Operations and Supply Chain Management
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Lecture - 20
Disaggregation – Time Varying Demand, Safety Stock – ROL for Discrete Demand Distribution

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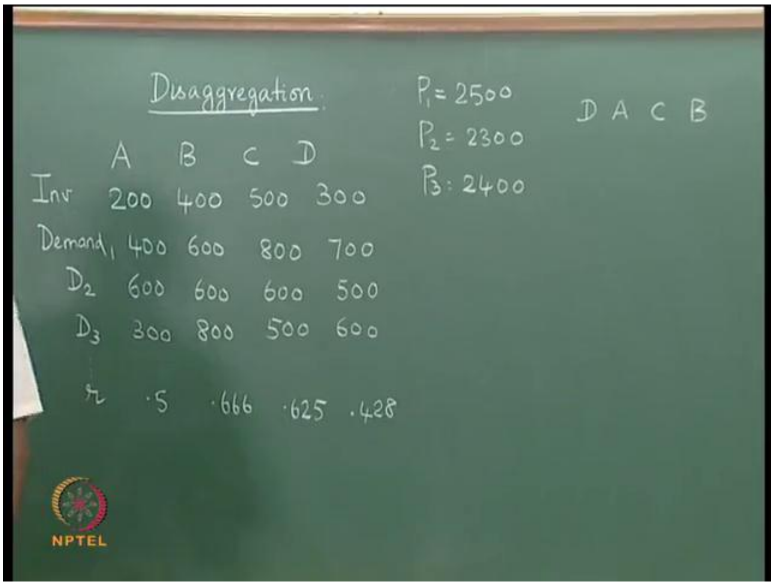

In this lecture, we continue the discussion on disaggregation models. We have already seen 2 models in disaggregation, we will look at the third model, where we consider that the demand varies in each period and the production capacity also varies in each period. So, let us explain that using an example, let us say there are 4 products A B C D. So, these are the inventory positions that are available for the 4 products and these inventories are given in man hours or person hours. And then there is a demand for these products. In the previous example, we assume that the demands 400, 600, 800 and 700 for the 4 products in all the periods, the demand did not change and in the previous model, we assume that the production capacity P is 2500, which happened to be the sum of the demands.

Now, in this model we are going to assume, that the demand is going to change in every period. So, we call this as demand 1 and then D 2 or demand 2, will be something like this 600, 600, 600 and 500 say demand D 3 will be 300, 800, 500, 600 and so on. So, we could have any number of months of demand that is known and most of the times these demands are actually forecasted values of the demand for each of these products. We also assume that P 1 production capacity available, in the first period or 1st month is 2500 and we would assume that P 2, this also change so this could be 2300 and let us say P 3 could be 2,400.

Now, the first thing that we did, in the earlier example was to calculate this r, r is the period up to which we need not produce and use the existing inventory to meet the demand of the item. For example, for product A or item A the inventory, current inventory is 200 and the first month's demand is 400. So, with this 200 inventory, we can meet 0.5 of the demand of the 1st month. So, 0.5 months we can use this inventory, we need not produce. But, we have to make sure that we produce before 0.5 months otherwise, we will be short of material or item. Similarly, inventory is 400 demand is 600. So, 4 by 6 is, 2 by 3, which is 0.666.

Inventory is 500 1st month's demand is 800. So, 5 by 8 is 0.625 and 3 by 7 is 0.428, based on these ratios, we also said in the earlier examples, that it is preferable and good to produce in the order of increasing ratios which means, we produce D first, then produce A and then C and then B so that before we have a stock out, we unable to produce these items. There is another thing to note here for example, if this were 500 instead of 200. How do we calculate this r? Because it was 200 it happened to be less than the first month's demand. So, we said half of the 1st month can be meet with this 400. In case this was 500, then we would say that out of this 500 the first, some 400 can be used to meet the 1st month demand and with the balance 100, we could meet 1 6th of the 2nd month's demand. So, this r would become 1.1666, 1 for the 1st month and 1 by 6 is 0.166.

So, we can calculate this r even proportionately or correctly even if, when the inventory is higher than the 1st month's demand. So, we also know that we have kind of decided on the order in which we will make it, which is D A C and then B. In the earlier models, we assumed that, all these values are the same as well as our each of these values the demand does not change with time. So, what we have to do now, is we have to first find out a cycle for the first cycle, essentially we need to find a first cycle. And then in order to find out the first cycle, we are going to kind of pick part of this data and then try to look at a demand up to a certain period.

For example, based on the experience of the two models, that we have seen earlier, we know that the cycle time is more than this 0.666 which is the largest of the ratios. The reason is that, in both the models our objective was to make T, the cycle time as large as possible and that would invariably happen when the last item, which means the item that has the highest r value. The inventory position comes to 0 and then we start producing that item. So, the T which is the cycle length is bigger than the maximum of the r.

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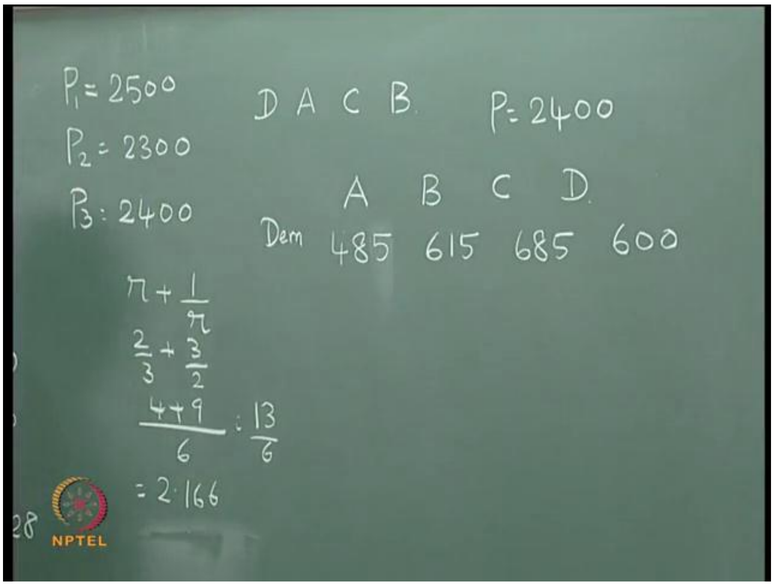

So, a quick thumb rule would be that, the cycle time T would be bigger than r and roughly of the order of r plus 1 by r, this is just a thumb rule and this is not a optimal in any way, with some kind of a guideline that the maximum cycle length T, will be of the order of r plus 1 by r. In this thumb rule of r plus 1 by r, we have to take the maximum value of r, which happens to be 0.666, 0.666 is 2 by 3. So, r plus 1 by r is 2 by 3 plus 3 by 2, which happens to be 13 by 6 which is 2.166. So, r plus 1 by r gives us a value of 2.166. So, what we do now is, we try and take the demand of each of these items for 2.166 periods and we also try and take the production for 2.166 periods.

So, let us write that so the 4 items A B C D and let us say equivalent demand for 2.166 period, (Refer Slide Time: 00:10) for A would be 400 plus 600 for 2 periods plus 0.166 of 300 which is 50. So, 1050, 1050 divided by 2.166 is 484 now, 484.63 we could even make it as 485. Now, this represents the equivalent or average monthly demand or average demand per period, (Refer Slide Time: 00:10) if we take 2.166 periods and average it out. So, this value of 485 is 400 plus 600 plus 0.166 into 300 divided by 2.166. Now, for B it will be 600 plus 600 plus 0.66 into 800, which is 132 plus 1200 divided by 2.166, which is 615.32.

