Operations and Supply Chain Management
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Lecture - 17
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Let us continue our discussion on inventory, by looking at inventory models, where the demand varies with time. So far, in all the inventory models, we have made two assumptions.

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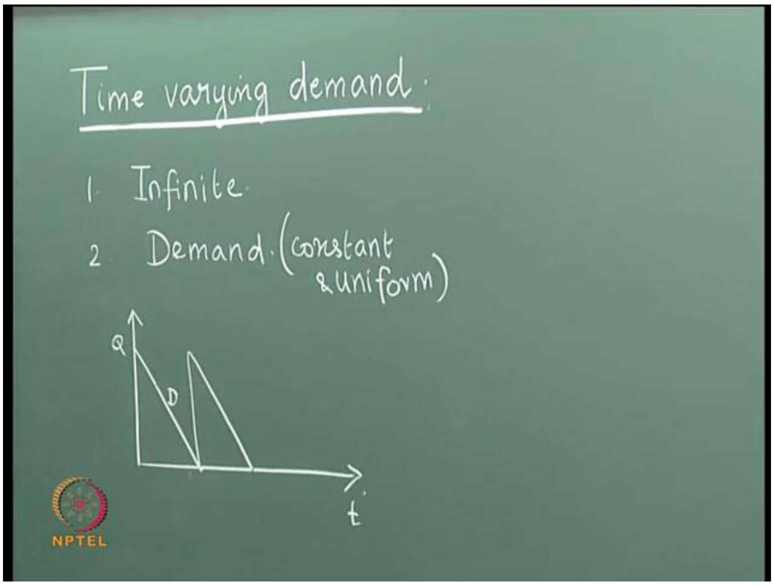

One is that, the planning period is infinite. So, we have an infinite planning period and 2nd is demand is the same, or constant and uniform demand. For example, when we looked at the basic inventory model, which is the saw tooth model and when we said we start ordering at Q. At time equal to 0, we order a Q and it is instantaneously replenished and then it is consumed. Till the inventory reach a 0 and then another order is placed and then this process continuous.

Now, in this we assume that this is going to go on forever and we are planning for an infinite period. And, because we are planning for an infinite period to evaluate the total cost we take a certain fixed period like a year and then we try and optimize the value of Q, for which the total cost is minimized. The other important assumption is that. The demand D, which is the rate at which the inventory is falling, the demand D, is the same at every period, more importantly it is a same at every instance.

Now, what happens to inventory problems when, the demand varies with time, which is a very practical thing that happens around; Production quantities are different in different periods, because demands are different for different periods and once the production quantities are different for different periods. The amount of material required to make them would also be different. Leading to inventory problems, where the demand varies with time. So, let us take an example and try and explain a few things about time varying demand.

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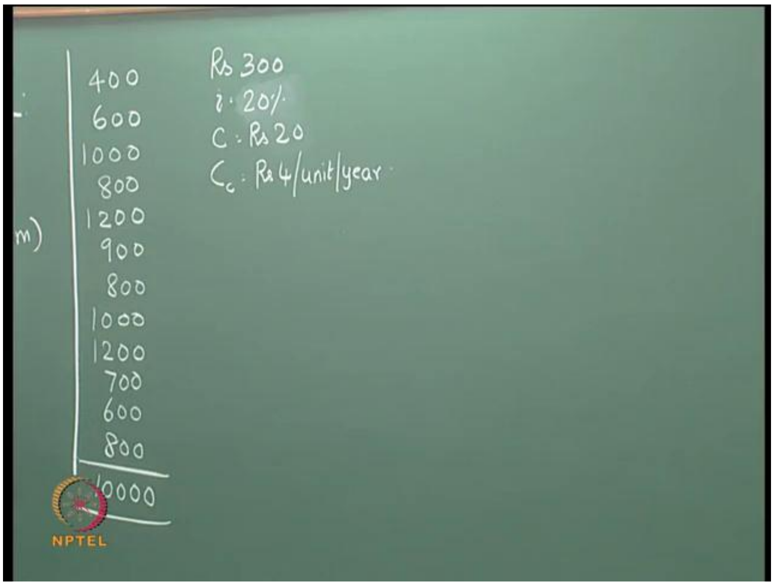

So, we take an example where we assume that, we have demand for 12 months and these demands are: 400, 600, 1000, 800, 1200, 900, 800, 1000, 1200, 700, 600 and 800. Now, these numbers have been taken such that, when we add all of them we get 10,000. So, now we assume that the annual demand of 10,000 is now split into 12 monthly demands of this. Now, several questions are there. Now, like in the earlier models (Refer Slide Time: 00:27) are we going to have the same order quantity, or is the order quantity going to be different? Or, like in the earlier models are we going to have the same order of frequency, or is the order frequency going to be different?

So, we assumed here again for the sake of comparison that order cost is rupees 300 per order. Inventory holding i is 20 percent, of unit price C, which is also rupees 20. So, that

C c is rupees 4 per unit, per year, which is the same number that we had assumed in our earlier examples and illustrations and the as I mentioned the other differences, we are considering 12 months, or 12 periods.

Now, each period is a month and our planning stops at the end of the 12 month. So, we are not looking at the 13th month demand in this particular situation, or illustration. One of the ways of solving this problem is by using integer programming and the motivation for that comes as follows. Now, if we take this month's period and this month's period. Now, we could either order 400, which would meet the demand of this period. Let us say we order 500. Now what happens is, if we order 500, then we order once here for 400. We order a quantity of 500, which is, out of which 400 is consumed at the end of the month and there is a 100 remaining at the beginning of this month, to meet this 600.

Now, with that 100 we will not be able to meet this 600, which means somewhere in between, we have to place an order for another 500. So, on the phase of it, if we order a quantity which is say, in between these two demands, it's quite likely that we end up making one additional order. So, if we really want to save that order, it is, it makes it easier to assume that, if we order here we would rather order a 400, or rather order a 1000 and not a number that is in between. So, this leads to a very important result and that result is the Wagner Whitin result, which we have actually seen earlier.

So, let me repeat those two results. So, the optimum solution to this, under certain conditions will satisfy two important properties which is, if I place an order in the beginning of a certain period, then the order quantity will be. Either: the demand of that period, or the demand of two period, from there, or demand of three periods from there and so on. So, the order quantity will be a summation of the demand, for certain k consecutive periods. 2nd important property is, I will place an order only when my stock at the beginning of that month is 0. So, when the stock is positive, then that stock should be enough for me to meet the demand of the entire month, at least the demand of the entire month.

