Operations and Supply Chain Management
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Lecture - 15
Inventory- Production Consumption Model with Backordering, Economic Lot Scheduling Problem

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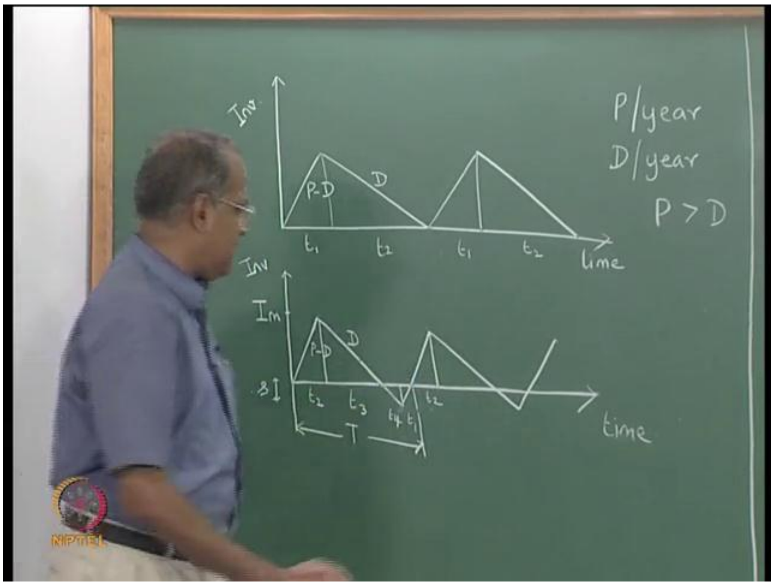

In the previous lecture, we looked at models which are production and consumption models, which means if we consider a particular item we can produce this item at the rate of P per year, while the demand for this item is D per year, so obviously in order to meet the demand our production rate P has to be greater than D or greater than or equal to D, or most of the times P is greater than D. So, when P is greater than D, we produce this item and while we produce we also consume and since, the production rate is higher than the consumption rate, there is an inventory buildup rate at the rate of P minus D here and we produce up to a particular time t 1. And then we stop production and consume from the inventory which has built up.

So, we stop production here after t 1, this is time, this is inventory but this graph is for the item and not for the facility that is producing the item. So, as far as the item is considered inventory is built up at the rate of P minus D for period t 1. Production stops and then this inventory is consumed at the rate of D, till the inventory position reaches 0 then the cycle starts all over again. So, once again from here it is again produced for a period t 1 and again consumed for a period t 2 and so on. So, we derive an expression for an economic batch quantity which was called E B Q very similar to E O Q, we also said that there is a set up cost that is involved every time we start producing.

Because, if we draw a similar figure for the facility that is producing, then the facility is busy making this item up to period t 1 and then the facility again starts making this item, once again at this point and is busy for a period t 1. So, in some sense the facility is not busy with respect to this item for this period t 2 and if we assume that, the facility will be used to produce some other item then while coming back to this item; at this point in time we need to change over. And therefore, there is a change over cost or a set up cost which was denoted by C naught to be consistent with the earlier notation.

So, we derived expressions for the economic batch quantity. Now, we will look at what happens to the economic batch quantity, if we allow back ordering. In the earlier version or in the earlier model we did not allow backordering. So, we started producing as soon as the stock reaches 0 so there is no back ordering. Now, if we allow backordering for the same item now, this is time, this is inventory. Let us assume that the inventory is at 0 so we produce, produce as well as consume so inventory is growing at the rate of P minus D. Now, we stop production at this point and start consuming let us say, we come to 0 unlike the earlier model, we allow a certain backorder in this case.

So, we do not produce immediately we allow a certain backorder and then start producing at this point so production once again begins at this point, this is parallel to this and then once again we consume so this will be parallel to this and so on. So, now let us define certain notation for this so let us call this as t 2 is the period where it is produced and consumed and there is a positive inventory buildup. Production stops here so there is a period t 3 where, the available inventory is consumed. So, at this point the inventory becomes 0, then there is a backorder that is built-up now, that happens for a period t 4 where, there is no production; there is consumption but there is no stock to consume therefore, there is backorder.

And then once a backorder is built up here, we start producing here which means there is a back order, we produce first for a period t 1 produce and consume for t 1 and also to meet the backorder and at this point the stock becomes 0, then there is a positive inventory buildup which goes on. So, the cycle essentially is from here to this point, which we can call as T which is the cycle. So, the cycle comprises of 4 time periods t 1, t 2, t 3 and t 4; where there is production during periods t 1 plus t 2 and there is consumption for periods t 3 and t 4. So, this production consumption model with back ordering.

Let us also add some more notation to it, inventory is built up at the rate of P minus D here, it is consumed at the rate of D here, it continues to be consumed at the rate of D here and is once again built at the rate of P minus D in the other case. We will also define I m as the maximum inventory that we have in this system and we will define s, which is this height as the back order that we build in this system.