So, this will be 615, for item C it will be 800 plus 600 plus 500 into 0.166 which is 83 plus 1400 divided by 2.166 which is 685. And for D it will be 700 plus 500, 1200 plus 0.166 into 600 divided by 2.166, would give us 600. Now, the P value will be 2500 plus 2300 plus 0.166 into 2400, would give us 2400. So, what we have effectively done here is to try and find out, what is the equivalent demand or average demand per period, if we consider 2.166 periods. Now, the production also has been subsequently averaged for 2.166 periods. Now, we could also get into a situation where, the average monthly production the equivalent term which is 2400, may not be equal to the sum of these 4. For example, sum of the average demands is 2385 which is less than 2400.

In such situation, if we utilize all the 2400 production capacity there will be a small inventory buildup, if this happens to be more than the 2400, then obviously the demand being more than production, at the end of the cycle, we would have consumed something from these values (Refer Slide Time: 00:10). That is precisely the reason, why these things are required, they serve 2 purposes, they serve one purpose where they allow us to have enough inventory, till the production of the item begins. And the second purpose is when the demand, some of the demands exceeds the production capacity at the end of the cycle, the total will be less than this total, but then this also acts as a cushion.

As long as at steady state or at the end of several cycles, if sigma P is equal to sigma D production capacity is equal to the demands, then at the end at any point, the inventory in the system will be a sum of these. So, these act also as a cushion to ensure to take care of two things, variation in the demand, initial inventory available till we produce the item. And there is a third aspect which is in situations, where demand is more than production, some part of this inventory will be consumed. But, at the end of it, it will level itself because finally, the average production will be equal to the average demand so it serves multiple purposes. So, it is also not necessary that these should add up to the 2400.

The question is do we have to compute r again considering the 4 demands of 485, 615, 685 and 600? The answer is we do not compute r again, because these demands represent the weighted or average demand. Over a period of 2.16 months whereas, (Refer Slide

Time: 00:10) the inventory that is available which is 200, 400, 500 and 300 is capable of meeting the demands of 0.5, 0.66, 0.625 and 0.428 months with respect to this data. Therefore, we retain the same r values and then we try and solve a linear programming problem that would try and maximize the cycle time or the run time. So, we will now sort A B C D in the order of r and we will do D first, followed by A and then C and then B. So, we now go back and formulate our familiar problem.

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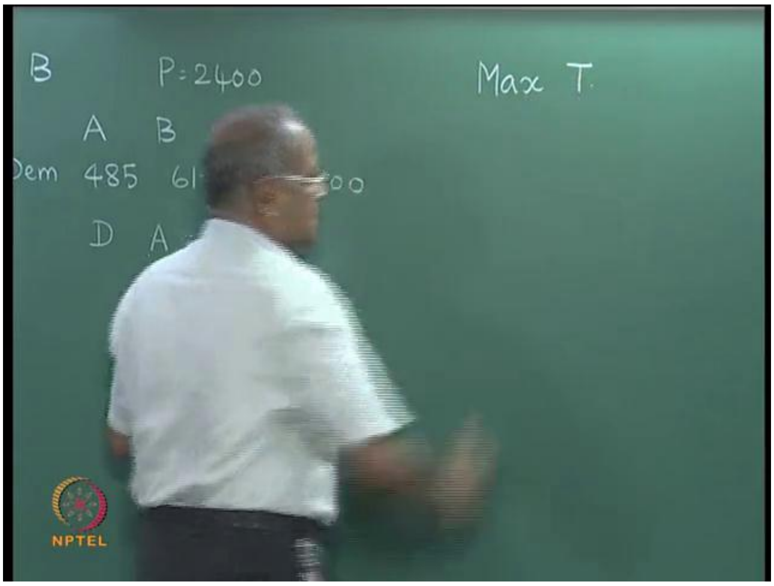

Where we try to maximize the cycle time T, so the objective will be to maximize the cycle time T.

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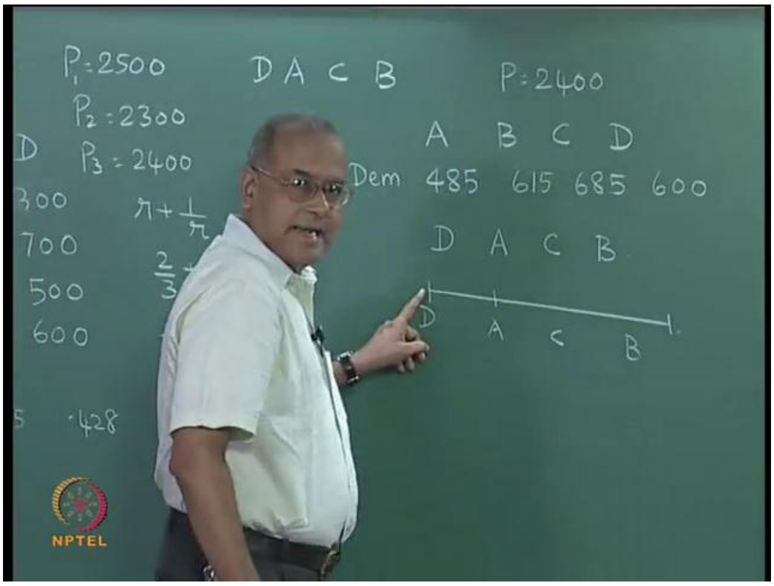

Now, we will start producing them, in the order D A C and B.

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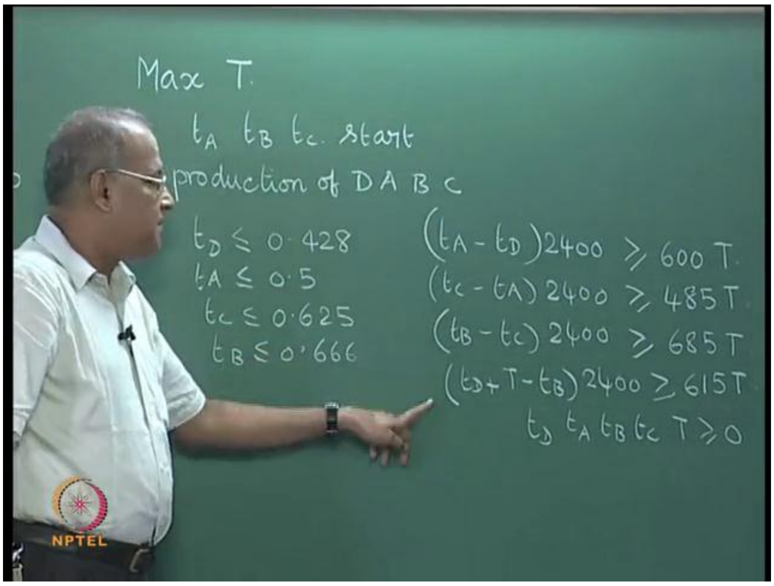

And then we will say let t D, t A, t B, t C represent, the start of production, of D A B C respectively. (Refer Slide Time: 14:36) Then we have to use these constraints t D less than or equal to 0.428, t A less than or equal to 0.5, t C less than or equal to 0.625 and t B less than or equal to 0.666. Then we produce the item D for a period t A minus t D. So, t A minus t D into 2400 is the production rate. So, t A minus t D into 2400 should be greater than or equal to, (Refer Slide Time: 14:36) the demand of A is 400 and demand of

D is 600 so that will be greater than or equal to 600 T. Similarly, t C minus t A into 2400 should be greater than or equal to (Refer Slide Time: 14:36) 485 T, t B minus t C into 2400 is greater than or equal to (Refer Slide Time: 14:36) 685 T.

And t D plus t minus t B into 2400 is greater than or equal to 615 T, t D, t A, t B t C, T greater than or equal to 0. So, this is our linear programming formulation. We have seen this formulation before; this is the first of the disaggregation models. Now, the explanation for this comes from the fact that first, we start producing (Refer Slide Time: 14:36) D at t D and then we stop producing D at t A so D is produced for the period t A minus t D.

