Operations and Supply Chain Management
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Lecture - 14
Multiple Item Inventory – Combining orders, Production Consumption Model.

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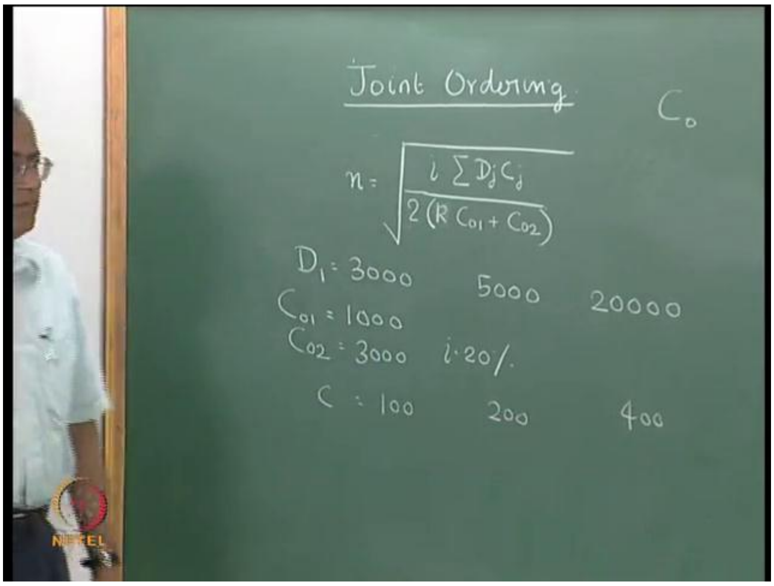

In the last lecture, we derived an expression for the number of orders n when we combine orders. The basic idea behind this equation for n is that, when we have multiple items and if we ensure that, the order quantities are such that they have equal orders, equal number of orders. If the order quantities are adjusted such that, both the items or all the items have equal number of orders, then it is possible to save on the order cost.

Now, the order cost which was originally defined as C naught in our earlier models. Now, has 2 components, one which is called C 0 1and the other is called C 0 2. Now, C 0 1 represents the admin cost and C 0 2 represents the truck cost.

Now, if the items are ordered such that, the number of orders per year for each item is the same, which means the orders are combined. Then, it is possible now, to save on the truck cost. So, there is only an admin cost for every item that is ordered. But, then we can assume that the same truck will bring all the material from the supplier, to the plant or the manufacturing facility. Therefore, the total order cost instead of becoming k times C 0 for each of the k items, assuming that there are k items. Now, it becomes C 0 2 plus k times C 0 1 because we assume now, that the truck has enough capacity to bring all the items together. So, there is a saving on the truck cost.

Now, this particular formula that we derived in the earlier lecture was for two items. Therefore, we had 2 times C 0 1 here, this 2 comes from the average inventory. Therefore, this 2 remains. But, then if in general if the number of items is k, then this can be rewritten with k times C 0 1 plus C 0 2. Now, we take a numerical example, to show the effect of this joint ordering. So, let us look at a situation where there are 3 items and the demands are 3000, 5000, and 20,000.We have the truck cost as 3000 and admin cost as 1000. So, C 0 1 which is the admin cost is 1000 and C 0 2 which is the truck cost is 3000 and then we take that, i is 20 percent, 200, 200 and 400. So, i is 20 percent and the unit price C is: 100, 200, 400 respectively. So, we look at the 1st situation, when the 3 items are ordered separately according to their economic order quantities.

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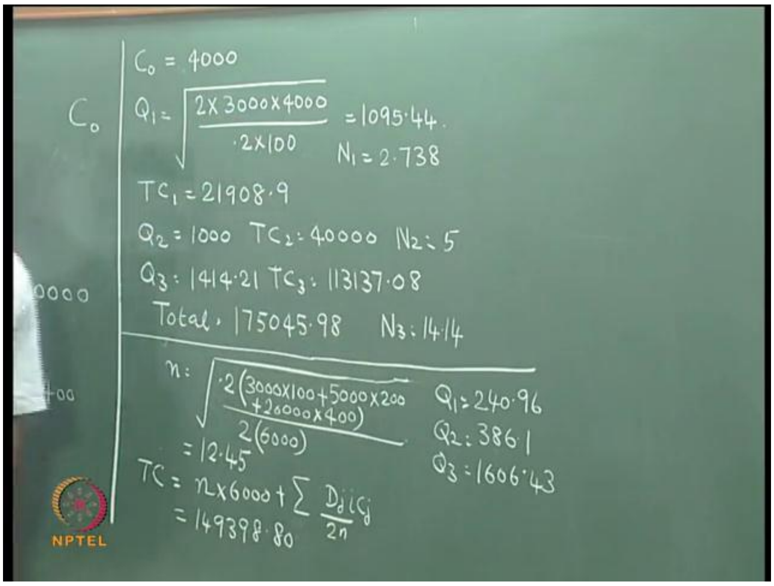

