Operations and Supply Chain Management
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Lecture - 12
Marginal Quantity Discount, Multiple Item Inventory – Constraint on Numbers of Orders

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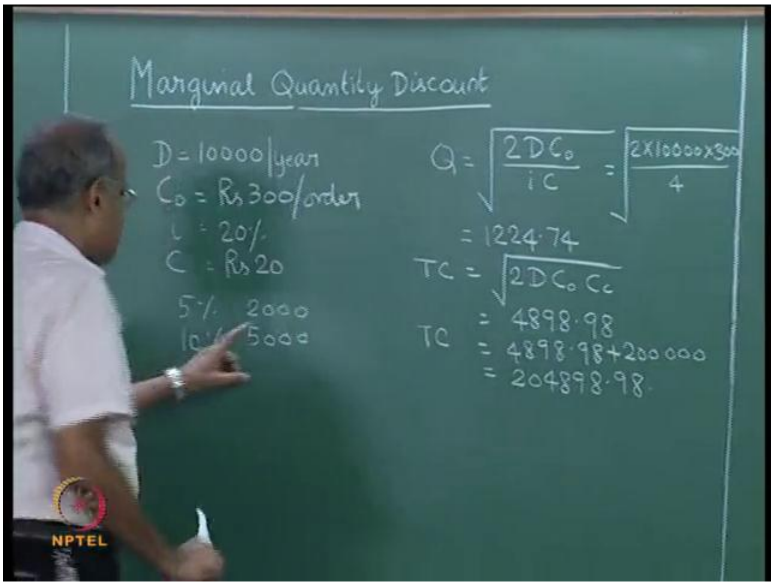

In the previous lecture, we were discussing marginal quantity discount through a numerical example. The example is as follows, there is a single item with an annual demand of 10,000 the order cost is rupees 300 per order. The cost of the item is rupees 20 and the inventory holding cost is 20 percent of rupees 20. If there was no discount then the economic order quantity Q is given by root of 2 D C naught by i into C, 2 into annual demand into order cost by carrying cost, which would give us 2 into 10,000 into 300 by 4, which will give us 1224.74 which we have already seen in the previous lecture.

The total cost associated with this will be, root over 2 D C naught C c which would give us 4898.98, which is a computation that we have seen in the previous lecture. Since, we are going to look at discount models we should also add the unit cost of the item, at the rate of rupees 20 for 10,000 items. So, T C in this case will be 4898.98 plus 200,000 which would give us 204898.98. Now, we apply the marginal discount to this problem. Now, there is a 5 percent discount from the 2000 item and there is a 10 percent discount from the 5000 item.

This means that, if we buy 2000 items then we will be paying 2000 into 20, the discount comes for the 2000 and 1st item onwards therefore, if we buy 3000 items the 1st 2,000 will be purchased at 20 and the next 1000 will avail or will get a 5 percent discount and will be priced at rupees 19. In a similar manner, if we buy 5000 items now the 1st 2000 will be priced at 20 the next 3000 will be priced at 19, the 10 percent discount or a price of 18 will come for the 5000 and 1st item onwards and therefore, if we buy 6000 then the 1st 2000 will be priced the 20, the next 3000 will be priced at nineteen and the remaining 1000 will be priced at 18, which is a 10 percent discount of 20. Now, in order to find the order quantities associated with these marginal quantity discounts are priced break, we use this formula to compute.

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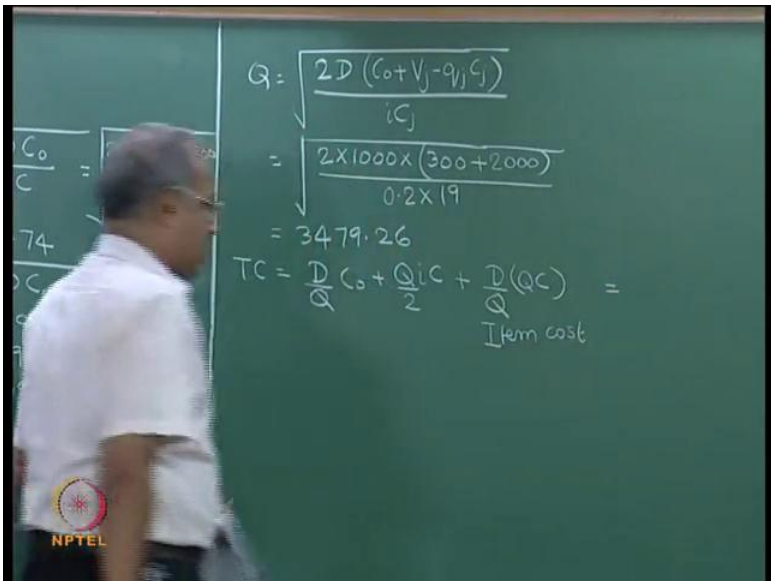