So, we look at these two basic ideas and then formulate this problem, as an integer programming problem. One could solve it using dynamic programming also, but at the moment let me formulate and solve an integer programming problem for this. So, the moment we take care of these two assumptions that. The order is placed at the beginning of the month and the order quantity should be the summation of k month’s demand. k could be 1, or 2, or 3 from that point. And secondly, we place an order only when the stock is 0. If there is stock at the beginning of that month, then there should be enough stock to meet the requirement of that month. So, the moment we assume these two thing; which will happen under certain conditions.

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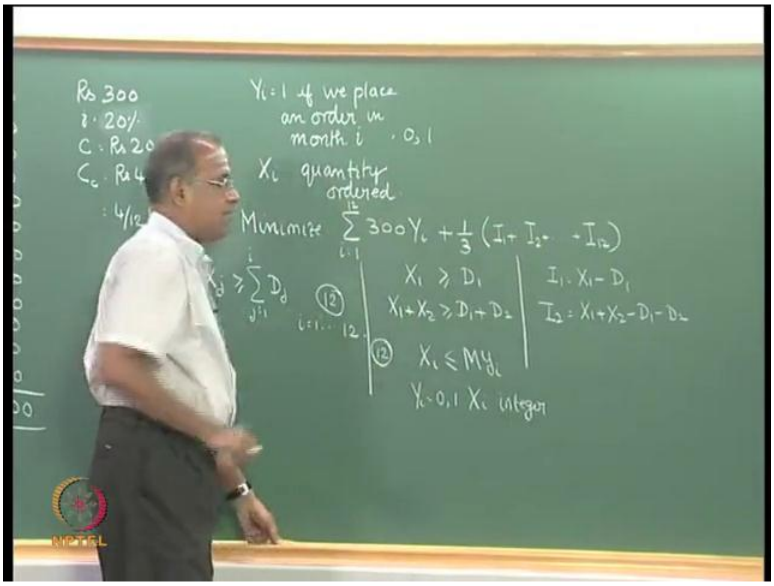

We now formulate a problem where, Y i equal to 1, if we place an order, in month i. When we say place an order, we mean place an order at the beginning of the month. We still assume instantaneous replenishment. So, the with the quantity that arrives at that time. We will be able to meet the demand of that month. Now, let X i be, the quantity ordered. So, Y i is a binary variable: so it is a 0 1 variable. We either place an order, or we do not place an order. X i is the quantity that is ordered. So, the objective function will be to minimize, the sum of the ordering cost as well as the inventory holding costs. So, we first write the objective function, we write part of the objective function. Then we write the constraints and then we write the remaining portion of the objective function

So, objective function will be to minimise the order cost plus carrying cost. So, minimise sigma, i equal to 1 to 12, 300 Y i. Because, every time when an order is placed, if Y i equal to 1 then we incur 300, which is the order cost. So, this is the total order cost component. Plus we also need to have, the inventory holding cost for the items. Now here, what we are going to do is, we have calculated that C c is rupees 4 per unit per year.

Our period is a month. So, C c has to be written as, so many rupees per unit per month. So, this will become 4 by 12, or 1 by 3, per unit per month. So, the inventory holding cost will be plus 1 by 3, into. Now, we are going to charge the inventory, we are going to make another assumption that, we are going to charge the inventory, for the ending inventory in that period. Normally, inventory can be charged in two ways, one is you charge it on average inventory, which means beginning inventory plus ending inventory divided by 2, or you could charge it only on ending inventory.

So, let us assume we are going to charge it only on ending inventory. So, let us call, I 1 plus, I 1, I 2, I 3 etcetera, as the ending inventories. So, this is 1 by 3 into, I 1, plus I 2 plus, I 12. Again we make an assumption that the beginning inventory is 0. We do not have any inventory, which means we have to place an order here. It is also fair to assume that I 12 also will be 0. So, it is enough to stop this at I 11, because we are trying to minimise the cost. So, at the end of the 12th period, or 12th month we are not going to leave behind an inventory. Now, the cost will be higher if we do so. Only for the sake of completion, we write this I 12 and let the solution say, that I 12 is 0. There is no inventory left at the end of the year.

So, this is the objective function. But right now, I have not defined the I 1 I 2 here, because I am going to eliminate the I 1 and I 2 at some point. Now, we have to look at the constraints. So the constraint, (Refer Slide Time: 02:38) the 1st month constraint will be, X 1, is the quantity that I order. So, it should be X 1 greater than or equal to D 1, because it should be greater than or equal to 400. So, that I meet the 400 demand. Now, the other way of looking at X 1 greater than equal to D 1 is, I am just writing it here. This is not an additional constraint. It is going to replace this. If I 1 is the ending inventory at the end of the period 1, then I 1 is X 1 minus, D 1. Or, when we convert this inequality into an equation, you can write X 1 minus, I 1 is equal to D 1, which would give us X 1 minus, D 1 is equal to I 1.

So, this is written as, from this we will know that I 1 is equal to X 1 minus, D 1. So, I am just going to write here as I 1 is equal to X 1 minus, D 1. Later we will substitute for I 1 here. Now, if I take the 1st two months the quantity that I order in two months is X 1 plus X 2. So, what I order in X 1 plus, X 2 now should be greater than, the sum of these two demands which is 1000. I should be able to meet them. So, I will have my 2nd constraint, which is X 1 plus, X 2 should be greater than or equal to D 1 plus, D 2.

So, if I 2 is the ending inventory at the end of period 2, then I 2 is obviously the difference between D 1, plus D 2. That has been ordered in the two months, I am sorry X 1, plus X 2, which has been ordered in two months and D 1, plus D 2, which has been consumed for two months. The difference is I 2. So, I 2 will become, from here X 1 plus X 2, minus D 1, minus D 2.

So, the moment we understand these two constraints, the rest of the constraints are easy. The 3rd constraint will be, X 1 plus, X 2 plus, X 3, is greater than or equal to D 1 plus, D 2 plus, D 3. From which I 3 will be X 1 plus, X 2 plus, X 3 minus, D 1 minus, D 2 minus, D 3. So like this, we have 12 constraints, for the 12 time periods. This can also be generalised and written in a single constraint, which is of the form sigma X j, j equal to 1 to I, is greater than or equal to, sigma D j, j equal to 1 to I for I equal to 1 to 12, which means if we take the 1st month I, then this will be X 1 greater than or equal to D 1.

Take the 2nd month i equal to 2. So, j summed from 1 to 2. So, X 1 plus, X 2 is greater than or equal to D 1 plus, D 2 and so on. So, there are 12 constraints, which are like this. Then we also need to link the X with the Y. X is the quantity, that is ordered and we can order an X j, or X i in the i-th month. Only if we choose to order, which means the corresponding Y i has to be 1. If X i is greater than 0. So, we need to relate the X and Y, which is done by the usual constraint, which is X i is less than or equal to M into Y i. So, when Y i is 0, which means I choose not to order in this month. Then M into Y i will be 0. This will force X i to be automatically 0. If I chose to order in a month, where Y i equal to 1. Then X i could be either 0, or non 0, which is what we want.