And if we do that, C naught for each item will be the sum of the admin cost which is 1000 and the truck cost which is 3000. So, C naught will be 4000 and for the 1st item Q 1 will be, root of 2 into demand 3000 into 4000 divided by 0.2 into 100 which on computation becomes 1095.44 and total cost T C 1 which is the sum of order cost and carrying cost for this item is 21908.9. In a similar manner, if we calculate Q2 and T c 2 we find that Q2 turns out to be 1000 and T C 2 turns out to be 40,000, Q 3 is 1414.21 and T C 3 is 113137.08. So, individually if we order the 3 items according to their economic order quantity. Now, these are the order quantities for the 3 items and the sum of order cost and carrying cost for the 3 items are given here. So, this gives a total cost for the 3 items put together which becomes 175045.98.

Now, this is the total cost incurred if they are ordered independently and separately. Now, if we apply this formula and assume that, they can be ordered jointly. And there is a saving on the truck cost if we order them jointly. Applying this formula (Refer Slide Time: 00:15) for k equal to 3. There are 3 items rest of the values are known here. We will compute n to be equal to root of i which is 0.2 into d 1 C 1 plus d 2 C 2 plus d 3 C 33000 into 100 plus 5000 into 200 plus 20,000 into 400 divided by 2 times 3 (Refer Slide Time: 00:15) into C 0 1 is 3 into 1000 3000 plus another 3000.

So, 6000 this on computation would give us, n equal to 12.45 and T C for each of these items, if we calculate the total cost and the total. Now, for n equal to 12.45, we can actually calculate the individual order cost, carrying cost and then do it. Another way of looking at T C is, if we have n orders. For each order the ordering cost is going to be this (Refer Slide Time: 00:15) 3000 plus 3 into 1000 so it will be 6000, so n into 6000 plus Q by 2 into C C. So, summation Q is D so D j by 2 n Q equal to D by n. We have 12.45 orders.

So, the order quantity for each item is the annual demand divided by 12.45. So, D j by n into Q into i C j which is the inventory carrying cost. So, this for n equal to 12.45. Now, becomes 149398.80. So, this example tells us that, if we are incurring a (Refer Slide Time: 00:15) 1000 rupees on the admin cost of ordering each item and there is a truck cost of 3000. And by joining the orders and by making sure that, all 3 items have the same number of orders per year. Now, we now have 12.45 orders for each of them and then we realize that the total cost has actually come down from 175045 to 149398.8.

There will be a reduction in the order cost because of the saving. But, then there will be a slight increase in the inventory carrying cost, because the order quantities will change. But, then the sum of the order cost and carrying cost comes down significantly. Now, what are the other issues related to this. Now, from our total cost point of view there is a considerable saving, if we can combine orders together. Provided there is a saving in the truck cost of 3000 every time items are combined together for example, when an item was ordered individually, the cost was 4000. But, then when two items are combined, the order cost was not 8000 but the order cost was only 5000. When three items were combined, the order cost was not 12000.

But, then it was 6000. So, that saving helped us in reducing this considerably. Then, there could be some other issues. For example, let us compute the order quantities for 12.45 so when you have 12.45 we realize that, the order quantity Q 1 is 3000 divided by 12.45. So, the order quantity Q 1 becomes 240.96 Q2 becomes 386.1 and Q3 becomes 1606.43 and all of them have 12.45 orders. Now, if we compare the order quantities in the unconstrained system this was 1095 and that has come down to 240.96.1000 has come down to 386.1 and 1414 has actually gone up to 1606. Let us now, calculate the number of orders for these values. So, N 1 is 3000 divided by 1095.44 which is 2.738 orders, N 2 is 5000 divided by 1000 which is 5 orders and N 3 is 20,000 divided by 1414.21,14.14 orders.

So, when we did not have this constraint the number of orders was 2, 5 and 14 very large difference. When we combined all of them now, came to 12.45 orders. So, we actually end up making more orders for these two items. Considerably more orders because 2.738 when we did not do this, 12.45 when we did this. Similarly, 5 when we did not have this joint ordering and 12.45 when we had joint ordering. So, when we assume that n is the equal number of orders for all of them. We now, tend to order more for items 1 and 2 and about the same number for item 3. May be slightly less for item 3, 14.14 becomes 12.45.