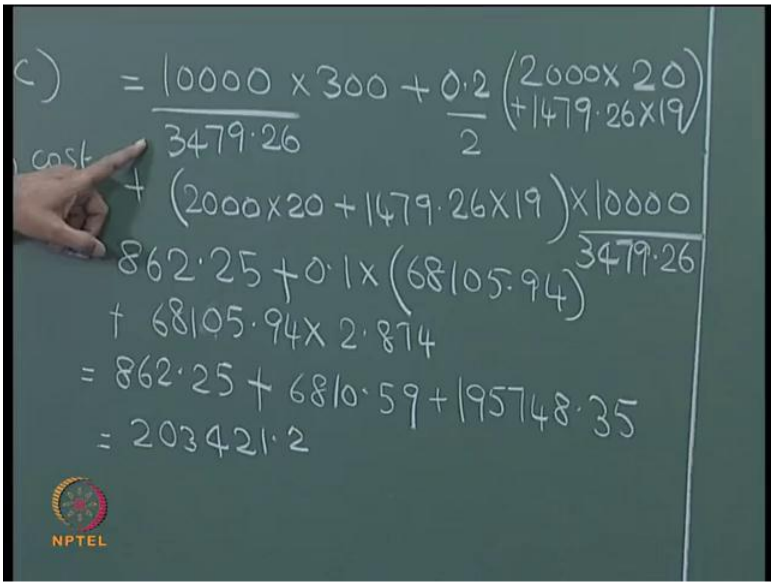

So, the first one which is 5 percent at 2000 onwards gives us Q equal to root of 2 D C naught plus V j minus q j c j divided by i c j. And we use this formula to compute the order quantity considering the marginal quantity discount. Now, we use this formula to get 2 into 10,000 into 300 plus now, we need to write the values of V j minus q j c j. (Refer Slide Time: 00:08) Now, we are considering the quantity 2000 so this quantity 2000 would be brought ordinarily at 2000 into 20 which is 40,000. And after a 5 percent discount the price becomes 19 so 19 into 2000 is 38,000.

So V j is 40,000 q j c j is 38,000 so the difference is 2000 divided by i into c j which is point 2 into 19, 19 is (Refer Slide Time: 00:08) a 5 percent discount on 20 rupees. So, this on simplification would give us Q equal to 3479.26. So, if we avail or wish to avail this (Refer Slide Time: 00:08) 5 percent marginal discount from the 2000th item, we need to order 3479.26 units, which also means that the 1st 2000 units will be priced at (Refer Slide Time: 00:08) 20 and the remaining 1479.26 will be priced at 19.

So, the total cost associated with this will be D by Q into C naught plus Q by 2 into i into c plus D by Q, which is the number of times into Q C or plus item cost. So, this will be 10,000 by 3479.26 into 300, this will be the total ordering cost. Now, this q by 2 into i c will be, now written as i into Q c i by 2 into Q c, which is 0.2 divided by 2 into the cost or prize of the 3479.26. which is 2000 into 20 plus 1479.26 into 19, will be the inventory holding cost plus the item cost will now be. Now, the cost of 3479.26 items is given by 2000 into 20 plus 1479.26 into 19, this is the money value of the item every time we buy that item and in a year the number of times we buy, is given by 10 000 divided by

3479.26.

Therefore, into 10,000 by 3479.26 will be the money value of the inventory. So, let us calculate these to get 862.25 plus 0.1 into 68,105.94 plus 68,105.94 into 2.874 this would give us so this quantity is 195748.35 the first one is 862.25 plus 6810.59, which would give us 203421.2. Now, we observe that, if we compare these numbers with these numbers (Refer Slide Time: 00:08) here the item cost is 200,000 because of the discount the item cost comes down to 1, 95,748. (Refer Slide Time: 00:08)

Here the order cost will be 2449 the order cost is only 862, that is because the order quantity has (Refer Slide Time: 00:08) increased from 1224.74 to 3479.26. So, there are fewer orders in a year so order cost has come down the carrying cost has gone up significantly, (Refer Slide Time: 00:08) 6810.59, because of a large quantity that we order here. So, total cost will be 203421.2, which is less than 204898.98 so it is advantageous at this point to avail the (Refer Slide Time: 00:08) 5 percent discount and order 3479.26.

