Operations and Supply Chain Management
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Lecture - 13
Multiple Item Inventory - Constraint on Money Value, Space, Equal Number of Orders

In the last lecture, we addressed multiple item inventory problems, and we specifically looked at a situation where there was a restriction on the total number of orders to be placed, considering multiple items. In this lecture, we will consider another restriction or constraint where there is a limit on the money value of inventory that is being bought or locked up. So, let us consider the same example and then let us derive the result and then apply it to the same numerical illustration.

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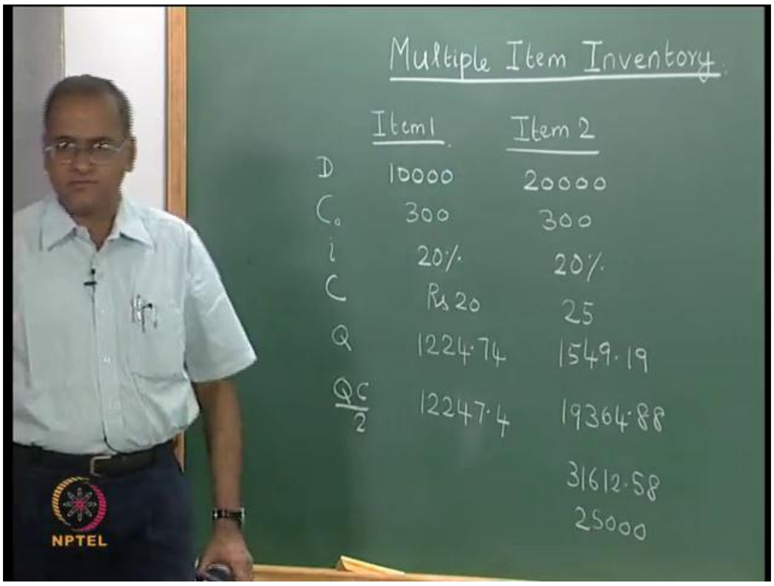

So, let us consider 2 items whose demands D are 10,000 and 20,000 per year. The order cost C naught is the same 300 per order. The inventory holding cost is 20 percent of the unit cost for both the items and the unit price of the items are 20 and 25. We have already computed the economic order quantity for both these items and the economic order quantity comes to 1224.74 for the 1st item and 1549.19 for the 2nd item.

Now, the numbers of orders are also different for each of them and in the last lecture we looked at a situation where there was a limit or restriction on the total number of orders.

And, at the end we also derived an expression and showed that if there is a restriction on the total number of orders and if that constraint is binding then the order quantities of both the items increase in the same proportion. Now, we are trying to place another constraint which is a restriction or limit on the value of money that is being locked up in the average inventory. Now, if the order 1224.74 of this item the average inventory is half of this, Q by 2 which we have already seen. And, the money value of the average inventory which we would call as Q by 2 into C, C is the unit price of the item so that will be 1224.74 divided by 2 into 20 which is 12,247.4.

And, for the 2nd item it will be 1549.19 into 25 divided by 2 which will work out to 19,364.88. So, this amount 12,247.4 and 19,364 represent the money value of the average inventory because we start the cycle with 1224.74. The cycle ends with 0 and we have already computed that the average inventory is Q by 2. So, the money value of that is so much. So, put together the money value of both these items comes out to 31,612.58. Now, these two items put together we observed that the average money value of the average inventory is 31,612.58. In large manufacturing organizations, there would be hundreds and thousands of items and the money value of the average inventory can be very high.

So, organizations place a restriction on the money value of the average inventory held at any point in time. And then the order quantities are adjusted such that the money value of the average inventory held at any point does not exceed a given predetermined or given value. So, in this case if we say that these two put together the average, if money value of the average inventory is 31,612. Now, we would pose a question and say that if the money value of the average inventory held is restricted to 25,000 instead of 31,612, what happens to the order quantities? So, this leads us to an optimization problem where we try to minimize.

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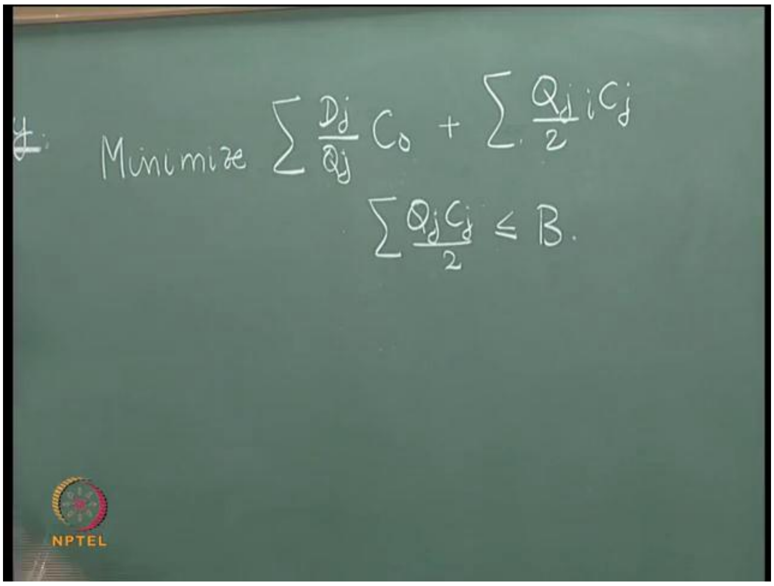

Now, D j is the annual demand for item j, Q j is the order quantity. So, D j by Q j is the number of orders into order cost plus sigma Q j by 2 into i C j. Now, Q j by 2 is the average inventory, i into C j is the inventory holding cost per unit per year. Here, the demand is also in years. So, this represents the cost of ordering plus cost of carrying subject to the condition that sigma Q j C j by 2 is less than or equal to some B. Now, Q j C j by 2 is the money value of the average inventory. Please note, that there is no i here whereas there is an i here. The moment there is an i here then this represents the inventory carrying cost per year. This represents the money value of the average inventory. So, Q j C j by 2 is less than or equal to B. Now, this is a non-linear optimization problem because Q j comes in the denominator here, comes in the numerator here with a linear constraint where the Q j comes in the numerator, in the constraint.