So, there are again 12 more constraints like this, which takes care of this part of it. Then of course, we have already defined Y i to be binary 0, 1 and we may define X i to be an integer, or we can define X i to be a continuous variable. So, let us assume we simply define X i also to be an integer, because by the Wagner and Whitin idea, we will have all X i’s to be integers because these demands are all integers. So now, the constraints are all written in terms of X and Y. But, the objective function has this I term. So, this I 1 has to be replaced in terms of X. So, I 1 was X 1 minus, D 1. I 2 is X 1 plus, X 2 minus, D 1 minus, D 2, I 3 is X 1 plus, X 2 plus, X 3 minus, D 1 minus, D 2 minus, D 3 and so on.

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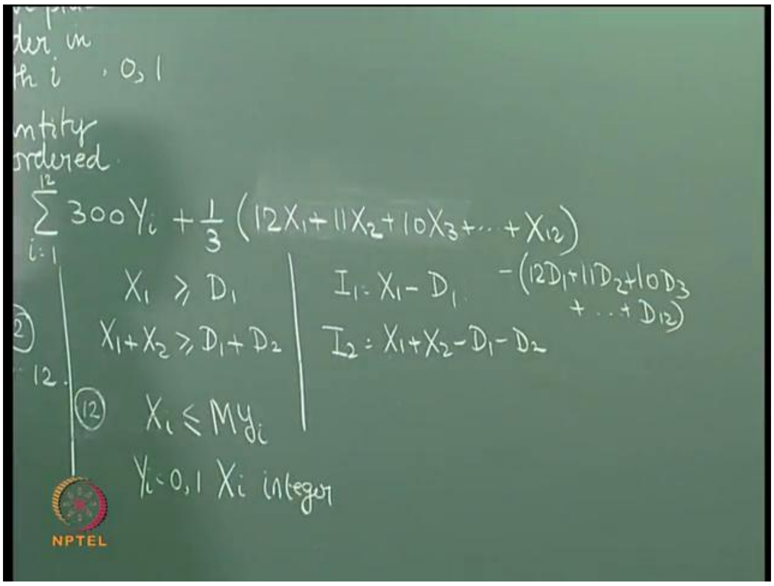

So, when we substitute this into the, I 1 plus, I 2 plus, I 3. What we will get is, 12 times X 1, plus 11 times X 2, plus 10 times X 3, etcetera, up to X 12. Because, for I 1 you are going to have X 1 minus, D 1, I 2 also has going to have a X 1 plus, X 2. So, it is going to have an X 1 term. I 3 also will have an X 1 term. So, all the 12 will have X 1 12 times. So, this will become 12, X 1. Now, X 2 is going to appear from I 2, I 3 up to I 12. So, X 2 will appear 11 times, X 3 will appear 10 times and so on. Minus, once again we will have 12 D 1 minus, or plus 11 D 2, plus 10 D 3, etcetera, plus D 12. The reason is this: D 1 is also appearing in all the 12 I’s. So, you will have minus 12 D 1. D 2 is appearing in 11 terms, from I 2 to I 12, so this will also appear.

So now, we have eliminated the I’s, the ending inventories. The I variables we have eliminated. And, it is also a customary in any linear programming, or integer programming, that if you have a constant term in the objective function. You remove it and then you solve it and later, you can add that constant into it. So, this is the constant term. So, we remove this term and then we solve this. So, we can now solve this integer, linear integer programming problem, using a solver or otherwise, to get the optimum values of X and Y.

So, there are 12 terms here, 300 into Y 1 plus, Y 2 up to Y 12. Plus another terms here that represent the inventory holding cost. There are 12 constraints, representing the demand. So, that the demand in every period is met. There is no shortage. Therefore, no shortage cost is added in the objective function. This model does not allow shortage, or backordering, Plus of course, the linking constraints between X and Y. So, when we solve this optimally, we will get a solution which is this.

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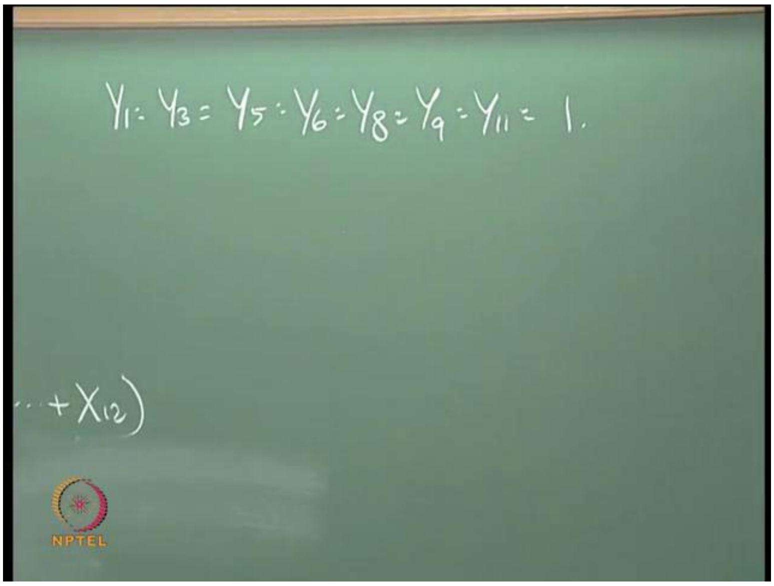

So, the solution to this will be Y 1, equal to Y 3, equal to Y 5, equal to Y 6, equal to Y 8, equal to Y 9, equal to Y 11, equal to 1. So, the solution says that we will order in months: 1, 3, 5, 6, 8, 9 and 11. So, 7 out of the 12 months, we will order. What are the order quantities? Of course, the solutions would also throw the excess as a variable which will be there. But, the moment we know the Y's we also know the X's, because I am placing an order here.