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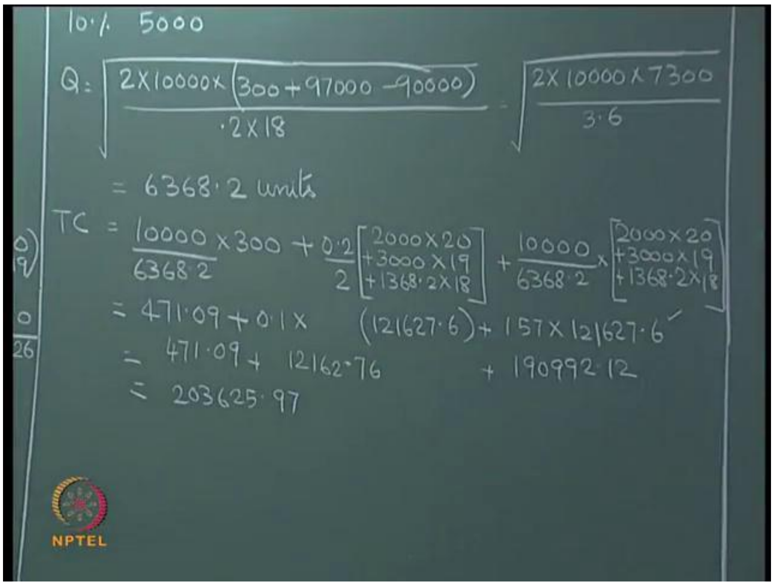

Now, let us look at the 10 percent discount from 5000 so Q will be equal to root of 2 into 10,000 into 300 plus. Now, we need to find out this (Refer Slide Time: 04:19) V j and q j c j. So, if we take 5000 items and if we (Refer Slide Time: 00:08) did not have this 10 percent discount then V j will be the price associated with the 5000 items that we would have bought. So those 5,000 items would be priced at (Refer Slide Time: 00:08) 2000 items at 20, which is 40,000 and 3000 items at 19 which is 57,000. So V j will be 97,000 minus for q j c j now, (Refer Slide Time: 00:08) 5000 is the quantity the price is 18 so 5 000 into 18 is 90,000 divided by 0.2 into 18. So, this would give us root of 2 into 10,000 into 7,300 divided by 3.6, this on simplification would give us 6368.2 units. So, if we wish to avail the additional marginal discount of 10 percent, which is 2 rupees from the 5000th item then we observe that we have to order 6368.2 units.

Now, the total cost associated with this will be sum of order cost plus inventory holding cost plus cost of the item. So, the order cost will be 10,000 divided by 6368.2 into 300, now, this term is this represents the number of orders in a year, multiplied by the order cost per order; this will be the annual order cost. The inventory holding cost will be 0.2, 20 percent of the average inventory so the average gives another by 2 into the money value of the 6368.2 units. So, the 1st 2000 will be priced at 20, the next 3000 will be priced at 19 and 1368.2 will be priced at 18. So, this is the money value of 6368.2 now, that divided by 2 into 0.2, the 0.2 is the 20 percent inventory plus the annual item cost. So, plus the annual item cost will be every time we place an order, we spend so much, the number of times we place an order is given here. So, plus 10,000 divided by 6368.2 multiplied by 2000 into 20 plus 3000 into 19 plus 1368.2 into 18.

Now, let us find out this number first, before we do that we get 10,000 into 300 divided by 6368.2 would give us: 471.09, which is the annual cost plus 0.1 into 2000 into 20 plus 3000 into 19 plus 1368.2 into 18 would give us 121627.6 so 0.1 into 121627.6 plus 10,000 divided by 6368.2, 1.57 into 121627.6. So, this term will be into 121627.6, 190992.12 this will be there is only 1.1. So, 12,162.76 plus 471.09, this will become plus 12,162.76 plus 471.09, which is 203625.96. So, this is 203625.97, 2000 20 plus 3000 into 19 plus 1368, 0.2 into 18 will give us 121627.6 into 10,000 divided by 6368.2, 190992.12 plus 12,162.76 plus 471.09; 203625.97 so we get 203625.97. Now, we have worked out the costs for a (Refer Slide Time: 00:08) 5 percent discount at 2000 and a 10 percent discount at 5000. Without a discount it was 204898.98 now, with the 5 percent discount it is 203421 and with the 10 percent discount 203625. This is slightly higher than this cost, based on the computations that we have shown. Therefore, we would choose a 5 percent discount.