So, since the number of orders is more here, the order cost component is also more. So, the other question that comes is. Now, instead of assuming an equal number of order is n, can we assume unequal number of orders? But, the number of orders for each item is a kind of an integral multiple and then can we check whether we can have some kind of partial saving? But, yet try and reduce this 149398 further. So, if we look at this, if we look at this alone. This is 2.738 this is like 5 and this is 14.14. So, this is roughly 1 is to 2 is to some number like 5. This is like 1 is to 2 is to 5 that is the number of orders.

Now, instead of using 1 is to 2 is to 5 suppose, we look at 1 is to 2 is to 4 instead of looking at 1 is to 2 is to 5.If we say that, the number of orders is roughly 1 is to 2 is to 4,2.7 into 5 is about 13.5. So, it is roughly 1 is to 2 is to 5 but then 1 is to 2 is to 4 is a very comfortable integral multiple. So, we could think in terms of 1 is to 2 is to 4.

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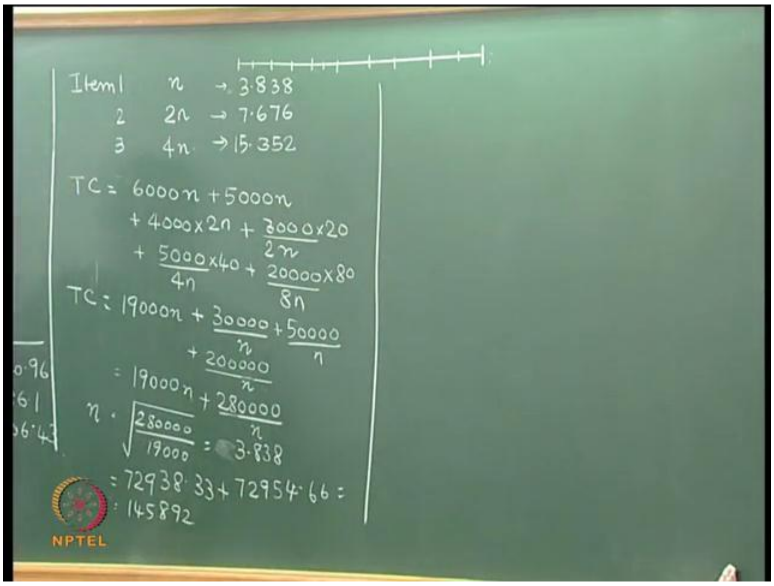

And then we say that item 1 will have n orders, item 2 will have 2 n orders and item 3 will have 4 n orders, which simply means that, for every order here, we will have 2 orders here and 2 orders here. So, this order n will synchronize with this and this. And this 2 n will also start synchronising with this 4 n. But, then the order quantities will be suitably defined and you can still exploit the basic idea of joint ordering, because the number of order is only an integral multiple of this.

Now, when we start doing this what is the gain that we have. So, the gain that we would have is this. Now, the gain will be the total cost for this system will be. So, totally there will be n orders in a year for this, there will be 2 n orders in a year for this and there will be 4 n orders in the year for this. So, for all these n orders so there will be a total of 4 n orders in a year, out of which this 2 n will be synchronized within this 4 n. And this n will also come within this 4 n.

For, example if we say that 4 n is equal to 16 and n is equal to 4 it, is like saying that the 1st item will be ordered 1 2 3 4 and then it goes, this is the next year. Now, this item will be ordered here, 1 2 3 4 5 6 7 and there is a 8. Now, this will be ordered 1 2 3 4 5 6 like this it will go. So, if there are 4 n orders, in all in a year. Now, for n out of these 4 n, all the 3 will be ordered together. For another n out of this 2 n, 2 will be ordered together. And for this 2 n out of the 4 n, there will be a single order. So, the total order cost will be, for n out of these 4 n the order cost will be 6000n, because when 3 items are ordered together, the truck cost is 3000 plus 3 times admin cost of 1000 will give 6000.

So, 6000 into n, for another n orders only these 2 will be combined. So, this will become 5000 into n and for the remaining 2 n orders this alone will go. So, this will be 4000 into 2 n so this will be the total ordering cost that we will have. Now, the inventory carrying cost will be. If I have n item, n orders for item 1. The order quantity will be (Refer Slide Time: 00:15) 3000 divided by n. This is Q so Q by 2 into carrying cost is C c which is i into C which is 20 percent (Refer Slide Time: 00:15) of 100 which is 20. For the 2nd item, the order quantity will be 5000 divided by 4 n. 5000 is the annual demand, 2 n is the number of orders. So, 5000 by 2 n is Q and Q by 2 is 5000 divided by 4 n into C C which is 20 percent of 200 which is 40 plus 20,000 divided by 8 n.

