Operations and Supply Chain Management
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Lecture – 33
Location Allocation Problems in Supply Chain. Layout

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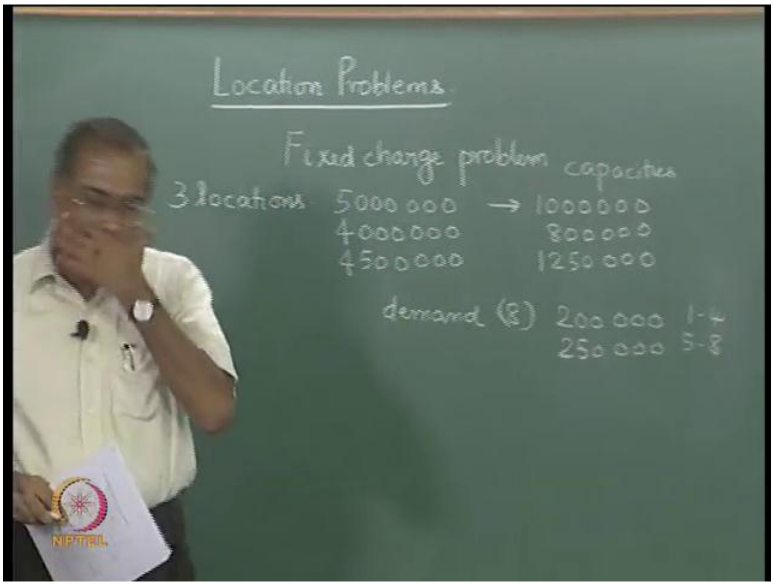

In the previous lecture, we saw the formulations for the fixed charge problem, where we looked at locating certain facilities and allocating the capacity of those facilities, to meet the requirement at some demand points. We saw 2 versions of the problem, one version is where a demand point can receive, from two facilities the other version where, every demand point is met from only one facility. So, we will take a numerical example to show, how this fixed charge problem works, now let us consider 3 locations and the fixed cost to locate in these would be.

So, the 3 facilities the fixed cost could be taken as five million, 4 million and 4.5 million, now the capacities or in these 3 facilities could be taken as, so they are taken as 1 million 800000 and 1.25 million are taken as the capacities. Now, will assume that, there are 8 demand points and the demand in 8 are 200000, for 1 to 4 and 250000, for 5 to 8.

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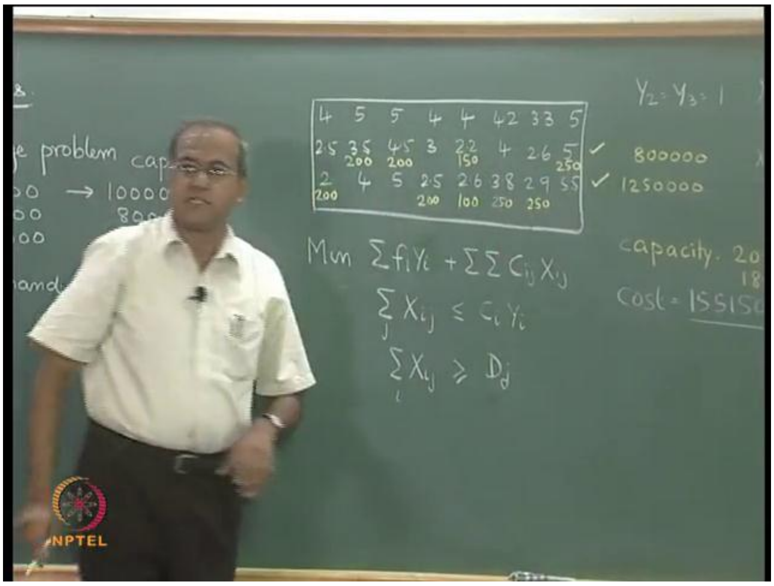

We will also assume that, there is a unit transportation cost from the 3 proposed sides to the 8 demand points and these are given as 4 5 5 4 4 4.2, so these are the unit costs of transportation, so if we use the formulation that, we saw in the previous lecture. So, the formulation will have an objective function of the type minimize sigma f i Y i plus C i j X i j, so the f i Y i will be 5 million into Y 1 plus 4 million into Y 2 plus 4.5 million into Y 3, the C i j X i j will be 4 into X 1 1 plus 5 into X 1 2 up to 5 into X 1 8 and so on.