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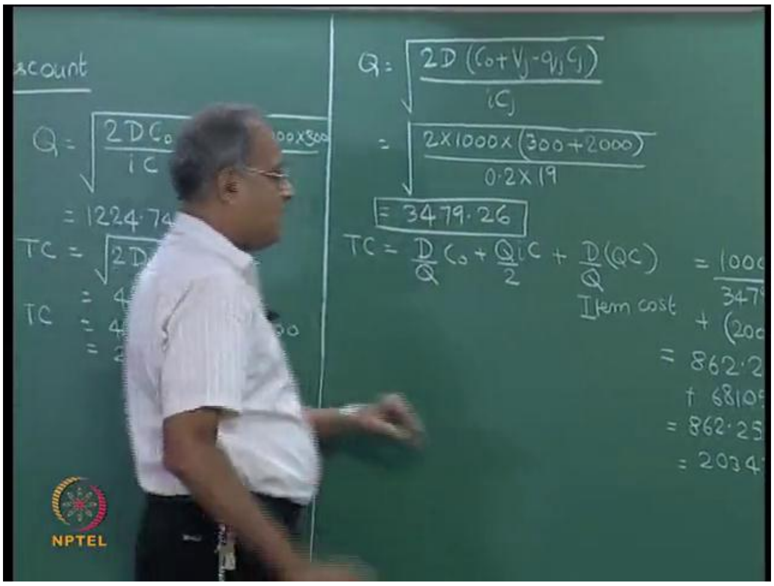

And we would order 3479.26 and incur an annual cost of 203421.2. Now, there are two other things that we have to see, the first thing is that this cost is not significantly higher compared to this cost. So now, do we consider ordering a larger quantity of 6368.2 because we would have more stock with us at any point in time? General thinking in inventory problems is not to build up a lot of inventory, and therefore this will we will not consider because this quantity is a very large quantity. Even if we had a situation where, this is slightly lesser than this; even if we have such a situation then it would still be wise to go for this because the larger the ordering quantity higher will be the inventory holding cost and more inventory will be built up in the system.

Now, we also need to understand that the order quantities have become bigger in (Refer Slide Time: 00:08) when we looked at the marginal quantity discount, even though the discount is available from 2000 plus onwards or 2000 and 1st item onwards. We realize that we actually benefit from the discount, only if we order a quantity that is higher than the price break. Therefore, a 2000 price break with marginal quantity leads us to 3479 and a 5000 price break with marginal quantity leads us to 6300. So, marginal quantity discount has a tendency to increase the order quantity because we can avail the benefit of the discount only when the order quantities are much higher than the price break or the point at which or from which that discount is applicable. So, it forces us to order larger quantities.

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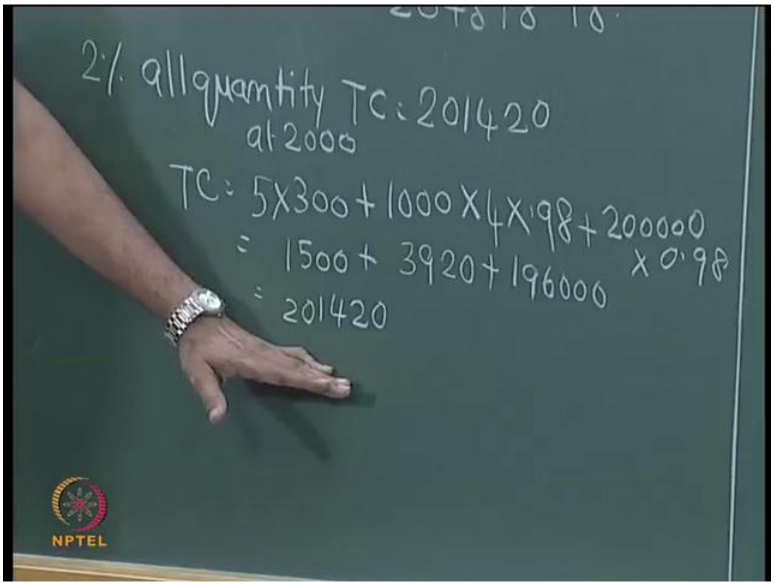

We also have to make a comparison with the all quantity discount, where we have earlier worked out that for a 2 percent all quantity discounts, the total cost turned out to be 201420. The computations are for a 2 percent all quantity discount so the price break at 2000, at 2000 units T C would be D by Q, there will be five orders because the order quantity is at 2000. There will be five orders, 5 into 300 plus Q by 2 into C c 1000 into there is a 2 percent all quantity discount. So Q by 2 into carrying cost is 4 into 0.98, which is your 2 percent discount and the item cost would be 200,000 into 0.98. So, this would give us 1500 plus this is 3.92 so 3920 plus 196000, which would give us 201420, which is cheaper than or less costlier than 203421.2. So, in this situation we could look at the all quantity discount if it is available and chose this, for two reasons that the total cost is lower and more than that the order quantity is also lesser.

