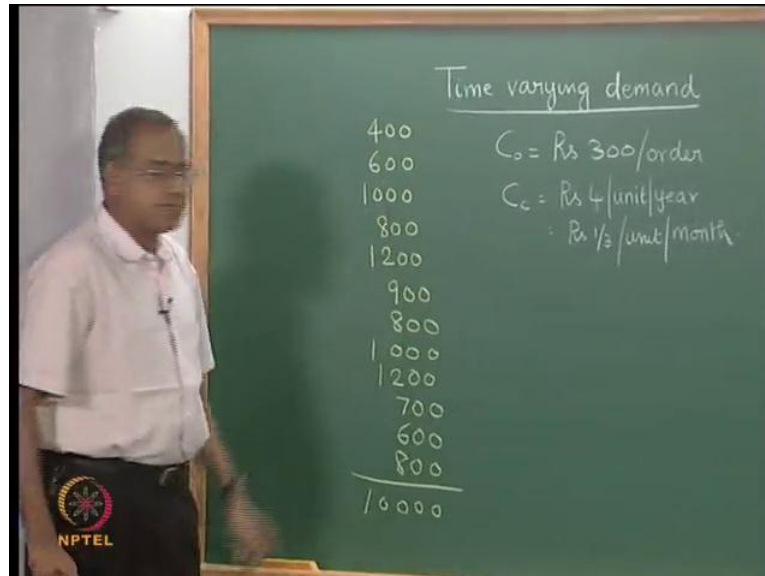


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**Lecture - 18**  
**Lot Sizing – Heuristics**

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In the last lecture, we looked at how to order when the demand varies with time. So, this was an example that we took where we took the annual demand to be 10,000 but the annual demand is not distributed equally to the 12 time periods. But, it changes with respect to each of these time periods. We saw a mathematical programming formulation for that problem and we also saw, why and how we may not be able to use the well known economic order quantity result in solving this type of a problem. We also ended by saying that, if we have a finite time horizon as it is in this case, integer programming can give us an optimal solution and dynamic programming also can give us an optimal solution.

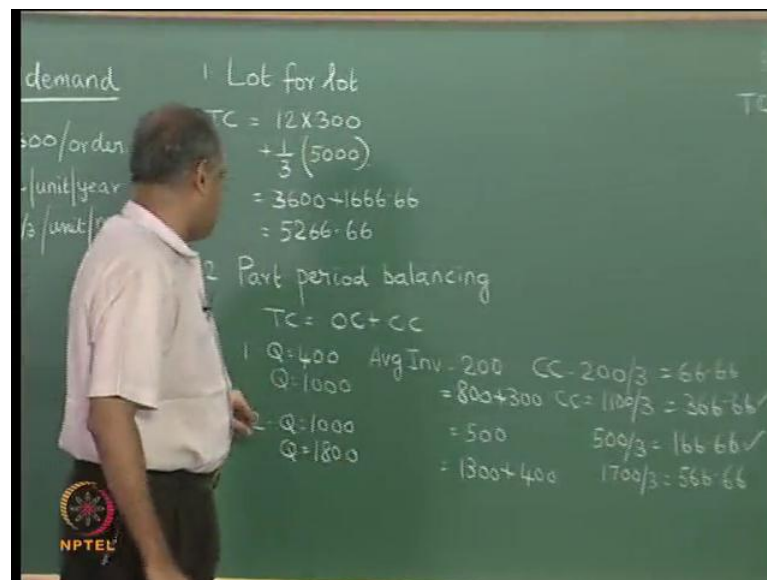
But then integer programming is hard and difficult and we require a solver. More importantly, in practice the time periods are not finite and time periods can extend. And we also have rolling forecast for the products which in turn gives us some kind of an estimate of the demand for each of the items that rolls over a time period. So, we have to solve a problem where this is not fixed, then this goes on and on and then we concluded

in the last lecture, that we could use Heuristics to solve this problem. Some other data that we used while doing the integer programming is that.

The order cost is rupees 300 per order and the carrying cost  $C$  is 300 per order. And the carrying cost  $C_c$  was rupees 4 per unit per year which becomes 4 by 12 or rupees 1 by 3 per unit per month. So, if you want to look at Heuristic solutions, if you use these Heuristic solutions do not guarantee optimality, they try and guarantee a solution which is close to the optimum. And since, the basic problem is a minimization problem a Heuristic solution would have an objective function value higher or equal to that of the optimal solution. So, the easiest thing to do is to if there are 12 periods.

Now, only for the sake of illustration of the Heuristic, we will take the same example so that they will be able to compare the final solution given by the Heuristic as well as the optimum. But, then we will also show how this Heuristic can be extended to meet the case where the planning period is not finite and keeps rolling. So, the easiest Heuristic is if we are looking at 12 months, the simplest thing to do is make an order every month. That is called lot for lot Heuristic.

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So, simplest thing is to make an order every month. If the demand time bucket is 1 month for which the demand is known, place an order exactly for that. So, if we do that we end up placing an order in each of these 12 months. So, total cost will be 12 times order cost because there is an order every month, 12 into 300 plus we are also computing

the average inventory that we have in each month so plus 1 by 3 which is the inventory holding cost. The average inventory for January will be 400 divided by 2 because you begin with 400 and end with 0. For February it is 600 divided by 2 so, average inventory for each month is demand by 2.

So, total sum of the average inventories over the 12 months is the total demand being 10,000. The average inventory, sum of the average inventory will be 10,000 divided by 2 which is 5000 so, 1 by 3 into 5000. So, this would give as a total cost of 3600 plus 1666.66. This would give us 5266.66. We also saw in the last lecture, that the optimal solution to this did not have an order in each of these months, it had 7 orders in the 12 periods and the optimal value or the best value was 5000 and this Heuristic is giving us a value a 5266. Now, this Heuristic is very nice because it is very easy to implement. Every month you look at the demand and you place an order to meet the demand of that month but it could be slightly costly because you end up making more orders than the optimum. The optimum had 7, now you have 12.