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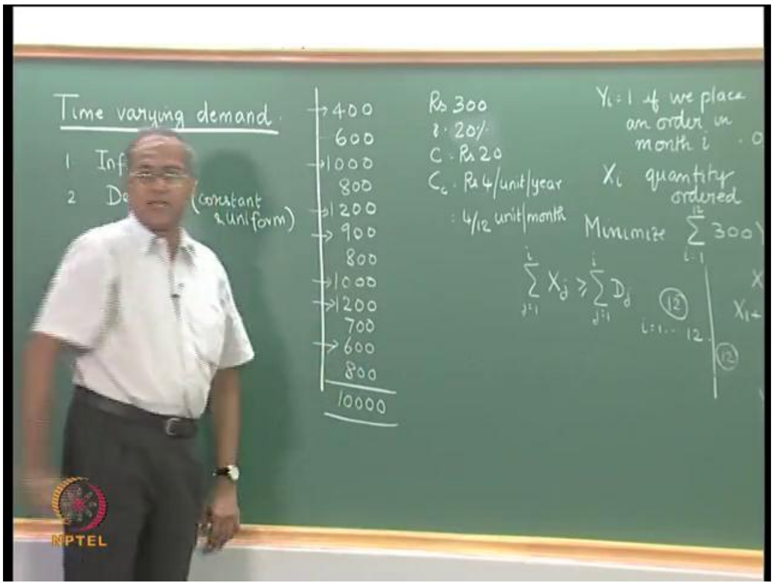

I am placing an order here at Y 3, placing an order at Y 5, placing an order at Y 6, placing an order at Y 8, placing an order at Y 9 and I am placing an order at Y 11.

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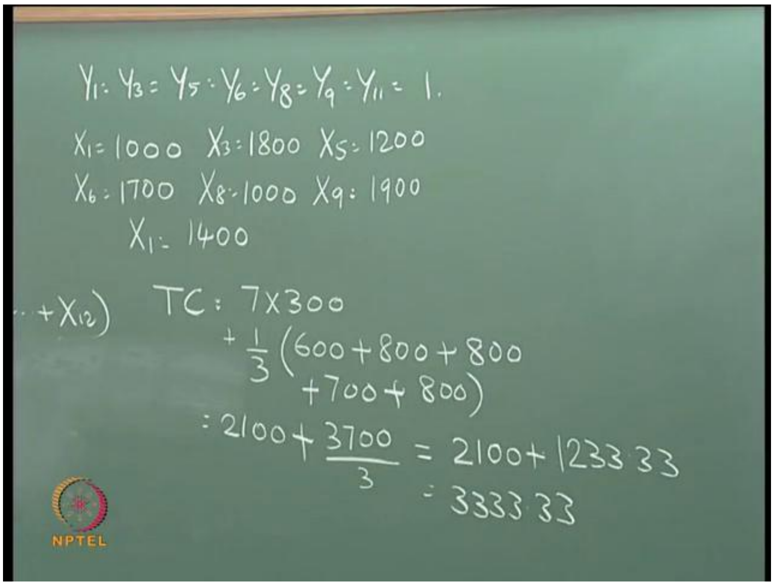

So, obviously my order quantities are X 1 equal to 1000. Because, I am not ordering in this month, I am not ordering here. So, the order quantity here is (Refer Slide Time: 20:12) the sum of these two. X 3 is 1800, X 5 is 1200, X 6 is 1700. So, 1700 comes because I have to order for this 900 plus another 800, so 1700. X 8 is 1000, X 9 is 1900 and X 11 is 1400.

Now, what is the total cost associated with this system? Total cost would be given by the objective function value here. This (Refer Slide Time: 17:07) would be the objective function value. Plus of course, that constant has to be added. So, what we will do now is, we will compute the total cost. So, there are 7 orders here. So, 7 into 300, the inventory holding cost is plus 1 by 3 into. Now (Refer Slide Time: 20:12) here we order 1000, at the end of this month we carry 600. So, 600 is carried. Now here it is 0.

So, we order 1800, so we carry another 800 here. Now, here (Refer Slide Time: 20:12) we order 1200. We consume all the 1200 here. Here we order 1700 we consume 900. So, we carry another 800 here, so plus another 800. We order 1000 we consume all 1000. We order 1900 we consume 1200. So, balance of 700 is carried, which is consumed at the end of this period. So, we order 1400, 600 is consumed here. 800 is what is carried, so 800. It is also interesting and obvious to note that there are 7 periods where there is an order. There are 5 periods where there is inventory holding, making up for the 12.

So, total cost will be 2100 plus, 6 plus 8, 14, plus 8, 22, 29 plus 8, 37, 3700 by 3 which is, so 2100 plus 1233.33, which is 3333.33. So, while writing this formulation I had mentioned that the objective function has two terms. One is the order cost term and the other is the inventory carrying cost term. (Refer Slide Time: 17:07) So, while writing the formulation I mentioned that the, inventory cost can be calculated either based on the ending inventory, or based on the average inventory.

Now, this expression was written, when we wrote it for the ending inventory in mind. So, we defined I 1, I 2 up to I 12 as the ending inventory in the 12 months and then we simplified to 12 X 1 plus, 11 X 2 plus etcetera, minus a constant. And then, we removed the constant, because in any mathematical programming problem. When you optimise a function, if there is a constant in the objective function, you can remove it and then you can optimize only the terms containing the variables.

Now, what we will do is we will try and see what happens if, we write this inventory (Refer Slide Time: 17:07) term, the carrying cost term. Considering the average inventory and not considering the ending inventory. Now, does the solution change, if we do that? So, let us go back and try to first compute this term (Refer Slide Time: 17:07) or write the equivalent of this term, if we consider average inventory and not the ending inventory. So, let us write that portion here. So, to begin with we will assume that, I 1, I 2 up to I 12, are the 12 values of the ending inventory.

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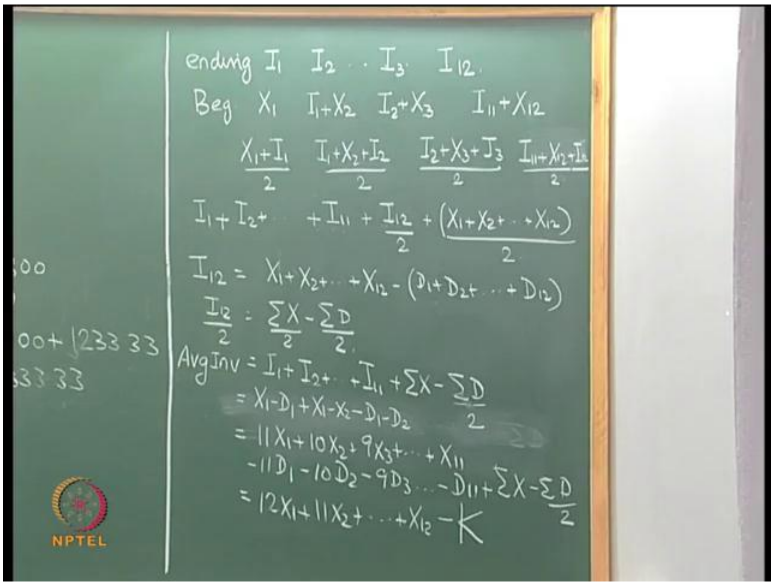

So, 12 values of the ending inventory are I 1, I 2 up to I 12. Now, if we take the 1st period, the beginning inventory is 0. So, the order quantity is X 1. So, beginning inventory now will be 0 plus X 1, which is X 1 in the 1st period. Now, at the end of the period, the ending inventory at the end of period 1 is I 1. Now, we order X 2, in the beginning of period two. So, beginning inventory will be I 1 plus X 2.

