Operations and Supply Chain Management
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Module - 1
Lecture - 11
Inventory - Models for all Quantity and Marginal Quantity Discount

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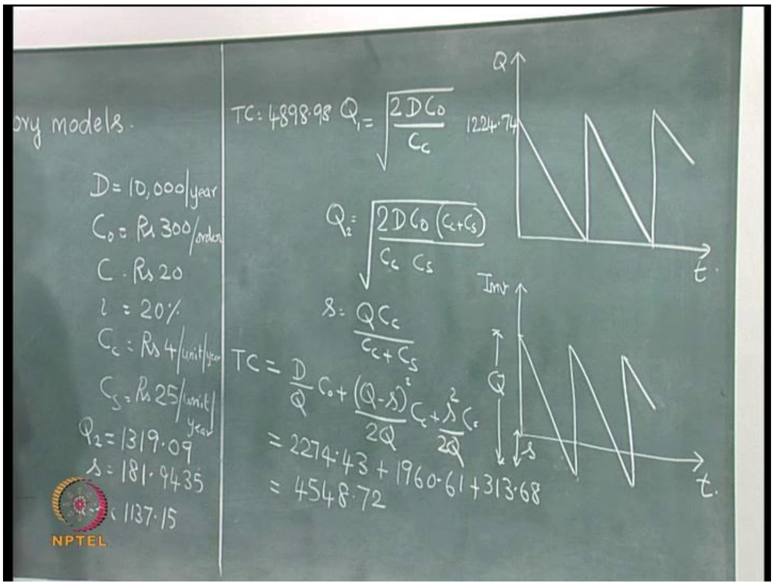

We continue our discussion on inventory models. We have so far seen two models. This is the diagram for the first model, where we consider continuous demand and no back ordering. In the second model, we considered back ordering and we derived expressions for the economic order quantity under both the assumptions. The economic order quantity for this model, without back ordering is given by Q is equal to root over 2DC naught divided by C c; where D is the annual demand, C naught is the order cost and C c is the unit holding or carrying cost.

Here, there are two variables, one is the order quantity Q which is actually this quantity and then there is a back ordered quantity which is called s. There are two variables. Then the formula for economic order quantity Q, under the assumption of back ordering with the back order cost of C s is given by Q is equal to 2DC naught divided by C c into C c plus C s divided by C s, and s which is the back order level given by QC c by C c plus C s. Both from this expression as well as otherwise, C c and C s have the same unit, which is rupees per unit per year. Here, it is rupees per unit per year to hold one unit of inventory. This is rupees per unit per year of one unit back ordered.

So, C c and C s will have the same unit, so that we can add them. The expression from which we derived these two is given by total cost is equal to ordering cost plus inventory carrying cost plus back order cost. This was simplified to D divided by Q into C naught plus Q minus s the whole square by 2Q into C c plus s square by 2Q into C s. Now, we will take a numerical example to try and show the computations involved in, using these formulae Q, S and TC.

Earlier we had taken a numerical example to illustrate this, where we had assumed D, annual demand is equal to 10000 per year. We had assumed C naught which is the order cost as rupees 300 per order. The unit cost of the item was assumed to be rupees 20 and the inventory holding cost i is 20 percent of 20 and therefore, C c becomes rupees 4. Using these we had calculated the economic order quantity for the first model and we had got a value of 1224.74.

Now, we use the same data and we also introduce a C s, which is the shortage cost and we use shortage cost of rupees 25 per unit per year. C c is 4 per unit per year. We now compute these values. We apply this formula to compute the economic order quantity. So, Q is equal to 2DC naught C c plus C s by C c, C s, since I have written the formula for both the models.

Let me call this as some Q1, which is the economic order quantity for this system, without back ordering, and let me call this as Q2 as the economic order quantity, for this system, which includes and considers back order. We are now interested in computing this Q2, we already know that this Q1 is 1224.74 which was done in an earlier lecture. So, we compute Q2. We know all the values 2 into 10,000 into 300 into 4 plus 25 divided by 4 into 25. This on computation would give us, Q2 is equal to 1319.09 and once we calculate this Q, 1319.09, we go back and substitute here. So this is 1319.09 into 4 by 4 plus 25 into 4 by 29 would give us s is equal to 181.9435 that would give us this value.

Now, let us also calculate the total cost associated with this model, and try and compare it with the total cost associated with the earlier model. The earlier model had only two components which is the order cost and the carrying cost, and then the economic order quantity was 1224.74 and the total cost in this case was found to be 4898.98. So, let us also try and find out what happens when the back ordering is allowed and C s is introduced.

