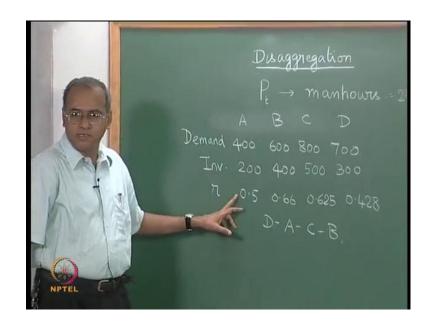
Operations and Supply Chain Management Prof. G. Srinivasan Department of Management Studies Indian Institute of Technology, Madras

Lecture - 19 Disaggregation

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In this lecture, we address the Disaggregation problem; the disaggregation problem was introduced in the last lecture. Disaggregation problem is also a certain variation of the economic lot scheduling problem; now one way of looking at the disaggregation problem is like this, when we solve the aggregate planning problem, we used a variable called P t which is the production time that is available in every period. Now, the next step is to try and see how we use that production time P t to produce various products. Now, in aggregate planning, when we studied aggregate planning, we assumed that there is an aggregate product, which adequately represents all the products that are made in the organization, and variable such as P t that is production time that is available in the period, which is a decision variable of an aggregate plan.

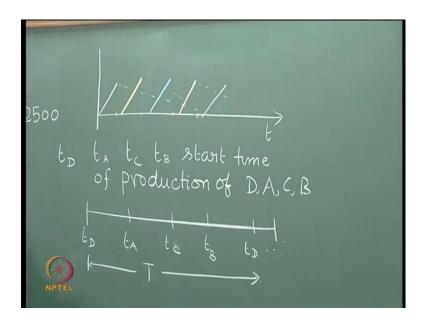
W t work force that is used in period t, which is also a decision variable, I t inventory at the end of period t, they were all defined in terms of a single unit called man-hours or person-hours. So that, all of them can be measured consistently using a single unit. So, when we say P t, we represent P t in terms of man-hours or person-hours available not in

terms of number of units produced. Because, at the disaggregation level we are going to break this P t into several products and therefore, it is convenient to define P t in manhours rather than in number of units.

So, let us define the disaggregation problem using an example, so let us assume that there are four products, which we call A, B, C, D and demand for these four products are 400, 600, 800 and 700 for A, B, C and D respectively. Inventory available is 200, 400, 500 and 300, and we also assume that P available is 2500 man-hours. In the first simple assumptions that we have made is that, the demand is also given in man-hours and the total demand is 1000, 1800, 2500, which is equal to P t that is a simple assumption that we have made.

So, we are not talking of either underutilizing this P t or demand being in excess of P t and not being able to meet the demand. So, we are looking at a situation where, the total demand is equal to the total capacity and demand is also given in man-hours. Similarly, the beginning inventory or inventory available is also given in man-hours, so all of them are in consistent unit called man-hours.





Now, let us go back and draw a very familiar figure that we had, which we have already seen, let us assume this is time and let us assume we are talking of producing some of these products. So, earlier we had used different colors to represent different products, so if we are looking at the production consumption model, which means we consume where we produce. And we also in such a situation know that P has to be greater than D, which is true in all the cases, the D's are like this and P is like this, so P is greater than D.

So, what we are doing is, we are saying that we produce a first item here and then we consume the first item and then we setup the second item and then we produce a second item let us say up to this and then we consume the second item. And then we again setup the third item and say we produce the third item and consume the third item, and then we go back and let us say there is a fourth item that we have. So, we setup the fourth item and then start consuming the fourth item and then we come back to the first item here and do it, which means we consume up to this point, so we are looking at something like this.

Now, we looked at this figure and we kept the cycle time T same in the economic lot scheduling problem, and in the economic lot scheduling problem, the objective was to try and minimize the sum of the setup cost as well as the inventory cost. Now, here what we are trying to do is this, now if I am starting at time 0, let us say I have inventory of 0 of this item I can produce and consume. So, I produce and consume, I buildup inventory up to this point and then I am going to consume out of the inventory till I begin the next set.

Whether, if we take the second, third and fourth items, there production is going to start only ((Refer Time: 05:47)) here, here and here, which means we should have enough inventory to consume when we are not producing. So, till the time we start producing items two, three and four, we need enough inventory to meet the demand of that item. And that is the reason we have these initial inventories which are here, which we will be using before we start producing that particular item.