So, this is the total production and the cycle time is T. So, this should be capable of meeting the demand of the entire cycle so (Refer Slide Time: 14:36) that is 600 into T. Similar explanations can be given for the production, this is for the production of A, this is for the production of C and production of B.

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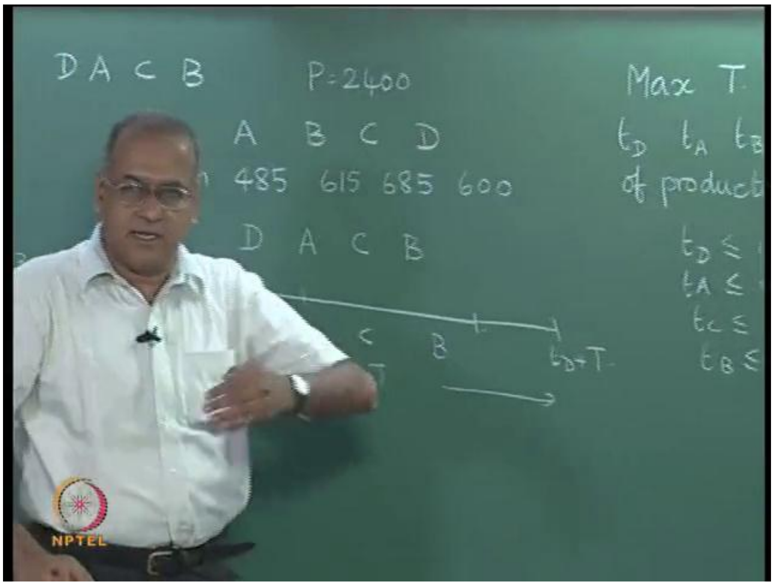

Will start at t B and then again the next cycle of D will happen and therefore, this is the entire cycle, which is T that goes up to this. So, t D plus T is the time at which D's production starts again. So, t D plus T minus t B this much is the length of time of production of B. (Refer Slide Time: 14:44) So, this is the total production of B, should be greater than or equal to the total demand of D during this period. We will follow model 1, we will not follow model 2, we will not follow the transient model. We will use the steady state model and try, which means we also will not try and adjust (Refer Slide Time: 00:10) these inventories in or in turn adjust the r j's and have a transient model. We will not do that; we will follow a steady state model. So, we try to maximize the T.

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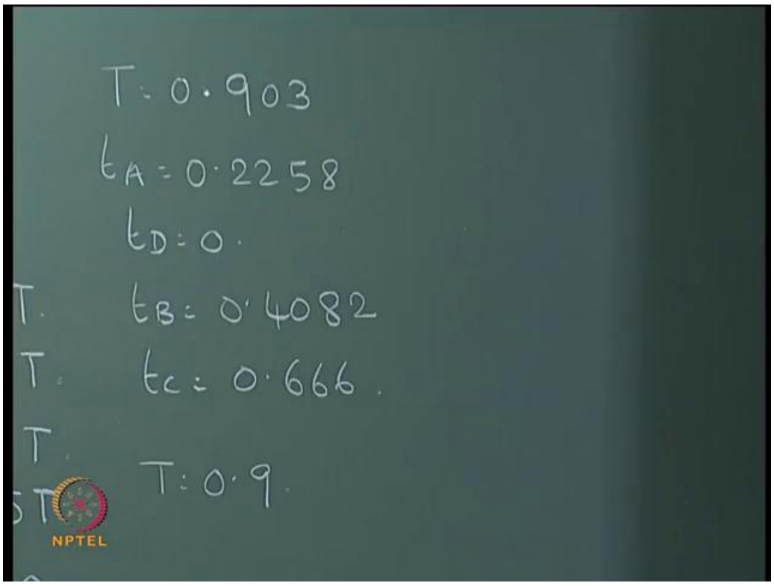

So, when we solve this linear programming problem. We would get t is equal to 0.903 months and we will have values which will be t A equal to 0.2258 so t A is equal to 0.2258, t D will be 0, t B and t C. So, t C is 0.4082, 0.666, 0.4082 and this will be 0.666. So, this is the solution once again. For ease of implementation, we will take T is equal to 0.9 instead of 0.903. And then we will say that we will implement the first cycle for a period of 0.9 months. So, when we do that we implement the first cycle for a period of 0.9 months. And in these 0.9 months, we will start producing D for a period of 0.2258. Then we will start producing A, which is between this and this. And then we start producing B which is the time period between this and this. And then the next production of D will start at time equal to point 9.

So, we will implement the first cycle by doing this way. Now, at the end of the cycle we will now, want to take stock of the inventory positions as well as the demand positions. Now, what can happen is ordinarily, (Refer Slide Time: 00:10) if this demand was deterministic which means this demand did not change. If this demand remained at 400, 600, 800 and 700, then (Refer Slide Time: 17:50) we would have produced D A C B in a certain way. And we would have consumed exactly (Refer Slide Time: 00:10) 90 percent of 400, 600, 800 and 700 in this period. We also had this initial inventory to begin with. So, there is an initial inventory, there would be some production for each of these items. And then there would be some consumption for each of these items.

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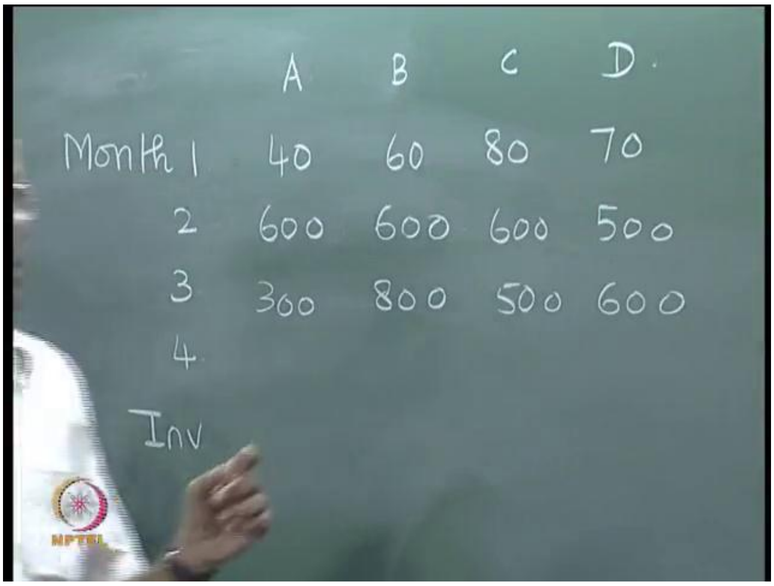

So, if we assume that the demand was exactly deterministic. Then we can go back and find out, then we would have consumed 90 percent of the demand of all of these. So, the unmet demand once again for A B C D, for A B C and D, the unmet demand for A B C D would be 40, which is 10 percent of 400, 40, 60, 80 and 70. The second month's demand would be 600, 600, 600, 500 and then D 3 would be 300, 800, 500 and 600. So, the data will now, move to something like this, where 10 percent of this demand has to be meet. Now, we are making an assumption right now that the demand is not going to change here (Refer Slide Time: 00:10).

Now, if for some reason within this 0.9 months, the demand had changed a little bit (Refer Slide Time: 00:10). Then we could have consumed some other number, other than 90 percent of this. And then these data also will change accordingly. So, if there is a change because of the change in demand during this period then we can update the 10 percent of the demand that is yet to be met. But, for the sake of this problem, we will assume that the demand is deterministic. And therefore, exactly 90 percent would have been consumed.

Now, again there is also a possibility that these demands (Refer Slide Time: 00:10) can change. In fact, we could even have updated the demand for the 4th month. We would assume that, by that time we would have an idea of the demand of the 4th month so we could have updated, so even some of these numbers may change a little bit, depending on the changes in the demand. Now, at time equal to 0.9, we will take stock and we will update these numbers. We will also update the inventory position at time equal to 0.9. Now, that inventory position would depend (Refer Slide Time: 00:10) on the initial inventory plus the production, less the demand will be the final inventory. I will just show the computation for the item D that, we would be doing first.