Now, this Heuristic can also handle time period, if it rose beyond that because it is a very simple Heuristic where every month you just place an order. The second Heuristic that we will see is called part period balancing. If we try and see what we actually optimized, in our mathematical programming model, what we optimized or try to minimize is a sum of the order cost and the carrying cost. So, the objective function value is total cost is equal to order cost plus carrying cost. Now, we want to make decisions regarding the purchase such that, we wish to optimize or minimize the sum of the order cost as well as the carrying cost.

We would also, if we see the optimum solution carefully we would also have realized that in periods where the demands were very high, you make order in that month and do not combine it with the next month and so on. Now, the basic idea is that, if carrying an item for an extra month for example, (Refer Slide Time: 00:18) if the tradeoff is between placing an order here for 400 and placing an order here for 1000. So, when we place an order here for 400 it automatically means we have to place another order here for the 600 so there will be 2 order costs. And individually we will be carrying 200 here and 300 here.

For example, if it is cheaper to place 1 order here and then find out the sum of the

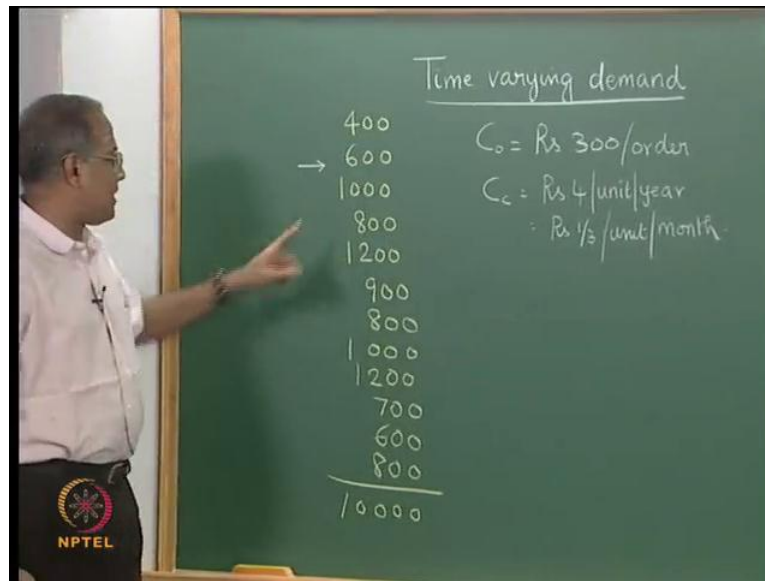
inventories that we will carry and if it is cheaper, then obviously we would like to order 1000 here, rather than ordering a 400 and a 600. Or in other words, if the cost of carrying the inventory of this in here also which is the additional thing that we do when we combine, if that cost of holding that inventory is actually cheaper than one order cost then we would end up ordering a 1000 here. So, if the cost of holding the 600 here exceeds the order cost then it is better to order separately one here and one here. So, it is a tradeoff between the inventory carrying cost of a particular month versus one order cost. That is the essential or central idea in the part period balancing Heuristic.

So, first thing what we do is (Refer Slide Time: 00:18) we look at this period and say order quantity  $Q$  is equal to 400. So, when order quantity  $Q$  is equal to 400 which means we are ordering only the first months demand. Then average inventory that we have is equal to 200 and the inventory holding cost is 200 divided by 3. Now, we try and see whether we can order 1000 here. So now, if  $Q$  equal to 1000 here which means we are trying to order for these two demands in this place. Average inventory will be equal to, 1st month (Refer Slide Time: 00:18) we begin with 1000 and we end with 600 so the average inventory is 800 for the 1st month.

Second month we would begin with 600 and end with 0. So, there will be another 300 here which will be and the carrying cost is 1100 divided by 3. So here, 200 divided by 3 is 66.66. 1100 divided by 3 is 3 threes are 9 20 366.66, 20, 3 366.66. Now, right here we observe that the carrying cost of bringing these two together is now exceeding an order cost of 300. So, the moment the order cost is exceeded we stop that and then we have to decide whether we chose this 66 or whether we chose this 366.66.

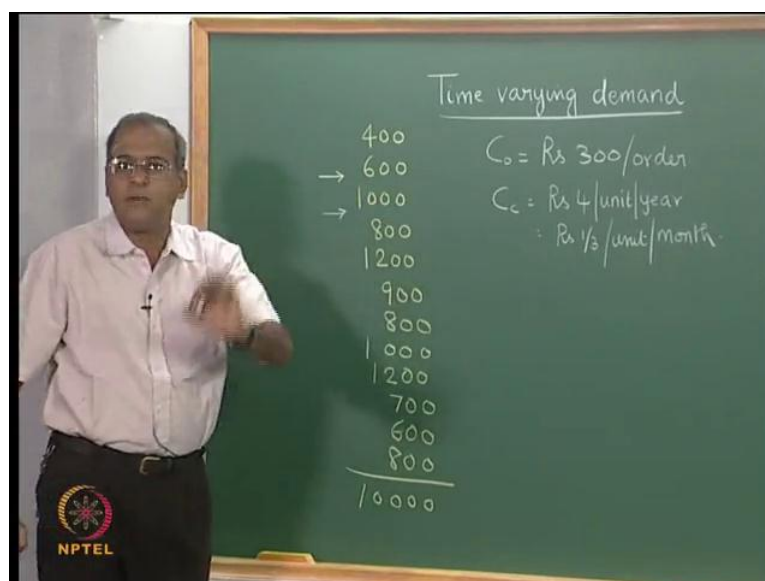
So, common sense tells us that we choose 366.66 because it is closer to 300 than the 66. So, when the carrying cost exceeds the order cost, stop the computation. And then, look at the last two values of the carrying cost, whichever is closer to the order cost you take it. So, now we would place, (Refer Slide Time: 00:18) we pick this and we say that in the 1st order we would order for 1000 which is bringing these two things together. So, let me explain the 2nd computation also and then you move to 2.