And once we start producing that item, we consume while production, buildup some more inventory and then use it once the production stops. Now, in this example I have used one, two, three, four and this example assumes that the order in which, we are going to take up these items for production is one, two, three, four. We have also mentioned that the changeovers are not sequence dependent, so any order is acceptable to us, as long as the changeover time, is only a setup time to produce.

In the economic lot scheduling problem, the setup time figured explicitly in the constraint, the setup figured in the objective function. Now, here we now have different levels of initial inventory, I think we also know that, while the first item can be produced

at time equal to 0, the second, third and fourth items, which means the items that we are going to produce as second, third and fourth. For example, we could start with A and then D, and then C, and then B, so whatever we produce as the second item, the inventory should be sufficient to meet the demand till we start producing that item.

So, we have to choose A, B, C, D or rank them or sequence them in an order such that, before they we start the production of any one of A, B, C, D, we should have enough inventory to meet the demand from time period 0, up to the time period at which we are going to start the production of any of them. Now, using this inventory I, we define a ratio called r and this ratio tells us, the amount of time buffer that we have. The inventory is now made into a time buffer, the man-hours it is already in time units, but then r is represented as, so many months of inventory that we actually have.

So, when we divide this 200 divided by 400 which is 0.5, so this is 200 is equivalent of 0.5 months of inventory of this item, this is equivalent of 0.66 months of inventory of B. So, let me write A, B, C, D, so ((Refer Time: 08:42)) this is A, this is B, this is C and this is D, so this is 5 by 8, which is 0.625 months and this is 3 by 7, which is 0.428 months. So, from this r, r tells us the number of months of inventory that we of these or the inventory that is available here is capable of meeting, so many months demand.

So, now, this r also gives us an indication of the order in which we can produce A, B, C, D, because the item whose inventory we are going to exhaust first is D. So, D has the smallest value of the ratio r, then it is going to be A and then it is going to be C and then it is going to be B. So, a good order in which they can be produced is D, A, C and B, because that is the order and which, we will exhaust the inventory if we do not produce them. I have to put it differently, we have to start the production of D, before 0.28 months, before 0.428 months, we have to start production of A before 0.5 months and so on.

So, now let us start defining the variables, let t D, t A, t C and t B be the start time of production of D A, C B respectively. So, it is like this is a time graph, let us say I am starting ((Refer Time: 10:53)) t D here, I am starting t A here, I am starting t B here, I am starting t C here, t B here and then once again I start t D and so on. Now, this is my cycle of production which is capital T, so I start from t D and one more up to this is my cycle. So, when I start producing D here at time t D and I start producing A at t A, then it

means I produce item D for a period t A minus t D. So, t A minus t D, first thing I have to ensure is these productions t D, t A, t C and t B should start, before I exhaust these inventories.

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So, the first constraint will be t A is less than 0.5, t B is less than 0.66, t C is less than 0.625 and t D is less than 0.428, then we are producing this item that is t D first, for a time period t A minus t D. So, t A minus t D is the time period for which we produce the item D, and that is produced at a rate of P 2500. So, that production should be able to meet the demand of the item D for the full cycle t D, so this should be greater than or equal to T into 700, should be greater than or equal to 700 T.

Now, this item A is produced for a period t C minus t A, so t C minus t A into 2500 is greater than or equal to, this is producing item A, so this is 400. t B minus t C into 2500 is greater than or equal to 800 T. And if this is t B, if we start producing T, we start producing D again here, so this is not t D, but this is T plus t D, where the cycle has come here. Earlier when I wrote t D, I was trying to say that we are going to start producing D again, but the movement we want to write the time at which we start producing D again, it becomes t D plus another T.

So, we are producing this item D up to t D plus T, so this is t D plus T minus t B into 2 1500 is greater than or equal to 600 T. Of course, we have all T j, T greater than or equal

to 0. So, these are the constraints that we actually have and then we have to write a suitable objective function.

