 12, 14 comma 5, 14 comes because there is a 2 here, and half of this 24 is 12 14 comma 5, these centroid will remain as 32 comma 15 D’s centroid will remain as 33 comma 5 C’s centroid will now have to be redefined, and that is computed as, now C is now taken as 2 rectangles. And we use the usual formula of a 1 x 1 plus a 2 x 2 by a 1 plus a 2 and b 1 y 1 plus b 2 y 2 by b 1 plus b 2.

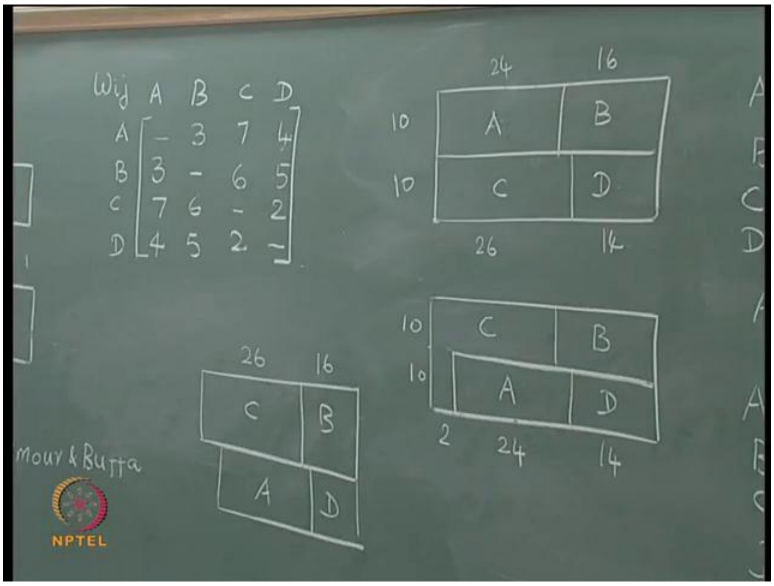
Now, when we use that this into centroid of this plus this into centroid of this divided by the area, will give us the actual centroid of C. We may even have a situation, where the centroid can go outside of the figure depending on the shapes that we actually get and therefore, in this case C’s centroid will now become 11.15 and 14.23. If we see carefully, the actual exchange has not happened 13 comma 5 has become as 14 comma 5, and 12 comma 15 has become 11.15 and 14.23.

Now, we have another layout which is like this, now when this layout we now try to make exchanges. Now, we can try and exchange A to B, we can do A to C, we can do A to D, we can do B to C, we will not do C to D, but we can do B to D. So, we can try 5 more exchanges by simply first exchanging the centroids, and then finding out of something is better. And after that actually making the exchange and updating the centroid.

So, this process can be continued till we actually get the best solution, and in this case the best solution would give us. Actually we realize that this would be the best solution, we are unable to get further improvement with this and therefore, we could actually terminate with this layout for this particular departments C B A and D. So, this would be the final output for the CRAFT algorithm, and we would now understand that we could have C and then B and then A and D, which are actually there.

Now, the algorithm would terminate with this kind of a layout, but then we also realize that having this kind of a layout is a little cumbersome. Because, C does not have a shape which is completely rectangular, and C has a different kind of shape, now for all purposes of control it is actually good to have all A B C D having rectangular shapes, and rectangle closure to a square. So, when it comes to actually implementing it we could think in terms of you know making some very small changes, and then checking out whether can we add some more area here and bring it, so that we can expand see a little bit.

(Refer Slide Time: 34:33)



So, we could have C occupying something like this, and then maybe we could think in terms of D being here, and A being here. Rather than have C come up to this, but then one has to understand that if we put C and B together we are looking at 26 plus 16 which is 42 instead of the 40 that is available. So, then we explore and see whether we could get some more area here for the department, and not have the entire thing as the rectangle. But, then each department actually has a rectangular area for itself.

Now, this is how the CRAFT algorithm works. CRAFT algorithm has a couple of advantages one is CRAFT is able to consider different area requirements for different departments, which was not explicitly considered by the quadratic assignment problem. Second CRAFT is a simple algorithm, it is a heuristic algorithm it is centered around pair wise exchange of areas of the various departments. So, it does not guarantee the optimal solution always, it is a heuristic solution.

Sometimes by the very nature of updating the centroids, we could get you know shapes like this. But, as I mentioned common practice is to convert them into manageable rectangles, so that a layout like this can be finally achieved even though the craft layout is like this. Now, CRAFT is a very significant contribution to the literature in operations management and layout because CRAFT also had a computer program written in the early 60's, which could actually generate and one could print a layout, using the computer program for the CRAFT.