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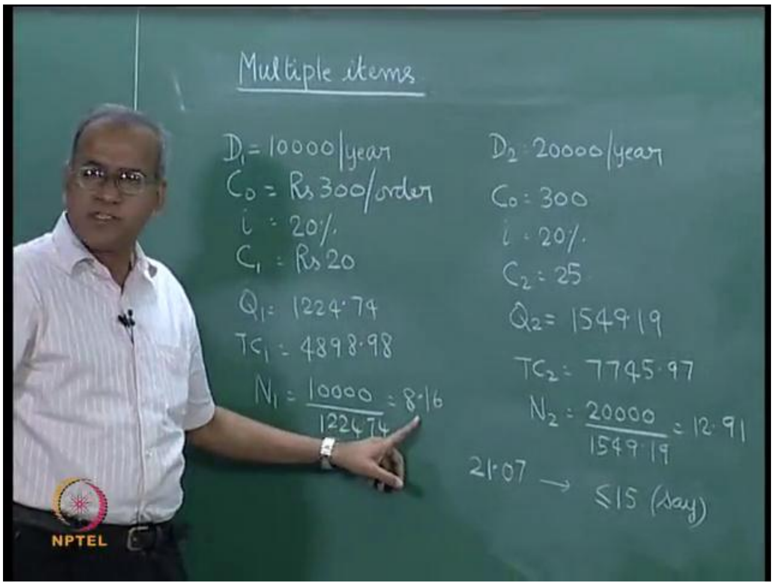

Now, after this discussion we move towards multiple item inventories. Now, in reality we will be looking at more than one item, though so far we have seen single item inventory problems. So, we now look at a two item inventory problem and try to understand, what the issues are with respect to multiple item inventories by simply considering two items at a time. So, we now look at the 1st item which has the demand of 10,000 per year we call it D 1 representing the first item and C 1, which is the unit cost of the 1st item. We assume that the order cost is 300 and i is 20 percent, we already know that Q 1 is 1224.74 and T C 1 is 4898.98.

Now, if we order 1224.74 every time, we make N 1 the number of orders per year will be 10,000 by 1224.74 which will become 8.16 orders per year. Now, let us consider a 2nd item with demand D 2 equal to 20,000 per year, we consider the same C naught equal to 300. We use the same i equal to 20 percent we assume that the unit cost of the item is 25, now, if we work out the order quantity then Q 2 for this item will become 1549.19, which is root over 2 into 20,000 into 300 divided by 5, 5 is 20 percent of 25 and the total cost associated with this will become 7745.97.

Now, for the 2nd item, with demand equal to 20,000 and order quantity equal to 1549.19 the number of orders per year N 2 will be 20,000 divided by 1549.19, which will become 12.91 orders. Now, if these two items are treated independently, then we will be making 8.16 orders for this item and 12.91 orders for the other item, which would give us a total number of orders as 21.07 orders, now, we realize that 21.07 may be a bit too large when we look at two items. And particularly, if we look at a very large number of items then we realize that, we will be making too many orders when all these items are put together. Therefore, we also build a restriction and a constraint that the number of orders will be less than or equal to 15 say and see, what happens to the ordering quantities, if we put this kind of a restriction.

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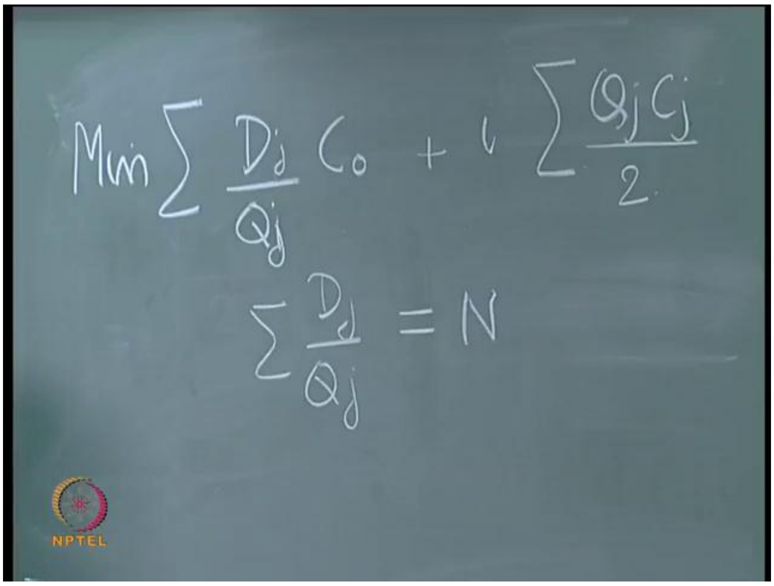

So, the problem will now reduce to minimizing total cost, which is D 1 by Q 1 into C naught or sigma D j by Q j into C naught plus i into sigma Q j C j by 2. If we take the 1st item the order cost will be D 1 by Q 1 into C naught, for the 2nd item D 2 by Q 2 into C naught. We have used the same C naught so C naught can be taken outside, we have used the same i so for the 1st item it is Q 1 C 1 by 2 into i for the 2nd item it is Q 2 into i C 2 divided by 2. So, the i is taken out, so we have this. Subject to the constraint that D j by Q j sigma is less than or equal to some N so in this case the number of order should be less than or equal to 15.