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SC + TC

Now, the usual objective function is to minimize some of the setup cost plus inventory cost, this is the very standard way of doing setup cost plus inventory cost. Now, one of the assumptions that we have here is that, we are going to assume for the sake of simplicity that, when we say that we are producing item D up to this period and then we are starting item A at t A and up to t C. So, we are going to make an implicit small assumption that the changeover times are negligible.

So, we do not have large changeover times, otherwise the changeover time will come into the constraint plus another K, if that is the changeover time which is that. Now, the movement we look at this constraint, what we are trying to say through this constraint is that, these four ensure that the starting times of A, B, C, D are well within the r, which means we should be able to start the production of A, B, C, D, before they are exhaust. But, when we write these constraints, we are able to write these constraints largely because the total 2500 is equal to exactly this ((Refer Time: 16:17)). So, the cycle can be so defined that, when we actually complete the cycle, we have consumed for the cycle based out of what we have produced, which kind of means that, these initial inventories are going to decide the t A, t B, t C, t D, but they do not play a major role in this.

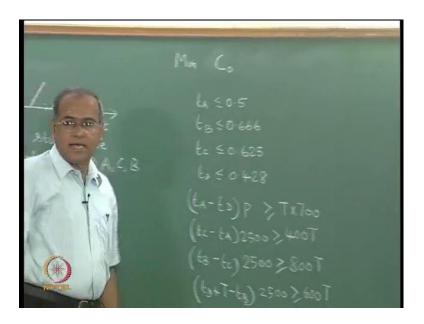
So, the general assumption is this, initial inventories will be used up, at the same time will not be very different at any point in time. The aggregate or sum of these inventories will not be very different at any point in time, because at steady state the way these constraints are written, we are able to meet the demand of the cycle based on what we produced in the shorter period of the cycle. So, the aggregate inventories will remain as they are, so at any point in time the average inventory will not deviate or will be more or less the same constraint.

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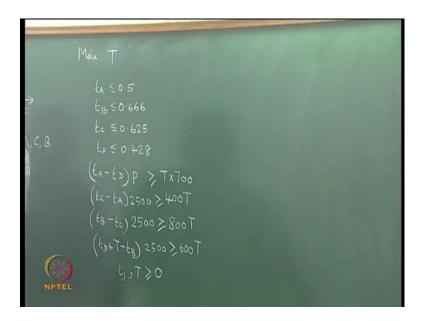
So, if we make that assumption that the average inventory is more or less the same constant, then the inventory cost will become a constant. Again assuming that the holding cost is the same for each item C c is the same for each item, so the question is to minimize the setup cost. Now, the total setup cost per year or per given time period is the product of the individual setup cost into the number of setups per year.

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So, individual setup cost if we call that as C naught, and number of setups per year each cycle is going to have four setups, so number of setups per year is four times, the number of cycles in a year. So, we want to minimize C naught into four times into number of cycles in a year, which means you are effectively minimizing the number of cycles in a year.

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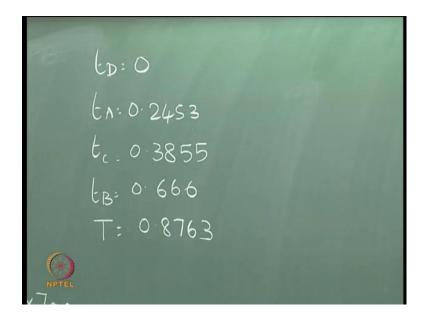


And if you want to minimize the number of cycles, it is like saying I want to actually maximize the cycle length capital T, so maximum the cycle length T, minimum number

of cycles in a given period, so this problem becomes this. So, this problem becomes a linear programming problem, because the objective function is a linear function of the decision variable. And all constraints are linear and then you have an explicit non negativity restriction on the variable.

If we had not made this assumption that the inventory holding cost is a constant, then we would get a non-linear term in the objective function, where this T will appear in the numerator here, and the T will appear in the denominator in the inventory. Then you will have a problem with a non-linear objective function and a set of linear constraints, so the solution methodology becomes a little more complicated. So, let us first take a look at this as a linear programming problem, and then let us say that if we solve this linear programming problem, we get an optimal solution which will be like this.

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The optimum solution to our example is t D is equal to 0, t A is equal to 0.2453, t C is equal to 0.3855 and t B is equal to 0.666 and capital T is equal to 0.8763, all these are in months. So, this cycle will extend for about 0.8763 months and then the cycle will repeat and so on.