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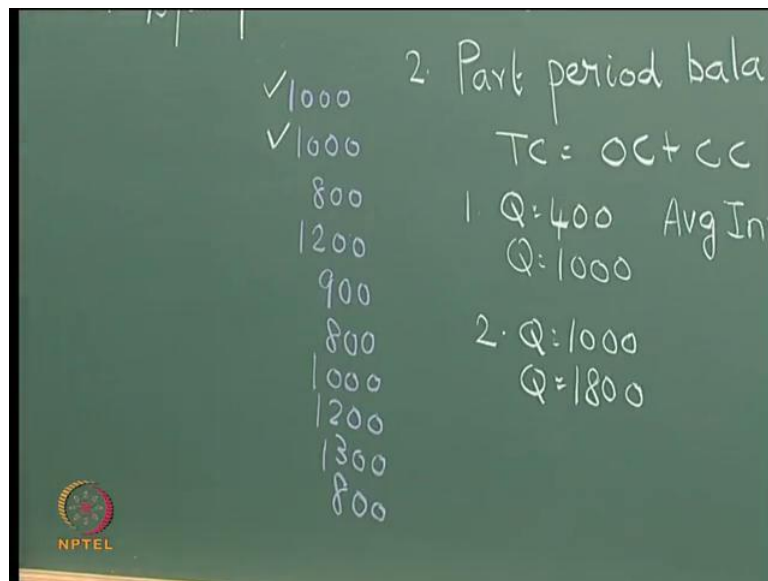
It means, right now we have finished ordering up to this. So, we will 1st look at Q equal to 1000. So, Q equal to 1000, average inventory is 1000 divided by 2 which is 500. So, the carrying cost is 500 divided by 3 which is 166.66. (Refer Slide Time: 11:20) Now, we consider 1000 plus another 800. So, we consider 1800 here. Now, the average inventory will be, 1st period here we begin with 1800, we end with 800. So, 1800 plus 800 divided by 2 which is 1300 which comes here. This is 1300 for the 1st period and for the 2nd period we start with 800 and we end with 0. So, plus another 400 so this is 1700 so, 1700 divided by 3 3 5s are 15, 566.66. Now, this 566.66 has exceeded the order cost of 300.

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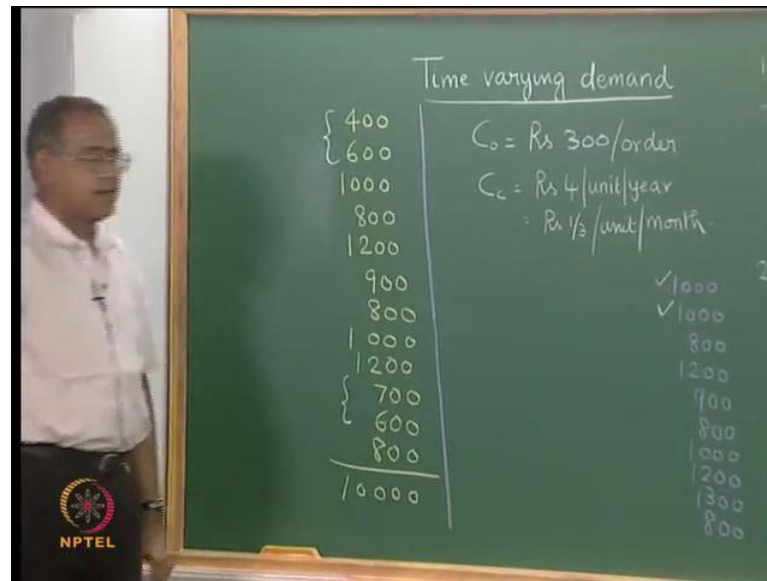
Therefore, we stop here and then we try and see which one is closer to 300. So, 166.66 is closer to 300, therefore we place an order here, which means your 2nd order is only for 1000. And now, your problem starts from here where you will 1st start looking at 800 and then you will look at Q equal to 2000 and so on. So, like that we can proceed till we finish orders for all the 12 periods and if we do so we would get a solution which is like this. So, we would get, when we complete this.

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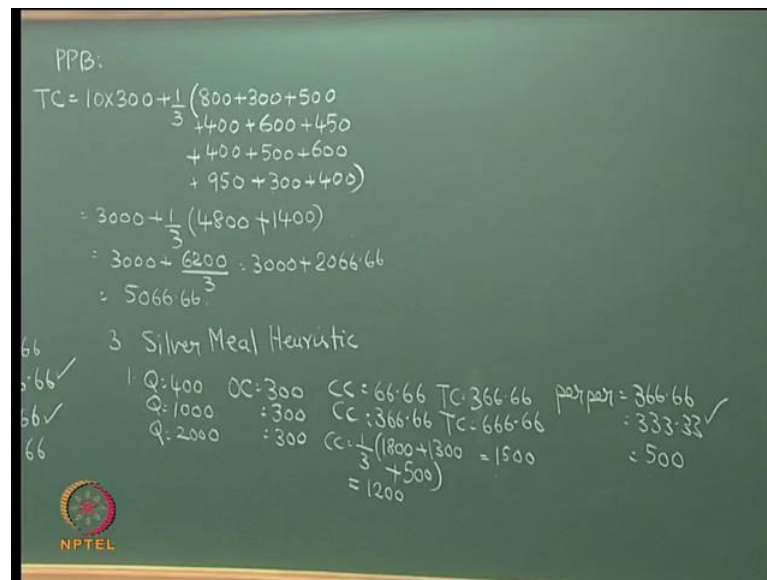
So, when we complete the part period balancing Heuristic our order quantities will be: 1000, another 1000, 800, 1200, 900, 800, 1000, 1200, 1300 and 800. (Refer Slide Time: 03:14) So, the part period balancing Heuristic, two of the things that we saw. We saw order quantity of 1000 and another order quantity of 1000 which I have shown here as 1000 and 1000. So, this would say or would ask us to order: 1, 2, 3, 4, 5, 6, 7, 10 times it would ask as to order in the 12 months.