If we do not take this constraint and we optimize this we will get the same answer here with the 1rst item having 8.16 orders and the 2nd item having 12.91 orders. Now, the unconstrained optimization solution would violate this constraint and therefore, this constraint will become binding. As shown in this example, if we do not include this constraint, the unrestricted problem will give us 21 orders, which violates 15 and therefore, the constraint optimization problem has to be solved. Now, this is a constraint optimization problem with an inequality as a constraint. Now what we will do now is, we will try and replace this inequality with an equation and we try to understand something through the properties of the total cost.

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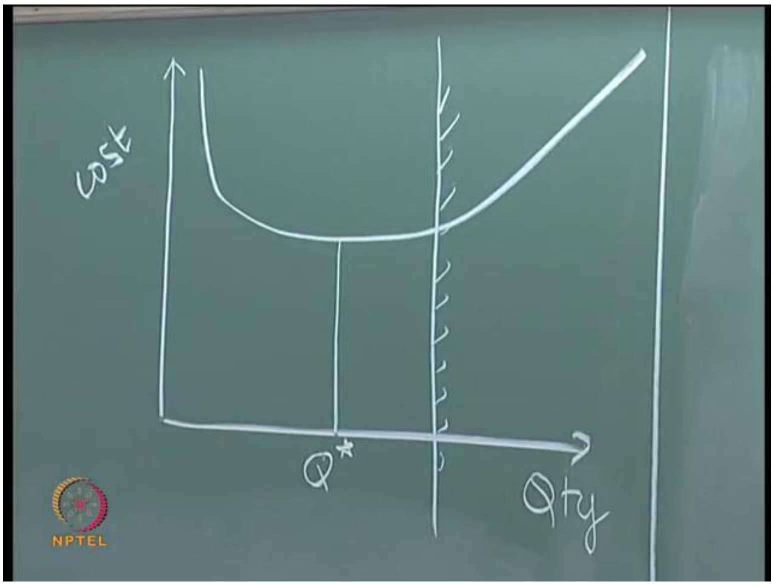

Now, we have already seen that if we have a single item where, we have quantity and we have cost the total cost curve will look like this and let us say the economic order quantity will be here. Now, we realize that if we take 1 item with order quantity 1224 and this equal to 8.16, 2nd item has 12.91. If this 21.07 orders has to be brought down to 15 orders then the order quantity for both the items will go up, which means this constraint if we take a particular item is like saying here is Q star, now, this constraint is like saying now we are going to increase the order quantity; so we are looking at a constraint which looks like this. Now, the objective function is a function of this type and we have a constraint which will try and put q greater than or equal to something.

Now, by the nature of the objective function the optimum will happen only here and will not happen at other points. Therefore, we make an assumption which is correct that, when the constraint is binding than the inequality can be replaced by an equation and we can solve it. Now the advantage of replacing this inequality by an equation is that we can use the method of Lagrangian multipliers and take this equation into the objective function and then we solve this. But, we should also note that only when the constraint is binding the inequality will become an equation.

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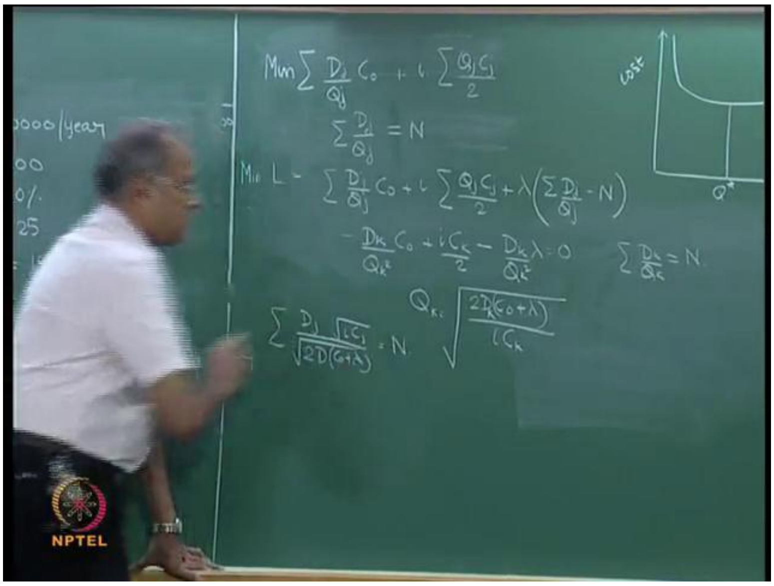

So, we take it into the objective function and introduce a Lagrangian multiplier, which will be like this. So, to minimize L the Lagrangian multiplier sigma D j by Q j into C naught plus i into sigma Q j C j by 2 plus lambda, which is the Lagrangian multiplier into sigma D j by Q j minus N. Now, partially differentiating this with respect to the k'th item, we will have minus D k by Q k square into C naught plus i C k by 2 minus D k by Q k square lambda equal to 0; from which Q k, general value for the k'th item will be 2 D into C naught plus lambda by i C k. Now, the effect of the constraint is the addition or inclusion of lambda into this value.