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Now, let us also try and look at the inventory positions during this time period, so what we do is we first keep item D we are producing it, in the order item D, A, C and B. So, at time t equal to 0, the initial inventories are 300 for D, 200 for A, 800 for C and 600 for B, 500 for C and 400 for B. Now, what we are doing is we start production of D at time equal to 0, and we produce D for 0.2453 months, so let us look at what are the inventory positions at t equal to 0.2453 months.

Now, this period we are producing D, at the same time we are consuming D while we produce, so we are producing at the rate of 2500 man-hours, we are consuming at the rate of 700 man-hours. So, the net inventory buildup is at the rate of one 1800 is the rate at which it is built up, and it is built up for a period 0.2453. So, the inventory at that position is 1800 into 0.2453 plus 300, which will be 741.54, which approximates to 742. Now, during this 0.2453 months, we are not producing A, we are only consuming A out of the 200 that we have.

So, we would have consumed 0.2453 into 400, which is roughly about 100 and so the inventory on hand will be about 100, so this is 102 on simplification. Similarly, we consume only C, so the inventory will be 500 minus 0.2453 into 800, which will be roughly of the order of 300 and the value is 304, and the last one can be calculated as 253. Now, we look at what happens at 0.3855, now here we produce and consume, here we consume out of 742, here we consume out of 304, here we consume out of 253.

So, quickly doing these calculations, we will have 643 here ((Refer Time: 23:50)), 396 here, 192 here and 169 here, now at time equal to 0.666, the inventories are 447 here, 284 here, 668 here and 0 here. And at t equal to 0.8763, we will get 300, 200, 500 and 400 again, let me just show only one of these calculations. Now, at 0.666, this is the inventory position of D, now between 0.666 and 0.8763, we are going to produce B, so B is inventory is going to go up, so B starts at 0.

So, we produce at 2500, we consume while production at the rate of 600, so 1900 is a rate at which inventory is building up, and that inventory is building up for a period this minus this, which is 0.2103, periods it is building up. So, 0.2103 into 1900 is 399.57 which is rounded off to 400, now if we take this D just for the illustration, we only consume from this inventory. So, we consume D at the rate of demand is 700, so 700 into 0.2103 is the consumption is 147 and this is 447, so balance inventory is 300.

So, we realize that at the end of the first cycle or at the beginning of the second cycle, the inventory position comes back to the same 300, 200, 500 and 400, and that is largely because of the assumption that some of the demands is 2500, while production is 2500. So, if we take a cycle of 0.8763, we have produced 2500, we have consumed 2500, so inventories will stay as they are, now we also observe that the total inventory at this point is 1400, ((Refer Time: 26:29)) this is also 1400, if we add this, it should be actually 1400.

But, it has become 1401, because of rounding off, so we could take this also as 1400, So, this is also 1400 and this is understandable, because at every point as I mentioned we have exactly the total consumption and total production will match. Therefore, the aggregate inventory will remain the same.

So, this would also validate our assumption that we left out the inventory cost as a constant and than we did this of course, another way of looking it is we left out the inventory cost, and then we optimized on that T and we observe that the sum of the inventories is a constant. So, the basic idea of this problem is to have a cycle of 0.8763 and proceed. Now, there are two other interesting aspects to this problem, now this formulation essentially us to maximize T, which means we want the cycles to be as lengthy or as long as possible.

So, the question is are we conflicting on our assumption, that if we want the cycle length to be long, then does it conflict with the basic ideas of shorter production runs, and just in time manufacturing. Just in time manufacturing requires that, we have very short production run lengths, and we are able to changeover very quickly, in a way we are not violating it and we will see why we are not violating it. After we look at one more modification of this, now let us go back to this we will address the other problem of conflict between just in time manufacturing a little later.

So, we go back and look at this problem, now let us assume for a moment that our objective is only to maximize T and then if we want to maximize T further what do we want to do? Now, one first thing we observe is that, if we start changing these inventory values, the r values will change, therefore these things will change and therefore, it will either shorten or lengthen the cycle. But, let us not change the inventory values, let us keep the inventory values as they are, let us keep these also as they are, the r values also as they are.