Now, for the 3rd period, the ending inventory will be I 3. But, the beginning inventory will be I 2 plus X 3, because I 2 is the inventory that is available at the end of period 2. And at the beginning of period 3 we are adding an X 3. So, I 2 plus X 3, is the beginning inventory for period 3. I 3 is the ending inventory for period 3. So, this way we can write all the beginning inventories and ending inventory term and for the 12th month beginning inventory will be I 11, plus X 11.

So, the average inventory for month 1 will be, X 1 plus I 1 divided by 2. The average inventory here will be, I 1 plus X 2, plus I 2 divided by 2 and one more term, it will be I 2, plus X 3, plus I 3 divided by 2 and for the 12th period. It will be I 11, plus X 11, plus I 12. The 12th period: the beginning inventory will be I 11, plus X 12, because X 12 is the quantity that we order. So, this will become I 11, plus X 12. So, this will become I 11, plus X 12, plus I 12 divided by 2.

So, if we sum all the average inventories, we will get the I 1 plus 2 appears here, I 1 plus 2 also appears here. Similarly, I 2 divided by 2 appears here, I 2 divided by 2 appears here. So, when we take only the I 1, I 2 up to I 12 terms, we will have I 1, because I 1 divided by 2 plus another I 1 divided by 2. So, I 1 plus I 2 plus up to I 11, the I 11 will have a term from the previous, apart from the previous term and apart from here, I 12 does not have a succeeding term because that is the last period.

So, up to I 11, plus we will have I 12 divided by 2. And then, we are going to have, X 1 by 2, plus X 2, by 2 plus X 3 by 2 up to X 12 by 2. So, this will be plus, X 1 plus, X 2 plus etcetera, plus X 12 divided by 2. Now, what we need to do is to have, to write this I 12 divided by 2, in terms of first write I 12 and then substitute for I 12 to see what happens. Now, what is I 12? I 12 is the inventory at the end of the 12th period, beginning inventory is 0 here.

So, I 12 is in principle sigma X 1, plus X 2, or sum of X 1 plus, X 2 up to X 12, which is what we have procured. Minus D 1 plus D 2 to D 12 which is what we have consumed. So, I 12 will now be equal to, I 12 is X 1 plus X 2, plus X 12 minus, D 1 plus D 2 up to D 12. So, let me write this as sigma X and let me write this as sigma D. So, I 12 divided by 2, is equal to sigma X divided by 2, minus sigma D divided by 2. So, this will become I 1. So, the sum of the average inventory will be, I 1 plus I 2 up to I 11, plus this I 12 divided by 2 is sigma X by 2, minus sigma D by 2. This is another sigma X by 2. So, this will become, plus sigma X minus sigma D. Because, there is a sigma X by 2 term that comes out of this. This is another sigma X by 2 term. So, they add up to give you sigma X minus sigma D.

Now, we need to go back. We can actually look at it in two ways. But, the more mathematical way is to try and define what are these I 1, I 2 up to I 11 and then see what they are. Now, what is I 1 plus I 2? I 1 is X 1 minus D 1, I 2 is X 1 plus X 2 minus D 1 minus D 2 and so on, then plus sigma X minus sigma D. Now this, now you get I 1 plus I 2 plus I 11, which is what I am writing. I 1 is X 1 minus D 1, I 2 is X 1 plus X 2 minus D 1 minus D 2.

Now, the total inventory is this term, this term is what I am writing here. Now, this term is I 1 plus I 2 up to I 11, plus I 12 by 2. Now, this portion I 12 by 2, is written as this. So, this portion is up to. So, this portion is, this is here plus I 12, let me write from here. So, that there is no confusion. So, up to I 11 I am writing from here. Now, this portion is I 12 by 2, plus sigma X by 2. Now, I 12 by 2 is sigma X by 2, minus sigma D by 2 plus sigma X by 2. So, there is a by 2 that comes here.

So, this will become X 1 minus D 1, plus X 1 minus X 2 minus D 1 minus D 2. We get up to 11 terms. So, the 12th term will not be there. So, the 11th term will be X 1 plus X 2 to X 11, minus D 1 minus D 2 up to minus D 11. So, this on simplification will give this, X 1 will come 11 times. So, this will give me 11 X 1, plus 10 X 2, plus 9 X 3 etcetera, up to X 11. Now, this one will become, minus 11 D 1, minus 10 D 2, minus 9 D 3 and so on, minus D 11. Once again this term will repeat.

So, plus sigma X, minus sigma D divided by 2. Now, as far as we are concerned, minus 11 D 1 minus 10 D 2 minus 9 D 3 up to minus D 11 is a constant. This is also a constant. So, this is 11 X 1, plus 10 X 2, plus 9 X 3 up to X 11. Sigma X is, X 1 plus, X 2 plus, X 3 up to X 12. So, when we add it we get 12 X 1, plus 11 X 2 etcetera, plus X 12, minus a big constant K.

So, we have to write, if we write average inventory term and bring it back to this (Refer Slide Time: 17:07) expression we should write, 1 by 3 into 12 X 1, plus 11 X 2, plus 10 X 3, plus X 12 minus some very big K constant. And, based on the same idea we can remove this constant and we can solve it. So, as far as the optimization is concerned, even when we look at average inventory, which is taken as beginning inventory plus ending inventory divided by 2. We get the same optimization problem (Refer Slide Time: 17:07) except that the constant that we are subtracting is slightly different. There is another way of looking at same thing. The reason perhaps is that, if you see this. Initially we started with ending inventory as I 1 plus, I 2 plus, I 3 up to I 12 is what we had.

Now, ordinarily we will have, I 12 will be 0. Because, beginning inventory is 0, pushing an I 12 will lead to higher cost. So, I 12 will be 0. So, it is enough if we do up to I 11 and by the same reason we will realise, that sigma X will be equal to sigma D, because when the initial inventory is 0. We will not buy and then finally, keep it as final inventory. So, this term will essentially become a constant. So, average inventory would also lead us to the same value for ending inventory. So, we have already seen the optimum solution to it. The problem does not change. So, this (Refer Slide Time: 19:32) is the solution for it. The only change is in our calculating the total cost.