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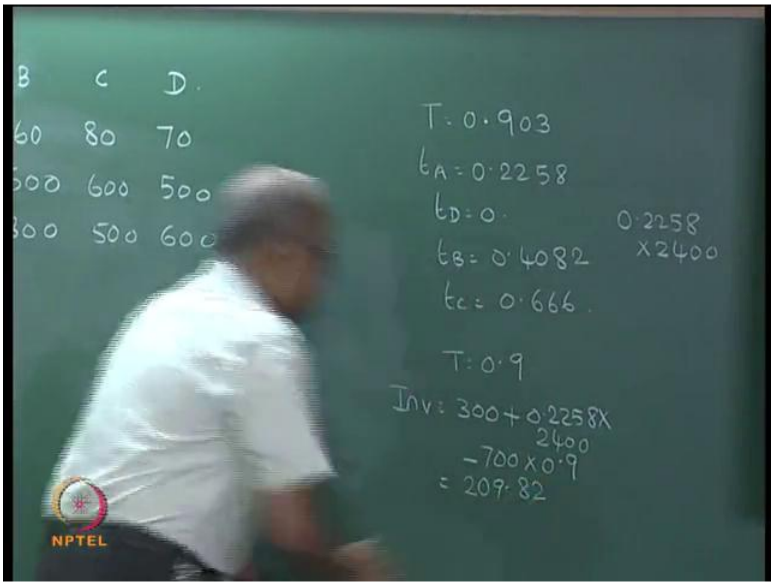

So, from this D is produced for a period of 0.2258 months. So, total production of D will be 0.2258 into 2400 so that would become so this is, this will be the production of D. So, the inventory of D will be, the starting inventory of 300 plus production 0.2258 into 2400 less demand of D, which is 700 into 0.9. So, this will be the inventory position of D and this works out to 209.82, 209.82.

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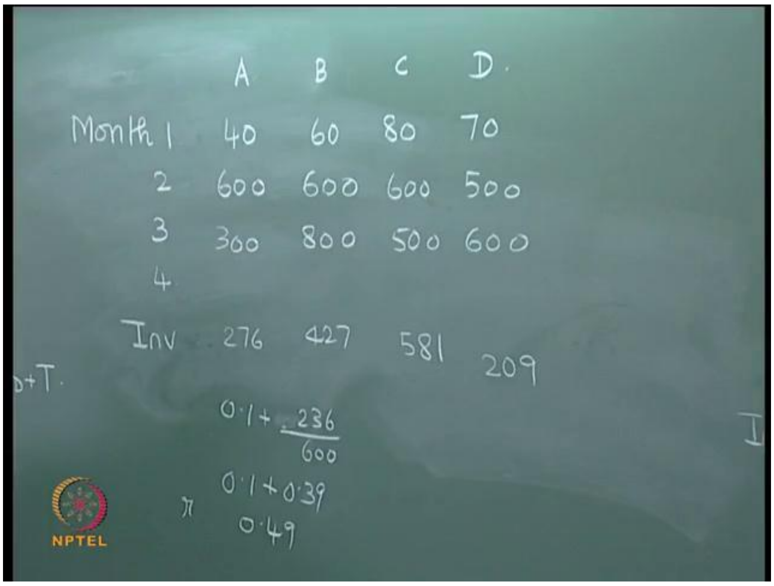

Now, like this we can update the inventories. So, this is updated to 209 and the rest of the numbers are 276, 427 and 581. Now, once again these numbers have been computed, assuming that the demand is deterministic. But, if there are some changes, then these numbers will also correspondingly change. So, now we are at time equal to 0.9 and then we can start doing this one more time by trying to understand, how many months can this inventory of 276? How many months demand can this 276 meet? So now, it will meet 0.1 month which would be 40. So, the balance 236 and the rest of them will be 236 by 600.

So, this will be 0.1 plus 236 by 600. So, this will meet so for this r will be 0.49. So, for this r will be 0.49 rest of the r's will be 0.49, 0.72, 0.94 and 0.378. Now, using this particular data now, we have to again find r plus 1 by r like we did earlier. So, we will have the new value of r plus 1 by r (Refer Slide Time: 07:04) and then for that new value of r plus 1 by r. (Refer Slide Time: 17:50) We will now, have to adjust and get these equivalent demands, as well as equivalent production. Now, the equivalent production numbers will be we have used for 90 percent of the times.

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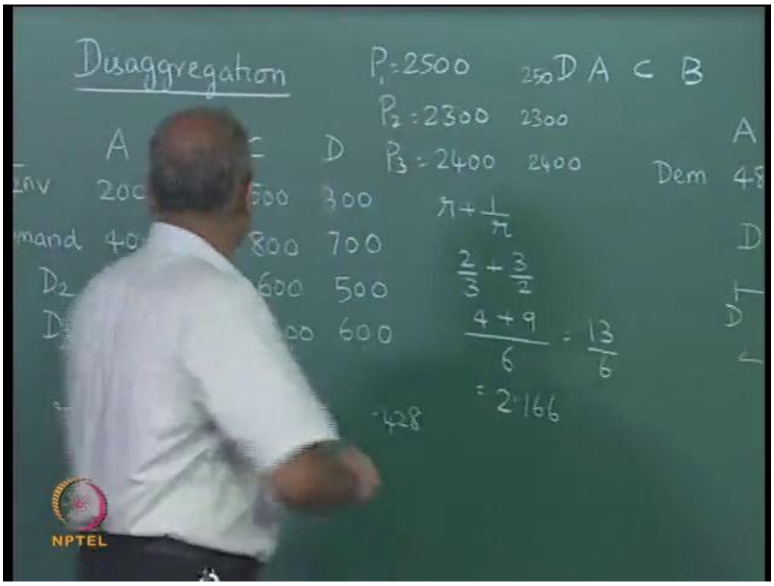

So this will be 10 percent so remaining production will be 250 here, then 2300 for this month and 2400 for this month. Now, for the new value of r plus 1 by r, we now have to find out the equivalent demands and the equivalent production like what we did. And then solve another linear programming problem, to get the length of the second cycle. The length of the second cycle need not be 0.903, it can be some other number. So, like this, we continue trying to have cycles of different lengths. So, this approach may not be the most optimal or the best approach but this is practical and this can help us in actually trying to solve the disaggregation problem with time varying demand.

Now, we also have these inventories, (Refer Slide Time: 00:10) we do not try to follow the transient model. We do not try to push these inventories to 0, because each cycle is now, going to be different. Also when we do not push this to 0 and use these inventories as they are. We will be able to absorb fluctuations in demand, over these months and then these demands get periodically updated and so on. So, that is how the disaggregation problem with time varying productions and demands, is actually solved in practice, using linear programming technique.