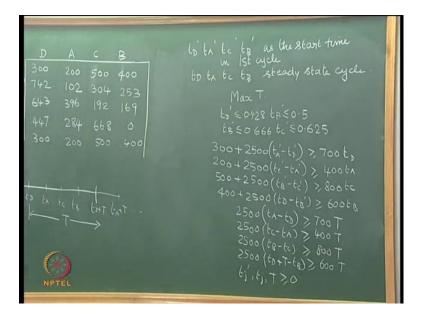
Now, if the cycle can be lengthened, then the first indication is here that B inventory is 0, when actually start producing this item, so the first thing I can do is, here I produce time equal to 0. So, when I produce a time equal to 0, I have an inventory of 300, but then end also have an inventory of 300, so I do not need to actually have this inventory of 300 at all, if I have 0 here, I would still be able to do it, so this will be 0 ((Refer Time: 30:16)), this will be 442, this will be 343, this will be 147 and this will be 0.

So, the next question is, if I ideally I can stretch my cycle further, T further if I produce an item when the inventory comes to 0. So, the question is can I keep a 0 here and can I distribute this 300 man-hours to other items, in such a manner that every time I start producing only when my inventory reaches 0. So, here you are producing when the inventory is 300, you are producing when the inventory is 102, you are producing when the inventory is 192 and then you are producing when the inventory is 0. So, thing is can I adjust it indirectly without explicitly changing it here such that, I can stretch the cycle.

So, this would mean that, we actually go through we look at this problem all over again and then say that, now let us assume I have 1 cycle where I go through and at the end of that cycle I am going to adjust the inventory levels, I am going to keep the 1400 constant. But, at the end of the first cycle, I will kind of adjust the inventories to all the four items such that, in what is called the steady state cycle that is going to follow, I would have an inventory of 0 here when I produce D.

I will have an inventory of 0 when I produce A, I will have an inventory of 0 when I produce C and I will have an inventory of 0 when I produce. Now, how do I reallocate it through at the same linear programming problem, at the end of the first cycle is the next formulation that we will see.

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So, in the next formulation what we would do is we call t D dash, t A dash, t C dash, t D dash as the start time, in the first cycle and t D, t A, t C, t D in the steady state cycle. So, it is like saying I have a t D dash, t A dash, t C dash, t B dash, t D, t A, t C, t B, t D plus t t A plus t and so on, t D, t A, t C, t B and t B in the real cycle. So, your actual cycle starts from t D to t D plus T, which is this as well as t A, t A plus T and so on. So, let me just write these constraints, so you will first have t D dash is less than or equal to you will have the same maximize T.

And then you will have t D dash is less than or equal to 0.428, t A dash less than or equal to 0, t B dash less than or equal to 0.666 and t C dash less than or equal to 0.625. So, the first cycle obviously, has to begin before these things run out of items, then we get into this, the first cycle should be such that, we also have three hundred plus, now I am looking at item D which is my first item. So, I want to stretch my t D to such an extent

that, I am able to meet that demand up to this point, using the 300 that I have, plus what I produce here.

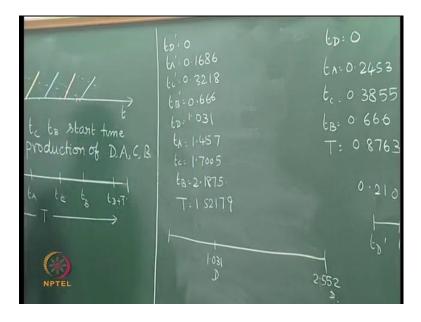
So, 300 plus 2500 into t A dash minus t D dash is greater than or equal to 700 t D. Now this constraint is going to tell me that, I am going to start producing t D in the steady state cycle or the actual cycle somewhere here. But, I have already produced a certain quantity for this period, so my production is 2500 into this length that should be equal and 300 inventory that I have. So, that should be capable of meeting my demand up to this point, when I actually start producing D again.

So, like that I write four constraints, which I write very quickly 200 plus 2500 into t C dash minus t A dash is greater than or equal to 400 t A, 500 plus 2500 into t B dash minus t C dash is greater than or equal to 800 t C and 400 plus 2500 into t D minus t B dash is greater than or equal to 600 t B. Now, the fourth constraint is important, because we are now looking at this, now if I look at item B, I am going to in the first cycle I am going to produce B up to this period t D minus t B dash at the rate of 2500. So, this the production in the first cycle, inventory that available with us is another 400.