The 8n comes because demand is 20,000. There are 4 n orders so 20,000 divided by 4 n is Q that divided by 2 - Q by 2 into 0.2 into 400 which is 80. So now, T C is equal to 6 plus 5, 11 plus 8 19,000 n plus this is 30,000 divided by n. This 2 gets cancelled into 10, this 4 also gets cancelled plus 50,000 divided by n plus 200,000 divided by. So, this is 19,000, n plus 200 and 80,000 divided by n, from which the optimum value of n can be found. By differentiating this with respect to n and setting it to 0. So 19,000 minus 200 and 80,000 by n square is equal to 0.From which n is equal to root over 200 and 80,000 divided by19,000 which would give us 3.838.

Now, this means item 1 will have 3.838 orders item 2 will have 7.676 orders and this will get 15.352 orders. So, the order quantity for this will be 3000 divided by 3.838, which is roughly about less than 1000. This will be 5000 divided by 7.676, 20,000 divided by 15.352 and now, when we substitute this n, into the total cost at the optimum. We now, realize that this n for this n, the total cost is 19000, 72938.33 plus 72954.66 which is 145892. In fact, this difference, slight difference comes more because of the rounding of errors. And at the optimum, we actually would realize that this 19,000 n will be equal to 280,000 divided by n.

These two, will have to be equal. The slight difference comes between 38 and 54 is in the rounding off, this 8.38. Otherwise, these two values will be equal and it will be 2 times. Now, this would ensure 1st thing we observe, is that the cost has further come down 145892 versus 149398. The other advantage that we have right now, here is that the number of orders are staggered and somewhat proportional to the annual demands of 3000, 5000 and 20,000. So, roughly all of them will have order quantities in some way or around 800 to 1200. The number of orders will be different. The average inventory that, we hold will also come down.

So, that is the advantage of using these kind of staggered values. But, then we now, ask ourselves another question. Now, we first worked out based on individual ordering quantities which is here 1750, then we worked out assuming that all of them will have equal number of orders which brought it down to 1493. Then, we worked out with unequal number of orders and then we found a small decrease and could perhaps help us in little more control of the order quantity. The number of orders will be different. So, the next question is, if we do say if we do 1 is to 2 is to 4, the cost is 145892.

Now, what is that optimal combination of say k is to l is to m or what is the ratio? Is it 1 is to 2 is to 4 or is it 1 is to 2 is to 2? Or if we try 1 is to 2 is to1, or 1 is to1 is to 2? What kind of answers would we get? And can we optimize this number further by trying different combinations of these ratios? Very basic idea comes from the fact that, when we have the unconstraint problem (Refer Slide Time: 04:01) 2.7, 5, 14. The first thing that comes out is that, the number of orders kind of increase in the 3 item. So, 1 is to 1 is to 1 gave us 1493, 1 is to 2 is to 4 would give us 1458. So, what is that ratio? What is that k is to l is to m, such that we can bring this further as well as, we can exploit this very idea of joint ordering.1 is to 2 is to 4 helped us in a certain way.

But, 1 is to 2 is to 4 actually increase this thing 19000, because out of the 4 n orders 2 n orders are done individually and separately which gave us which brought this term 4000 into 2 n. Now, if we do 1 is to 1 is to 2 or if we do 1 is to 2 is to 2, then we realize that actually this part will come down. Because, if we do 1 is to 1 is to 2, there will be 2 n orders, out of which n orders will be joined and the other n orders will be individual. So, we could try some of these combinations. There are some intelligent ways of trying, to find out. What is the ratio? Is it 1 is to 1 is to 2? Or is it 1 is to 1 is to 4 and so on. Generally multiples are advantageous 1 is to 2 is to 4 is always advantageous compared to 1 is to 2 is to 5.

Because if we do, 1 is to 2 is to 5 then we would not be able to overlap orders. When the number of orders is an integral multiple then it is always easier and if you have multiple items, if the last one is an integral multiple of all of them, it is like an l c m of all of them. Then it is actually advantageous, something like 1 is to 2 is to 5 or 1 is to 3 is to 5 is not of great advantage whereas, 1 is to 1 is to 2 or 1 is to 2 is to 2 can be of certain advantage. Actually we can go on expanding on it but let me just give you one simple solution and leave it at that.