We also have the constraints, we have 3 constraints for the capacity of these 3 that come in, there will be 8 constraints of the demand, so there will be 3 constraints for the capacity, the capacity constraint would be like this summed over X i j, j is less than or equal to C i Y i. So, if facility i is chosen then, there is a capacity C i and things can be transported from i, so this will be the capacity constraint, this will be the demand constraint, for every j summed over i will be greater than or equal to D j, which is the demand for that item. So, we can actually solve this problem, optimally and I will give the optimum solution to it.

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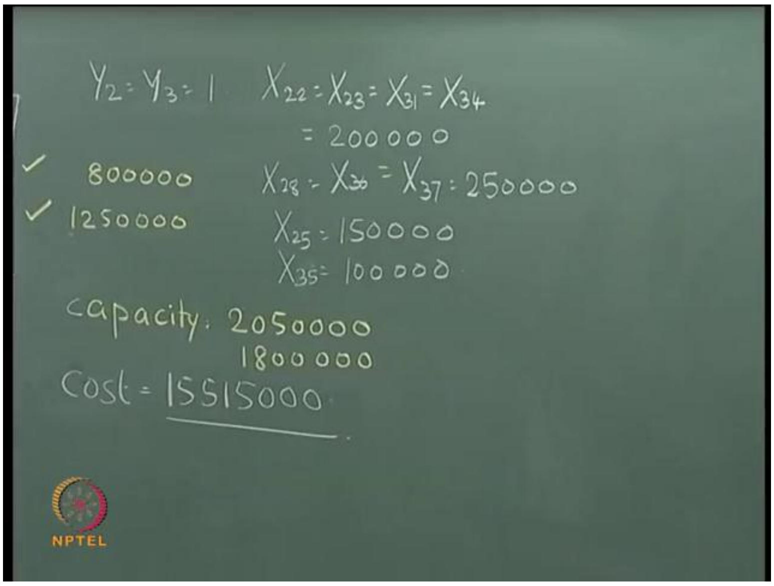

So, the optimum solution would be Y 2 equal to Y 3 equal to 1 with X 2 2 X 2 3 X 3 1 3 4 is equal to 200000 X 2 8 X 3 6 X 3 7 is equal to 250000 X 2 5 equal to 150000 X 1 5 equal to 100000. So, if you look at this solution, the optimum solution has picked Y 2 and Y 3, so it has picked this as well as this, so there is a cost of 9.4 plus 4.5, 8.5 million will be the fixed cost and the locations are now these 2 are chosen, so this and this are chosen.

So, the capacities are 800 plus 1 2 5 0, so 2 0 5 0 0 0 0, so available capacity equal to 2 0 5 0 0, the requirement is 200000 into 4, which is 800000 plus 250000 into 4, which is one million, so the requirement will be 1 8 0. So, total requirement is 1.8 million whereas, the total capacity is 2 0 5 0 0 0 0, so we have enough capacity to meet the demands of these 8 places and let us see how the capacities located to the various demands.

So, we have X 2 2 equal to 200000, so this will give 200, I am just writing 200, but it is 200000 X 2 3 is 200, so there is another 200 here, X 3 1 is 200, so X 3 1 is 200, X 3 4 is 200, X 2 8 is 250, X 3 6 is 250 and X 3 7 is 250, 2 5 is 150 and X 3 5 should be 100000. So, this should not be 1 2, this is X 3 5, X 3 5 should be 100000 1 5 would not come here, because facility one is not chosen only 2 and 3 are chosen, so the transportation can happen only from facilities 2 and 3.

Now, we realize that, if we see this capacity, this capacity is 800000, this capacity is 1 2 5 0 0 0 0, now we realize that, this 800 capacity is utilized like this 200, 200 remember these are in 1000s, so 200 plus 200 400, 550 plus 250 is 800, 800000 is utilized. Here, it is 200, 200, 400 plus 100, 500, 500 plus 1000 is 1500 plus 500 is 1000, so there is a excess capacity that is left here, so the demand for all those is also met by this solution.

So, the cost corresponding to this solution has 2 components, one is the fixed cost component, the other is a variable cost component, the fixed cost component is a sum of these 2, which is 8.5 million. The variable cost component will be the multiplication of these and the final value, when we find out is total cost at the optimum is equal to 1 5 5 1 5 0 0 0 comes as the optimum cost of this.

Now, there are this is the solution that, we get if we formulate this problem as an integer programming problem and solve now sometimes heuristic solutions are also sought for this problem, where the heuristic solution can be of 2 types. One is to try and find out in terms of cost that is to try and locally optimize this portion, which is the total fixed cost that is required, which means to choose facilities that have lower fixed cost that together can meet the total demand.