Now, the 1st thing we need do is, to solve this problem as an unconstraint problem and then try and solve it, and if it so happens that the solution to the unconstraint problem satisfies this constraint then it is optimal to the constraint problem. So, we follow the same approach and try to solve this unconstraint problem. Now, when we solve this unconstraint problem, this unconstraint optimization problem turns out to be two independent problems, one for each item where the optimum solution is the economic order quantity itself.

We have already calculated the economic order quantity and we have said that (Refer Slide Time: 00:54) the values are 1224.74 and 1549.19. The value of Q j C j by 2 summation for the economic order quantity we have already calculated and that is (Refer Slide Time: 00:54) 31612.58. And now the, which is the limit on the money value of the average inventory is 25,000. Therefore, the solution to the unconstraint problem which is the economic order quantity while it is the constraints and the constraint is binding and the solution to the unconstraint problem is not optimum or feasible to the constraint problem. So, we have to use this constraint and then solve it again. Like, we did in the previous illustration by the very nature of the total cost function, if this constraint is binding and this has to be satisfied or if this is violated by the E O Q then when we solve the optimization problem.

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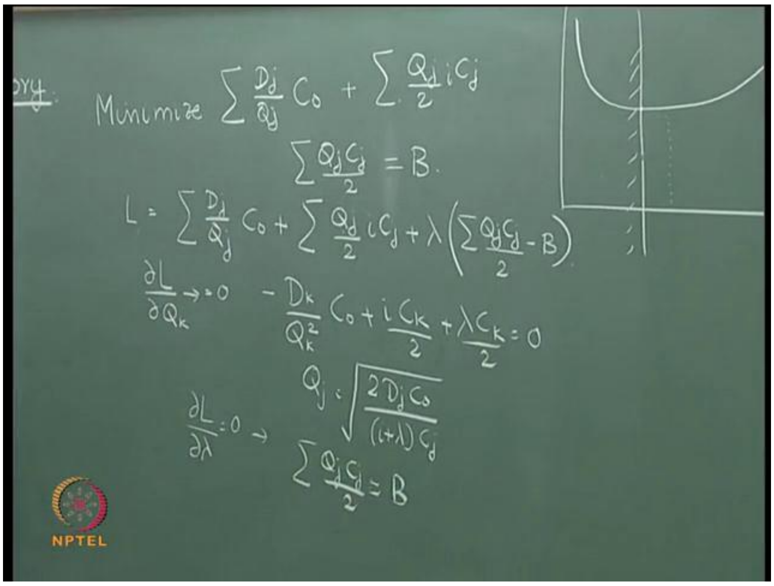

The order quantities will adjust themselves such that Q j C j by 2 will become exactly equal to B and will not be less than B by the, very nature of the total cost curve. We have already seen that the total cost curve is like this. The optimum is somewhere here and a limit on the money value of the average inventory is like adding a constraint which is like this. And therefore, the optimum will come here and the optimum will not come to the left of the point of intersection. Therefore, when this constraint is binding it will be an equation and it will not be an inequality. Now, the moment we understand that this is going to become an equation then we can use the method of Lagrangian Multipliers as we did in the earlier case and then solve it. So, the Lagrangian problem will become L is equal to sigma D j by Q j C naught plus sigma Q j by 2 into i C j plus lambda into sigma Q j C j by 2 minus B.

Now, we can partially differentiate the Lagrangian function with respect to Q and with respect to lambda to get the solution. So, partially differentiating with respect to a Q k and setting it equal to 0 would give us minus D k by Q k square C naught plus i C k by 2 plus lambda C k by 2 equal to 0. Now, this on simplification would give us Q j, I am writing it for a general j, as root over 2 D j C naught into i plus lambda C j. Now, we can compute Q j which is what we want only if we know the value of lambda because right now in this equation all other values other than lambda are known. So, we need to find out lambda and in order to find out lambda we partially differentiate L, the Lagrangian function with respect to lambda. So, dou L by dou lambda equal to 0 would give us sigma Q j C j by 2 is equal to B. So, in order to find lambda we substitute for Q j from this and therefore we get:

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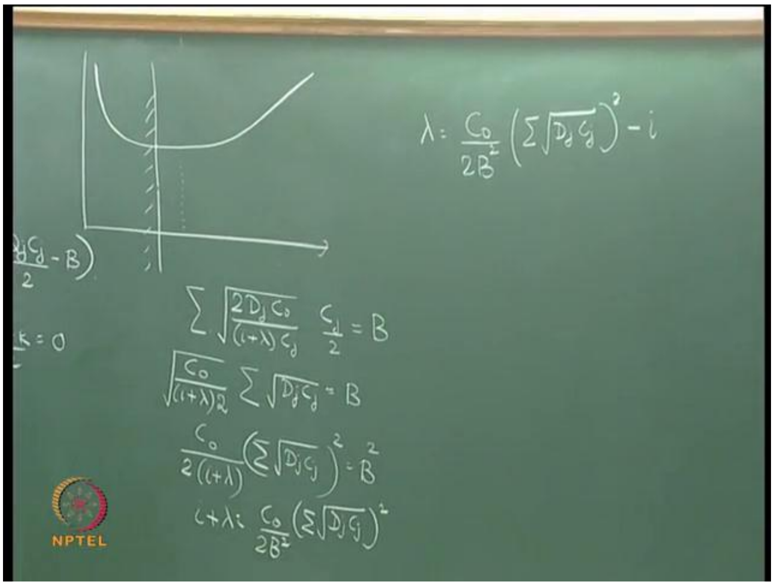

Something like sigma, now Q j is root over 2 D j C naught by i plus lambda C j into C j by 2 equal to B. So, we can take root over C naught by i plus lambda, C naught root over C naught comes out root over i plus lambda comes out. There is a root 2 here, there is a 2 here. So, there is a root 2, so which comes under this square root. Then sigma root of D j C j is equal to B. There is a root C j here, there is a C j here so root over this. So, squaring we get C naught by 2 times i plus lambda into sigma root over D j C j square is equal to B square, i plus lambda is equal to C naught by 2 B square sigma root over whole square from which lambda is equal to C naught by 2 B square sigma whole square minus i. So, this is the expression for lambda and then we can now substitute for the values here to try and get lambda and then we can use this lambda substitute it here to get the value of the Q j.

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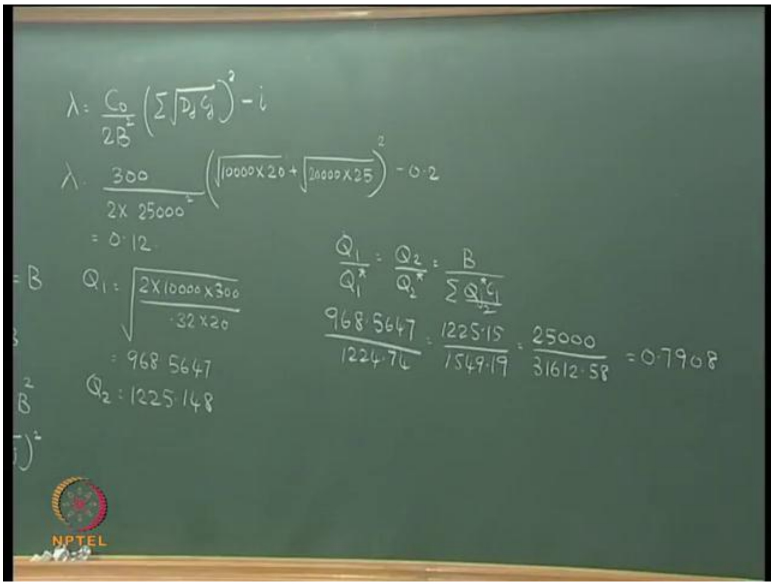

So, now we apply this to the numerical illustration that we have. Therefore, applying this numerical illustration lambda is equal to 300 divided by 2 into 25,000 square. We have now kept the B as 25,000. So, 25,000 square to root of 10,000 into 20 plus root of 20,000 into 25 the whole square minus point 2. And, this on simplification would give us lambda equal to 0.12 and once lambda equal to 0.12 we can go back and calculate Q 1 and Q 2 out of this. So, Q 1 will be equal to root over 2 into 10,000 into 300 divided by 0.32 into 20. The 0.32 comes to out of i plus lambda. (Refer Slide Time: 09:17) So, i is 0.2 lambda is 0.12, so 0.32 into 20. So, Q 1 on computation is 968.5647 and Q 2 is 1225.148.

So, we observe that the order quantities have come down. They have become 968.5647 and 1225.148 as against (Refer Slide Time: 00:54) 1224.74 and 1549.19. We can also do this simple calculation and say that 968.5647 into 20 divided by 2 which is Q 1 C 1 by 2 plus Q 2 C 2 by 2 will add up to exactly 25,000 which is the restriction that we have put here. There is also an interesting rule very similar to what we did in the last time.

Now, we want to bring (Refer Slide Time: 00:54) a total of 31612.58 to 25,000 it is enough to proportionately adjust the order quantities. So, we can also prove that Q 1 by Q 1 star is equal to Q 2 by Q 2 star is equal to B by sigma Q j star C j by 2. For example: 968.5647 divided by 1224.74 is equal to 0.7908, it is 0.7908. Now, this is the same as 1225.15 by 1549.19 and it is the same as 25,000 divided by 31,612.58. Now, we realize that the ordering quantities get proportionately adjusted and the factor is 0.7908. It is also possible to derive through this that the ordering quantities do get proportionately adjusted.

Now, there are two other issues that we need to look at. One is we are able to get a closed form solution for lambda. We are able to get an expression for lambda which if we substitute all these values we can get the correct value of lambda which would take us there. That has come largely because the term, (Refer Slide Time: 09:17) the constraint term is Q j C j by 2 minus B as it goes into the Lagrangian and there is also a Q j i C j by 2. So, this term and this term are more or less similar and with the Lagrangian multiplier I get an i and a lambda. Therefore, I get an i plus lambda which goes into it and therefore, we are able to get a solution. In the previous case where we put a restriction on the number of orders the constraint was of the type D j by Q j equal to n. So, there is a C naught and there was a lambda. So, we could get a C naught plus lambda term that comes ((Refer Time: 20:01)).