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So this, these two are coupled together for a 1000. 1000 remains 800, 1200, 900, 800, 1000, 1200, there is another bunching here 1300 and another 800.

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So, what is the cost associated with part period balancing? So P P B, cost is equal to, there are ten orders so, 10 into 300 plus 1 by 3 into (Refer Slide Time: 14:47) So, here 1st period is 1000, 1000 plus 600 divided by 2. So, this is 800 for the 1st period, 300 for the 2nd, 3rd will be 500, 4th will be 400, 5th will be 600, 6th will be 450, 400, 500, 600. (Refer Slide Time: 14:47) Now, here there is a bunching. So, we would start with 1300

finish with 600. So, 1900 divided by 2 which is 950. This will be 300, we begin with 600 end with 0 so average is 300 and here the average is 400 so, plus 300 plus 400.

So, this is 3000 plus 1 by 3 into 8, 11, 16, 20, 26, 30, 35, 41, 44, 48, 4800 plus 450 plus 950 is 1400. So, this is 3000 plus 6200 divided by 3, this is 3000 plus 2s are 6, 0, 6 are 18, 66.66, so this is 5066.66. So, you can see the difference, now the optimum was 5000. Now, part period balancing is giving a solution with 5066.66. Lot for lot was much more expensive. In some sense, lot for lot did not try to optimize anything. It just said do it as it is or take it as it comes. In some sense there is no optimization here, there it is only an evaluative algorithm where, if I make an order every period what is going to be my cost?

Now, here there is a certain amount of optimization because we found at least by bunching (Refer Slide Time: 14:47) these two we are able to save one order cost, which means the cost of carrying this 600 along with this 400, that is far more important for another period seem to be lesser than one additional ordering cost. So, there was a bunching here. Similarly, there was another bunching here. So P P B is able to give a better solution in this instance when compared to lot for lot. And we could generalize and say that part period balancing would anyway give a better solution than lot for lot because it is trying to optimize something somewhere.

Now, the third one that we will see is the most popular Heuristic. It is called Silver-Meal Heuristic, after the paper from Silver and Meal who are the researchers who made that Heuristic. It is somewhat similar to part period balancing but slightly different. We explain the difference using a certain computation. Now, let us start with like we did in part period balancing. So, we begin with Q equal to 400. (Refer Slide Time: 14:47) Let us assume that we are, let me remove this and let me also remove this.

So, we start first with this, so we start with Q equal to 400. So, when we place an order we have to incur an order cost of 300. There is a carrying cost of, we begin with 400 we end with 0 so, average inventory is 200. So, 200 divided by 3 which is 66.66. So, total cost is 266.66, let me keep it there. (Refer Slide Time: 14:47) Then we consider combining these two together so, Q equal to 1000. So, when we combine these two together we still try and have 366.66, we will make that change. The order cost is 300, carrying cost is 66.66 so, total cost is 366.66. Now, when Q equal to 1000 we still try and order 1000 in a single order. So, order cost is 300, carrying cost we have already



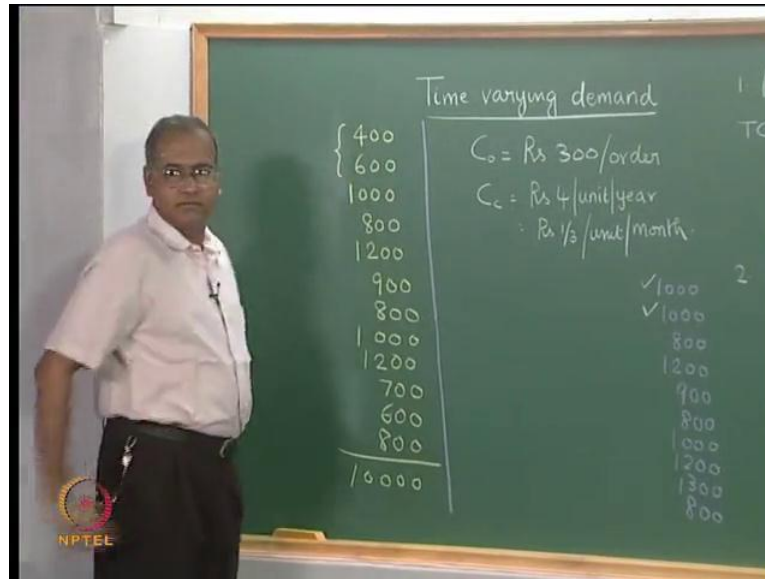
computed. We have just kept it here, carrying cost is 366.66.

So, this is 366.66, total cost is 666.66. Now, it is not fair to compare this 366.66 with 666.66 because this 366.66 is the cost for the 400, while 666.66 is the cost for 1000 but essentially it is a cost to meet the demand for 2 periods. So, what we do is we try and compare what is called a per period average cost. So, we know write another one called per period, will be equal to this is 366.66. This is 666.66 divided by 2 which will give us 333.33. So, now we realize that the per period cost is coming down. So, we try and see if we can optimize further on the per period cost.