So, which would give us these 2 facilities and once these 2 facilities are chosen, this one is not chosen, take only these 2 and solve a transportation problem with the capacity of these facilities as the supplies and the demand taken there and we would get a solution to that. Another way of solving it is, to try and solve a transportation problem first, looking at all of these and then, we will try and get some facilities being used, some facilities not being used. We may in this case get a solution, where if we solve it as a transportation problem first we may still have some allocations from here, here and here.

Which means, if we solve the transportation problem optimally, it might ask us to send something from this, to send something from this and to send something from this, it may happen. In this particular example, it may not happen, because these 2 put together have enough capacity to meet and all the costs here are greater than or equal to the cost except this one. So, it may still not happen, but then if we have different types of data and if we solve a transportation problem, it might give some allocations here, some allocations from here, and some allocations from here.

But, then it will be costly to create all the 3 in all the 3 places, because there is going to be both excess capacity and very high cost, then from the transportation solution, we can do some local optimization, to try and reduce this cost further. But, for smaller sized problem such as this, which involves only 3 potential locations and 8 demand points, it is always easy to solve this problem optimally, using any solver and try to get the best solution to the problem.

Now, the second variation of this, is a case where, a demand met entirely by one facility and not by more than one facility, if we see this particular example, the demand of this is met only from here, only from here, only from here, only from here, 5 is met both from 2 and 3. Otherwise 6 8 7 out of the 8 demands, the demand points the demands are met from supply from only one facility and not from 2, except demand 0.5, which gets 150 from location 2 and 100 from location 3.

So, if we put an additional restriction that, the entire demand is to be met, only from 1 supply, we have seen the formulation for that, we have said C i j D j X i j, X i j equal to 1, if facility I meets the requirement of demand point j. And we also said in this case that, this should be D j X i j should be less than or equal to C i Y i and X i j should be equal to 1, so it comes only from one facility. We have already see this formulation in the previous lecture, we will now show only the solution to it, so in this case, what will happen is since, there is additional capacity, this entire demand will be met from here.

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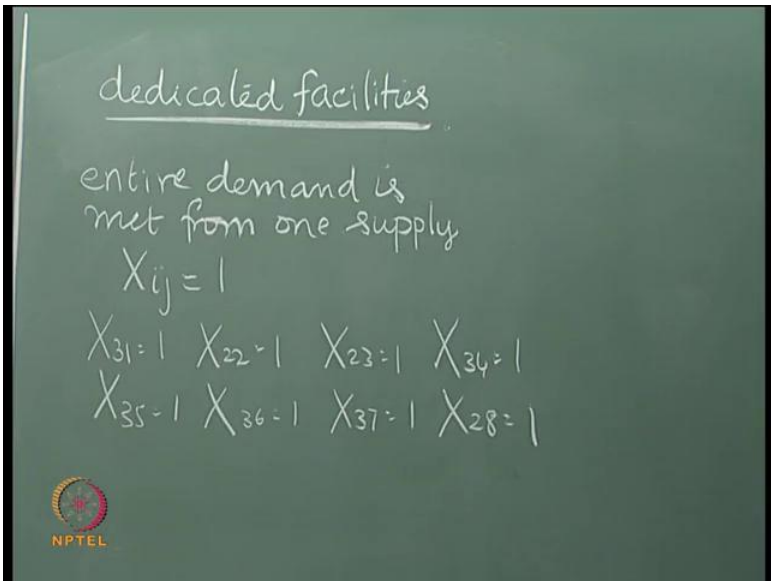

So, the solution would now become, the only change in the solution would be that, X 3 5 will be equal to 1 and the entire demand of 5 will be met only from this, so there will be, so this will have enough capacity to meet. This 800000 will be spent as 200 plus 200 plus 250 about 150 will not be used here, this entire 150 will move here to get 200 200 250 250 and 250, it would still be within the 1 2 5 0 0 0 0.

So, the solution there will be still X 3 5 equal to 1, which means all the demand of 5 will be entirely met from supply 3, because the when, we consider dedicated facilities, entire demand is met from 1 supply X i j will be equal to 1, if supply I meets the entire demand for j. So, the solution would be X 3 1 equal to 1, X 2 2 equal to 1, X 2 3 equal to 1, X 3 4 equal to 1, X 3 5 equal to 1, X 3 6 equal to 1, X 3 7 equal to 1 and X 2 8 equal to 1.

So, there will be an additional cost and that additional cost will be this 100 and 50000 into 0.4 will be the additional cost that, we will incur, if we solve this problem with dedicated facilities. So, this is a numerical example, which kind of tells us how the fixed charge problem works, there is a location component, there is an allocation component to the fixed charge problem, there are still a couple of interesting issues in solving the fixed charge problem.