So, with this I should be able to the meet the demand starting from here, up to the period t B where I start producing again, so write these four constraints that will try to stretch these things to the extent possible. Plus now I go back and write the steady state cycle equations, now this is the period where I produce item D at steady state. So, now, I go back to 2500 into t A minus t D is greater than or equal to 700 into T, because now at steady state when I start making this item, now this ((Refer Time: 38:26)) is the production, this is the demand period; so production should be greater than or equal to the demand.

Similarly, I write four more equations, now t C minus t A, 2500 into t C minus t A is greater than or equal to 400 T, 2500 into T t B minus t C is greater than or equal to 800 T and 2500 into t D plus T minus t B is greater than or equal to 600 T and then we also have all the t j dash t j T greater than or equal to 0. So, this gives us a much slightly larger linear programming problem, which can be solved optimally to try and get the maximum cycle length that is possible by playing around, or adjusting the inventories.

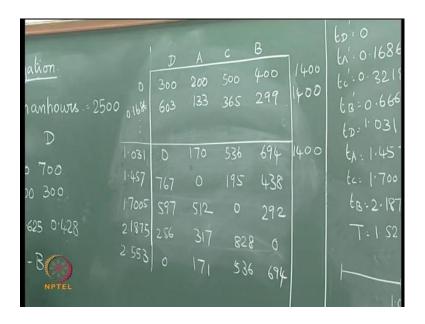
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Now, when we solve this problem optimally, so let me write the optimal solution to this problem somewhere here, t D dash is 0, t A dash is 0.1686, t C dash 0.3218, t B dash 0.666, t D 1.031, t A 1.475, t C 1.7005 and t B 2.1875 and capital T 1.52179 that is the capital T cycle which goes. Now, the first thing that is happened is, the cycle time T has increased from 0.8763 to 1.52179 almost doubled. And then if we go back and draw this kind of a chart here, the first cycle ends and the steady cycle begins at 1.031 that is when the steady cycle begins.

Now, D is produced again the second cycle will be 1.031 plus 1.521, which is roughly 2.552, so 2.552 D is produced again, D is produced again. Now, it is also possible for us to draw a similar chart here, to understand the inventory positions and let me just quickly give you the values.

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So, time equal to 0, it is 300, 200, 500, 400 at time equal to 0.1686, it is 603 133 365, 299 we go through this, and point that is most important for us is 1.031, it is 0 170 536 694, 1.475 767 0 195 and 438, 1.7005 597 512 0 292, 2.1975 256 317 828 0 and 2.553 it is 0 170 or 171 536 690. There are some minor rounding of errors, this is 2.552 is written as the starting time of the next cycle of D, the last 3rd digit is the rounding off here, it is given as 553.

Similarly, the 170 and 171 is due to round off, now what you observe out of this, the movement the steady cycle begins, we produce D when the inventory is 0 Now, at this time when we actually start producing A the inventory has come to 0, when we start producing the next item C the inventory has come to 0. And when we start producing the next item B the inventory has come to 0, whereas when we went here, which was the steady cycle in the earlier case when we started producing D the inventory was not 0, but 300 and so on, only here it was 0.

So, by adjusting the inventories, in such a manner that at steady state we are getting 0, we are able to stretch our T as much as we can. Now, once again back to the beginning of the next cycle, it is 171 536 694, now let us do another quick calculation, which is fairly obvious. Now, the sum of these is 1400. Now, inventory here is 1400.

So, inventory is not going to change, the total inventory is going to remain the same largely because the total demand is 2500, total production is also 2500. So, what we have

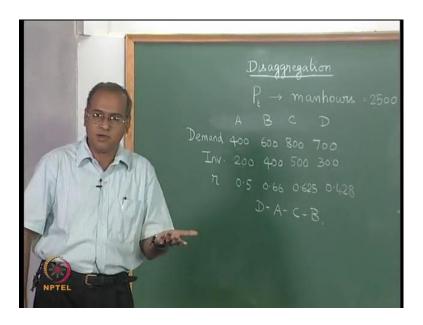
effectively done is through the second problem, we have redistributed this 1400, which was at 200, 400, 500, 300. Now, that has been redistributed to 0 170 536 694 to add up to 1400, so that the cycle time T can be made as large as possible. So, there are two aspects to this problem, one is obviously, the cycle T depends on the total amount of inventory that you have number 1.