So, we look at the 3rd possibility which is to try an order, (Refer Slide Time: 14:47) 400 plus 600 plus 1000 so try an order 2000. So, order cost is again 300, carrying cost we first have to find out the average inventory. So, to find the average inventory, 1st month we start with 2000 and we end with 1600. So, 2000 plus 1600 divided by 2 is 1800. So, this is 1 by 3 into 1800 for the 1st month. (Refer Slide Time: 14:47) 2nd month we begin with 1600 and end with 1000 so, 2600 divided by 2 is 1300. 3rd month we would begin with 1000 and end with 0 so, average inventory will be 500.

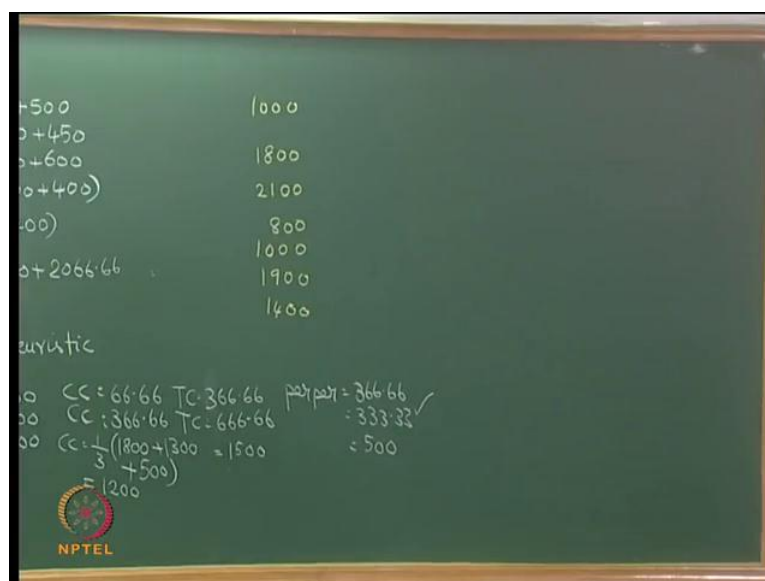
So, this alone will be 1800 plus 1300 is 3100. 3100 plus 500 is 3600, 3600 divided by 3 is 1200. So, total cost will be 1200 plus another 300 which is 1500 and the per period will be 1500 divided by 3 periods which would give us 500. Now, we realize that the per period cost was 366.66 when we considered 1 period, 333.33 when we consider 2 periods and 500 for 3 periods. So, it is started, it went down and it came up. Therefore, this is the place where it is giving us a local benefit the best out of the 3. Some kind of a local optimum is happening here so, the decision is to try an order for 1 and 2 together, combined these 2 together.

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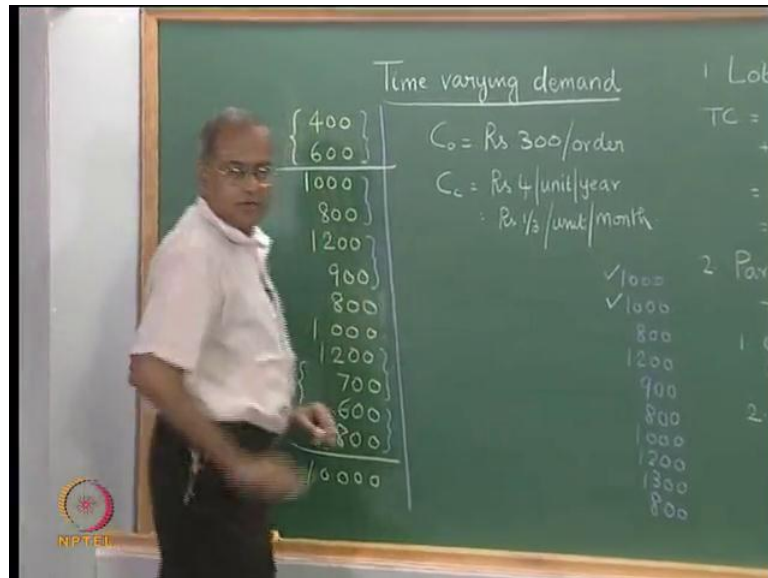
Now, we have to repeat what we have did. The reason I said, it similar to part period balancing but different is that, both of them essentially try to tradeoff between the order cost and the carrying cost. Part period balancing explicitly tries to see if the additional carrying cost exceeds the order cost. Silver-Meal tries to do the combine cost for multiple periods, average asset and see that period for which the average is showing a local minimum or the average is optimize. Now, we have to continue like what we did here (Refer Slide Time: 24:41) starting from this. So, we will start with Q equal to 1000, Q equal to 1800 and so on till we find yet another local optimum and then we proceed.

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Now, if we do the Silver-Meal Heuristic, continue to do it for all the 12 periods, the solution that we will have is, so let me write down the solutions for the Silver-Meal. So, the solution for the Silver-Meal will be 1000, 1800, 2100, 800, 1000, 1900 and 1400. Let us go back and make this comparison once again.

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So, if we do Silver-Meal, let me use a different color, so when we use Silver-Meal we order these two together. Then comes 1800, we end up ordering these two together whereas, in part period balancing they came as two different orders. Then we do 2100, we end up ordering these two together. Then, we have an 800 which is here. Then we have 1000 which is here, then we have 1900 which comes here and then we have 1400 which actually comes here. I think in part period balancing the combination happened like this.

So, we realize that the solution is slightly different in Silver-Meal compare to part period balancing in fact, we could even show. Take it exactly here to show how a Silver-Meal is optimizing, combining these two together whereas, part period balancing is trying to look at these two separately. (Refer Slide Time: 03:14) We have the part period balancing calculation which said that, go for 1000 here. So, let us just do the Silver-Meal calculation to show that it is actually moving to one more. So, let us just start from here and then look at this.