One is this f i represents the fixed cost of setting of the facility, whereas C i j X i j represents the cost of transporting items from the chosen facilities to the demand points. Now, there is also a D j, which comes here D j is the demand during period j, now what is the period that, we are talking about now this is like a onetime cost of allocating or locating a facility whereas, this is incurred every time there is transportation. So, the best way to define f i and C i j is like this, f i is should be defined as some kind of a equivalent cost per year in locating it.

For example, if this facility has a realistic life span or a period of use of say 10 years and whatever is the total f i should be brought down to equivalent 1 year, now d j should represent the demand at point j for 1 year and C i j is the unit cost of transportation. So, that both these actually represent cost corresponding to a certain time period and corresponding to the same time period. So, if D j is demand per month in point j then f i should adequately represent, the equivalent monthly cost, if we were to locate this facility in this place i.

So, one should not say that f i is the cost that is incurred over a 5 year life of the plant whereas, D j is a monthly demand, the assumption of course, what we make is there is a single item or D j represents the demand of an equivalent single item. Another assumption that we make is that, this D j is not going to change with time, so D j represents the demand at point j of an equivalent item, for the equivalent period.

So, both f i and D j particularly, these 2 parameters one is the cost parameter for location and the other is the demand parameter, the one has to make sure that, they are factored suitably to represent the period, for which this cost is computed. So, it is customary to compute it as an annual cost, so f i is often taken as the equivalent amount per year and D j is taken as the demand of an equivalent product for 1 year.

So, that we are consistent with the notation and the use of the algorithms to solve this problem, now we can actually expand this fixed charge problem in the context of a supply chain to consider intermediate facilities also. So, when we looked at the basic fixed charge problem, we said there are plants or production facilities and there are demand points or distribution points, now we can expand this to consider intermediate facilities.

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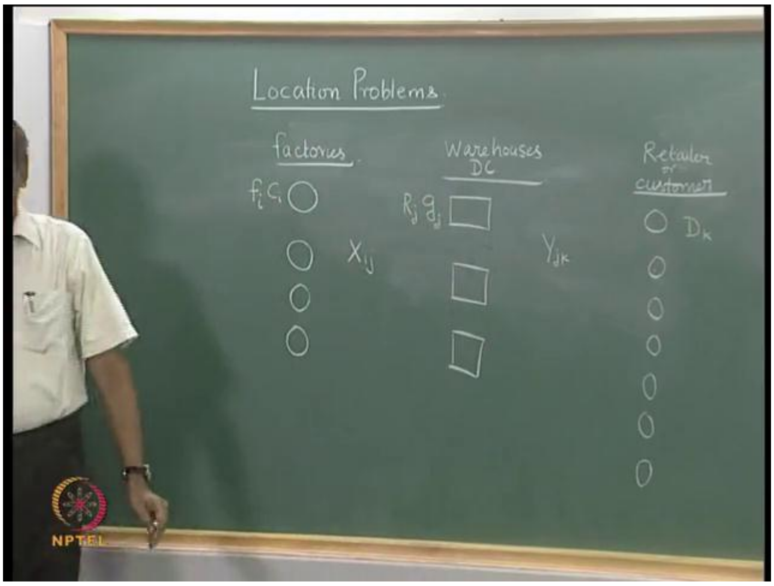

So, there could be factories here, there would be warehouses here or distribution centers and then there would be retailer or customer or customer, so we would have multiple locations for these factories. Now, there could be different possible locations for the distribution centers and then, there could be several customers, which are here, now as far as this. So, we could say that, there is a fixed cost f i of locating a factory here, there could be a fixed cost of g j of locating a warehouse in this place.

So, then we would and there will be a capacity of C i here and there could be a capacity of some R j here and there will be a demand of D k from customer k. So now, the decision variables will become X i j are the quantities transported between the factories and the distribution centers and Y j k are the quantities that are transported between the distribution centers and the retailers. So, there is a certain fixed cost of locating the facilities and there is the variable transportation cost.