It is also important to note that such a closed form solution was possible because C naught and i (Refer Slide Time: 00:54) are assumed to be the same for both the items. If C naught and i were not the same for both the items and if we had a C 0 1 and a C 0 2 which were distinctly different then it is not possible to straight away get this closed form solution for this. It will become difficult to do that. Then it comes to the issue of, are we justified in giving the same value of C naught and the same value of i for both the items? In a way giving the same value of i can be understood and explained because i represents the interest on the money that is borrowed to procure these items. And, we can assume that irrespective of the item that is procured, the interest remains the same and therefore, the carrying cost becomes i into C and i is the same for all of them.

Now, are we justified in keeping C naught the same for all of them?? In some sense the answer is yes because of may be two different reasons. One is: it is also extremely difficult to individually compute the C naught for various items and if we do so at the end of it we also realize that they are not very different from each other. The difference may be very small and negligible. The other way to look at it is C naught essentially, the components of C naught are: inspection, transportation, order placing and so on. And, if the same people are going to do the inspection then the inspection cost cannot be different.

But, if the inspection requires different types of equipment or if the inspection requires a difference skill set of the person inspecting then C naught can be different. Similarly, the transportation is essentially going to be through trucks and if we assume that irrespective of the item that is transported, the truck cost is the same. Then it is not unrealistic to assume that C naughts the same. So, C naughts can be assumed to be the same. However, if some of these items require special features for example, if we require an air conditioned truck to carry something while for some other item you do not need an air conditioned truck then the order cost can vary.

But, for the purpose of this computation we assume that order costs are the same, i is the same and because these constraints (Refer Slide Time: 09:17) are of very similar nature we are able to get an expression for lambda (Refer Slide Time: 14:29) which we can use. And, at end of it we are also able to prove through this though we have not taken the trouble of showing it that Q 1 by Q 1 star, star is the economic order quantity, this is the new order quantity. So, (Refer Slide Time: 14:29) new order quantity by economic order quantity follows this proportion.

So, it gets proportionately adjusted as there is a limit on the total number of items that are there. The other aspect that needs to be looked at is also somewhere here. (Refer Slide Time: 09:17) We said that if this constraint is violated by the economic order quantity then this constraint will be satisfied as an equation and not satisfied as an inequality. Now, the understanding that this is satisfied as an equation helps us use Lagrangian Multiplier as a methodology to solve it. So, the very idea of using Lagrangian Multipliers is centered around the fact that we are solving a constraint optimization problem where the constraint is an equation.

So, for example if we just, for the sake of argument or for the sake of completion if we say that instead of (Refer Slide Time: 00:54) 31,612 we solve this problem with 25,000. Suppose, we solve this problem with 35,000 say. If the limit on the money value of the average inventory was, 35,000 instead of 25,000 then what happens? If it was 35,000 and if we had blindly use this formula to get lambda, then we would get a negative lambda here. If we substitute B as 35,000 instead of 25,000, we would get a negative lambda. A negative lambda means at this point, a negative lambda means that, the constraint is actually satisfied by (Refer Slide Time: 09:17) the economic order quantity. Therefore, one should not blindly use this lambda. One should use this lambda, only after ascertaining that the economic order quantity violates the condition (Refer Slide Time: 09:17) Q j C j by 2 is less than or equal to B.

If we forget that and we still use the formula blindly and if we get a negative it is an indication that, the economic order quantity actually satisfies the condition or satisfies the upper limit on the money value of the average inventory. If we continue to make the mistake of using a negative lambda and substituting it here into this (Refer Slide Time: 09:17) then we will realize that since lambda is negative the economic order quantity will go up and the economic order ((Refer Time: 25:39)) or the new order quantities will be such that, the sum of the money value will be 35,000. Because, this lambda formula comes from satisfying this (Refer Slide Time: 09:17) as an equation. So, this will always push to be equal to their number that we give here (Refer Slide Time: 00:54). So, the correct procedure is to solve for the economic order quantity and only when, the economic order quantities violate the condition (Refer Slide Time: 09:17) Q j C j by 2 is less than or equal to B, then we make it equal to B and then use the method of Lagrangian multipliers.

So, Lagrangian multiplier method should not be used blindly, for every value that is given here. (Refer Slide Time: 00:54) It has to be used only in the cases where, the value given here is less than the value given here. Otherwise, in its anxiety to make this (Refer Slide Time: 09:17) as an equation it will end up pushing the order quantities up, which is not desired. (Refer Slide Time: 00:54) And, if this were 35,000, then the economic order quantity which gives 31,612 is indeed optimum and therefore, there is no question of this constraint coming into play at all.

So, all these constraint multiple inventory problems we first solve for the economic order quantity and only when the economic order quantity violates the condition, we use the method of lagrangian multipliers and we solve them. Other thing is, when (Refer Slide Time: 00:54) the constraint is binding as in this case when we compute, you find here that lambda is 0.12, i is roughly 0.2. So, in all these, in this particular problem when there is a restriction on the money value and if the constraint is binding, lambda will usually be in the order of i, for example, we will not get lambda equal to 400 and so on. It will be of the order of i.