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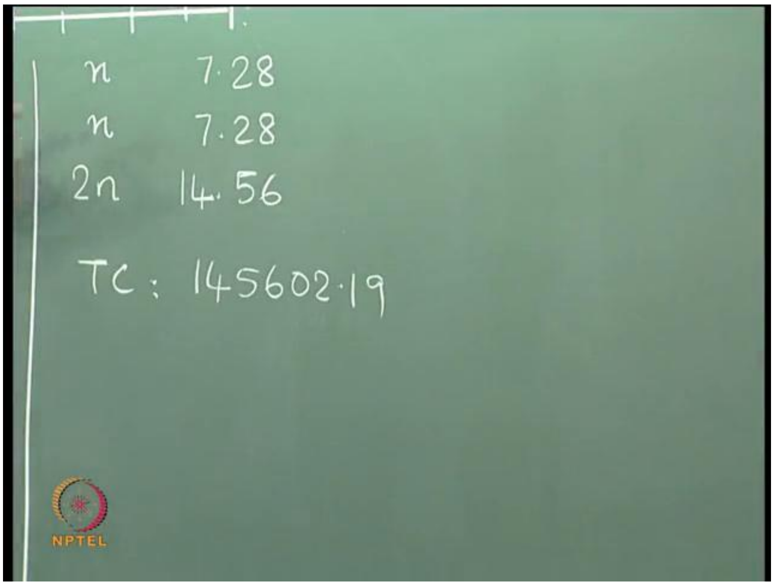

In fact, if we do 1 is to 1 is to 2 which means n, n and 2 n.We do 1 is to 1 is to 2 for this particular problem, we will have n equal to 7.28 and total cost actually comes out to 145602.19, which is a slight decrease compared to 145892. The other thing that, we can observe though it is difficult to generalize is that, once we reach this number like 1458.92, we were able to show a slight decrease. But, then we will not get too much of a gain. It its little unlikely to have a solution which is 1375 or whatever, so the decrease is only very minimal and one can try and look at you know, other alternatives like 1 is to 2 is to 2 or 1 is to 2 is to 3 and so on and try and see whether we could make some gains, though 1 is to 2 is to 3 is not a great advantage compared to 1 is to 2 is to 4.

But, there are intelligent ways of trying to find out the ratios, we define something called m j where m 1, m 2 and m 3 are the number of orders per year, as a multiple of n and then they can try and optimize that m to a limited extent and try and optimize this cost further. But, many times a very commonsensical approach of trying to find out these values for example, (Refer Slide Time: 04:01) this 2.7385 and 14.14. The simple ratio of these three rounded to the nearest integer, an integral multiple like 1 2 and 4 (Refer Slide Time: 15:44) invariably gives a very good answer, though from an optimality point of view, one would choose this compared to this. It is also easy to play around here, from this to try and get 1 is to 1 is to 2 from 1 is to 2 is to 4.

So, these are strategies that are being used particularly, in the context of supply chain management, the reason supply chain management is because we are now, combining orders. So, that we can save on (Refer Slide Time: 00:15) the truck cost today, with supply chain management and logistics becoming increasingly important. The transportation is usually outsourced to 3rd party service providers, take up the task of transporting from vendors to the manufacturers and since, they can combine orders from the same vendor to different destinations or from different vendors to the same destination and they can do some kind of a milk run and so on.

So, the whole idea of supply chain management and logistics leading as towards 3rd party service providers, an outsourcing of the transportation function allows organizations to now, go more for joint ordering (Refer Slide Time: 00:15). So, that the truck cost can be saved and the total inventory cost can be brought down. Now, in such a process sometimes, we may end up getting a large order quantities for some items and small order quantities for some other items. Large order quantities for some items would mean quite a lot of money, is locked up in inventory. And that also can be offset by trying to order like (Refer Slide Time: 15:44) 1 is to 2 is to 4 and 1 is to 1 is to 2.

So, with this we have now covered certain aspects of deterministic inventory. Now, we would look at another aspect which is production as well as consumption. So, we will get into models, where we produce and consume. So, far in inventory we have seen, two very basic inventory models and then we looked at discount and then we looked at constrained inventory problems and then we looked at trying to optimize on total cost by, saving on the truck cost. Now, let us spend some time on what are called production and consumption models.