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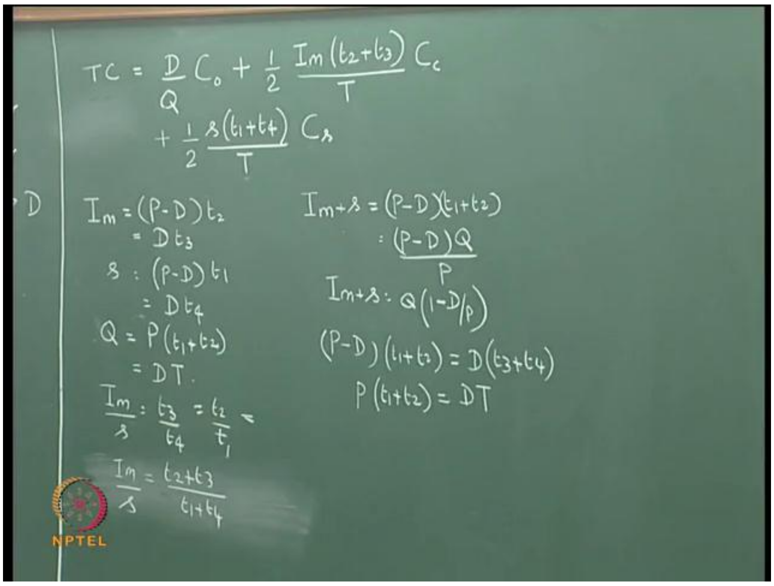

Now, let us try and write down the necessary total cost equation for this. There are three costs, there is a cost of set up, there is a cost of inventory and there is cost of shortage. So, if we produce in this cycle T, if we produce a quantity Q, let Q be the quantity that is to be produced. So, total cost is made up of 3 terms which is a set up cost, the inventory holding cost, as well as the backorder cost. Now, the cycle time is T and every time we produce, we produce a quantity Q so number of times we set up will be D by Q, that is the number of times this item is set up in a year is D by Q because D is the annual demand. C naught by our definition is the setup cost. So, D by Q C naught is our total annual setup cost, then we have to model the inventory holding cost, as well as the backorder cost. (Refer Slide Time: 00:09) So, we said the total inventory that is held in a cycle is the area under this curve, which is half into base into height so half into I m into t 2 plus t 3 that is the total inventory.

So, total inventory held is half into I m into t 2 plus t 3 but in every cycle, the cycle length is T so this much inventory is held for a period (Refer Slide Time: 00:09) t 2 plus t 3, 0 inventory is held for the period t 1 plus t 4. So, average inventory will be half into I m into t 2 plus t 3 plus 0 into this divided by T so we get divided by another T. Average inventory is total inventory by time so total inventory of half into I m into t 2 plus t 3 is the total inventory held; that is held for a time period t 2 plus t 3, 0 inventory is held for that t 3 plus t 4.

So, total inventory is half into I m into t 2 plus t 3, total time is: t 1 plus t 2 plus t 3 plus t 4 which is T so average inventory is so much. So, average inventory into C c which is the cost of holding, this average. So, this is the inventory holding cost plus we have the back order cost (Refer Slide Time: 00:09). So, total quantity backordered is area of this triangle, which is half into s into t 1 plus t 4 so plus half into s into t 1 plus t 4. Now, if we look at this cycle this is the total back order quantity so that is held for a period t 1 plus t 4 back order is 0 for a period t 2 plus t 3. So, total average back average backorder is, half into s into t 1 into t 4 divided by t 1 plus t 2 plus t 3 t 4 which is T so divided by T.

So, this is the total, this is the average backorder that is held in a cycle or at any point in time so that is multiplied by C s which is the backorder cost. We should also note that backorder cost C s and inventory holding cost C c have the same unit of rupees per unit per year. So, C s is also rupees per unit per year backordered, this is the average backorder quantity at any point in time. So, this is the function that we have to optimize but then this function has then several unknowns: so Q is an unknown, I m is an unknown t 1, t 2, t 3, t 4 and t are all unknowns, s is also an unknown.

What are known to us are D C naught C c I m is also an unknown, C c and C s are the known quantities. Now, from this diagram we also realize that several of these unknowns are related to each other therefore, we try and get equations relating these unknowns and try to reduce the expression in terms two unknowns, which are Q and s as we did in earlier models. And then try and partially differentiate with respect to Q and s. So, first we have to write a whole lot of them in terms of the other known and unknown quantities. So, from this particular (Refer Slide Time: 00:09) figure now, first thing is we can write few equations.

Now, we have now this quantity which is I m which is the maximum inventory. I m will be P minus D into t 2 because (Refer Slide Time: 00:09) inventory grows at the rate of P minus D for a period t 2 and the same I m is equal to D into t 3 because it is consumed at the rate of this so D into t 3. Now, s which is the (Refer Slide Time: 00:24) quantity that we have here, now from here s is this quantity so s the quantity backordered is P minus D into t 1. So, s is equal to P minus D into t 1 which is also equal to D into t 4 because this is the consumption (Refer Slide Time: 00:09) for demand is at the rate of D for a period t 4 that is the backorder so s is equal to D into t 4. We also have Q which is the quantity that is produced.