Already we have seen that the order quantity increases from 1224.74 to 1319.09, a back order quantity of 181.94 is allowed. Let us calculate the total cost based on this. So this component would give us 10,000 into 300 divided by 1319.09, 2274.43 plus Q minus s is equal to 1137.15. So, this quantity will become 1960.61 plus 313.68, and the total is 4548.72.

So, when we allow back ordering and if the back order cost is rupees 25 per unit per year, we observe that the order quantity Q increases from the old value of 1224.74 without back ordering to 1319.09. Quantity back ordered is calculated as 181.94. The total cost comes down from 4898.98 to 4548.72. So, the first observation is that the total cost comes down; second observation is that, there are three components to the total cost. The order cost component is 2274.43, carrying cost component is 1960.6 and the shortage or back order component is 313.68.

We also observe that, this 2274.43 is exactly half of the total, which is 4548.72. This contributes to half of it, this contributes to the other half of the total. So, in this model we are essentially trying to find out a Q or order quantity such that the total ordering cost is equal to the sum of the inventory carrying cost and the back order cost. In this model, we found Q the order quantity such that the order cost component is equal to the inventory holding cost component. So, each was equal to half of the total cost.

In this case, this is half of the total cost, these two put together will contribute half of the total cost. Because we allowed back ordering, the order quantity went up from 1224.74 to 1319.09. So, number of orders per year came down compared to the earlier one. The number of orders per year came down. Therefore, total order cost came down. The total cost is two times the order cost; here also total cost is two times the order cost. Therefore, the total cost also comes down.

Other way of describing it is, even though we order 1319.09, because of instantaneous replenishment, we will place an order, when we were here. Then place an order for 1319.09, and as soon as the item arrives, which is instantaneous, the back ordered quantity of 181.94 is immediately given. Therefore, the inventory on hand, quickly reduces to 1137.15, which is the difference between this and this.

So, the maximum inventory that we will hold is 1137.15 as against 1224.74. Also the period for which we will be holding the inventory, will be far less in this case, because when we divide by 1224.74, we have a certain period. When we divide by 1319.09, the whole period comes down. This period comes down and we also realize that this maximum inventory is not held for this period, but it is held for a smaller period. Therefore, the average inventory comes down and the inventory holding cost comes down. So, the order cost comes down, the inventory holding cost comes down, but there is a back order cost, which is not there in this model, which comes by holding the quantity of 181, for a much smaller period.

So, the back order cost contributes to another 313.68, but as I mentioned earlier, the total cost effectively comes down. In this model, we are trading off with order cost on one side and inventory holding plus back order cost together will be equal to the order cost at the optimum. This actually points us to a very interesting question. This is a model which did not allow back ordering. This is a model that allowed back ordering. So, when we allow back ordering, we seem to get a lesser total cost. We seem to be holding lesser quantity, for a lesser period in inventory.

So, on the phase of it, by merely look at these numbers, one would be tempted to agree and believe that, the model with back ordering is desirable. If we have to make a decision only based on the total cost, then we might say, yes, the model with back ordering has lesser total cost. This also happens because we look at it as an optimization problem. This is a problem that allows back ordering. This is the problem that does not allow back ordering. So, this is a restricted version of this problem, where you are putting an additional restriction that, back ordering is not allowed.

This is a relaxed version, where back ordering is allowed, and it is only obvious that in a minimization problem, the restricted version will have an objective function value higher than the relaxed problem. So, the first model will have total cost higher than the second. Then we need to answer this question, should we always go for this model or should we go for this model?

Now, we can try and do a little more experiment with this model, try and show some more computations and then we would actually realize that if we keep increasing the C s for example, the shortage or back order cost was defined at 25 per unit per year. Suppose, we make it 100 per unit per year, what will happen? I am not going to show the computations, but what I am going to tell here is that if we increase this C s, then automatically what will happen is, this cost will also increase and this will come closer to this 4898.98. If we increase C s, the order quantity will decrease from 1319.09. The total cost will increase and come closer to 4898.98.

As we keep on increasing this C s, this quantity will keep coming down, this will come down, and this total cost will go up. When C s is equal to infinity, this will take a value 1224.74, this will take a value 0 and this total cost will be equal to this total cost. This Q will be equal to that Q. That is also clear because we can rewrite this as 2D C naught by C c into 1 plus C c by C s. So, when C s is infinity, C c by C s will become 0. So 1 plus C c by C s is 1. Therefore, when C s is infinity this will become 2D C naught divided by C c which is the same model.