In the earlier example where, there is a restriction on the number of orders lambda would be of the order of C naught because the term here (Refer Slide Time: 09:17) is i plus lambda. So, lambda will be of the order of i. In the earlier illustration the term was C naught plus lambda. So, lambda would be of the order of C naught that is another little hint that we have. So, lambda would be of the order of i. So, let us now move on to another type of constraint problems.

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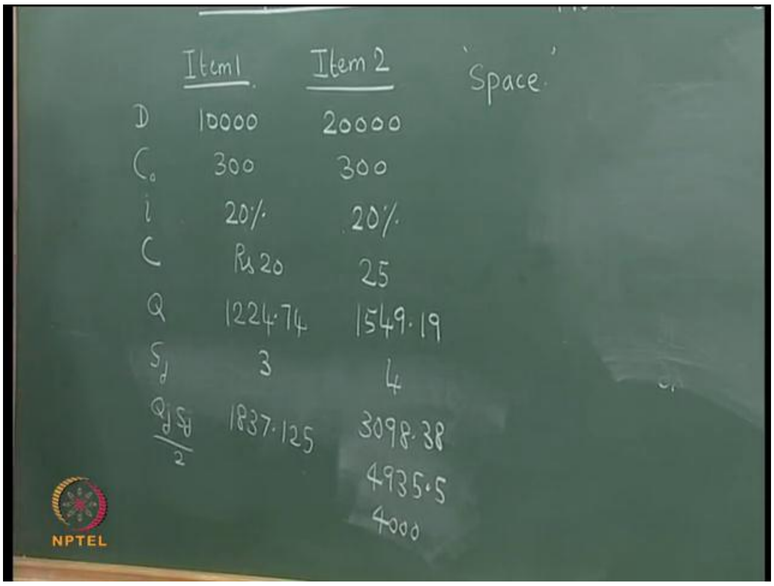

Now, let us look at the 3rd type of a constraint, where we place a restriction on space. Let us assume that there is a space restriction to store the items. Now, let us assume right now, that the spaces required are 3 and 4 respectively. So, let me call this as S j which is the space required to store a unit of this item and this is space required to store another unit of this item. So, let us assume that these are in say cubic feet and this requires space of 3, this requires space of 4.

So, what we are going to do now is the average space required for these two items. So, we will do Q j S j by 2, which represents the average space required for each of these items. So, Q j S j by 2 on computation would give us, 1837.125. So, this would give us

1837.125 and Q j S j by 2 for the other item would be 3098.38. So, this gives us 3098.38 and the total space that is required is 4935.5. So, let us assume that the space required for, storing the average inventory value of both these items is 4935.5 and let us assume that, we have space of only 4000 to store these items. Now, how do we solve this problem? Now, before we actually solve the problem, let us look at one small aspect of the whole thing. Now, here we have assumed the space requirement for the average inventory and we have not looked at this space requirement for the entire inventory that arrives.

So, there is this question whether, the space requirement is for storing the average inventory or whether the space requirement is for storing the entire inventory that we have. Now, we use average inventory simply because there are multiple items and while some of these items will have very large inventories, some of them will have smaller number of inventory. And therefore, we are justified in assuming that at any given point in time, we are only going to have a certain average inventory in the system, whose storage space is what we are looking at. Sometimes, we may take an extreme view and say that we should have enough space, when the entire consignment comes and which means we need Q j S j and not Q j S j by 2. So, all these numbers get doubled but then the methodology of solving it is not affecting by that decision. So, we will proceed by assuming that these space requirement is for the average inventory, which means on an average we require 4935.5 cubic feet or unit subspace and let us assume that we have a constraint of 4000. So, now we need to solve this problem because the economic order quantity again does not satisfy this.

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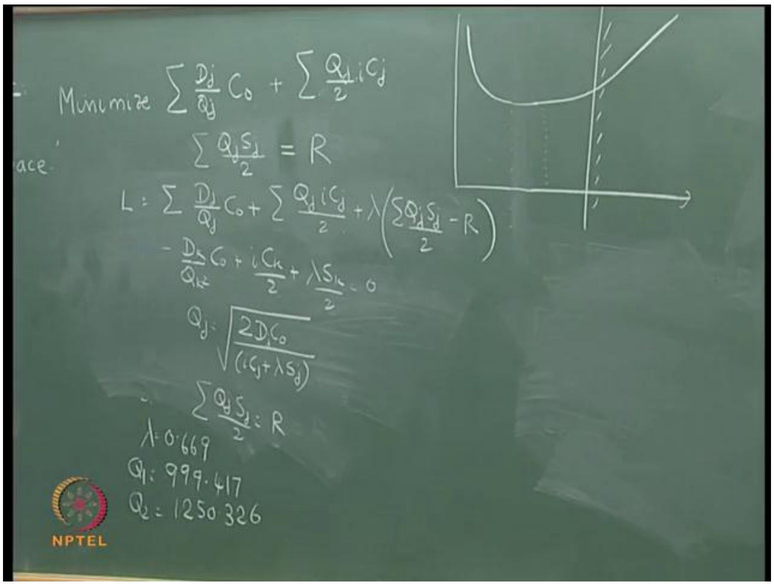

So, we will say that I now want to minimize the sum of ordering cost and carrying cost subject to the condition that the space required by the average inventory is less than or equal to, some R. So, as usual we solve the unconstraint version. So, the unconstraint version gives us the economic order quantities (Refer Slide Time: 28:48) and average space requirement of 4935.5. Now, this is higher than the available space of 4000. So, we once again do the lagrangian and say L is equal to and once again say that this inequality at the optimum will become an equation, by the very nature of this. Obviously a restriction on the space is going to increase the order quantity somewhere here, please note that this line does not exist. So, it is going to increase the order quantity.