Now, whenever we produce, we produce for a (Refer Slide Time: 00:09) period t 1 plus t 2 so Q is equal to P into t 1 plus t 2. (Refer Slide Time: 00:09) And in a cycle the for example, the cycle starts this is a cycle and there will one more cycle here. So, whenever the cycle starts actually the inventory is 0 therefore, what is produced in the cycle is consumed in the cycle. Therefore, Q is also equal to D into T; Q is also equal D into T and we also have from similar triangles, (Refer Slide Time: 00:09) this is I m this is s so I m by s is equal to t 3 by t 4.

So, I m by s also (Refer Slide Time: 00:09) this is t 1 this is t2 so once again from this similar triangles I m by s is equal to t 2 by t 1, I m by s is also equal to t 2 by t 1. Now, let us write a few simple equations from this so from this we can write now, these are all the relationships. So, from this we can write I m plus s is equal to now, I m plus s is equal to P minus D into t 1 plus t 2 so P minus D into t 1 plus t 2. Now, t 1 plus t 2 is equal to Q by P so this is equal to P minus D into Q by P because from here t 1 plus t 2 is equal to Q by P. So, I m plus s is equal to Q into 1 minus D by P now, this is a very important equation which, we will use to relate I m s and Q so this is one important relationship.

Now, we also have P minus D into t 2 is equal to D into t 3 so P minus D into t 2 plus P minus D into t 1; so P minus D into t 1 plus t 2 is equal to D into t 3 plus t 4 D into t 3 plus t 4. Now, we take this D into t 1 into t 2 on the other side so P into t 1 plus t 2 is equal to D into t 1 plus t 2 plus t 3 plus t 4. So, we also know that this T is t 1 plus t 2 plus t 3 plus t 4. So this is equal to D T. From this we can also write another thing so if a by b is equal to c by d, which is equal to a plus b by c plus d, that is 1 thing. We can also do one more thing here which is I m plus add 1 to everything. So, I m plus s by s is equal to t 3 plus t 4 by t 4 which is equal to t 1 plus t 2 by t 1. We can actually write another thing let us not do this, let us do here. So, this is equal to t 2 plus t 3 by t 1 plus t 4 is equal to I m plus s.

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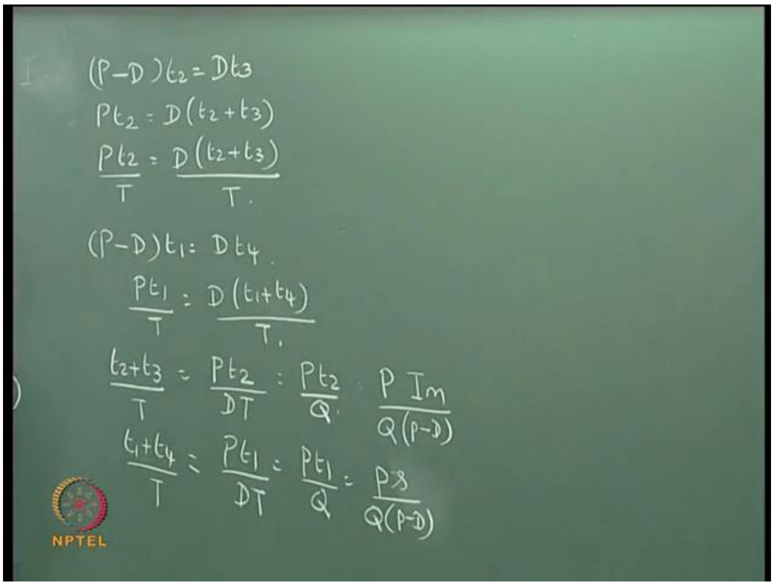

We can write another thing from here. So I m is equal to P minus D into t 2 which is equal to D into t 3 so this is P t 2 from this P into t 2 is equal to D into t 2 plus t 3 so P into t 2 by T is equal to D into t 2 plus t 3 divided by T. So, we have an expression for t 2 plus t 3 divided by T here, we also need an expression for t 1 plus t 4 divided by T, which comes from P minus D into t 1 is equal D into t 4 from which p t 1 by T is equal to D into t 1 plus t 4 by T so we have that. Now, from this you can write t 2 plus t 3 by T is equal to P t 2 by D T now, D T is Q, so P t 2 by Q.

Now, we also have from here, from here we can have another relationship (Refer Slide Time: 00:09) t 2 plus t 3 by T is P t 2 by T. From here we know that t 2 is equal to (Refer Slide Time: 07:32) I m by P minus D so this will be P into (Refer Slide Time: 07:32) t 2 is I m by P minus D so p I m by Q into P minus D. Similarly, t 1 plus t 4 by T will be equal to P t 1 by D T which is P t 1 by Q, which is P I m now, from here t 1 is equal to t

1 is equal to s by P minus D. So P I P into from here P t 1 P t 1 by Q so from this (Refer Slide Time: 07:32) t 1 is s by P minus D so P into s by Q into P minus D.