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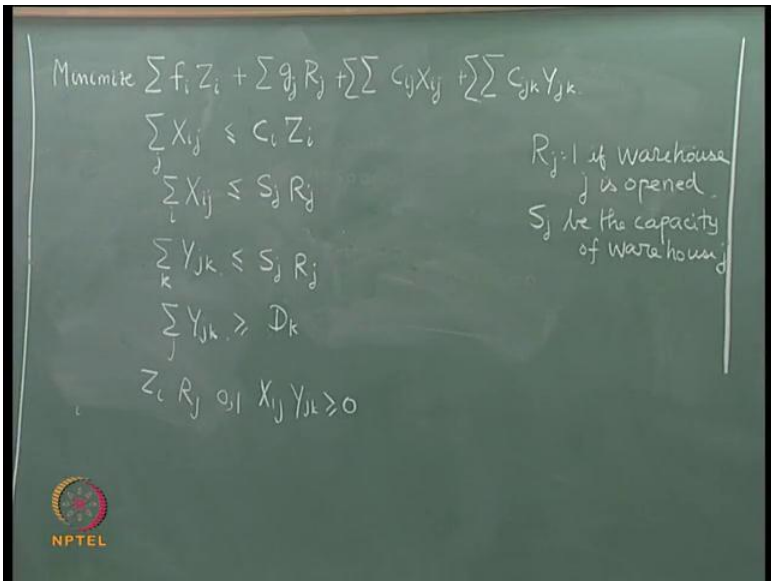

So, the problem will be to minimize f i Y i, now let us not call it Y i, let us call it as X Y Z i, so Z i equal to 1, if we locate a factory in place i plus g j say R j, where R j equal to 1, if we locate a warehouse in place j plus C i j X i j, one more summation. C i j is the unit cost of transportation between i and j and X i j is the quantity transported plus C j k Y j k, where C j k is the unit cost of transportation from this to this and so on. So, this will be the objective function that has the location cost, as well as the transportation cost.

Now, the first thing that we require is this, so what goes out, so sigma X i j, X i j from a particular i summed over all j, so from this whatever goes into this and there is a second summation also here, because 2 facilities are involved. So, whatever goes out, if this should be less than or equal to its capacity C i, so sigma X i j summed over all j should be less than or equal to C i into Z i, Z i comes in only, when the facility is chosen I can move something out of it.

So, whatever that goes out is less than or equal to C i Z i, now as far as these warehouses are concerned, whatever it receives should also be first two things will have to happen, it can receive and some quantity X i j only when, the warehouse is opened, otherwise it cannot receive. So, as far as every warehouse is concerned, whatever it receives X i j summed over i, whatever it receives, one way is to write it as should be less than or equal to big m into R j.

So, R j is 1 only when there is a warehouse that is opened here, so only when a warehouse is opened, it can receive something, if the warehouse is not opened then this R j will be 0, Big M is large and positive, therefore it cannot receive it. If this warehouse is opened then whatever it can receive right now an infinite amount of quantity, but if there is a capacity with respect to this, which is, now I defined here.

Now, R j is a variable, now let me write it R j equal to 1, if warehouse j is opened and let S j be the capacity of warehouse j, so one other way is to say that instead of saying big m. So, whatever it receives should be less than or equal to S j R j. R j is the variable S j is a known capacity that it is there, now similarly whatever leaves this warehouse can leave only from a warehouse that is opened, so sigma Y j k summed over k should also be less than or equal to S j into R j.

Now, whatever leaves it can leave only from a opened warehouse and whatever leaves should be within the capacity of the warehouse, so Y j k summed over all k, so whatever leaves from a chosen warehouse should be less than or equal to the capacity of the warehouse and whatever reaches the customer. So, sigma Y j k summed over j should be greater than or equal to D k should be greater than or equal to the demand of the customer.

Now, we will have Z i R j as binary and X i j and Y j k greater than or equal to 0, which represent the transportation quantities, so this is a generic formulation of a multi facility or a multi level location allocation problem, where we consider 3 levels here in the production distribution network. And we say 2 stage this is one stage, this part is another stage or essentially, there are 3 levels and 2 sets of interactions between the factory and the warehouse and between warehouse and the retailer.

Now, one can expand it by including one more level in between, which could be factories warehouses separately distribution centers, as well as retailer and the location will be restricted to factories warehouses and distribution centers. So, depending on how many levels are there in the supply chain the formulation can get bigger and bigger, right now, the formulation talks about only one item or a single item that is being transported.

So, D k is defined only as demand at customer k or at retailer, k which is for a single item, for an equivalent item, then we can add number of items into it and this demand will now have a second dimension, which is at customer k, for a particular item l. In which case the production capacities will now have to be defined, for producing each of the items, it would not be C i, it will be C i l. Then, we would also have time as another dimension, where the demand can be given for various time periods, so D will now become D k l m demand at customer side k for item l in time period m.