So, the new order quantity we will be able to find out only if we know the value of lambda, we know for the 1st item we know we will have a D k here. So, for the first item we know its 2 into 10,000 into 300 plus lambda by point 2 into 20 but then we do not know the value of lambda. So, in order to find the value of lambda we partially differentiate this with respect to lambda and then we will get sigma D k by Q k is equal to N. Now we already know this Q k is equal to this now, just to be consistent with the rotation I am going to replace k by j. So, now we will have sigma D j by Q j. Q j is root over 2 D into C naught plus lambda into root over i C j, so D j by Q j is equal to N.

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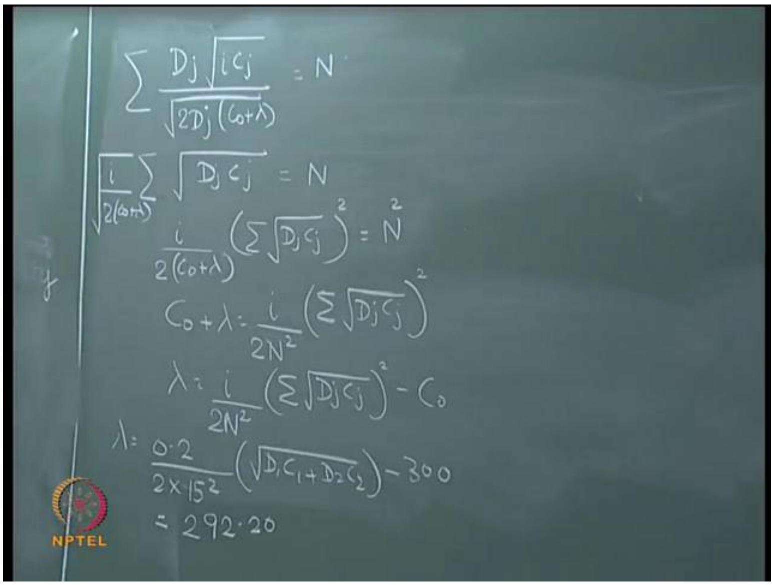

So we simplify here, now we have sigma D j into root over i C j divided by root over 2 D j into C naught plus lambda. So, D j by Q j is equal to N, so now, we simplify here to get root over sigma root over D j C j, we have root of i by 2 times C naught plus lambda is equal to N. Now, we take C naught plus lambda; first we square it to get i by 2 times C naught plus lambda sigma root of D j C j square is equal to N square, from which C naught plus lambda is equal to i by 2 N square sigma root of D j C j the whole square, from which lambda is equal to i by 2 N square sigma root of D j C j the whole square minus C naught. So, that is the value of lambda and now for our example, we will now substitute lambda is equal to 0.2 by 2 into 15 square into root of D 1 C 1 plus D 2 C 2 minus 300, which on substitution, would give us 292.20.

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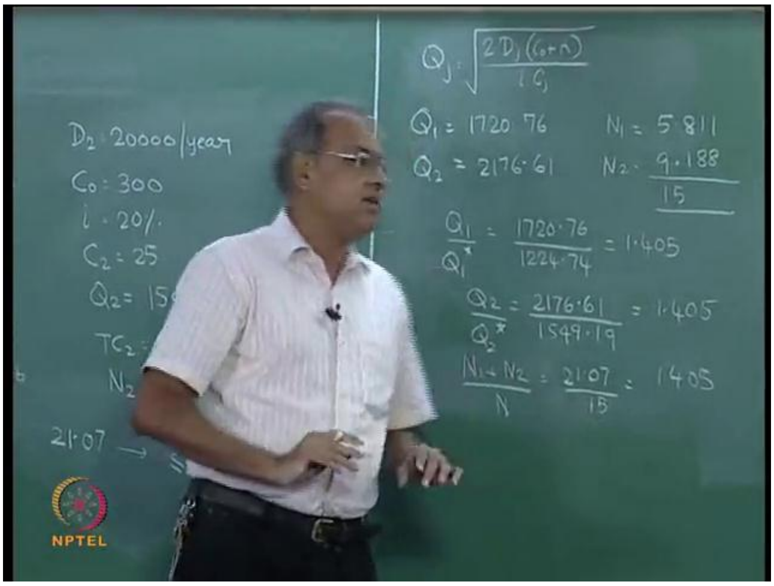

And then we go back and substitute this lambda into this and if we do this substitution so let me, Q j is equal to root of 2 D j into C naught plus lambda by i C j. Now, the new Q 1 will be root of 2 into 10,000 into 300 plus 292.20 divided by 4, which would give us 1720.76 and Q 2 will be 2176.61. Now, Q 2 can be obtained as 2 into 20,000 into 300 plus 292, which is C naught plus lambda divided by 5. So, these are the 2 order quantities now, for these order quantities N 1 will be, the number of orders will be 10,000 divided by 1720.76 which is 5.811; other one will be 20,000 divided by 2176.61, which is 9.188 which when added will become 14.999 or 15 orders.