So, only when the back order cost is kept as infinity which implies that back order is not allowed, we will automatically get the other model. As long as the back order quantity is anything smaller than infinity, however large it may be, this model will always give a total cost, slightly lower than this model. But the actual way to look at it is, unless we are absolutely sure about the correctness in the computation of C s, and we know that this is exactly the back order cost and let us say we have included the loss of good will and all those intangible or costs that cannot be measured easily, let us say we have measured all the cost and if the organization is very clear that this is my back order cost and this is exactly what I am going to incur, there is going to be no delay to the customer.

Therefore, there is no cost associated with this. If we sure about all of that, only then this model can be used straight away. If we are not sure and if there are some more uncertainty, where there would be a delay because of this back order, and we are not able to estimate the cost of such delay accurately, we can either give a very large value for this, or to be on the safe side if we do not want to have back order at the planning stage itself, we would rather give value equal to infinity and go towards the first model.

The second model is very important and has to be used very carefully. Unless we are sure of the assumptions of the second model, it is always good to go back to a situation where the back order is not allowed at least during the planning stage. So, that is the reason why look at two types of inventory models particularly for bought out items. Now look at another aspect of it, which is called discount.

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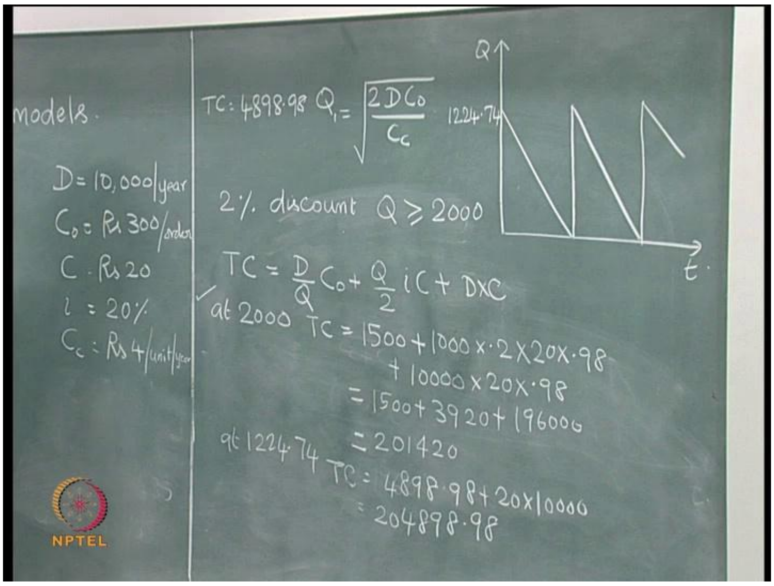

In order to explain the discount model, we will be looking at this model and we will not be considering this model. When we have a single item and we buy this item, let us assume, we are working with this data, where the annual demand is 10000, order cost is 300 and so on. We have already found that the economic order quantity is 1224.74 and the total cost is 4898.98. When we consistently buy the same item from the same vendor or when we are buying a larger quantity, we would ask for a discount and the vendor would offer a discount invariably when the buying quantity is large.

Let us look at a situation where the vendor is offering a discount for this, and then we need to evaluate, whether we will accept the discount or we will reject the offer of discount. Let us assume that the vendor is willing to give a 2 percent discount when the order quantity Q is greater than or equal to 2000. If we go back to the derivation of this Q1, we first said that we would consider there were 4 costs; the order cost, the carrying cost, the back order cost and the cost of the item. This model does not allow back order.

So we look at the 3 costs and wrote the expression TC is equal to D divided by Q into C naught plus Q by 2 into C c plus D into C.

This is the ordering cost component, this is the inventory holding cost component, and this is the cost of the item per year. Since, the variable was how much to order and the variable did not figure in this D into C in our derivation, we left it out because D into C is a constant. We left it out and then we derived this expression. But now if we want to consider this order, this discount offer, for a quantity of 2000 or more. Let us consider that we are looking at whether to accept the discount at 2000, we will then address the situation, whether we will accept the discount at a quantity greater than 2000.

If we are interested in quantity of 2000 and we want to avail the discount, at any order quantity which is different from 1224.74, the sum of these two D divided by Q into C naught plus Q divided by 2 into C c which represents the total cost of ordering and holding, will definitely be more than the 4898.98 because we have derived 1224.74 as the optimum value, where this total function is minimum. This D into C is also a constant. So, when we increase the order quantity 1224.74 to say 2000, this part is going to go up. And if this were a constant it would never be economical to avail it. But then there is a discount. Because of the discount the C now takes a new value, it is not 20 anymore but, it is 98 percent of 20 with a 2 percent discount.