If total demand is equal to total production, then the cycle time T number 1 is going to depend on the total amount of inventory that we have, as secondly, and more importantly, it also depends on the way in which the existing inventory is distributed, so the existing inventory can be distributed in such a manner that your T exceeds. When I say distribution one has to quickly understand that, it is 1400 man-hours of inventory, it is not substituting A with D, it is about saying these many man-hours are used or available there, so it can be redistributed.

So, distributing is helped us achieve a larger value of cycle time. Now, then we get into another question, now would we use this model or would we use this model or would we use a model that is somewhere in between. Now, the advantage of this model is that, if my objective is to maximize capital T which is the cycle length, then I have really stretched it to the limit, where I am going to have exactly 0 inventory, while I produce. Whereas, here I certainly had a certain buffer or a cushion for at least three items when I started production.

So, if the demand is purely deterministic and it is not going to change at all, then one can take the risk of redistributing it, in such a manner that you achieve maximum cycle time. And every time you reach 0, you produced or every time you produce when you reach inventory, but reality is very different from these, we have made very simplistic assumptions in both the models.

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the first one is of course, is that the total P 2500 is sum of the demands, and we have also made an assumption that this demand is going to be constant for every period, and it is going to be continuous. Because, when we made these calculations, we have subtracted, we have just multiplied the time by the demand, so it means the demand is stated as continuous and so on. The movement the demand is not continuous and the demand start showing some variations, then this model has to be very carefully looked at, because we may get into shortage.

Now, we do not want shortage in reality, so either we have to put a certain safety stock over this model, or essentially we use something similar to this, so that we stretch ourselves only at the last point and for all other places, we have some cushion that we have. So, it effectively boils down to playing around with the available inventory, in such a manner that if we want a certain cushion to handle variation and uncertainty, it is actually good to distribute it in a particular way. But, then if the demand is purely deterministic, then we can try and stretch it to the extent possible that comes out of this.

Now, let us answer the next question, now both the linear programming models the objective was to maximize T, which is to maximize the cycle time and make the cycle time as large as possible. When we raised a question, whether it would conflict with the principles of smaller run manufacturing and so on, the answer actually comes here. We

know by now that the T depends not only on these numbers, but also depends on the way these numbers are distributed.

So, automatically if we want the cycle time to be smaller, then keep the total amount of initial inventory on hand keep it smaller. If this 1400 total becomes 400, then automatically T is going to move from 0.87 to somewhere like 0.5 or 0.25 or what, which is like a weekly cycle. So, the answer still lies in playing around with these inventories, or if organizations follow the principle of zero inventory, by which they want to have lesser and lesser inventory.

Automatically the mechanics of the problem or optimization of the problem will lead us to solutions, which have smaller values of cycle times. Another dimension to it is also to see the tradeoff between inventory holding cost and changeover cost. And if we changeover times are smaller, then automatically more changeovers are possible and therefore, we also be able to make more variety, and then keep the cycle time as small as possible.

So, even though the objective is to maximize T, in some sense we are not trying to do it at the expense of today's thinking, the model essentially tries to maximize T, but the model still takes us to something that is practical and something that is the requirement of today, which is to minimize the cycle. So, that we are able to have shorter runs of larger variety, the answer lies in the amount of inventory that we keep on hand.

So, automatically if these values are smaller, the r values will becomes smaller and the cycle time will be smaller. Similarly, if the changeover times and changeover costs are smaller, then it is possible to have more changeovers bringing more variety, and produce at shorter runs and smaller T. Another relaxation that we have to look at is, now can we handle a situation where this demand varies with time right now, this has been both the models have assumed that this demand is the same in every period; and this demand is uniform.

Now, can we change these modules to meet situations where, these demand can be different in different months, and this production capacities are also different in different months. So, aggregate planning could give us solutions where the production capacity P available is different in different months. So, we solve such a problem using a slight

approximation and a heuristic solution, which essentially leads us to another linear programming problem, and we will see that model in the next lecture.