Now, when we consider ending inventory, the total cost was computed for order cost plus ending; carrying cost of the ending inventory. Now, what we have to do is, we have to compute this cost again. Considering the average inventory and there will be 12 terms, instead of 5 terms. The 5 terms came here, because we ordered for 7 periods, which meant that 5 periods is demand was carried. So, when we computed based on ending inventory, we got 5 terms here. Now, when we compute it based on average inventory we will have 12 terms. So, let us compute those 12 terms again and see what happens to the total inventory cost. So, there are 7 orders. Order cost is taken as 300.

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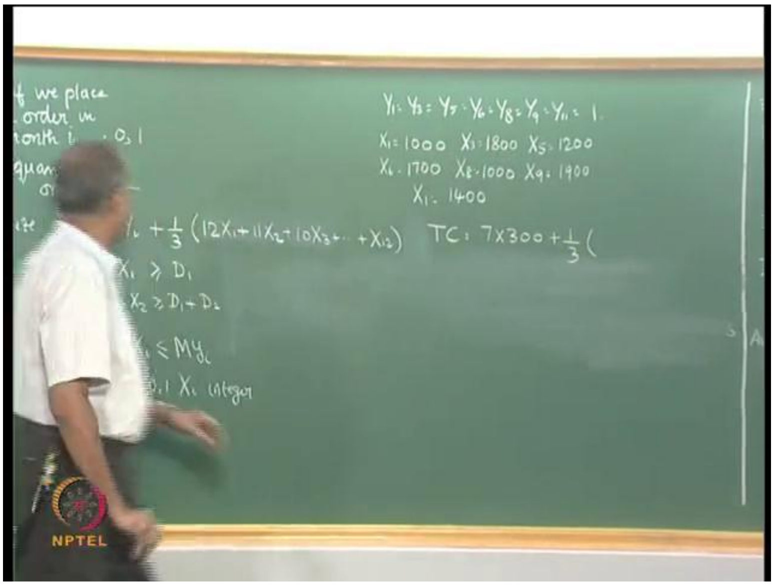

So, 7 into 300 plus 1 by 3, 1 by 3 is our C c, (Refer Slide Time: 02:38) 4 per unit per year becomes, 4 divided by 12, per unit per month, so 1 by 3 per unit per month. So now, what do we do? We start the 1st month by ordering 1000. So, 1st month beginning inventory is 1000. So, let me write down the points what we order, using a different colour perhaps yellow.

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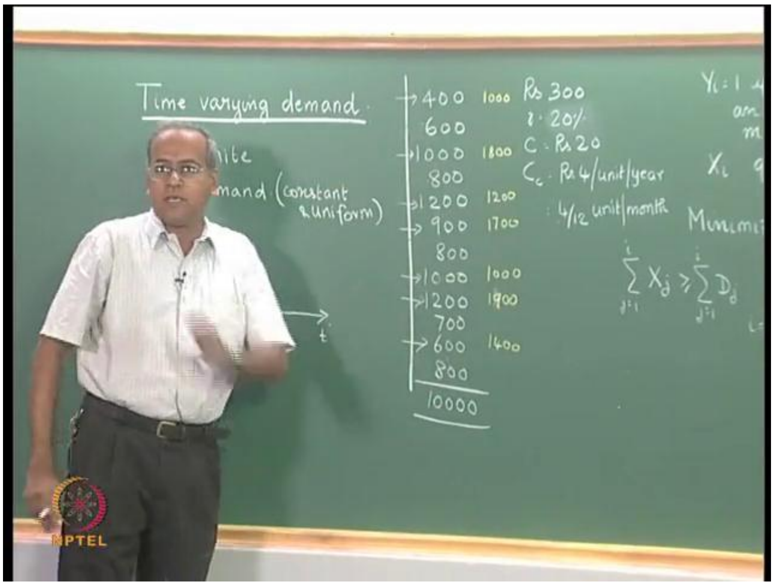

So, we will have 1000 ordered here. Y 3 is 1800. This is 2100; I am sorry 1200, 1700, 1000, 1900, 1400. So, 1st month beginning inventory is 1000, ending inventory is 600.

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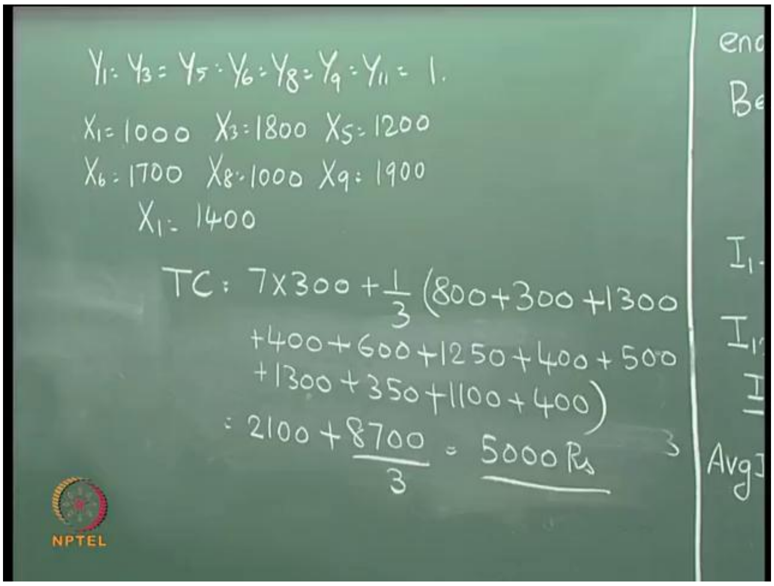

So, 1000 plus 600 divided by 2, is 800. 2nd month beginning (Refer Slide Time: 38:42) inventory is 600, ending inventory is 0. So, average inventory is 300. 3rd month beginning inventory is 1800, ending inventory is 800. So, 2600 divided by 2 is 1300. 4th month beginning inventory is 800, ending inventory is 0. So, you get 400. 5th month 1200, ending inventory is 0, so we get 600. 6th month, 1700 plus 800 divided by 2; so

1250. 7th month will be 400. 8th month will be 500. 9th month is 1900, plus 700 divided by 2, 1300. 10th month is 350. 11th month is 1100 and 12th month is 400.

Now, 400 comes, 1100 comes because we order 1400. At the end of this month ending inventory is 800. 1400 plus 800 is 2200 divided by 2. So, this will become, 2100 plus, now let us add only the 50 terms first. 1250 plus 350 is 1600. 2400, 2700, 4000, 4400, 5000, 5400, 5900, 6200, 7200, 8300 plus 400 is, 8700 divided by 3.