So, the increase in this, if it is offset by the gain here or if the gain here is more than the increase in this, then it would be advantageous for us to avail the discount. There is also a very small component, which needs to be looked at; it depends on how we are going to define this C c. We have already seen that this C c can be defined in two ways; one is as C c or as i into C.

If we now define this C c as i into C, and if we are interested in an order quantity of 2000, at 2000 this carrying cost will also come down slightly because this C also changes slightly. So there will be an overall increase in D by Q into C 0 plus Q by 2 into i C but, that overall increase will also have a very small decrease because of this decrease in C. But then this increase has to be offset by the gain because of the discount.

So, let us first look at, can we avail this discount at 2000. Before that let us answer this question, are we going to avail? We are not going to get this discount at all, for any quantity up to 2000 and for any quantity up to 2000; the total cost will be the lowest at the economic order quantity of 1224.74. So, we should look at 2000. When we look at 2000, at 2000 T C is equal to, now D by Q is 5, 10000 divided by 2000 is 5. 5 into C naught is 1500 plus Q by 2 is 1000, i C is 20 percent of rupees 4 into 98 percent because at quantity of 2000 the unit price is not 20, but it is 98 percent of 20.

Therefore Q by 2 into i into C will become 1000 into Q by 2 is 1000 i is 0.2 into 20 into 0.98 plus D into C 10000 into 20 into 0.98. This on computation would give us 1500 plus 3920 plus 196000 which is 201420. This cost has to be compared with what is the cost that we would incur if we had ordered 1224.74. At 1224.74 T C cross is equal to 4898.98 plus 20 into 10000 which is 204898.98. The total cost at economic order quantity is 204898.98. The total cost at an order quantity of 2000 and availing at 2 percent discount would give us a total annual cost of 201420. This is lesser than 204898.98.

Therefore, it is profitable to avail the discount at 2000. We have answered the first question as to, whether we will accept it 2000. Now, we have to answer another question whether we will accept it at a quantity at a greater than 2000. For example, even at 3000 will get the 2 percent discount. So, is it advantageous to avail it at a quantity of 3000 and in order to do that we should go back and do 10000 by 3000 into 300 plus 3000 by 2 into 0.2 into 20 into 0.98 plus 10000 into 20 into 0.98. So, this portion is going to be remaining the same. But then we have to check, whether this one decreases, so that we could avail. In order to do that, let us go back to another curve that we have already seen, which is called the total cost curve.

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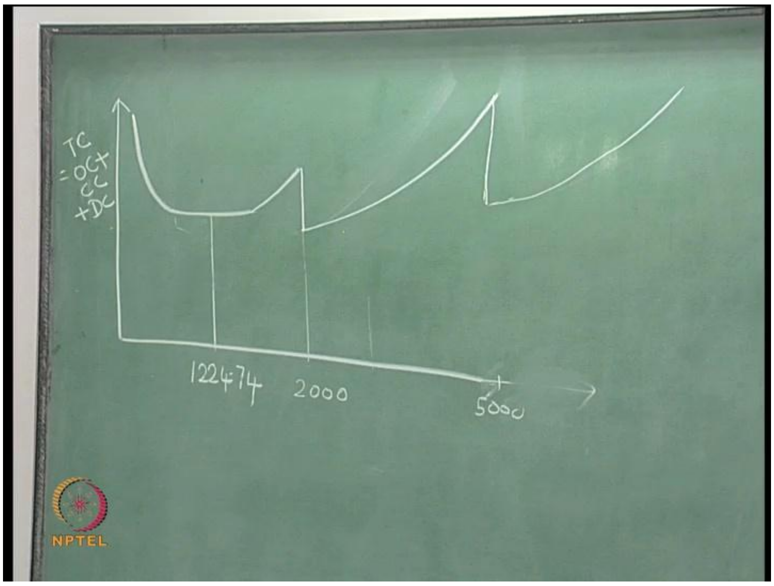

We have already seen that the total cost curve is very flat near the economic order quantity. Let us say this is the economic order quantity and this is 1224.74 and then at 2000 let us say we are getting this discount. We are going to get this discount. So, it comes down a little bit. I have not actually given you the y axis cost, the y axis now is the order cost plus carrying cost plus cost of the item. So, total cost is equal to order cost plus carrying cost plus D into C, which is the item cost.