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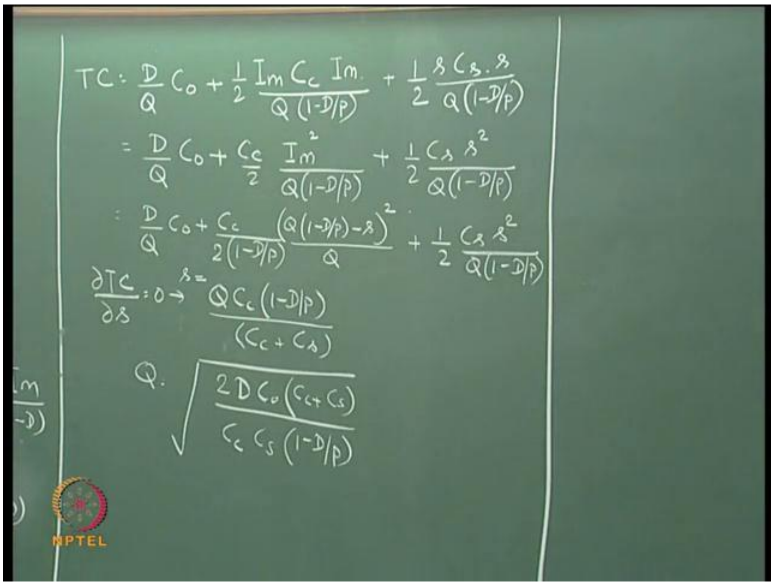

So, now we have derived all these expressions that relate all of them. So, all we now need is to substitute them so when we substitute you get T C is equal to D by Q C naught, (Refer Slide Time: 07:32) that comes from here. Then we have half into I m into t 2 plus t 3 by T into C c, so half into I m into (Refer Slide Time: 07:32) t 2 plus t 3 by t into C c so I m into C c into t 2 plus (Refer Slide Time: 19:30) t 3 by T which is P I m by Q into P minus D. This is written as I m into I m divided by Q into 1 minus D by P, (Refer Slide Time: 19:30) this P is brought back into a denominator as division by P so you get 1 minus D by P plus half into (Refer Slide Time: 07:32) now, the 3rd term half into s by T into s by t so t 1 plus t 4 by T we write from here.

So, we have s by 2 into C s plus half s into C s, s by 2 into C s into t 1 by t 4 by T which is P s by Q into P minus D. So this is written as, once again into s by Q into 1 minus D by P. Now, we have written the this on simplification will give us D by Q C naught plus C c by 2 I m square by Q into 1 minus D by P plus half into C s into s square by Q into 1 minus D by P. Now, we still have three variables here which are I m Q and s. Now, these three are also related and we have the relationship which is I m plus s (Refer Slide Time: 07:32) is equal to Q into 1 minus D by P. Now, we go back and substitute for I m as Q into 1 minus D by P minus s.

So, this will become D by Q C naught plus C c by 2 into 1 minus D by P. Now, I m (Refer Slide Time: 07:32) square will become Q into 1 minus D by P minus s the whole square. So, this is Q into 1 minus D by P minus s the whole square, by Q plus half into C s into s square by Q into 1 minus D by P. Now, this is written in terms of only two variables which are Q and s. Now, we have this expression, we can partially differentiate this expression with respect to s first and then get a equation or value for s. And then we partially differentiate this with respect to Q and then substitute the value for s and try to get the final one.

Then we have a lengthy derivation which can be done so when we partially differentiate this with respect to s and set it to 0 dou T C by dou s equal to 0 would give us, these are the terms where we will have s. Finally, we will have an expression s is equal to Q C c into 1 minus D by P Q C c into 1 minus D by P divided by C c plus C s. And then we have to partially differentiate this with respect to Q and Q actually comes in all the three terms and then we need to substitute for this s into that and solve it. We will finally get, Q is equal to root over 2 D C naught into C c plus C s by C c C s into 1 minus D by P. So, this will be the final expression for Q as well as for s, this will give s is equal to Q C c into 1 minus D by P C c plus C s.

Now, once we know all the values of s and Q we can calculate s and Q because all the other things are known and once s and Q are known we can go back and calculate t 1, t 2, t 3, t 4 (Refer Slide Time: 00:09) and the total cost and so on. So, that is how this formula works. So, let us take a numerical example and show, what is the effect of this? Take a small example. So, if we consider the same example that we used for the earlier model. The earlier model (Refer Slide Time: 00:09) does not involve back ordering, this model involves backordering so we take the same numerical illustration and then we give a value for C s and find out happens to the total cost as well as to the economic batch quantities.