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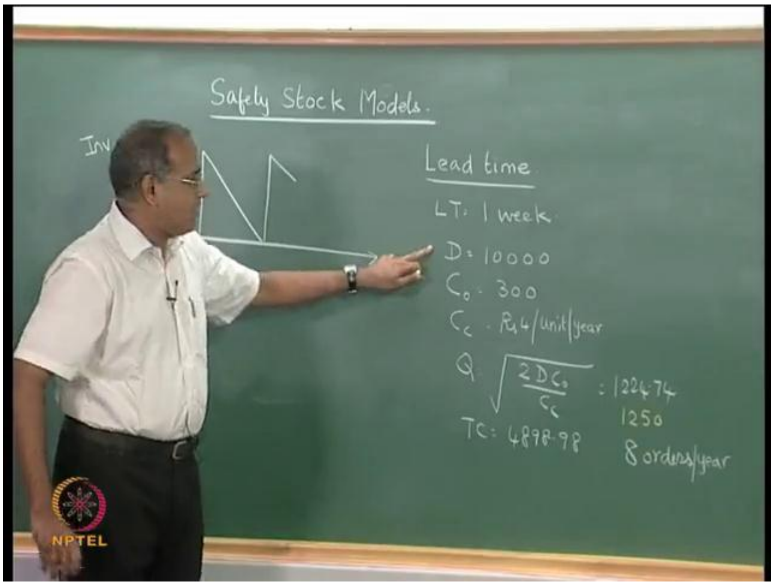

So, far we have seen several models that involved inventory of items. We started with very basic inventory models, where items were bought out. We looked at models where we allowed backorder deliberately. We also had models of where, we avail discount, where we looked at constraint inventory problems for multiple items. And then we looked at production consumption models, then we looked at time varying demand models. We looked at economic lot scheduling problem, which have production and inventory. And then we also looked at the disaggregation problems, where we try to maximize the cycle length with certain restriction on the inventory.

Now, in all these models that we have seen the demand and more importantly is the demand was taken to be deterministic. The monthly demands were known many times, the demand was the same, it was constant and continuous. There were some models where the demands changed at different periods. But, nevertheless the demands were known. So, let us go back to the basic model and try to relax one or two of the assumptions, to see, what kind of models such relaxation of assumptions lead us to. So, we go to the very basic inventory model, where we buy an item. So, we have this as the time axis, we order a quantity Q. So, let us assume at the beginning so this is time, this is inventory.

So, we begin with stock equal to Q and then we consume this Q at the rate of D till it comes to 0 and then we place an order here. So the stock comes up here back to Q. Once again, we consume it at the rate of D, you can take these 2 to be parallel and then it reaches 0 and then an order is placed and this goes on now, this is called the Saw tooth model of inventory. Now, this is T the length of the cycle. Now, we made certain assumptions here the first assumption of course is that, the demand is continuous and the demand is the same D per year at every instance. Second and the most important assumption is that of instantaneous replenishment, which means the moment, we place the order we get the items. Now, in reality these 2 things are not the way we have assumed.

So, first thing we need to look at is, the demand need not be deterministic and continuous. We will come to that a little later but then there would always be a time between placing an order and receiving the item. Now, that time is called the Lead time. And then in this model we have assumed that, the lead time is zero. So, the moment the order is placed, the items arrive. Now, we look at several cases of what happens to the lead time. So, the first one is, let us say lead time is not zero. But, lead time is a known value say, let us say let us look at a situation where lead time is equal to 1 week. Now, let us go back to the numerical example that we have used earlier.

So, we have used a numerical example where, the demand for the item is 10,000, the order cost C naught is 300, the carrying cost is rupees 4 per unit per year, this gave us the economic order quantity Q as root over 2 D C naught by C c which is 1224.74 and we would take total cost as 4898.98 which is the sum of the order cost, as well as the inventory carrying costs. Now, let us assume that we are not exactly implementing 1224.74. Let us say we are implementing a more comfortable number of 1250, which would also give us exactly 8 orders per year. Now, if we say that this 10,000 is actually based out of, let us say we work for 50 weeks in a year.

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And therefore, we would say weekly demand is equal to 200. So, that the total demand is 10,000 per year. Now, if we say that our lead time is 1 week, then what we do is, we will not place an order when the stock is equal to 0. We will look at, since they it is going to take 1 week for us to get the items. And we know that the weekly demand is 200. Let us assume that the 1 week stays and there is no change, it is exactly 1 week and every week the demand is exactly 200. So, in such a situation, we would place the order when our stock position reaches 200 so that exactly in 1 week the 200 will be consumed. The new order will arrive (Refer Slide Time: 28:56), so stock position will be zero exactly when the order arrives and its get replenished to 1250.

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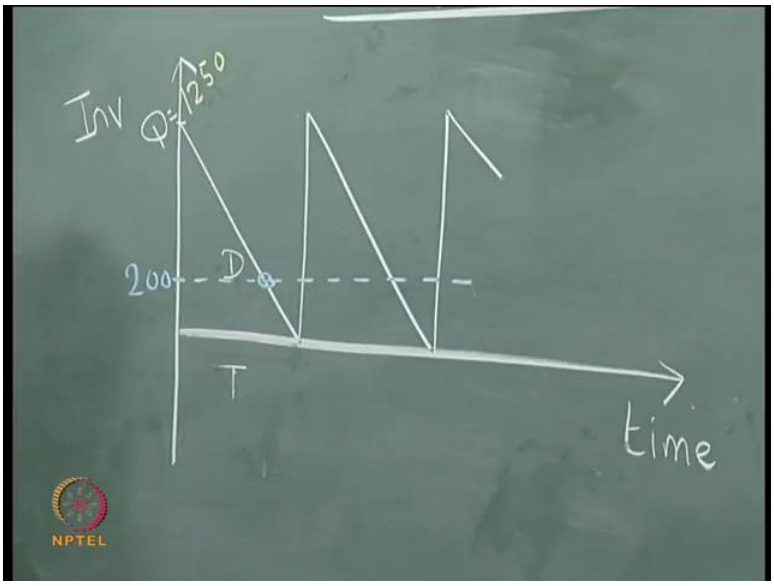

So, let me write this as Q equal to 1250 and then let us say, that this is the position where we have stock equal to 200. Let me use a different color so this is the position where stock is equal to 200 so this is 200. So, if this stock comes to 200 so at this place at this point, we will place the order. And then we start consuming so after a week the stock will come to zero. And exactly at that time, the new consignment will arrive.

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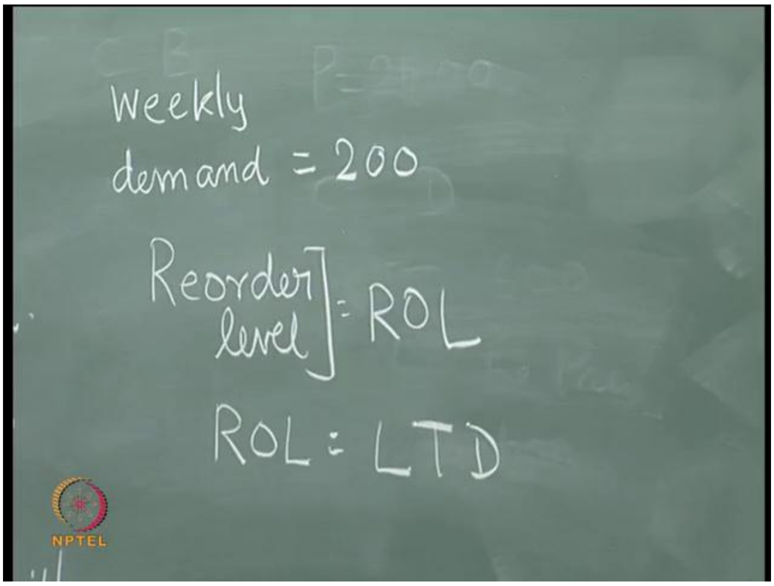

So, the point at which, we place the order is called the Reorder level, which is abbreviated to R O L. And the reorder level is equal to the lead time demand in the case of the lead time and the demand, being deterministic. If lead time is not 1 week but if the lead time is exactly 2 weeks, (Refer Slide Time: 35:08) then instead of placing the order at 200 we would place the order at 400. More importantly the definition of reorder level is, the stock position at which we place the order. Now, we need to compute the reorder level, under certain assumptions of the lead time, the demand and the lead time demand, lead time demand is called demand during the lead time. So, if the lead time is exactly 1 week, lead time demand is 200 in this particular example.