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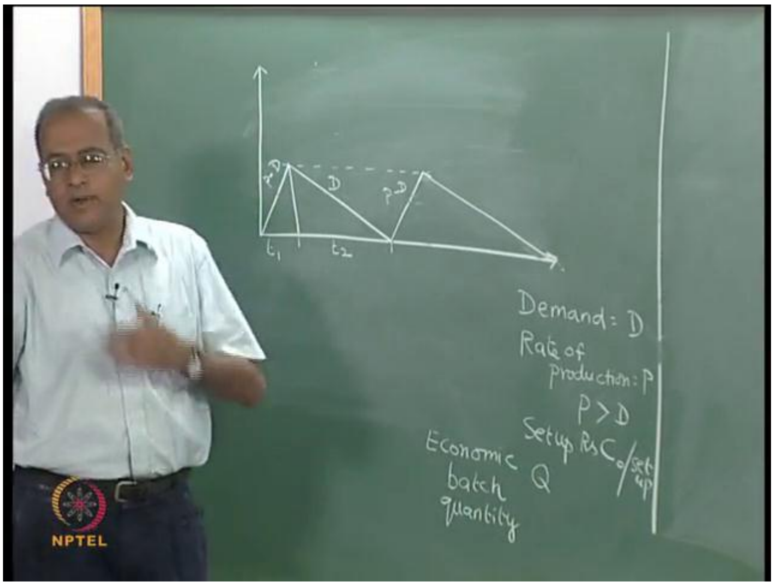

Now, let us look at a single item which is not bought out, but it is produced. Now earlier; in the earlier inventory models we assumed that, we are going to buy the item from a vendor and therefore, every time an order was placed there is an order cost and then the items come in and then the items are stored. So, there is an inventory carrying cost or a holding cost. Now, let us assume that we are not going to buy this item, instead we are going manufacture this item internally and then consume it. So, let us assume that a single machine or a single facility is going to make this particular part or this item. And let us assume that the annual demand for this item is D, the rate of production is P per year.

Now, if we have to meet the annual demand of this item, obviously our production rate has to be higher than the demand. So, produce at P to meet the annual demand of this item D. And since we have to meet the annual demand D, it is only fair to assume that the production rate is higher than, the demand and therefore, P is greater than D. Now, let us assume that P is slightly higher than, D and in fact much bigger compared to D. So, let us assume we start producing this item so we produce this item and as we produce, we consume this item. Therefore, the inventory is going to increase at the rate of P minus D, because we produce at the rate of P and while producing we consume a D. So, the inventory build-up is going to be at the rate of P minus D.

So, let us say we build up this inventory, up to a certain point and then we stop producing this item and then we start consuming this item, only from the inventory that has been built up. So, then from this, let us assume we stop production here, at this point. and then we start consuming this item. So, we consume it at the rate of D and then the inventory comes to 0 and we assume that the moment, the inventory comes to 0, we start producing this item again. So, when we start producing this item again, we will again have this up to P up to a particular point and then we use this and this is how the inventory cycle goes. So, note that these two points are actually the same.

So, and so now, we assume that, in between during this period, when we stopped production and then we again start production this is called t 2. This is called t 1, t 1 is the period where we produce, which means we produce and consume from an inventory point of view, t 2 is the period where we only consume the item from the inventory point of view from the machines point of view t 1 is the period, where we produce this item and once again. There is going to be another t 1 after a gap of t 2, where we produce this item again. So, we will assume that this machine would be used to produce some other thing, during this period t 2.

And therefore, when we come back and start making this item again on this machine somewhere, here we need to setup this machine, which means every time we start production of the item, we have to setup the machine to produce the item. And therefore, there is a setup cost associated with this system. Now for the sake of uniform notation, let us assume that, the setup cost has the notation C naught per setup, Rupees C naught per setup. So, every time this machine is setup to produce this item, there is a C naught per setup. This C naught, is very similar to the ordering cost C naught. Except there it is the cost occurred internally, when the machine is being setup for the sake of uniformity to show some other things.

We assume the same notation C naught as the setup cost, every time this machine is setup. Now, we have to find out whenever, we produce a certain quantity Q and what is the economic batch quantity. When we were ordering this item the Q was called economic order quantity. Now, when we produce this Q is called economic batch quantity, it is called E B Q it was called E O Q later. Now, let us assume that we produced a quantity Q. Every time we setup this machine the annual demand is D.