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Handwritten calculations on a chalkboard:

- $Q=1000$ :  $OC=300$ ,  $CC=\frac{1}{3}(500)=166.66$ ,  $TC=466.66$ ,  $TC/period=466.66$
- $Q=1800$ :  $OC=300$ ,  $CC=\frac{1}{3}(1700)=566.66$ ,  $TC=866.66$ ,  $TC/period=473.33$
- $TC(\text{Silver Meal}) = 7 \times 300 + \frac{1}{3}(800+300 + 1300+400 + 1500+450 + 400+500 + 1300+350 + 1100+400)$   
 $= 2100 + \frac{1}{3}(8000+800)$   
 $= 2100 + \frac{1}{3}(8800) = 2100 + 2933.33$   
 $= 5033.33$
- $C=366.66$ ,  $TC=666.66$ ,  $period=366.66$ ,  $period=333.33$ ,  $period=500$

So, we know look at the case two when Q equal to 1000. Order cost is 300, carrying cost is 1 by 3 into 500 which is 166.66. So, T C is equal to 466.66 and T C per period is equal to 466.66. (Refer Slide Time: 26:26) Now, in Silver-Meal we try and combine this 1000 and 800 together so, Q equal to 1800. Order cost is 300, carrying cost is 1 by 3 into (Refer Slide Time: 26:26) 1800 plus 800 divided by 2 which is 1300 plus 400 so, 1700. So, 1 by 3 into 1700 which is 5100 divided by 3 so, 5100 divided by 3 is 700 by 3 1700 by 3 is 566.66 so, T C is 866.66. Now, T C per period is 433.33.

So, Silver-Meal straight away tells you that based on the Silver-Meal way of optimization, combining 1000 and 800 is cheaper than doing 1000 separately. Whereas, part period balancing based on its own way of optimization is a now says somewhere here that, if you do 1000 it is here (Refer Slide Time: 03:14) 1800 is here 1000 is cheaper. (Refer Slide Time: 26:26) Now, when we do 1000 plus 800 plus 1200 we would realize that the part period cost is higher so, Silver-Meal would have combined 1000 and 800 whereas, part period balancing separated them from 1000 and 800.

So, let us try and do the total cost for Silver-Meal. So, total cost for Silver-Meal is equal to, Silver-Meal is giving us 7 orders as against (Refer Slide Time: 13:48) part period balancing had 1 2 3 4 5 6 7 8 9 10 orders. So, 7 into 300 so, Silver-Meal order cost is 7 into 300 plus 1 by 3 into so, for the 1st month I am combining this 1000 so 1st month I begin with 1000 and I end with 600. So, the first month will be 800, 1st month is 800

2nd month is 300. Now, 3rd and 4th I am combining. (Refer Slide Time: 26:26) So, 1800 plus 800 divided by 2 is 1300 plus another 400 so, 1300 plus 400.

Similarly, 5 and 6 I am combining so, I begin here with 2100 and I end with 900, 3000 divided by 2 is 1500, 1500 plus 450 is 1950 so, 1500 plus 450. (Refer Slide Time: 26:26) Keeping this separately, this is 400, this is 500 so, plus 400 and 500. (Refer Slide Time: 26:26) Combining these 2 so 1900 I begin with plus 700 is 2600 so it is 1300 is the average, 1300 another 350. So, 1300 350, the last one again I am combining. (Refer Slide Time: 26:26) So, start with 1400 and end with 800 so 2200 divided by 2 is 1100 plus another 400. So, 1100 plus 400 so, this is 2100 plus 1 by 3 into 8 plus 3 11, 24, 28, 43, 47, 52, 65, 76 plus 4 80, 8000 plus 450 plus 350 which is another 800.

So, this is 2100 plus 1 by 3 into 8800 which is 2100 plus 2933.33 which would give us 5033.33. For this particular example, there is a slight gain when we do Silver-Meal compare to when we do part period balancing. In some sense, both Silver-Meal and part period balancing give solutions which are very close to the optimum, the optimum is 5000. If I have the problem is also such that the optimum 5000 was not very far away from the EOQ is 4898.98, though we discussed the issues involved in using or approximating the EOQ in the earlier lecture.

So, the optimum is 5000, part period balancing is giving a solution that has a cost of 5066.66, Silver-Meal is giving cost of 5033.33. Both are very good Heuristics, Silver-Meal is used is much more popular and used little, perhaps a little more than part period balancing. But, invariably both of them are used for a given problem and then the best one is chosen depending on what solution they give to the particular instance of the problem. Both of them are intuitive in their own way, the difference is very subtle but both of them are centered around optimizing the total cost. The only difference is Silver-Meal computes the total cost and then divides it by the periods to optimize it.

Part period balancing essentially looks at the moment the inventory cost exceeds the order cost, which one is closer. So, mostly in that decision to find out which one is nearer, part period balancing solution is different from Silver-Meal solution. We did see a situation (Refer Slide Time: 03:14) here where 166 566 because order cost was 300 this was chosen, while Silver-Meal is way of optimizing allowed us to combine both of these. So, both the methods can be used and both of them are used extensively. Two other

issues before we close this and move to another topic.

The first issue is, how do we measure the goodness of a Heuristic? There is a Heuristic there are more than one Heuristic in fact, there are many more Heuristics there are other Heuristics also which we are not covering in this lecture series, there are few more Heuristics to do that. Then how do we measure the goodness of the Heuristic is number 1 and number 2 is are there instances where the Heuristic can behave slightly poor, are there problem instances where a particular Heuristic may not give a very good solution. So, people address both these issues we are not going to dwell into much more detail on these. I will make a very brief mention of both of these.