In which case, the capacities may be defined as different, for different time periods, but same capacity for each time period, but for different items, so this would have one more subscript that comes. Now, these capacities, which are our S j's ,now can also be defined item wise or can be kept as a generic upper limit on the capacity of the warehouse. Sometimes the item capacities come into the picture, if different items are of different sizes or if different items and products require some kind of a specialized either equipment or infrastructure to be preserved and kept, in which case the capacity will now become item specific in addition to the capacity in the place. So there will be one more dimension that would come in. So, we add the number of dimensions the problem becomes far more bigger and complicated in terms of number of variables and in terms of number of constraints. Right now the cost or the cost of location and the cost of distribution, sometimes when we consider multi period problems, we would even look at cost of holding inventory and cost of not meeting the demand in that period.

So, we could add terms that are for inventory cost, as well as shortage or back order cost, if we consider multiple time periods in this formulation. So, the formulation can become extremely big and large, which would be would become a little difficult to solve particularly, when we are considering a large number of facilities. We are considering multiple items and we are considering multiple periods. So, in such cases getting the optimum solution can be bit difficult and people would resort to using heuristic solutions to solve this problem. Nevertheless, this is the broad frame work with, which we formulate and solve location allocation problems or fixed charge problems in the contacts of the supply chain, we now move on to the next part, which is about the layout.

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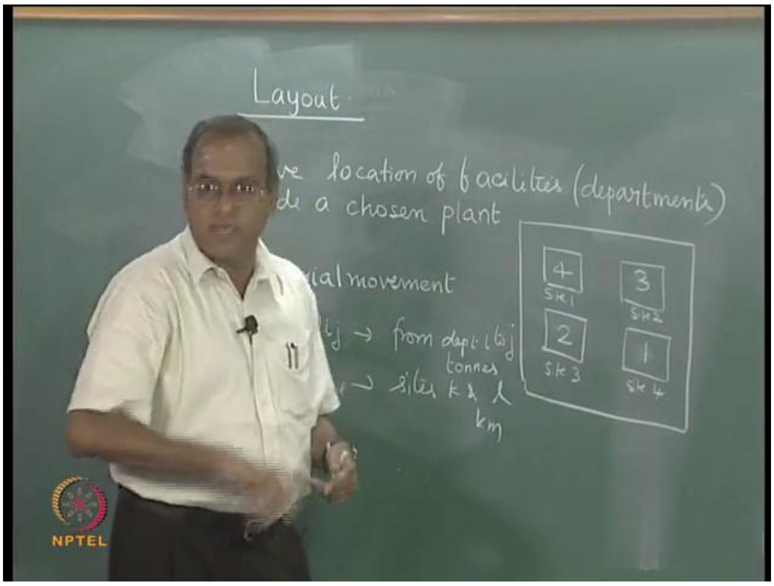

Now, so far in location, we also spoke about where to locate a facility, location decisions are one of the most important decisions, for the success of any organization. Where does one locate the factories, where does one locate the central distribution points etcetera are strategic decisions, which actually dedicate the success of the supply chain in manufacturing. Now, from location, we move to layout in talks always about relative location of facilities, so the word relative is very important, when it comes to layout.

So, layout is relative location of facilities inside a chosen plant or location, essentially it is like, we have decided to locate a factory somewhere, now within that certain departments have to be located. So, layout normally talks about location of facilities or departments within a chosen plant, so there could be 4 or 5 departments and how does one relatively locate, these 4 or 5 things inside. Now, why does a question of relative location come into the picture, now if for example, we have 4 departments that have to be located within a certain place or within a certain factory?

And suppose, we locate department 1 here, department 2 here, department 3 here and department 4 here, now there are 4 places, where these 4 departments can be located, now this is called location 1 or is usually called site 1, site 2 site 3, site 4. So, in this example, we are talking about locating department 1 in site 1, department 2 in site 2, department 3 in site 3, department 4 in site 4.

We could have another situation where, department one is located in site 4, department 2 is located in site 3, department 3 is located in site 2, department 4 is located in site 1, this is another possibility. Now, why are we concentrating, so much on the relative location the reason is within this is let us say the overall facility and 4 departments are located, what we assume in all layout problems is that, there will be a material movement, among the departments.

So, let us call some w i j, as the amount of material movement from department i to j and this material moves within various places, now there is also a distance between sites k and l, which we call as d k l. So, if we have this kind of a layout, then the total material movement will be this is usually given in tons, this is in kilo meter, so this is a weight measure, this is a distance measure. So, the material movement has a unit of ton kilo meter or weight into distance measure, so if we have a, this kind of a layout.