Now, if the lead time demands, let us say, let us look at a situation where the lead time is exactly 1 week but the demand is not deterministic, it is not exactly 200, it could be 200 plus minus something. So, in certain now, what happens is. There are exactly 8 cycles in this year (Refer Slide Time: 37:19) because what we do right now, is we assume that, the moment we compute the reorder level. (Refer Slide Time: 37.33) Let us say we are now, going to compute the reorder level. Now, the moment we compute the reorder level, we would place and, as soon as the stock comes to the reorder level, we place an order.

Now, we assume that with the quantity which is called the reorder level, which is with us, we will be able to meet the lead time demand, if we are able to meet the lead time demand, there is inventory before the next cycle, next order, next consignment comes. If you are not able to meet the lead time demand with the reorder level quantity, there could be some shortage or some backorder. Now but on an average even though the demand fluctuates, let us say every week. But, on an average, the demand is going to be 200 every week. So, the expected demand for the year is going to be 10,000 and since (Refer Slide Time: 28:56) we are going to have 8 cycle of 1250 ordered, then we will have exactly 8 cycles where we will be ordering 1250 in each cycle. But, then the reorder level has to be found out so we have separated two things, we have separated the order quantity and the reorder level.

So, in this particular way of analyzing the problem, we define the order quantity separately based on economic order quantity model or a value that is suitably rounded from the economic order quantity which is 1250. So, this model would be like saying that every time we order, we place, we place order for 1250. We expect to order 8 times an area and we place an order when, the order quantity is, when the stock that we have is 200, or if we compute the reorder level differently now, whenever we reach the reorder level, we will try, we will place an order. Now, say the lead time is exactly 1 week (Refer

Slide Time: 28:56). But, let us look at a case where the (Refer Slide Time: 35:48) demand is not exactly 200. But, the demand follows a certain distribution.

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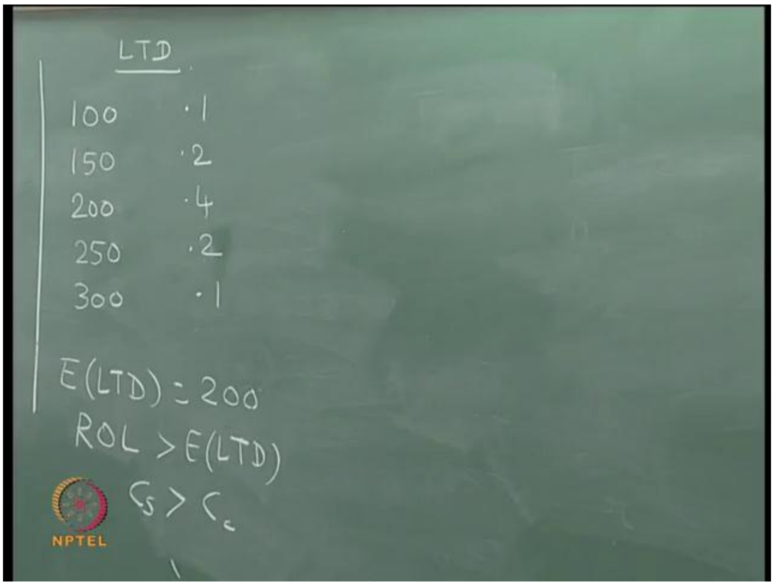

So now, we would say that the demand can be anything from 100 to 300. So, demand can be 100 with the probability 0.1, demand can be 150 with probability 0.2, demand can be 200 with probability 0.4, demand can be 250 with probability 0.3 or 0.2 and demand is 300 with probability 0.1. So, let us say this is the weekly demand and we will take this as lead time demand because lead time is exactly 1 week. Now, we have to find out the reorder level that optimizes our total cost. Now, this is the demand during the lead time and lead time is exactly 1 week. So, let us assume that we have placed an order, when we have reached a certain reorder level, which we are going to calculate using this data.

So, once we our stock reaches that reorder level, we place an order and we believe that the reorder level quantity which we have, which is the stock on hand when we place the order, is capable of meeting the lead time demand. And we want to find out that reorder level. Now, if we keep now, the expected value of lead time demand, will be 100 into 0.1 plus 150 into 0.2 plus 200 into 0.4 plus 250 into 0.2 plus 300 into 0.1. Now, this will add up to 200 because it is 0.1, 0.2, 0.4, 0.2, 0.1 and the gap is that, is symmetric about the mean so 200 is the expected value of the lead time demand.

Even before, we proceed further; we could ask ourselves how did we get these numbers? How do we get these probabilities? These probabilities can be thought of as we go back in time collect data, historical data has to, how the lead time demand has behaved over several past cycles? Let us say someone has collected such a data for 10 past cycles and we have realize that the lead time demand has been 100 in 1 out of the 10 times 150 in 2 out of the 10 times 200, 4 out of the 10 times and so on. So, a proportion automatically represents the probability of occurrence under the assumption, that future will replicate the past.

So, that is how we get these numbers and the expected value is 200. Now, if we take the reorder level to be expected value of lead time demand. (Refer Slide Time: 35:48) In this case we took exactly equal to lead time demand, because lead time is deterministic, demand is deterministic, if we take expected value of lead time demand based on this thing. Now, about keep this roughly about 70 percent of the times, we will be able to meet the demand and 30 percent of the times, we will not be able to meet the demand because of 200. So, the thing is we will be able to meet the demand more times then we will, than the situation where we will not be able to meet the demand.

But, then if we look at from a symmetric point of view, say roughly 50 percent of the times we will be able to meet at, 50 percent of the times you will not be able to meet it. If this is not 100, 150, 200, If this were 100, 1 naught 1, 1 naught 2, 1 naught 3 up to 300, then we will say 50 percent of the times, we would have met 50 percent of them we would not met. So, whenever we are meeting the demand there will be excess inventory, where we are not able to meet the demand. We incur a backorder cost or a shortage cost we assume backorder, which means as soon as the stock arrives, we meet the demand and then we continue.

Now, when we have excess inventory, we incur inventory cost. When we have shortage or backorder, we incur backorder cost. So, backorder cost is always higher than the inventory cost. Therefore, we want to avoid backorder. And therefore, we end up having this reorder level always as greater than the expected value of lead time demand. If C s is greater than C c, C s is shortage cost, C c is inventory cost, shortage cost is higher than inventory cost. So, reorder level is always greater than or equal to the expected value of lead time demand.

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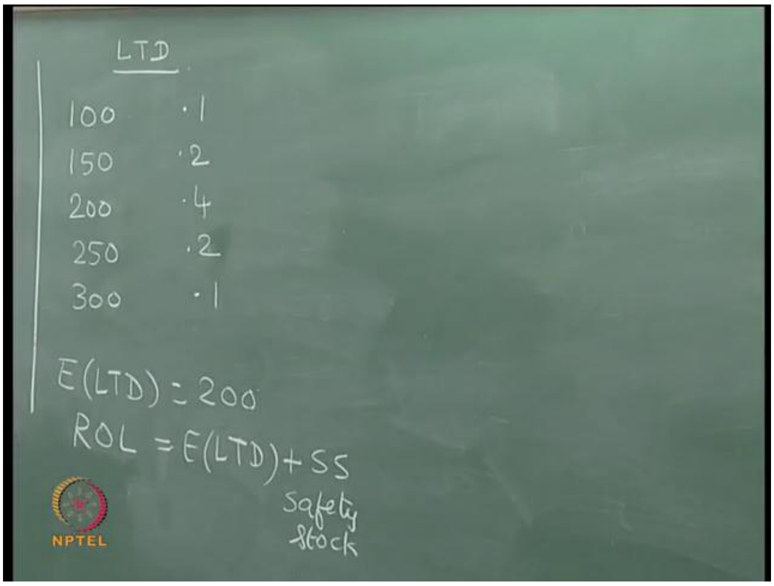

So, reorder level is now, equal to expected value of lead time demand plus safety stock. This S S is called safety stock so we could say either we want to compute the reorder level or we would say we want to compute the safety stock. Now, the simplest thing to do here is, to say that we want to find out the reorder level. So, we can look at all these 5 cases, where reorder level is equal to 100, reorder level is equal to 150, 200, 250 and 300.