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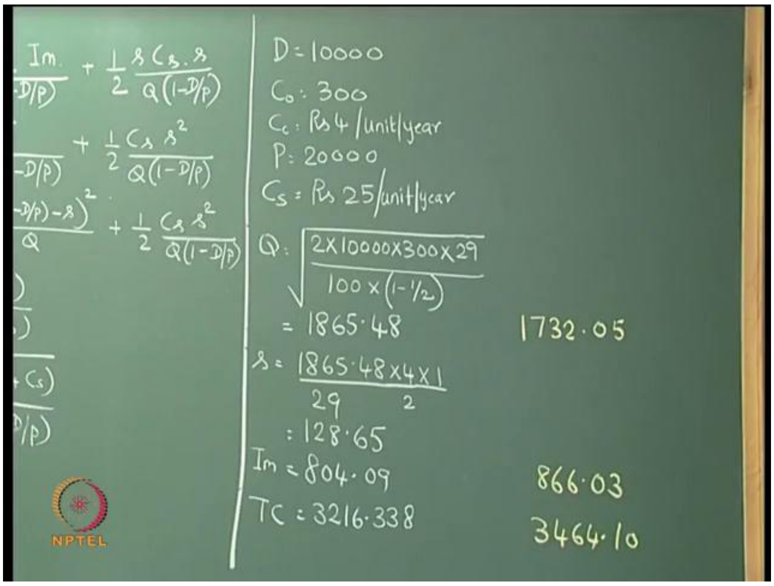

So, we go back and work out the same problem as such so we consider D is equal to 10,000 D; C naught is equal to 300, C c equal to 4 and P equal to 20,000. Now, in addition we consider C s which is the shortage cost in this case, So, C s is equal to rupees 25; equal to rupees 25. So, once we consider this the first thing we do is we need to find out Q. So Q is equal to root over 2 into 10,000 into 300 into 29 which is C c plus C s 25 plus 4, note that, both of them have a unit of rupees 4 per unit per year rupees 25 per unit per year. C c plus C s divided C c into C s which is 100, 25 into 4 into 1 minus D by P which is 1 minus half 1 minus 10,000 by 20,000 which is 1 minus half.

So, this on simplification would give us a value of 1865.48, is what we get for this and then we also have s the moment we know Q we can get s, s is equal to (Refer Slide Time: 23:57) Q C c into 1 minus D by P divided by C c plus C s. So this is 1865.48 Q C c into 4 Q C c by C c plus C s is 29 into 1 by 2, 1 minus D by P is half so into 1 by 2 so this gives us s equal to 128.65. Now, we can calculate I m, I m which is the maximum inventory, from this equation in I m plus s is equal to Q into 1 minus (Refer Slide Time: 07:32) D by P D by P is half. So, q into 1 minus D by P is 932.7, 932.7 minus 128 would give us something like, 804.09.

Now, once we know Q s and I m, we can substitute here, (Refer Slide Time: 23:57) to get the total cost or we can substitute either here or here, they are one and the same. So, this is easy to substitute here or here to get the total cost and the total cost for this would come to on substitution the total cost would come to 3216.338. Now, in order to see the effectiveness of the back ordering, the only thing we could immediately do is to see what happens if did not allow backorder (Refer Slide Time: 00:09) this model allowed backordering, this model did not allow backordering.

So, we could go back and compare with numerical result of the numerical illustration, if we did not have back ordering or is C s is equal to infinity. So, when C s is equal to infinity this model will automatically become this model. We have anyway solved the same numerical illustration for the 3rd model or for the other model so when we do this we get Q is equal to 1732.05 I am just writing the value here, now this is the economic batch quantity when we did not allow backordering or when C s is equal to infinity.

Now, in that case the maximum inventory was 866.03, I m was 866.03, in that case and total cost in that case was 3464.10. Now, what are the, what do we gain by comparing? The first thing that strikes us is when we have C s or when we can allow backordering the total cost comes down 3216 versus 3464 when C s was infinity. The same result when we go back in time to compare the first two inventory models for items that we buy where, we showed that if we allow backordering, it actually works out cheaper per year than the case without back ordering. The case without back ordering can be seen or assumed to be C s is equal to infinity so the cost comes down.

Now, what happens to the total production quantity per cycle now, when we allow backordering the quantity that will be produce is 1865.48. So for the same annual demand T which is the cycle becomes smaller because this quantity is higher. We can also show even though we have not shown it explicitly we can show that in this model actually the cost is a tradeoff between this and (Refer Slide Time: 23:57) these two put together. In fact at optimum this cost will be equal to those two costs for example, we can easily show that now, when Q is equal to 1865.48 the number of times we order a we produce in a year is D by Q which is 10,000 divided by 1865.48 which is 5.36 into 300 is 1608.

So, this component is 1608 (Refer Slide Time: 23:57) the total is 3216 so obviously this is equal to this. So, what happens here is the production quantity per cycle is higher and slightly lower, the cycle length or duration comes down slightly. Secondly there is even though the production quantity is slightly more 1865.48 there is a backorder which is also eventually cleared and then the average inventory here, actually comes down to 804 against 866 in the earlier case. The average, the I m maximum inventory that is there in the system comes down because of the backorder and the total cost comes down because this I m is essentially held for a slightly short period, compared to 866.03 being held.