So, lagrangian is D j by Q j C naught plus sigma Q j i C j by 2 plus lambda sigma Q j S j by 2 minus R. So, partially differentiating with respect to Q j, we get minus D j by Q k, let us do it with some Q for some Q k, C naught plus i C k by 2 plus lambda S k by 2 equal to 0, from which a general Q j is equal to root of 2 D C naught by i C j 2 D j C naught i C j plus lambda S j. Now, we do not know this value of lambda. So, in order to find lambda, we partially differentiate L with respect to lambda to get sigma Q j S j by 2 is equal to R. Now, in this particular case we, even if we substitute this Q j into this and try to write this equation, we will not be able to get a unique expression for lambda because of this term i C j plus lambda S j that comes in.

So, we do not get a closed form solution. You do not get an expression for lambda which we can use. So, in this case we can solve for lambda only using a trial and error method. So, we can try various values of lambda and then check for a chosen value of lambda, you can calculate Q 1 and Q 2 and then substitute here and that value of lambda for which l h s equal to r h s, is the best value of lambda. So, when we apply this to this numerical example we need to try out different values of lambda and then we observed after an extensive amount of computation, that lambda is 0.669 in this example. So, lambda equal to 0.669. This gives us Q 1 is equal to 999.417 and Q 2 as 1250.326. So, Q 1 and Q 2 are now different.

They decrease because (Refer Slide Time: 28:48) 1224.74 becomes 999.417 because there is a restriction on the space so the order quantity has to come down. So, 1224.74 becomes, 999.417. Let me again repeat that for the economic order quantity the space required is 4935, which is higher than the available space. Since, the available space is lesser a restriction on the space will only reduce the ordering quantities. So, order quantity will reduce from (Refer Slide Time: 28:48) 1224.74 to 999.417 and from (Refer Slide Time: 28:48) 1549.19 to 1250.326.

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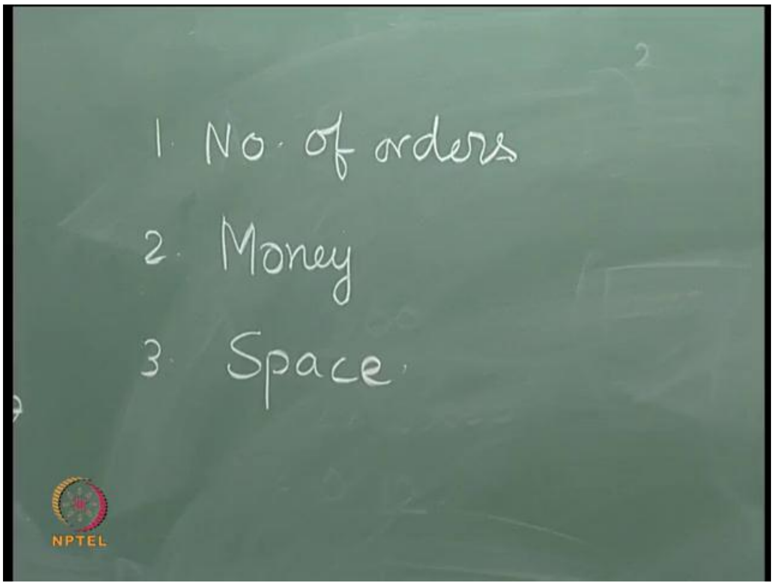

So, we have seen three types of constraint inventory problems till now: one is the constraint on the number of orders, the second is on money value and the third is on space. And, we derived three different or we solved the problem for three different instances individually, we did not look at two constraints at a time. Now, suppose we had both this constraint: number of orders, as well as money. Say, in this (Refer Slide Time: 28:48) numerical example we said that the number of orders should be 15 and the money should be less than 25,000. Now, if we have two such constraints then how do we solve such a problem? Now, an obvious thing to do is to try and say that, we write two constraints (Refer Slide Time: 32:34) and we could bring two Lagrangian Multipliers and bring them into the objective function and then solve it so that is methodologically correct. But then let us also try and see what happens. So, if there is a restriction on the number of orders, then the order quantity will increase.

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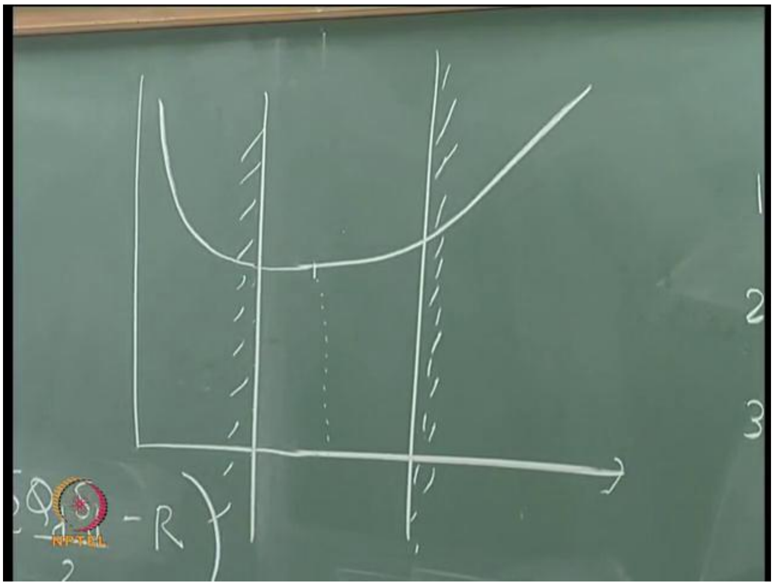