May be, in some sense it is also in fact the Heuristic is complete when we are able to say what is called or able to define what is called the worst case performance of the Heuristic. Now, if  $L$  naught or the  $C$  naught in some sense let us call it  $L$  naught as the optimum or  $o$  as the optimum (Refer Slide Time: 15:05)  $o$  in this case is 5000,  $h$  is the Heuristic solution. In still in part period balancing it is 5066.66. Now, based on this numerical example one would say that  $h$  divided by  $o$ , which is Heuristic divided by the optimum would be of the order of 1.000 something because you are dividing 5066 divided by 5000 whereas, the same as 5033 divided by 5000 would be little smaller for the Silver-Meal.

So,  $h$  by  $o$  the smaller  $h$  by  $o$  is the better the Heuristics. (Refer Slide Time: 26:26) one is to try and look at numerical simulations like this. Take examples and show find out average  $h$  by  $o$  and so on. The other is also to derive some kind of theoretical expressions. After all, the basic idea is known so depending on how large or small this value is with respect to the previous one. Part period balancing is going to decide whether this is going to be separate or whether this is going to be clubbed because it is going to check the closeness to 3000.

So, it is possible to do what is called a worst case analysis and show that for example, whatever be the problem, whatever be the number of time periods part period balancing will be within 1 plus  $x$  of the optimum or Silver-Meal will be 1 plus  $x$  of the optimum, actually between the two researchers have found that part period balancing is little better amenable to that kind of an analysis. So, there are some results which talks about the goodness of the part period balancing. Now, there is another question that can also come

is, how do these Heuristics behave? (Refer Slide Time: 26:26)

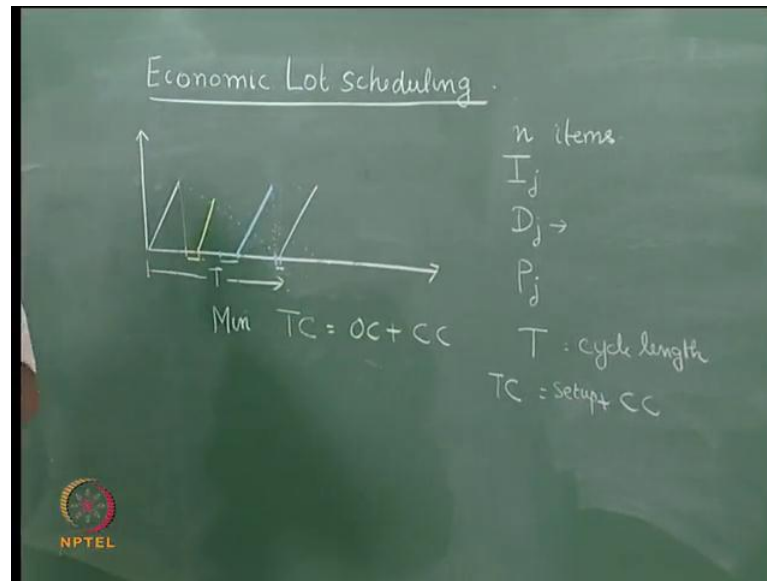
If for example, there are lots of peaks and valleys in this data, we did briefly mention it in the earlier lecture the impact of the peak. We did give an example where if the 1st month's demand itself is 2000 out of the 10,000, then rounding off economic order quantity is not going to help us. The other issue is if there are time periods where the demand is 0 for certain items, how does a Silver-Meal behave? How would part period balancing behave? Would something behave better than the other consistently if there are 0 demands in certain periods?

So, some of these questions have also been addressed in the later step and there are some problems instances where, Silver-Meal does not behave extremely well when there are too many 0s coming in the middle and particularly if those 0s are constitutive and so on. But, leaving out all those exceptional situations both are very effective Heuristics to solve this problem and both these Heuristics are not bound by the number of periods. By the very definition, the very nature, the way the Heuristics run we are not bound by a finite number of periods. (Refer Slide Time: 26:26) For example, if we are right here let us assume we are addressing a practical problem and we are right here after 10 months.

But, by that time we would have forecast for the next 6 or 7 months whereas, when we are solving this particular instance where we have to close this. And let us say for some reason the Silver-Meal optimizes up to here because we are looking at a finite time period this has to go into one order. But then, if you have a rolling time period by the time you come here, you would have another four 5 pieces of data for the next few months. And then, you can continue to apply part period balancing or Silver-Meal and you will not be bound by having one order exclusively for this. You may be able to combine this and the next one and so.

So, in fact in the situation where the demands are rolling and you do not have a finite time period so, like these work much better than integer programming from a practicability point of view because integer programming is going to bind you to a finite number of time periods. So, these Heuristics are used extensively to solve. So, let us try and introduce one more problem in this lecture and then let us see how we are able to handle this.

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Now, let us revisit the Economic Lot scheduling problem. So, let me write the Economic Lot Scheduling problem again. Now, when we define the Economic Lot scheduling problem, we did something like this. We said, we looked at a particular machine or a facility that is making multiple products. If you take a particular product or an item it produces at the rate  $P$ , the demand is at the rate  $D$ . So, what you normally do is you produce for a certain period, consume while you produce so, inventory is built up at the rate of  $P$  minus  $D$ . And then with the built up inventory you consume it for a certain number of period and the cycle begins again.

So, let us say that if we are going to do this we consume here and let us say the cycle begins again. We also said that in between this period when we do not produce this item say we may produce another item which we are going to show by the yellow color which is item number 2. And we also said that there has to be a certain changeover time between the end of this and the beginning of this. So, let us say we have a small changeover time here and then the yellow begins. Let us assume the yellow is produced up to this period.