So, inventory cost comes down, backorder cost goes up but eventually the order cost balances (Refer Slide Time: 23:57) these two costs and the total cost come down. So, when we have the production consumption model if we can allow backordering, then we allow backordering. The result is very much dependent on the value of C s that we have. So, only if we know C s accurately or if there is enough protection in the system in terms of additional inventories and so on, then one could think in terms (Refer Slide Time: 00:09) of applying this model. But, if we are not sure about C s and if do not have the extra question of some additional help, then it is better to assume C s to be infinity.

And then use the earlier model and not try and optimize further on the cost, by allowing for backordering and assuming a certain C s. The strength of this model lies, or the usability of this model lies in the exact computation or exact estimation of C s. So, if C s can be estimated accurately and it is not going to hurt us, if we allow a backordering then (Refer Slide Time: 00:09) this model can be used, otherwise this model has to be used.

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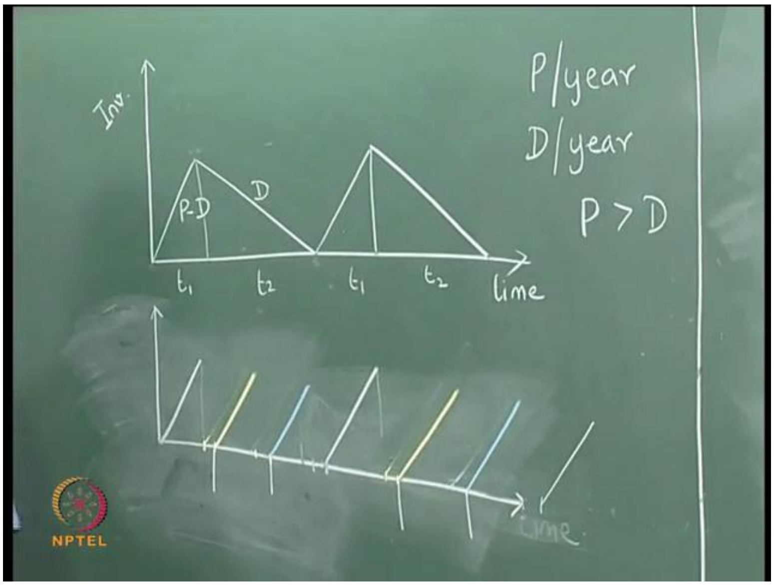

Now, let us comeback to the old model, where we do not allow backordering which is this model. And let us do a few more things now; let us assume that we have this for one particular item. Now when we worked out this model we also said that for a particular item, the production starts from here, stops at time period t 1 that is an inventory buildup. Then the production for that item stops. The consumption takes place from the inventory that has been built up. Now, if we start looking at the same graph, from the facilities point of view, then it will look like this. Now, from zero to t 1 I am producing this item. Right now I do not do anything then once again from here I produce this item and so on.

Now, for this period t 2, where we consume from the built up inventory, earlier I had mentioned there it is quite likely that this will not be kept idle and this will be producing some other item. So, let us assume that it produces a 2nd item and then so let us assume that the 2nd item actually starts from here and the 2nd item cycle is this, so which also means that, the 2nd items next cycle will be like this. Now, if you consider two items the yellow is the second item and white is the 1st item. Now, once again the 2nd item is produced up to a certain time period and then from this period to this period at the moment our equipment can be idle.

So, what we do is we produce the 3rd item let us say, that we have here. So the third item comes from here and say the 3rd item is produced up to this so the 3rd item is produced up to this. So, once again the cycle now comes here may be for this period the facility is idle and then the next cycle begins. So, if we look at this facility and three items this is how the utilization of the facility will look like. Now, we also said that when we set this product; this item again there is going to be a set up cost. The cost to changeover from this to this, when we make a second cycle of yellow which is the 2nd item there will be a cost to changeover from white to yellow. But, we also have to note that in addition to the changeover the cost there is also a changeover time.

So, we have to assume now, that this gap now has to be for the cycle to be effective this gap should be enough, for the time of three changeovers. Because, somewhere we have to changeover from white to yellow, yellow to blue and then blue to white, which simply means that, if we are able to do something like this, now, this much is the changeover time to yellow and then the yellow cycle happens, now this much is the changeover time for the blue, then blue cycle happens. And then this much is the changeover time once again for white and the white setup happens. Now, if we start looking at this problem, from the point of view of the facility that is making if I am making certain number of products then I should be able to produce the cycle time for the each product in it is desirable that the cycle time is the same.

So that, this cycle time would be the same as this cycle time we have to draw this thing again little clearly. So, there is a small time which is there and then the yellow and then there is some time here and then the blue and then there is some more time here and then the white. Now, we would like to have this cycle time same, this cycle time same and so on. So, the T has to be the same for all the products so that we need to find that T and then there will be individual times, which you may call this is as t 1 this as some t 2, this as some t 3 and so on. The time you produce each of these items then I should be able to buildup inventory and then I should be able to consume and so on.