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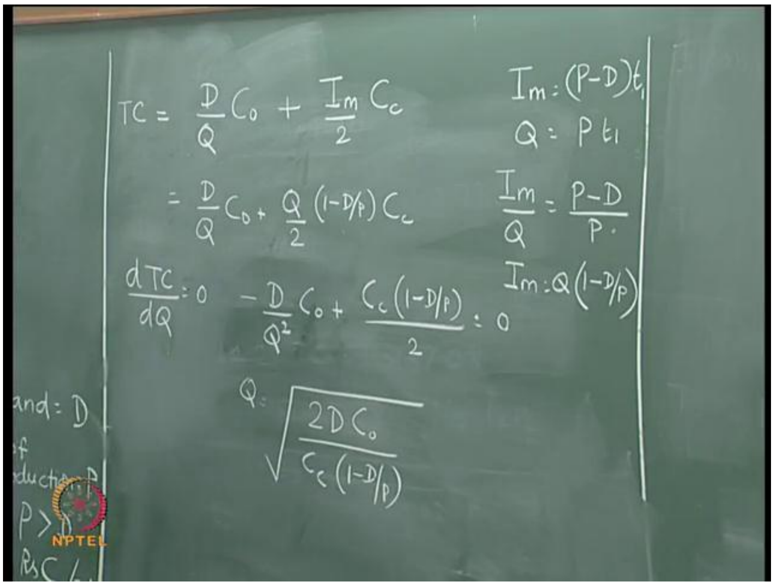

Therefore, the number of times we setup this machine will be D by Q or the number of batches we produce would be D by Q. Every time we setup we incur a setup cost of C0so, we incur a cost of D by Q into C 0 which is a very familiar term which we have used earlier. Now, this is the total setup cost plus there is going to be an inventory cost associated here. Now, this is the cycle starting from 0 to a cycle T (Refer Slide Time: 33:31) which is t 1 plus t 2 another cycle. So, inventory total inventory in the cycle as area under the triangle, average inventory will be area under the triangle, divided by the base. So, average let us call this quantity as Im which is the maximum inventory that we have built.

So, area of the triangle is half into I m into t, divided by another t is the average. So, the average inventory will be I m by 2 and the inventory holding cost will be rupees C c per unit per year. So, the total cost associated here, is the sum of the setup costs as well as the sum of the inventory holding costs. Now, we need to write this I m as a function of Q. Q is the only variable we need, to write this I m as a function of Q. Now, Q is the total quantity that we produce, I m is the maximum inventory, that we actually built it. (Refer Slide Time: 33:31) So, we also know that, as we produce at the rate of P, there is a consumption at the rate of d.

So, this inventory is building up at the rate of P minus D. So, I m is equal to the maximum inventory, that we have which will be P minus D into t 1.So, I m is equal to P minus D into t 1, t 1 is the time period for which we produce. Inventory is building up at the rate of P minus D. So, I m is P minus D into t 1. Now, Q is the total quantity that we produce. Now, we produce at the rate of P for a period t 1. So, Q is equal to P into t 1 right P into t 1. Now, I now, dividing 1 by the other Q by I m or I m by Q is equal to P minus D by P or I m is equal to Q into1 minus D by P. Now, substituting for I m, we get D by Q C naught plus Q by 2 into 1 minus D by Pinto C c. Now, we need to find out the best value of Q which minimizes the total cost.

So, this is got by differentiating d T C by d Q equal to 0, would give us minus D by Q square C naught plus C c into 1 minus D by P divided by 2 equal to 0, from which Q is equal to root over 2 D C naught by C c into 1 minus D by P. So, this is the best value of the economic batch quantity, such that we optimize the total cost. Now, once we have this equation, we can find out the economic batch quantity. Given values of D C naught Cc etcetera. So, let us take a numerical illustration and show the computation of the economic batch quantity and the associated total cost.

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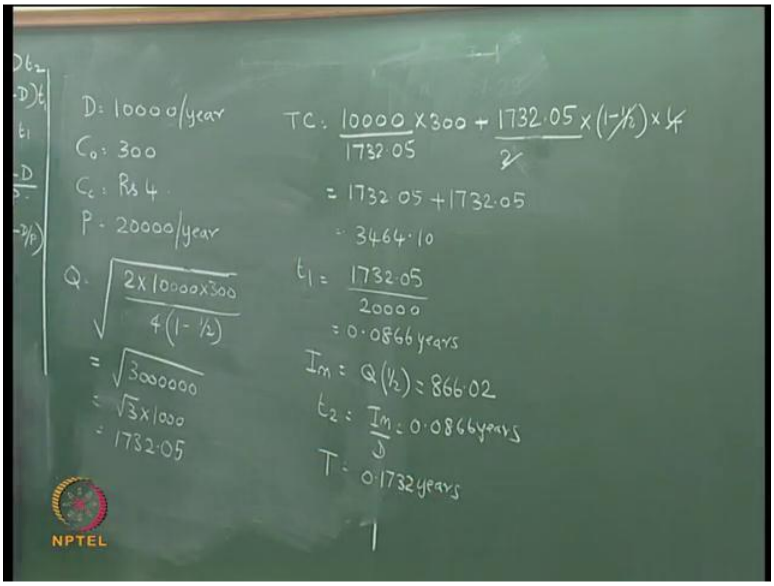