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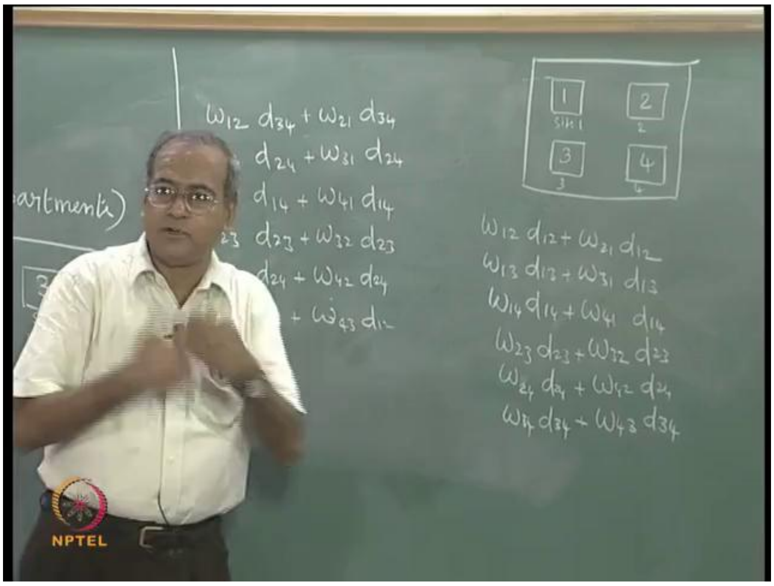

Then the total material movement will be W 1 2 from 1 to 2, there is a material movement, into d 1 2 w 1 2 into d 3 4, because department 1 is located in site 4 department 2 is located in site 3, so w 1 2 amount of material moves from 1 to 2 and moves a distance of d 4 3, from 4 to 3 it moves. We will assume by symmetry that d 4 3 is a same as d 3 4, similarly whatever moves from 2 to 1 will also move the distance d 3 4, so we will have w 2 1 into d 3 4.

Now, between n 1 and 3, there is going to be a w 1 3 d 2 4 plus w 3 1 d 2 4 moves from here and whatever moves from here, similarly w 1 4 d 1 4 plus w 4 1 d 1 4, this is the interaction between these 2. Now, the interaction between these two will be w 2 3 d 2 3 plus w 3 2 d 2 3 interaction between these 2 the interaction between these 2 will be w 2 4 d 2 4 plus w 4 2 d 2 4 and lastly the interaction here will be w 3 4 d 1 2 plus w 4 3 d 1 2. But, if on the other hand, we do not follow this layout, but instead we follow some other layout like this, where say 1 goes to 1, 2 goes to 2, 3 goes to 3 and 4 goes to 4.

This is site 1 site 2 site 3 site 4, then the weight into distance ton kilo meter measure will become w 1 2 d 1 2 plus w 2 1 d 1 2, w 1 3 d 1 3, w 3 1 d 1 3, w 1 4 d 1 4, w 4 1 d 1 4, w 2 3 d 2 3, w 3 2 d 2 3, w 2 4 d 2 4, w 4 2 d 2 4, w 3 4 d 3 4, w 4 3 d 3 4. So, if we follow this layout, this is the total ton kilo meter, if we follow this layout, this is a total ton kilo meter, now whichever is cheaper or less costly, we would use it. So the problem of relatively locating them with respect to each other becomes significant.

So, in layout we normally have 2 types of layout models, some of which are called qualitative layout models and we have quantitative layout models. Now, in a in a typical quantitative layout model, such as what we have seen, we will have a w matrix, which is the material movement matrix and then, we will have distance matrix. And then, we would like to try various combinations of putting them into various sites and then evaluating the total ton into ton kilo meter or weight into distance and trying to find out, which is the best.

So, quantitative models concentrate on that qualitative models on the other hand approach the problem in a slightly different, way which we will see right now. So, if we have 4 departments. Let us say, we have 4 departments and they have to be placed in certain places relatively, now what we do in a qualitative model is we try to define a letter or an index, which captures the relative importance of these facilities being located to close to each other.

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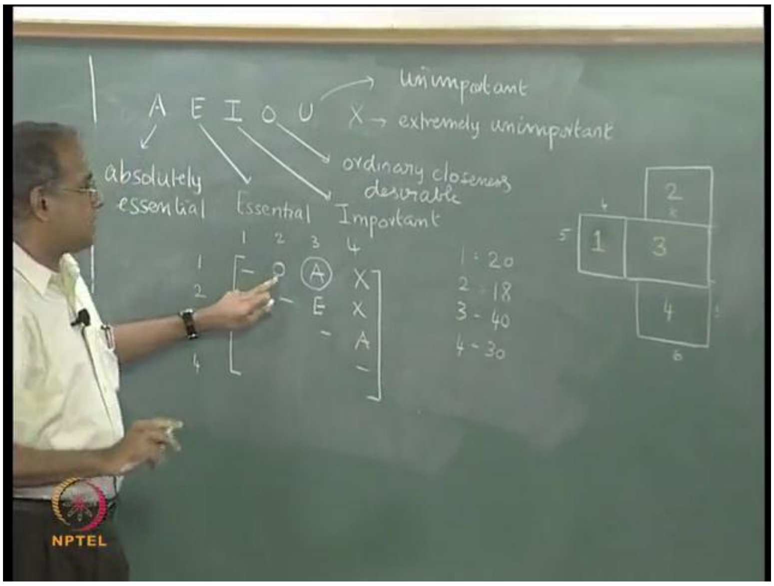