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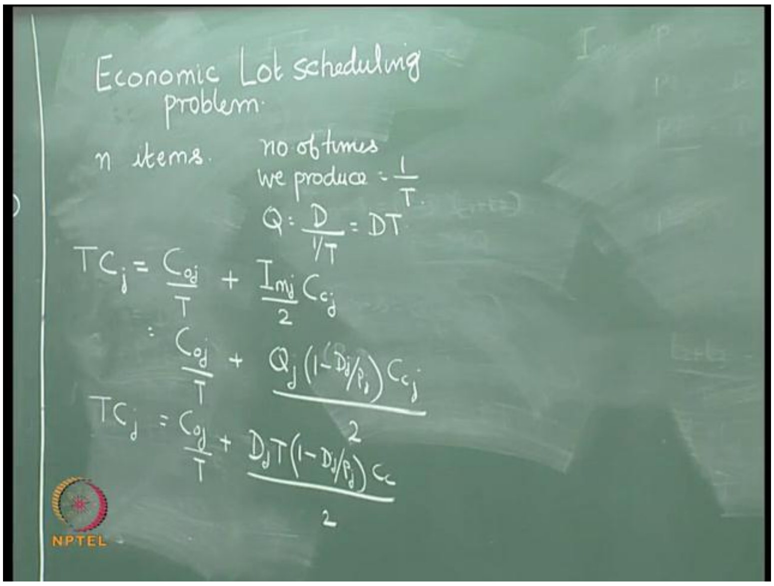

Now, can we find out such a T is the next problem, if we are given multiple items. Now, this problem is called the economic lot scheduling problem, And then we will try and formulate this economic lot scheduling problem and try to solve it. Now, if there are n items that we have to produced, there are n items that we have to produce, (Refer Slide Time: 40:08) in this example we show three items the white, the yellow, as well as the blue. For each item there is a set up cost, there is a inventory holding cost right. So, for example, each item will show a figure like this but the equipment will show a figures like this.

Now, if we take a particular item the only condition is the cycle time T is the same for all the items, T is the cycle time for all of them. So, if the cycle time is T, T years then the number of times we produce is 1 by T so many times in the year we produce. So, number of time we produce is equal to, 1 by T the production quantity is the total demand divided by number of times you produce. So, production quantity Q will be equal to D divided by 1 by T which is D T. So, production quantity Q is equal to D T. Right? So, for each item, so for the first item if I find total cost is equal to set up cost plus inventory holding cost. So, set up cost is number of times we produce which is 1 by T and every time we produce we incur a setup cost of C naught.

So, for a particular item let us say T C 1 or T C j for the j-th item, then the number of times we produce will be 1 by T, so C naught by T is the total setup cost that we will have. Now, if C naught is same for all the items then we call it as C naught if the j-th item has C naught j, then this the total setup cost for the j-th item C naught j by 2. Now, what is the inventory holding cost? The inventory holding cost is the average inventory into C c, average inventory from here we are as I m by 2 I m by 2 total inventory is half into base into height so I m by 2 into t 1 plus t 2 divided by T which is I m by 2 into C c. So, this is I m j by 2 into C c of j carrying cost of the j-th item. (Refer Slide Time: 40:08) We also know from this I m is equal to P minus D times t 1 for this item and it is equal to D into t 2 for this item.

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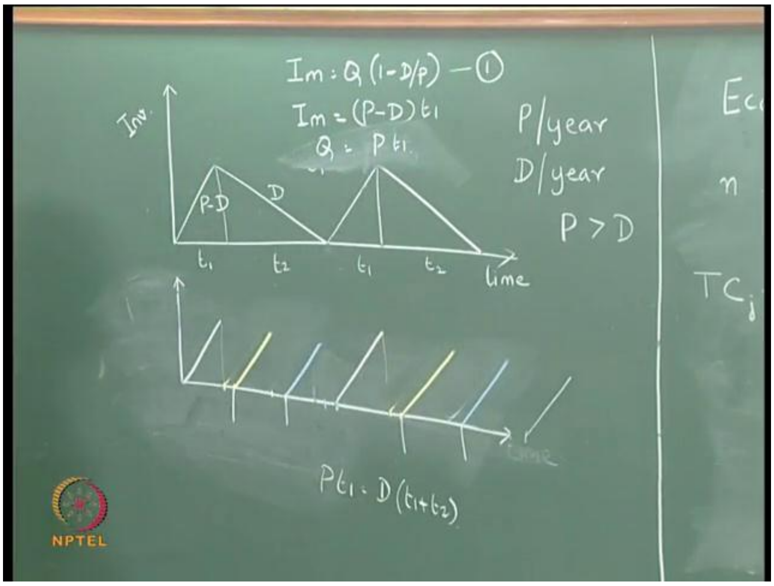

So, from here we also know that I m is equal to Q into 1 minus D by P because P minus D, I m is equal to P minus D into t 1 which is here which is equal to D into t 2, that we have here. So, I m by and from similar so from this we have, I m is equal to the rate at which the inventory grows which is and Q is equal to D into T or D into t 1 plus t 2, so from this we would have. So, p into P minus D into t 1, so P t 1 is equal to D into t 1 plus t 2 Q is equal to P t 1 so dividing 1 by other would give us I m is equal to Q into 1 minus D by this is an important equation.