So, let us look at a illustration where we have D is 10,000 per year, C naught is 300 C c is 4 and P is 20,000 per year. Now, note that D and P should have the same unit, because we have a 1 minus D by P coming in. So, if D is per year, P is also per year. So, Q economic batch quantity is given by root over 2 into 10,000 into 300 divided by 4 into1 minus D by P or it is 1 minus 1 by 2,1 minus 10,000 by 20, 000, so one minus one by 2.

So, this will give us 4 into 10,000 into 300 divided by 4, which is root over 10,000 into 300 this will be root 3 into 1000 this is 1732.05. So, every time we produce, we produce quantity equal to1732.05.

The total cost will be D by Q 10,000 divided by 1732.05 into 300 plus Q by 2 1732.05 divided by 2 into 4 into 1 minus D by P, 1 minus half. So, this part will be equal to 1732.05 plus into Q by 2 into C c into 1 minus D by P. So, this is half. So,2 into 2, 4 and this 4 will get cancelled plus another 1732.05 which is 3464.10. So, when we produce, we produce a quantity of 1732.05. We also produce for a time t1. t 1 is equal to from here t 1 is equal to (Refer Slide Time: 39:28) Q by P which is 1732.05 divided by 20,000 which is 0.0866 years. Now, this can be converted into hours to know, what is the number of hours, that we produce. Similarly, t 2 can also be calculated and the cycle time t (Refer Slide Time: 33:31) can also be calculated.

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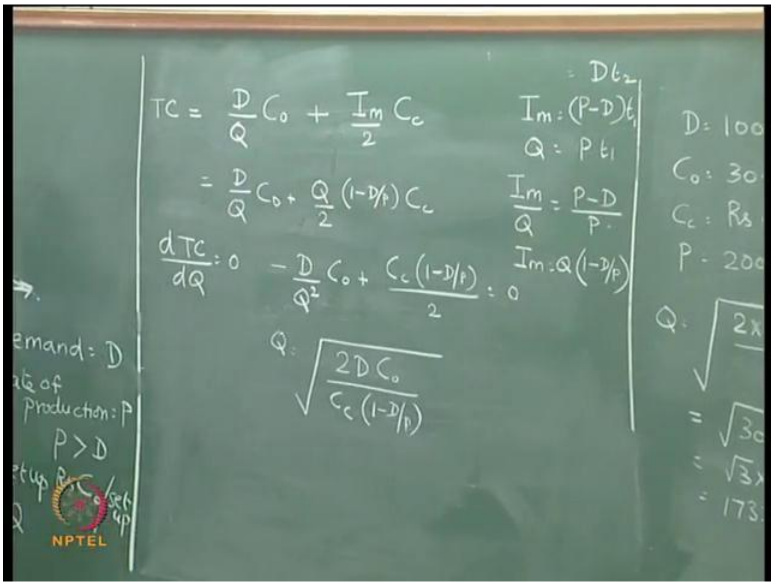

Now, t 2 it is very easy to write, t 2 is P 1 is equal to P, I m is equal to P minus D into t 1 I m is also equal to D into t 2. The reason being this, much inventory (Refer Slide Time: 33:31) that is built up as I m is now, consumed over a period t 2 at the rate of D. So, I m is equal to D into t 2 from which t 2 can be calculated and the cycle time t can also be calculated. Now, t 2 now, in order to calculate t 2, we first need to find out, I m. So, I m is equal to Q into 1 minus D by P. So, Q into half so I m will be 866.02 that is, that is not t 2. So, I m is equal to Q into 1 minus D by P. So, I m is 866. t 2. So, t 2 is equal to I m divided by D which will be another 0.0866 years and the cycle t will be 0.1732 years.

So, in this particular example, when we draw the graph you realize that t 1 is equal to t 2. Need not be the same for all other kind of situations. So the next model that we should be looking at is, can we allow for some backordering here like we did in the bought out items? Can we allow for some back ordering? Build up a backorder and then produce use the, satisfy the backorder first and then build up the inventory. So, that would be a production consumption model, with backordering which we will see in the next lecture.