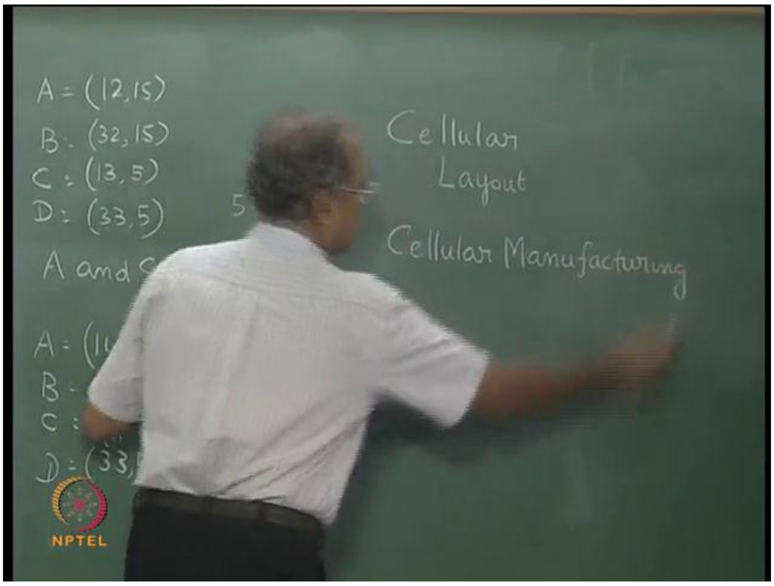
For this reason CRAFT became an extremely popular algorithm, even though CRAFT has certain simple limitations. Like the fact that it is not an optimal algorithm, it is essentially a heuristic algorithm, and sometimes it would give us some kind of complicated shapes like this. CRAFT also can be modified and has been modified to include three way exchanges, and several modifications to CRAFT have also happened.

In this lecture series we have touched upon very basic algorithms in the area of location and layout, location and layout as an area is extremely important and as mentioned takes care of strategic decisions, in the context of manufacturing and supply chain. There are several algorithms both qualitative and quantitative that researchers, and practitioners have created over the last 50 years or so which will try and meet the requirements of location and layout.

But, then we have addressed very basic algorithms, which capture the very essence of the ideas in facilities location, and facilities layout. So, let us now have a quick recap of what we have actually covered, and are there few other things which can be touched upon before, we kind of wind up this part of operations management in this lecture series. Now, the other things that one could look at is in terms of layout, what we have actually done here is we have tried to model what is called a functional layout into a given area.

Now, when we say that there are 4 departments A B C D, we assume that there is departmental specialization. And these departments have interactions amongst them, so a typical functional layout, will be laid out this way, where A B C D would be four departments in a functional layout. Much later if we see the advances, the different types of layouts are would be the line layout, and the functional layout. Line layout is also called the product layout, where according to product the machines are laid out. Functional layout is called the process layout, so the machines are laid out according to the manufacturing process.

(Refer Slide Time: 38:57)



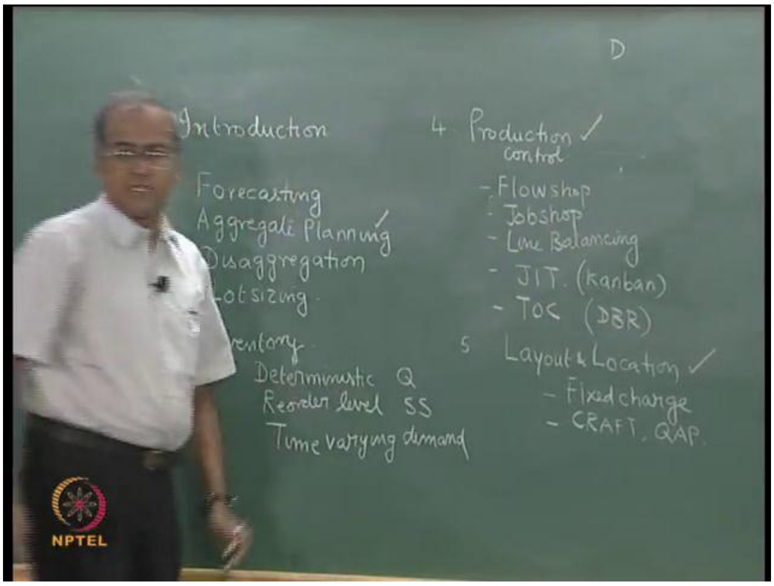
Now, much later people moved on to what is called cellular layout or cellular manufacturing, about which we saw briefly in the introductory lectures to this lecture series, where machines different machines are grouped together, in order to process a set of parts which are called part families, so in the cellular manufacturing and cellular layout, a set of parts which can be made by a set of machines, the parts would be similar the machines would be dissimilar functionally.