So, this is 2900 plus 2100, which is 5000 will be the optimum, the value of the objective function at the optimum, if we consider the average inventory. So, important assumptions as to, for a quick recap the important assumptions are, that the Wagner and Whitin assumption that, we have placed an order, when the inventory is 0. And, when we place an order, we place an order for that period or some of demands of certain k number of periods. When there is inventory at the beginning of the period, we will not place an order and that inventory should be, enough to meet at least this month's demand and in principle to meet the demand of an integral number of months, starting from now.

So, under those two assumptions, we formulated this problem. The problem could also be solved by dynamic programming. But we solved by integer programming by defining Y i as minus. Now, let us make a very quick comparison with what we know. Now, let us go back and (Refer Slide Time: 38:42) take this 10,000, as the annual demand. The important deviation of this problem from the economic order quantity problem is that. The demands are now given over 12 periods.

Whereas, there is annual demand of 10,000 was given as an annual demand and the demand is the same at every instance of time. Now, demand varies here. The original e o q, we assumed in infinite horizon. Now, there is a finite horizon of 12 time periods. Nevertheless, let us make a comparison. Now, just in case we have taken this 10,000 as the annual demand and then use the economic order quantity formula, to find out the economic order quantity and the associated total cost. What happens? So, when we do that.

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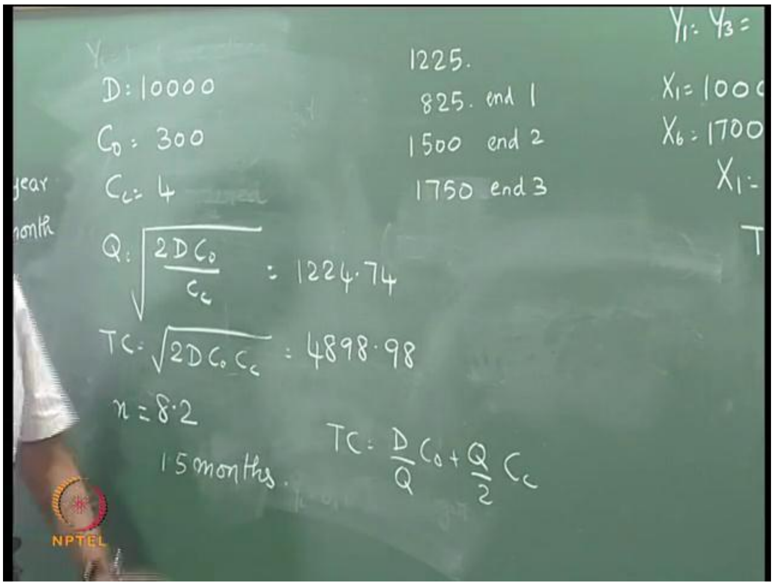

So, we could write D equal to 10,000, C naught equal to 300, C c equal to 4 per unit per year. Now, Q is equal to, root over 2 D C naught by C c, which we have already calculated in an earlier lecture, 1224.74 and then we also know that the total cost for this is, root over 2 D C naught C c, which will give us 4898.98.

Now, we have to first understand. This is an optimal solution to an integer programming problem, under a certain assumptions, which gives us 5000. The best value of economic order quantity, assuming that this is an annual demand of 10,000, is 4898.98 and it is slightly less than that 5000. It is very clear as to why this was marginally higher. That is because; this problem is further constraint, by many reasons.

Now there is an, this is (Refer Slide Time: 39:16) has a finite period, this has an infinite period. That has demand that varies with time. Here you have demand that does not vary with time. So, in principle that is a constraint problem. So, it gives us 5000, but then why should we not use this, if this is usable? Now, let us go and see why we need this and why we would not be able to use this. Now, to do that let us go back and find out. If the order quantity is 1224.74, then the number of orders per year n will be 10,000 divided by 1224.74 and we quickly compute that to be, 8.2.

So, let us call it 8.2 orders, actually 8.17. Let us call it as 8.2 orders. Now, there are going to be 8.2 orders over 12 months. So we, let us call the ordering frequency per month. How many orders is 12 divided by 8.2, which is roughly about 1.5. Let us approximate it to about 1.5 orders per month, 8.2 orders over 12 months then, we have an order every 1.5 months, per order.

Now for example, we are going to order, let us say 1224.74 lets kind of approximate it to 1225, which is not a very wrong approximation. So, we 1st order 1225. Now, let us try and see what we do (Refer Slide Time: 38:42) with this. So, we can meet the 1st month's demand of 400 through it. So, at the end of the 1st month, we will have 825. But, then somewhere in between, we after 0.5 months we will have an another 1225 coming in.

So, the (Refer Slide Time: 38:42) 2nd month's 600, we can actually meet. But, somewhere in between we will get the 1225 that comes in. So, at the end of it we will have, 225 plus 1225. So, we will have 1225, plus 225, which is 1500. So, at the end of the 2nd month, we will have 1000 every one and half months we will place an order. So, we will have 1500. This is, beginning of the 1st month. So, end of 1st month, end of the 2nd month, we will have 1500.

Now, meet this 1000 here. (Refer Slide Time: 38:42) So, we would have 500 at the end of this month. Now, half way, the next order is going to come half of, now the 1st order has actually come here. The 2nd order will come at this point. So, 500 plus 1250, will be 1750 will be the order. So, we will have 1750 here, at the end of 3. That will be the inventory at the end of 3 and so on.

So, actually we would realise that, we will be able to meet the entire demand by doing this. Actually, we will be able to do that in this case, if we continue this kind of calculation. We will be actually able to show that, we can meet the 12 months demand by suitably ordering this. There are no issues on that. The only place, why? Even though this will be a feasible solution, if we show, this will be a feasible solution, with cost equal to 4898.98. But, then the optimal solution here (Refer Slide Time: 39:16) will show, will ask us not to order, somewhere in the middle here (Refer Slide Time: 38:42) and so on.

The optimum solution will ask us to order, at these points, with the slightly higher 5000. So, we need to explain, why that difference happens. Essentially the difference happens because when we made this calculation. This calculation was based on a total cost, T c is equal to D by Q, C naught plus Q by 2 C c. Now, the average inventory there was taken as Q by 2, that comes from here, which was based on the assumption, that the demand is

D in every instance. Now, the moment we look at problem with time varying demand this way, (Refer Slide Time: 38:42) the demand is not D at every instance. The demand is different. Therefore, the correctness of Q by 2, as representing the average inventory comes into question. So, Q by 2, is not a correct estimator of this.

Therefore, if we use this Q by 2 instead and then optimize, the optimization portion is right differentiating and setting it to 0 and getting it as right. But, Q by 2 into C c, is not the correct way to represent the average inventory and because the average inventory was represented in an incorrect way here, it ended up giving us a 4898.98. Whereas, the best way to do it, is the way we have done it in this formulation (Refer Slide Time: 39:16) to get this answer, to get a value of 5000. So, using the economic ordering quantity per say and implementing it, even though it is implementable, as in this example it is implementable, it is fine.