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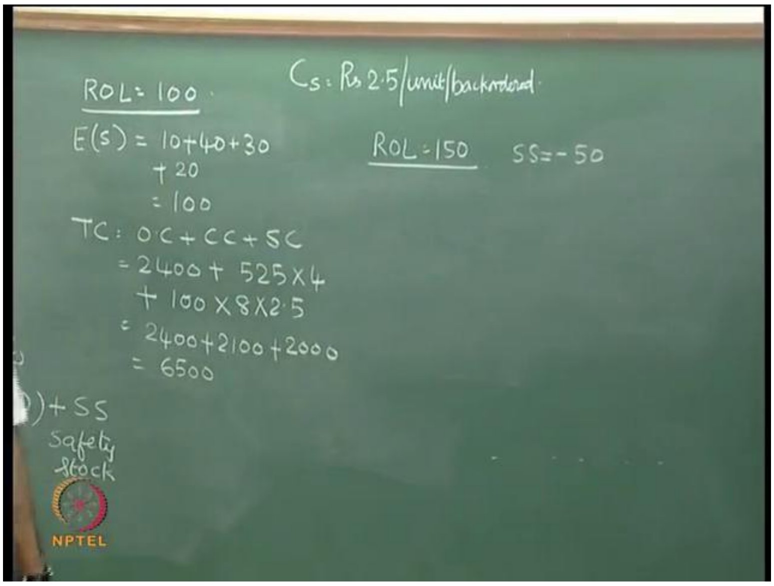

So, let us do those calculations. So, let us have a situation where reorder level is equal to

100 so when reorder level is equal to 100 which means when the stock level is going to be 100. We are going to place an order and the demand is either 100 or 150 or 200 or 250 or 300. Therefore, there is going to be no inventory at the end of the period because whatever is the demand, we have to consume the 100. So, there will be no inventory in the period so then what will happen is, there will be only shortage. So, expected value of shortage in a cycle will be, if the demand is 100 there is no shortage. If the demand is 150 shortage is 50. So, 50 into 0.2 so 50 into 0.2 is 10 plus 100 into 0.4 is 40 plus 150 into 0.2 is 30 plus 200 into 0.1 which is 20, this is the expected shortage.

Let me explain, the computation again, if the reorder level is 100 which means our stock is 100. If the demand is 100, then we are able to meet all the demand, no problems but when the demand is 150 (Refer Slide Time: 44:27) we will meet only 100 out of the 150 so shortage will be 50 which happens with the probability of 0.2 so 50 into 0.2 is 10. Similarly, 100 into 0.4 is 40 and so on. So, expected shortage is 100, there is no expected inventory out of this. Now, total cost will be order cost plus carrying cost plus shortage cost. Now, order cost is going to be constant, because we are going to order 1250 eight times. (Refer Slide Time: 28:56) So, 8 into 300 is 2400, we have assumed order cost as 300 so 8 into 300 is 2400.

Now, what is the inventory carrying cost? The cycle inventory that we will carry is Q by 2 that is the expected inventory, normally expected inventory is Q by 2. Now, in this case what will also happen is the actual cycle inventory will be Q by 2 plus the safety stock that we have. So, in this case we will have Q by 2 minus 100 because there is an expected shortage of 100 every time so Q by 2 minus S S. So, it will become 1250 minus safety stock of 1250 divided by 2 Q by 2 is 625 minus 100, would give us 525 that is the average inventory into C c, C c will be 4 per unit per year plus expected shortage is 100. Now, let us take the shortage cost in this example, as rupees 2.5 per unit backordered. So, C s is taken as 2.5 per unit backordered.

So, 100 is the expected shortage cost in a cycle, there are 8 expected cycles in a year. So, into 8 into per unit is 2.5. So, this is 2400 plus 525 into 4 is 2100 plus 800 into 2.5 is 2000, this is 6500 is the total cost. Now, let us do one more computation if R O L is equal to 150, if R O L is 150, first we calculate safety stock, is equal to minus 50. Now, this comes because expected value of lead time demand is 200. So, from this definition (Refer Slide Time: 44:27) R O L is equal to expected value of lead time demand plus safety stock. Now, for the sake of computation we are showing the safety stock, is minus 50.

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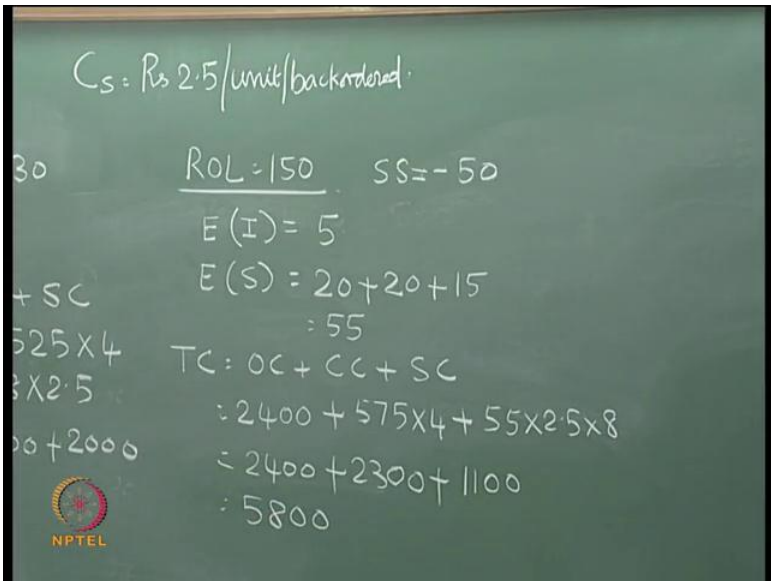

So, now once again expected inventory will be now, the R O L is 150. (Refer Slide Time: 44:27) So, we will have inventory, if the demand happens to be 100. So, there will be an inventory of 50 in this cycle, there will be an inventory of 50 this cycle, with the probability of 0.1 so expected inventory is 5. Now, expected shortage will be (Refer Slide Time: 44:27) R O L is 150 so there will be shortage if the demand is 200 or 250 or 300. So, this is 50 will be the shortage if the demand is 200 so 50 into 0.4 which is 20 plus 100 into 0.2 which is another 20 plus 150 into 0.1 which is 15 so expected shortage is 55.So, total cost will be, order cost plus carrying cost plus shortage cost so order cost is 2400 plus carrying cost is Q by 2 minus safety stock as a general rule. So, 625 minus 50 is 575 into 4 plus 55 into 2.5 into 8. So, this would give us 2400 plus 2300 plus 1100 this is equal to 5800.

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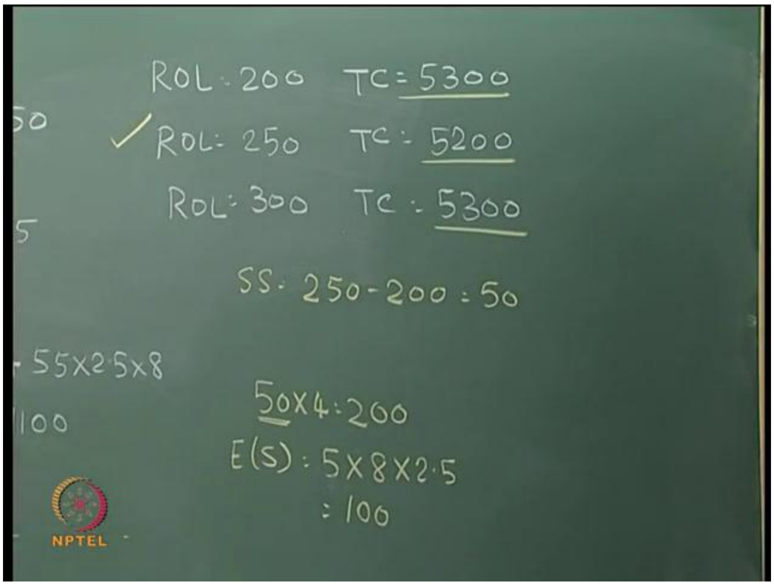

(Refer Slide Time: 44:27) So similarly, if you we compute for 200, 250 and 300. If we do those computations we get, if R O L is 200 we get, we would get total cost equal to 5300. If R O L is equal to 250, our total cost is 5200 and if R O L is 300, our total cost is 5300 per year. Now, based on all the total costs that we have, the total costs are 6500 (Refer Slide Time: 45:07), 5800 (Refer Slide Time: 50:43) 5300, 5200 and 5300. If we go by minimum total cost, then we would look at R O L is equal to 250. So, if we go by minimum total cost our R O L is 250 and our total annual expected cost is 5200, which is the cheapest.