So, from here we know that this is C naught j by T plus I m is equal to Q into 1 minus D by P so Q j into 1 minus D j by P j into C c j divided by 2. And we also know that Q is equal to D T so C naught j by T plus Q j is d j T into 1 minus D j by P j into C c divided by 2 this is your T C j for all these items. So, C naught j by T plus C c j D j into 1 minus by 2 P j into C c this is what we have.

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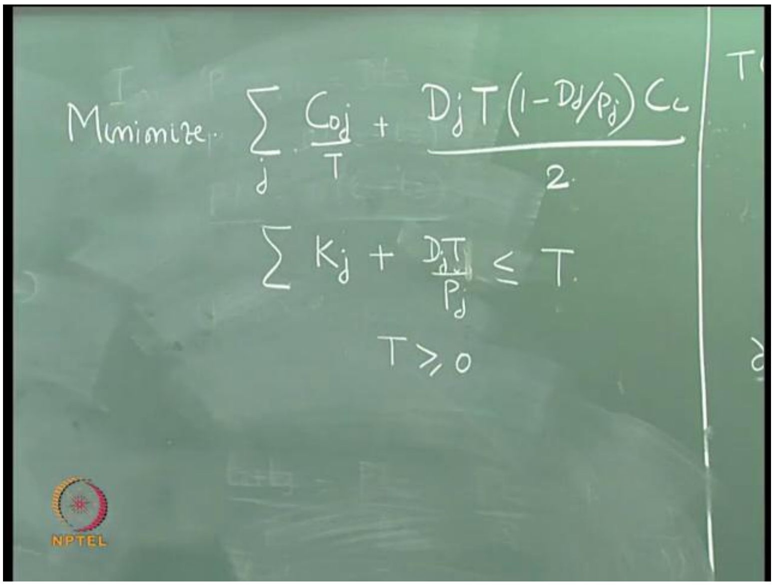

Now, we want to find out T such that we minimize the sum of costs for all the items, this is for the j-th item so we want to minimize summed over j for all the items, C naught j by T plus D j t 1 minus D j by P j C c divided by 2. So, we want to find out a T such that we minimize this, actually we observe that what we have written here is very similar to normally writing as D by Q into C naught plus is I m by 2 into C c that is what we have done here. Except that we have written in terms of T. But, the T should also satisfy another important thing that, the time that we have here in this cycle is say between this start and this start should be, should take care of the production times of all the items plus the changeover times of all the items.

So, the cycle should satisfy an additional constraint that it should be able to take care of all the production times as well as all the setup times. So, subject to the condition sigma if K j is the setup time for the j-h item, note that C naught j is the setup cost for the j-th item K j is the setup time for the j-th item plus..

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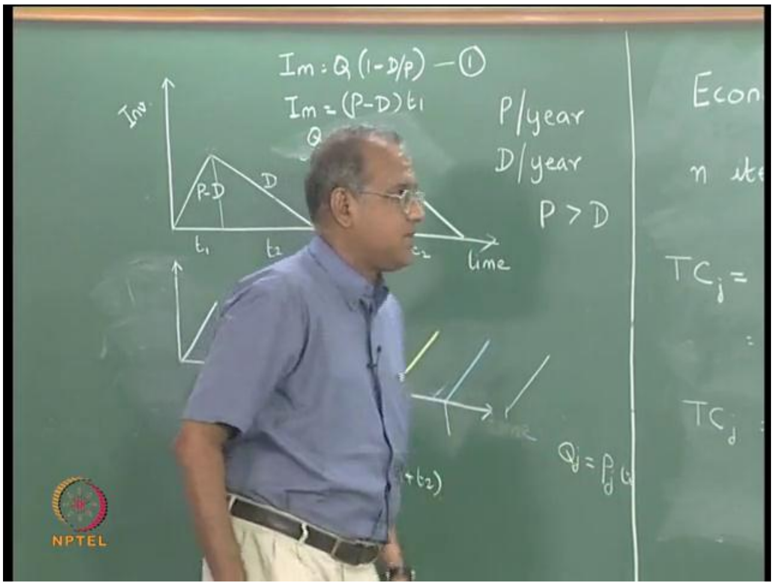

What is the production time for all of them? Production time production quantity is Q j Q j, j is equal to P into t 1, t 1 is the production time. So, P j into t 1 so t 1 is equal to Q j by P j so production time is Q j by P j and Q j is D T j so this is D t j by P j, is less than or equal to D j into T by P j less than or equal to T. So, this is the objective function this is the constraint. The only catch in this constraint is we have to rewrite this constraint because the T term is here as well as here plus of course, we have a situation that T greater than or equal to 0. Now, this problem is called the economic lot scheduling problem. In the next lecture, we will see how we solve this problem to get the optimum values of T.