Now, we also observe that the original 1224.74 Q 1 star is 1224.74 and Q 2 star without the constraint was 1549.19. Now, we realize that the constraint has increased 1224.74 to 1720.76 and it has increased 1549.19 to 2176.61. We also observe now, that Q 1 by Q 1 star is equal to 1720.76 divided by 1224.74 which is 1720.76 divided by 1224.74, which is 1.405. Q 2 by Q 2 star is 2176.61 divided by 1549.19, which is 2176.61 divided by 1549.19; 1.4049 which is this. And we also realize that 21.07 orders has become 15 therefore, N 1 plus N 2 divided by the constraint N will be 21.07 by 15 is 1.4046, which is 1.405 subject to the rounding of errors.

So, we also realize that a simple formula would have helped and the Lagrangian multiplier method leads us to the simple formula. So, if a total of 21.07 orders has to be brought down to 15 then multiply the individual order quantities by a quantity 21.07 by

15 which is 1.405 so 1224.74 into 1.405 will give the new order quantity for the 1st item and similarly, for the 2nd item. So, this is how we solve a multiple item inventory problem with constraints on the total number of orders. But, before we wind up this discussion on the constraint problem with total number of orders, we again go back to this and then we look at this number of lambda equal to 292.20. Now, in order to understand that let me write the formulation again the formulation is to minimize sigma D j by Q j into C naught plus sigma i Q j C j by 2, subject to sigma D j by Q j less than or equal to N.

Now the unconstrained problem here, gave us 21.07 to the left hand side which made the constraint binding and then because we understood that when the constraint is binding the constraint can be converted to an equation so that we could use Lagrangian multipliers and solve it. Now, suppose we had capital N is equal to 25, which meant that this constraint is not binding; 21.07 will be less than or equal to 25, so the obvious thing is not to worry about this constraint and continue to order 1224.74 and 1549.19 so that the total cost is minimized to 4898.98 plus 7745.97.

It is also obvious that when we compute the total cost for these 2 the total cost will be higher than 4898.98 and 7745.97. So, when the constraint is satisfied as in the case of n equal to 25, we will not use this method so we will first solve the problem independently like we did here check whether the constraint is satisfied for N equal to 15 it is violated; for N equal to 25 it is satisfied. When it is violated we could use the method of Lagrangian multipliers get a lambda and compute these values or we could use these ratios to get it ratios are also theoretically correct.

But, if N was 25 and therefore, this constraint is already satisfied, then we will not use the method of Lagrangian multipliers, we would simply take these unconstrained values as optimum values. In spite of that, if we go back and substitute N is equal to 25 for this, instead of N is equal to 15 substitute 25 square now, we will realize that we will get a negative value of lambda. Now, this negative value of lambda, when which the moment we get a negative value of lambda we need to understand that the constraint was actually not violated by the original values.

Therefore, there is no need to do the Lagrangian multiplier method but we simply blindly have used this formula and we will get a negative value of lambda if we do that. Now, such a negative value of lambda will now come back here and that will force the order quantities to be lower than 124.74 and 1549.19 so that, the total will be forced to 25 now, we should not allow that to happen. So, we have to understand this, first we solve the problem without a constraint, check whether the constraint is violated and only when the constraint is violated we will use the method of Lagrangian multipliers to solve this.

Now, if the constraint is satisfied as in the case of N equal to 25, we will not use this method, we would simply use the unconstrained one and get the minimum cost of 4898.98 plus 7745.97. In spite of this, if we make the mistake of using this formula, we would get a negative value of lambda because; this formula was derived for an equation here. Therefore, it will give a negative value which will force this to be an equation and it will force this to get 25 orders in a year and it will reduce these order quantities and increase the cost to more than 4898.98 and 7745.97, which we will not use.

So, the moment we get a negative value there, we have to check the calculation and if we are sure that you get a negative value it means, we should not have used the Lagrangian multiplier method the original unconstrained solution itself is optimal and there is no need to use this method to get the new values of the order quantities. Now, other aspects of multiple item inventories such as restriction on the money value of inventory etcetera, we will see in subsequent lectures.