But, they will be capable of making everything required for these parts are all grouped, and each machine in part group is called a cell. So, the layout move towards what is called cellular manufacturing, where the cells where created and cells were also kept separate. The difference is in this context of cellular manufacturing, we do not consider each cell like a department here because material movement among the cells is far minimal compared to the material movement, among the departments in a functional layout.

So, ordinarily we do not try to optimize the ton kilometer distance amongst the various cells because the very principle of cellular manufacturing the material movement among the cells is very, very less. So, the basic ideas in cellular manufacturing is more to do with cell formation than to do with how to layout the cells; however, depending on the unidirectional flow in the cell, depending on some intracellular movements, depending on the flows of various parts that go in the cell.

Cell layout is also an important aspect to be touched upon, though we do not attempt to go deeper into cell layout in this lecture series. So, let us come back from the beginning and see what are the topics that we have actually covered in this lecture series.

(Refer Slide Time: 41:31)



So, we started off with an introduction to, we started off with an introduction to manufacturing and operations management. We also described what was the basic challenge that today's manufacturing organizations face, and challenges that are there faced by organizations, which are part of any supply chain. And then we moved into basics of operations management, we started with forecasting and we explained a lot of time series models for forecasting.

And then from forecasting we moved to aggregate planning, where using the demand for the products, we made decisions on how much of regular time, overtime capacity that is being used. And from aggregate planning we moved to disaggregation, which gave us how the time that comes from an aggregate plan is going to be used to make different products. Then we also saw a little bit of lot sizing here, and then we moved to inventory problems, where we saw deterministic inventory, order quantities.

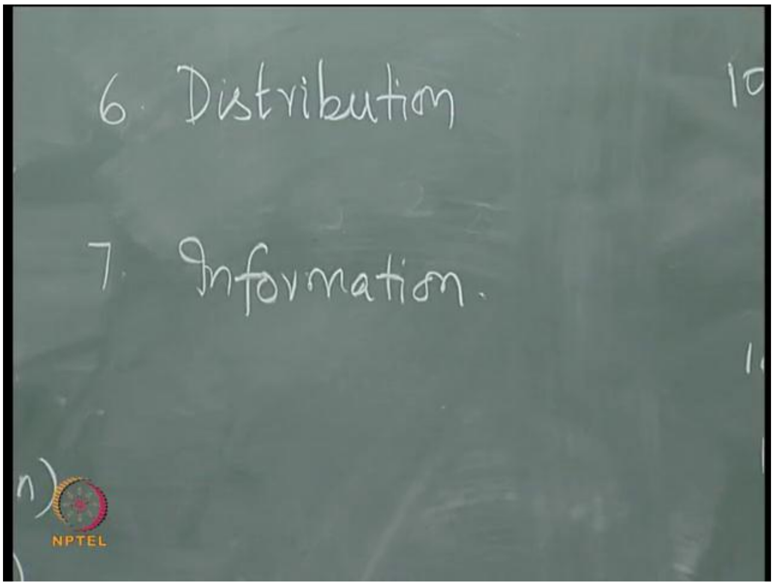
And then we saw the reorder levels, and the safety stocks, and we discussed some probabilistic inventory models, which also spoke about the safety stock that is required. We then moved into production control, within the inventory we also saw inventory with time varying demand, we saw some aspects of production control, we saw flow shop scheduling, we saw job shop scheduling, we also saw line balancing. There are two other aspects of production control, which we have not touched upon in this lecture series which we would possibly in different courses under NPTEL.

And those two aspects of production control are called JIT or Just in Time manufacturing, which also deals with kanban controlled manufacturing systems. And the other is called theory of constraints or synchronous manufacturing, where the production control mechanism is called the DBR mechanism or Drum Buffer Rope mechanism. So, these two parts of production control, we have not touched in this lecture series.

Then we saw some aspects of layout and location, we saw the fixed charge problem or location allocation problem, which is also called the network design problem. And we have seen some aspects of layout, such as the craft algorithm and the quadratic assignment problem. So, what we have seen, so far are from the context of supply chain we could call major decisions in supply chain as location decisions, production decisions, inventory decisions, distribution decisions and information related decisions.

So, from a context of supply chain we have seen location decisions here, where we spoke about different aspects of location and layout, we seen production decisions. Production planning decisions, as well as production control decisions, and we have seen aspects of inventory, so we have seen inventory decisions.

(Refer Slide Time: 46:08)



And what we need to see are distribution decisions, and information related decisions in the context of a supply chain. So, we will spend some time on the distribution as well as information related decisions, in the context of supply chain management in the subsequent lectures in this lecture series.