Ordinarily, we use terms like A E I O and U, sometimes X is also used, where A means it is absolutely essential, E would means essential, I is important O is ordinarily ordinary closeness desirable, U is unimportant and X is not at all required or extremely unimportant. Now, what we do is there are only let us say, there are 4 facilities departments that have to be allocated, so what we do is we try and create a matrix, which is made up of A E I O U and ask the experts to define, how important it is to keep these, these two facilities close to each other for example.

We would have 4 departments 1 2 3 4 1 2 3 4, now somebody might say might fill an O here, somebody might fill an A here, somebody might fill an X here, somebody might fill an E here, somebody might fill an X here, somebody might fill an A here. So, here we are not quantifying the relative importance through, the weight or material movement and the distance. Here we ask the experts to say based on what they have seen and based on their view as to how important, it is 2 keep these 2 facilities close to each other.

So, they would mark and an A or E or A I or A U inside this matrix, sometimes this is symmetric sometimes, it need not be symmetric, in general it is not a wrong thing to assume that it is symmetric. But, if we go purely by the w i j’s, there can be situations, where the direction of flow of material would be from 1 to 2 and not from 2 to 1 and therefore, they may not be symmetric.

But, by and large symmetry is assumed, so if someone marks and A or E or I or O or U or X then, out of these A is the biggest thing. So if there is an A, you want these 2 to be very close to each other, if there is a U or an X, you do not wanted to be close at all, you do not want them, you want them to be far away. Now, based on these, we could have some kind of relative location of these departments, now we look at this matrix and try and find out there is a U between the highest is a there is an A between 1 and 3.

Now, we might also assign some kind of areas to these 4 departments, suppose we say that after all these departments require space and let us say the department 1 requires 20 units of space, department 2 requires 18 units of space, department 3 requires 40 units of space, department 4 requires 30 units of space. Now, we realize that 1 and 3, there is an - so one is 20 units of space, so we could try and have a rectangle as close to as a square, whose area is 20, so we could think of a 5 by 4.

So, we could have facility a here with 5 by 4 here, so this is a with 5 by 4 and then a 1 to 3, this is department 1 department 1 with area 25 by 4 department 3 is the next best, so here it is 40, we could think in terms of 8 by 5 rectangle, which is close to a square. So, we could think in terms, this is 5 and this is 4, so we could have another 8 by 5, which is facility 3, so this is 8 by 5, this is 4 by 5, let me use a different color to show that, these are facilities, so 1 and 3.

Now, there is another between 3 and 4, which is here, so 3 and 4 has an so 4 requires 30, so we could think in terms of five into 6. So, there is a 5 here, we could think in terms of now we could try and put. And now 3 and 4 as an A, so we could think in terms of the 4 coming here or 4 can come here or 4 can come here, if we want this entire thing to be a very compact square or a rectangle then, we would put department number 4 somewhere here with 30.

So, we could have 5 into 6, so department 4 will come here, so this is department 4 with 5 and 6, now the only one that remains is 2, so 2 as an O E and X, so E is the next best 2 to 3 is E. So, we would think in terms of putting 2 somewhere here, 2 is 18, so 2 could be a 6 into 3 or it would be a 5 into 4 minus a small quantity, so we could think in terms of 2 coming somewhere here, because 2 to 4 is also an X 2 1 to 2 is O. So, we could do this or we could think in terms of putting 2 above here, which could also be possible, because 2 to 3 is E 1 to 2 is also O.

So, putting it above would bring 1 and 2 closer to each other, so typically 1 would say that this is how, we would have the layout for departments 1 2 3 and 4, now there is a certain amount of subjectivity in the whole thing. But, there is still a certain reason and a scientific way of relatively placing them, to take care of some of these, there are better versions of the algorithm that, we just now described.

But, most qualitative algorithms are around a basic principle of trying to generate a matrix, which captures how close, we want the departments to be and then, essentially trying to have a layout of these, which fits as close to possible, as what as what is represented in this matrix with regard to some of the quantitative models for layout, we will see them in the next lecture.