So, this is the economic order quantity. So, order quantity increases and you have something like this. So, the optimum will be shifted to the right, which means the order quantity increases. If there is a limit on the money value and the economic order quantities do not satisfy that, then that would make the order quantities decrease from this side and the order quantity will shift to the left. So, if we have a situation where the economic order quantity is violating (Refer Slide Time: 37:36) both this restriction and this restriction, then we cannot have a feasible solution to the problem. So, we need not take the trouble of writing two constraints and taking it into the lagrangian and differentiating it. If the economic order quantity violates both these, then it is obvious that the problem does not have a feasible solution at all.

(Refer Slide Time: 37:36) Now, we could think in terms of number of orders and space. Space will behave the same way as inventory behaves. Because, if there is a restriction on the money value, (Refer Slide Time: 28:48) the economic order quantity, the order quantities come down. If there is a restriction on the space the order quantities come down. (Refer Slide Time: 37:36) So, if we have a restriction on the number of orders and then another restriction on the space and the economic order quantity violates both of them, then once again by the same argument the solution will be infeasible because this restriction will try to move it in this direction, (Refer Slide Time: 37:36) space will try to move it in the other direction and therefore, you will not have a feasible situation.

So, the only thing that we could think, in terms of is if we have a restriction on the money and if we have a restriction on the space (Refer Slide Time: 37:36) both try to move it in the same direction. And therefore, we will have a solution, when we have restriction on the money, as well as restriction on the space. When we have such a situation it is actually easier to solve (Refer Slide Time: 37:36) for this 1st and then we try and see whether this solution is satisfied by this. If so it is acceptable. If not, we have to solve for this and the solution will automatically satisfy this. Because, solving the problem with the money restriction is easier because we have closed form solutions, the proportionality rule also works.

So, it is easy to solve (Refer Slide Time: 37:36) for this 1st and then check whether that solution satisfies this. Then it is optimal to both, otherwise we have to solve for this and it will automatically satisfy the other one. So, that is the way we take care of multiple items. We do not prescribe to have a situation where (Refer Slide Time: 32:34) you have more than one constraint and then you take more than one Lagrangian multiplier. The labor required is very high and it gets highly complex. Whereas, always solving it as a single constraint problem, one at a time is a lot easier than building two constraints into the problem. Now, let us look at one more version of this problem. Now, particularly when we have multiple items, we would also like to consider another aspect when we have multiple items.

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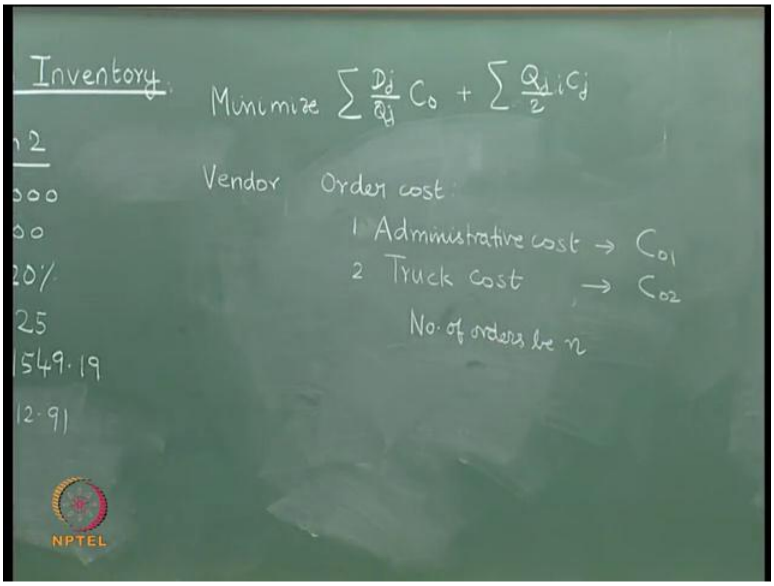

Now, let us assume that multiple items essentially come from the same vendor. Now, let us assume that these two items come from the same vendor who supplies both of them.

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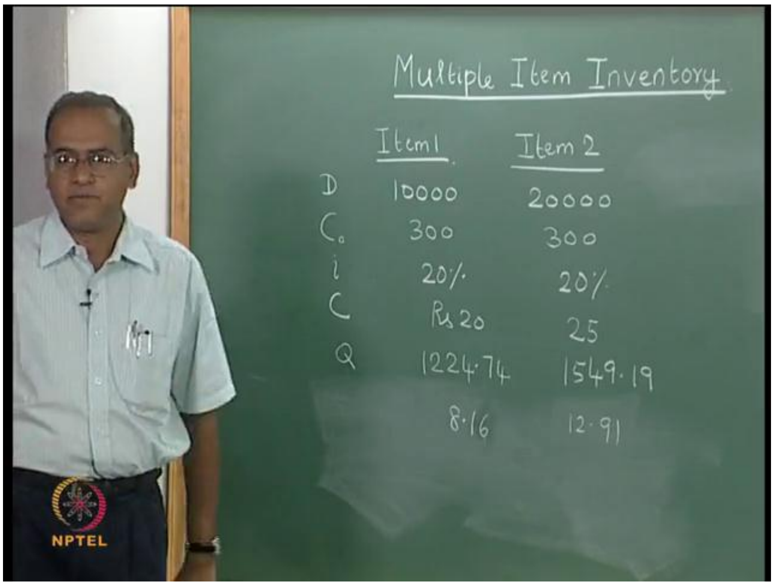