Now, if we have reorder level of 250, then we will have a safety stock of 250 minus 200 which is 50, safety stock of 50. And we also realize that this 50 safety stock is something that we will need not end up consuming at all. Because if we take every week or every cycle, the expected demand is 200 but then we have an additional 50 which is the safety stock. So we have a 250 so in some weeks the demand can be more than 200. Some weeks the demand can less than 200 but when we average out all the demand, the demand is going to be 200 per week. But, then the safety stock of 50 is going to be with us all the time. And we will not consume this safety stock at all. Now, this safety stock of 50 to put it in a different way this safety stock of 50 would incur, a cost of 50 into 4 which is 200 that we have.

Now, this cost for 1250 (Refer Slide Time: 28:56) would actually give us 5000. So, we have this 5200 which is there, plus a very small shortage cost that is also involved. So, roughly the figure is close to that 5200 that we actually have, plus if we calculate the for (Refer Slide Time: 28:56) 1250, if we calculate D by Q C naught plus Q by 2 C c, then the total cost will become 8 times 300.

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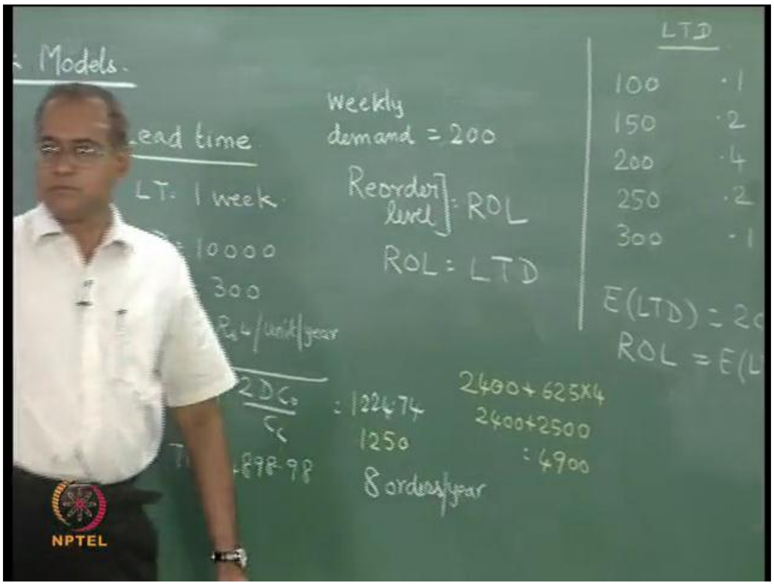

Which is 2400 plus Q by 2 into C c which is 625 into 4 so 2400 plus 2500 will be 4900. The additional inventory cost is 200 which is 5100. Now, the expected shortage is, if reorder level is 250. The shortage is 50 (Refer Slide Time: 44:27) so 50 into 0.1 is 5. So, expected shortage cost will be 5 is the unit short 8 times a year, into 2.5, which is 40 into 2.5 is 100. So, the order cost is 2400. The normal inventory safety inventory carrying cost which is Q by 2 into C c is 2500. (Refer Slide Time: 53:15) The excess 50 safety stock incurs a cost of rupees 4 and note that we do not put safety stock by 2 here we put Q by 2 in the calculation because of the average inventory. The safety stock is not going to be consumed at all.

So, the safety stock of 50 will be carried forever and it would incur an additional cost of 200 plus a shortage cost of 100. So, 4900 plus the extra 300 is the expected additional cost, if the demand instead of being exactly (Refer Slide Time: 44:27) 200 every week, follows a distribution like this so the effect of that is an increased inventory of 50 which is our safety stock and an increased cost of expected increased cost of 300 per year. (Refer Slide Time: 44:27) Now, with an additional safety stock of 50 and a reorder level of 250 which means here, then every time or every cycle, we will be able to meet the demand 80 percent of the times. So, the demand in a cycle is met 80 percent of the times that is called service level is, 80 percent. It is the number of percentage of times, we will be 90 percent of the times. So, this is 0.1, 0.3, 0.7, 0.9.

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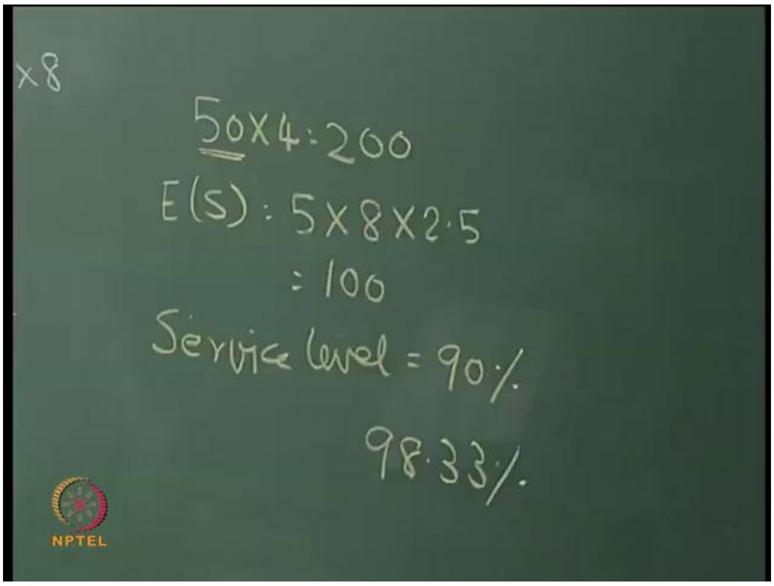

So, we are able to meet the demand in 90 percent of the cycles, we will be able to meet the demand. There is also another quantity called fill rate, which is like this. Now, if we have a (Refer Slide Time: 44:27) reorder level of 250. If the demand is 100, 150, 200, 250 we will be able to meet all the demand during the lead time. So, 90 percent of the times, we are able to meet all the demand. (Refer Slide Time: 44:27) And if the demand happens to be 300, we will actually meet 250 out of the 300, 50 is the only thing that is short. So, we will be able to meet 250 out of the 300 which is 5 by 6, 250 by 300 is 5 by 6 which is 83.33 percent of the times, we could meet and that is going to happen 0.1 of the time.

So, the fill rate will be 98.33 percent of the demand I will be able to meet. 90 percent of the times I will be able to meet 100 percent of the demand 83.3 percent of the demand, I will be able to meet with the probability of 0.1 so if the fill rate is like 98.3. So, the fill rate is higher than the service level, many times we are concerned about the service level, which is our ability to meet all the demand. There are times; we also look at the fill rate, where we are able to meet part of the demand. So, if the demand follows a discrete distribution like this now, we have found the way by which we can calculate the safety stock as well as the reorder level. Now, what are those computations, if the demand follows different distributions or the demand follows a continuous distribution such models, we will see in the next lecture.