And then, inventory is built up. Now, let us say yellow is also consumed up to a certain time period and then in the remaining period we could again set it up for another product which is say the blue color product. And then, the blue color is produced say up to this and then, the blue color production is over. Let us say the blue is produced up to this. So,



the blue color production is, let me say blue is produced up to this. So, the blue color production is up to here then we changeover and go back to the white color product which is the original product and then the cycle begins.

The other thing in the Economic Lot scheduling problem we said that, between this and this the consumption the cycle time is  $T$  and then we try to find out what is the value of  $T$  that optimizes the changeover cost plus the inventory holding cost for all the periods taken. Now, in some sense when we did worked on the Economic Lot scheduling problem we are not unduly varied about the order in which we are going to make this white, yellow and blue products. Though, the example that I am giving right now follows the order: white then yellow and then blue.

Let us assume these three colors represent three distinct products. Once we solve the Economic Lot scheduling problem whether we make it in the order white, yellow, blue or white, blue, yellow it is the same because the cycle time is the same for all. The reason why it is not order dependent is because the time to changeover or setup as we call which is this time to changeover to yellow, this much time to changeover to the blue, this much time to changeover to the yellow and this much time to changeover to the white, depends only on the succeeding job and does not depend on the procedural job.

This is not defined as changeover between white and yellow. This is not defined as changeover between yellow and blue. It is defined as changeover to yellow, changeover to blue irrespective of what the earlier product is. If the changeover times were sequence dependent which means it depends both on the current job as well as the next job to be done then the problem gets far more complicated because sequencing them to in the order that minimizes the sum of changeover times. When the changeover times are sequence dependent becomes a traveling salesman problem which by itself is a very difficult problem to solve.

So, in the Economic Lot scheduling problem we do not assume at least at the moment that the changeover times are sequence dependent. So, changeover times are sequence independent and then the Economic Lot scheduling problem essentially tries to minimize total cost which is a sum of order cost plus carrying cost. Now, let us look at a certain variation of this problem where we are going to make some more simplistic assumptions and then we are going to solve another type of a problem here. Let me introduce this that

problem in this lecture.

Now, let us assume while we started, when we started doing this we said, now I am building up inventory for this at the rate of  $P$  minus  $D$  and then when I consume this which is shown by this white dotted line, I have built up inventory here. Similarly, when I start this yellow I am building up inventory here and then I stop producing the yellow and I consume the yellow for a while. Now, what am I doing to meet the demand of the yellow and blue for this period and for this period. In the same graph if you see that I start producing when the inventory of white is 0, I produce and consume so why it is completely taken care of.

If my entire planning for yellow is going to start only here or here I set it up and then I produce. Then this part is ok because to meet this demand of the yellow product during this period I have built up this inventory. Now, what have I done to meet the demand during this period? So, if we start the problem here at some time equal to 0, we definitely need some inventory of yellow and blue on hand so that by the time we stop producing this or by the time we start producing this yellow, that inventory should be enough to meet the demand of that period.

Similarly, that inventory of blue should be enough to meet the demand of that period. So, the problems will begin, will be redefined as I have a certain initial inventory of all the items may be: the white, the yellow and the blue. I have inventory of all the items so when I have some inventory of all the items now let us assume that we are going to make certain number of items. So, let us say that I have  $n$  items or  $n$  products which I have to make. I have a certain inventory of each of them so  $I_j$  is the inventory that I have of each of them on hand.  $D_j$  is the demand for item  $j$  that I have,  $P_j$  is the rate at which I can produce the item.

Now, I want to define what is the cycle length  $T$  which is a same decision variable as in the Economic Lot scheduling problem. What is the cycle length  $T$  such that I minimize a total cost, this is cycle length such that I minimize a total cost right, such that I minimize a total cost. Now, ordinarily this total cost is going to be order cost plus carrying cost. Now, this problem in some sense will become the Economic Lot scheduling problem if the  $D_j$ 's are the same for every period for each item right. If we take annual demand for item  $a$ , it is a same its one number which means you are not using the case of time

varying demand. Demand is the same at all periods.

And, if we consider the order cost as well as the carrying cost. The only difference between this and the Economic Lot scheduling problem is the presence of  $I_j$  which essentially means that we are ensuring that there is no shortage, right up to here. One way of looking at the economic lot scheduling problem is we are looking at it at some kind of real time, which can start from any point and there is sufficient inventory to meet this the point that I raised right now to meet the demand of yellow for this period and the demand of blue for this period.

The modification here is if I have a certain amount of inventory my initial inventory is also going to help me in some way to stretch the  $T$ , the cycle. It will help me in defining the cycle. And then, in some sense I would like to have longer cycles so, I would like to have longer cycles. When we have longer cycles obviously the order cost is going to come down and the carrying cost is going to go up. So, from a purely inventory point of view or just in time manufacturing point of view, we would like to have small lot production which simply means that we do not want longer  $T$  but then, let us try and look at this problem at least to begin with that how long can we make our cycle. Then we start defining how we play around to make that cycle as small as it can be.

So, let us first define the problem to try and have longer cycles so that the shortages are not there as well as the order cost or setup cost component because its manufacturing. It is not order cost, it will be setup cost. So, this will be setup cost though we have used the same  $C$  naught as the noted. So, the problem will now boil down to trying to maximize the cycle length, not to try and maximize the total cost will also show how the moment we assume that we want to maximize the cycle length this objective automatically moves to maximizing the cycle length. For a given amount of inventory, what is the maximum cycle length that I can achieve? And then, we modify the problem by saying if I play around with this inventory, can I maximize my cycle time further? And third is if the demand is time varying, how do we bring that, modify that into this problem. So, these problems are called disaggregation problems and these problems we will try and address them in the next lecture.