Now, if we chose to order the economic order quantities of each of these, then we have already found out that this will be some 8.17 orders and the other one will be roughly about 8.16 and 12.91 orders. So, 8.16 orders and 12.91 orders per year and we also know that, the ordering cycles do not synchronize. For example, we will be placing orders every 47 days here and roughly for about 30 days or 29 days in this one, when the order cycles do not synchronize.

So, if we look at the ordering process, the ordering process particularly it is the order cost in particular is largely dominated by, the cost of transportation and the cost of inspection. Now if we, since both these are coming from the same vendor, if we are able to order them together and make the number orders equal so that the ordering frequencies are the same. Now, both these can come in the same truck from the supplier, to the factory. The inspection costs still have to incurred for this as well as this but the transportation cost can be significantly reduced because both can come in same truck. If they are ordered differently, we know that this has to come separately, this has to come separately. So, the cost of transportation is twice, where as if we do this the cost of transportation is single. Therefore, what we do to take advantage of multiple items that go to the same vendor? (Refer Slide Time: 42:38) We now split the order cost into two components: one is called the administrative cost and the other is called the truck cost.

So, if we combine two orders we still incur two administrative costs but then we incur only one truck cost. Now, there is a saving in the total order cost when items are order together and when they come from the same vendor. Now, is this economical or if it is economical, can we implement it is the other question that we would like to look at. Now, let us assume that, the order cost is made up of two components. Now, this is called the administrative cost which we call as C 0 1 and this is called the truck cost, which is called as C 0 2. Now, let us assume that we order both these items together. Now let there, let the number of orders be n. So, there are n orders that are going to be there.

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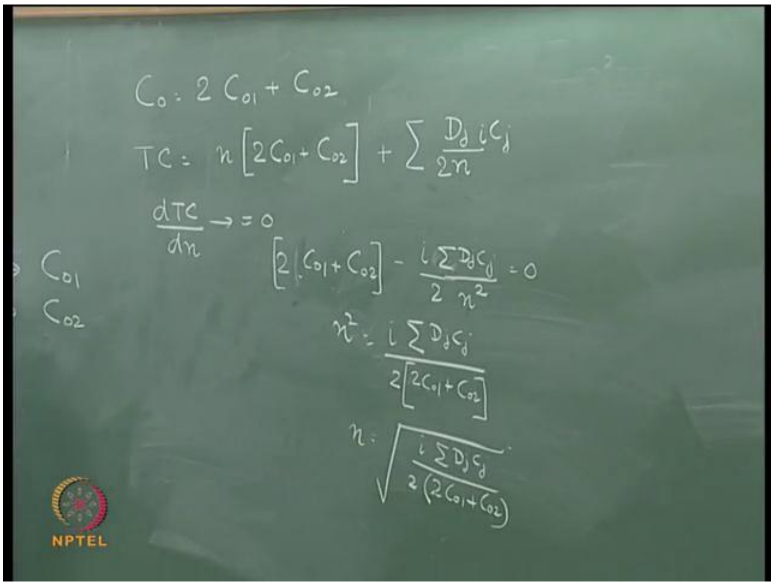

Now, when we order them together the total order cost C naught will be 2 times C 0 1 plus C 0 2 because there is going to be (Refer Slide Time: 42:38) only one truck cost, there will be two administrative costs. So, if we order both together and there are n orders the total cost will be, (Refer Slide Time: 42:38) if there n orders per year, then the total cost will be n into 2 times C 0 1 plus C 0 2. This is the total ordering cost per year. The total carrying cost per year will be, if the demand is D j for item j, then the order quantity Q j is D j divided by n because there are n orders per year. So, Q j will be D j by n and the inventory holding cost is Q j by 2 into i C j so Q j by 2 into i C j, summation of this. Now, we need to find out the best value of n. So, that can be got by differentiating d T C by d n and setting it equal to 0. So, this would give us 2 times C 0 1 plus C 0 2 minus i C j by 2 or i sigma D j C j by n square is equal to 0. Now, this n will go in the differentiation so you will get 2 C naught 1 plus C naught 2, i can be taken out, i by 2 can be taken out sigma D j C j is a constant.

So, it will remain here, 1 by n will give us minus 1 by n square is equal to 0. From this, we will have n square or this is equal to so n square will be i into sigma D j C j by 2 into 2 times C 0 1 plus C 0 2. Take this to the other side, n square goes here, this quantity comes down. So, n square is i sigma D j C j by 2 times this and n is equal to root over i sigma D j C j by 2 times 2 C naught 1 plus C naught 2. Now, in this D j is the annual demand for item j, C j is the unit cost for item j. The order cost has two components, which is the administrative cost and the truck cost. By combining the orders we incur 1 truck cost and 2 administrative costs and n is the equal number of orders that we have. So, we can use this formula to get this n and then we explain this through a numerical example in the next lecture.