But, then the correctness of the average inventory will come into the question and when we actually correct this term, for the solution, which is 1225, every one and a half months and if we correct the term while doing this calculation. We correct this term when we do this calculation, then you realise for example, that the average inventory would not be Q by 2. It will be something slightly more, which would I eventually give us a number; there is slightly more than 5000. So, we need to use this method (Refer Slide Time: 39:16) when we actually work with problems on, time varying demand and not approximate the economic order quantity. Now, another area where the economic order quantity can suffer is this. Now, suppose instead of the, this is the total demand let it be 10,000, but then if we say that the 1st periods demand is not 400, but for the sake of argument.

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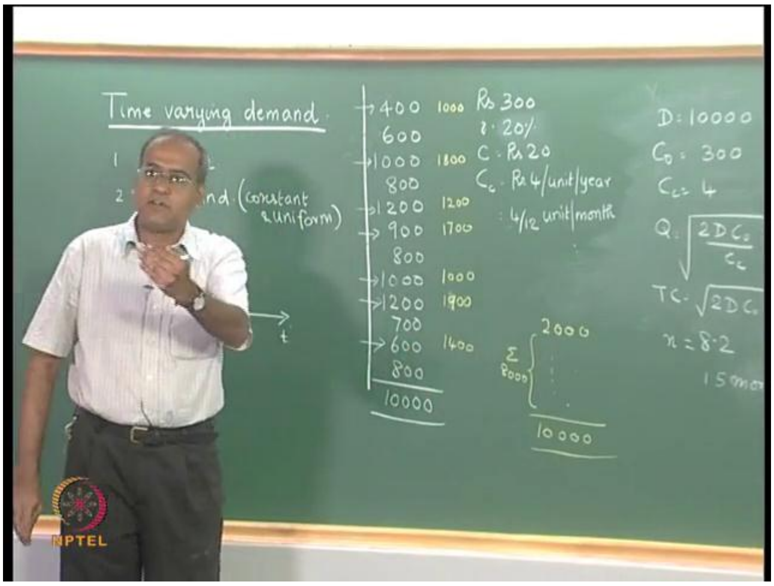

If the 1st period’s demand is itself is 2000. Now, the rest of the demands are such that, the total annual demand is 10000, which means, the demand for the remaining 11 periods on summation, is actually 8000. So, that the total demand becomes 10000. Then instead of using this model, which is a Wagner Whitin algorithm, if we had gone by the economic order quantity and said we would use it, then the economic order quantity tells you to order 1224.74, the same thing based on the same 10000.

So, we initially begin with 1224.74 and then our next order is going to happen after one and a half months, because of this 8.2 and 1.5 months. So, if we assume that in the first day of January, you have ordered 1225 and their next order you are going to make around 15th of the February, which is one and a half months from now. So, you have to meet the one and the half months demand, with the 1225, that you have.

Now, you would not be able to do that, because the 1st month’s demand itself is 2000. So, we will be forced to have shortage and then we have not included shortage in doing this. So, for another reason you cannot directly use the economic order quantity formula and then say that I will suitably modify it to take care of time varying demand. In an example like this, where it is not you know, you do not have a real peak. One month taking a demand of 2000, while the other 11 months having a demand of 8000. Clearly shows there is a peak here. Whereas, if we look at these numbers, you see that they are reasonably spread out, around the average value of 1000, or slightly less, the average value of 833. It is kind of spread out, even though you have a 1200 somewhere here.

But, if one of the demand shows a peak, then implementing a modification of the economic order quantity, would lead us to shortage. Therefore, this approach (Refer Slide Time: 43:19) is not recommended, while this approach (Refer Slide Time: 38:11) is recommended. Now we do, having seen this, let us look at just a couple of more instances of this.

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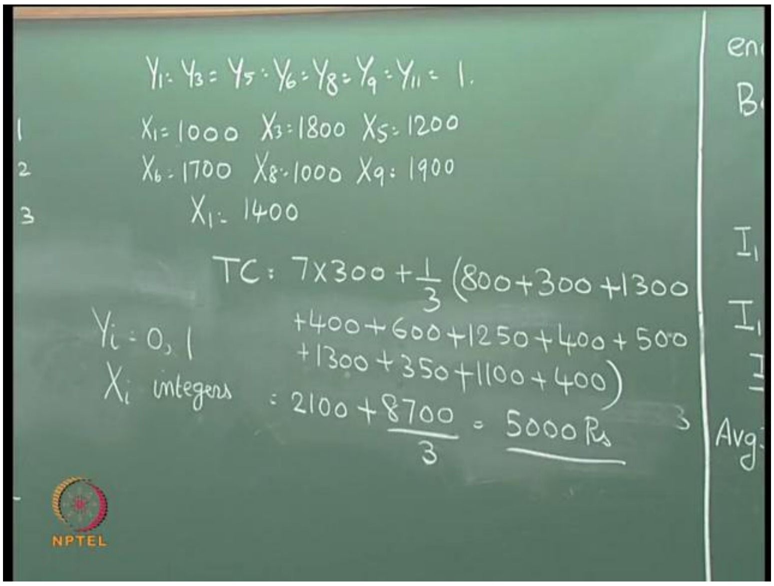

One is, this problem is formulated and solved as a integer programming problem, where the Y i's were binary variables and X i, were defined as integers. I also mention the little earlier that, it is not necessary really to keep them as integers, because the demands themselves are integers. But, the Y i's have to be binary.

Now, this would require an integer programming solver and if we are doing it for large time periods, integer programming may take a little longer than, we normally anticipate and we may, require more sophisticated solvers. So, the question is? Can we try and solve this problem heuristically and not really worry about the optimal solution, but try and get a good near optimal solution. 2nd issue that comes is, now this integer programming formulation can be made. If we have a finite number of periods: if we have a situation, where we do not want finite time period.

If we look at practice, about demand of a particular item and let us say the forecasting happens continuously. And then there is a rolling kind of a forecast, where initially the forecast is made for 1 year and then after a quarter, after 3 months the forecast is made for another 1 year. Therefore, the forecast is rolling and in some sense there is no end to this number. So if this, there is no end to this number. We cannot make that formulation, because that formulation is for a finite time period. So, the question is how do we extend this to look at situations, where we do not have finite time periods?

Now, can we solve that optimally? The answer is no, because we need to know how many periods. All these are summations. So, you need to know how many periods you require. So, in order to handle situations, which are more practical, where the time period is not fixed and defined. We need to look at heuristics, which give good solutions comparable to this optimum solution. So, we look at a couple of such heuristics, in the next lecture.