We realize that at 2000 it actually comes down, becomes lower than this, this is your 204898.98. This value is your 201420. This is here and then if you see carefully, the total cost will go up. Therefore, from the very nature of this graph, we will understand that it is not necessary for us to see whether we are going to avail it at 3000 or any value higher than the point at which there is a discount. So, the point at which there is a discount is called the price break point. The price break point is 2000 and therefore, we need not evaluate it at 2001 or 2 or 3000 or any value greater than that price break point, to avail that discount quantity.

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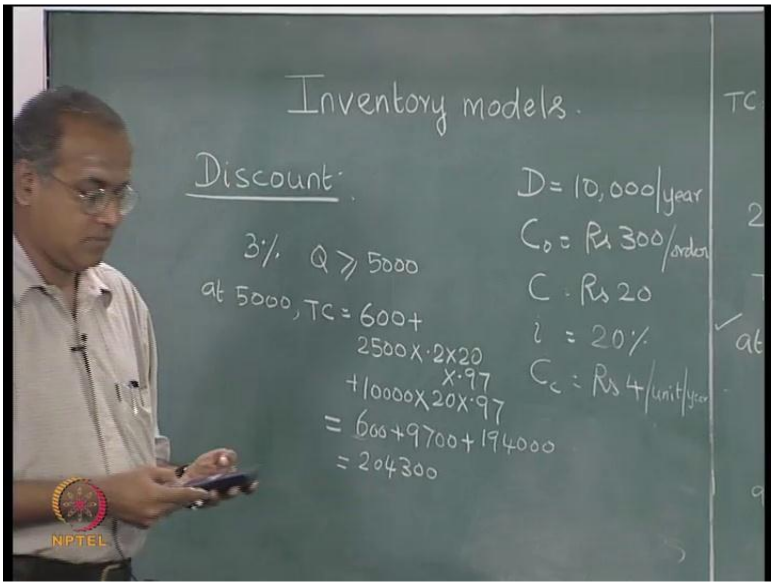

Now, let us assume that the vendor is giving us another price break point and the vendor says that he can give a 3 percent discount, if the order quantity Q is greater than or equal to 5000. So, we have to do the same calculations. At 5000 what is the total cost? T C is equal to D by Q into C naught, so there will be two orders 10000 divided by 5000 is 2. 2 into C naught is 600 plus Q by 2, 2500 into C c which is 0.2 into 20 into 0.97 plus 10000 into 20 into 0.97. This will become 600 plus 9700 plus 19400. So this is 204300.

To show it in this graph, let us say 5000 is somewhere here, it goes up. Say 5000 is here so it goes up now, there is another price break, so it comes down and somewhere in the middle, this is still minimum, this is 4898. It comes somewhere up to here and then it keeps moving like this. This is still the optimum price break point and therefore, we will not accept this we would accept a 2 percent discount at 2000 and we will not accept a 3 percent discount at 5000.

Wherever there is a price break, it is necessary to compare the total cost at the price break, not at any other point. Only other part is the case where there is a price break before the economic order quantity. Let us say there is a half a percent discount at 1000, then in such situation, we will try and avail that half a percent discount not at 1000, but at 1224.74. So, if there is a discount at a quantity before the economic order quantity then we will try and avail that discount at the economic order quantity. If there is a price break after the economic order quantity, we will try and avail it, at the price break point

It is generally customary that price break points are for quantities, higher than the economic order quantity, but it so happens that there is a price break point before the OQ. For example, there is a half a percent discount at say 1000 which means, it is going to come down here, it is going to further down, till reaches the economic order quantity and proceed. The other minor adjustment there is that the economic order quantity can also shift a little bit because if there is half a percent discount at 1000, then the quantity changes from 2 into 10000 into 300 divided by 4 which is the original one which gave us value of 1224.74.

Now if there is a half a percent discount at 1000 that we wish to avail at the economic order quantity, then the economic order quantity will shift a little bit. It will become 2 into 10000 into 300 divided by 4 into 0.995, if there is a half a percent discount. It will become slightly more than 1224.74. It may come to around 1224.85 or may be closed to 1225. Physically there would not be too much of a change, but if we look at it from the computation point view, there will be a slight increase in the economic order quantity and we will avail the discount, at the new economic order quantity.

This is how we handled the discount. The discount problem always involves computation of a total cost, which also includes the cost of the item, because the gain in any discount comes from the reduction in D into C, which was otherwise a constant because C was not a function of Q. In the discount problem the item price C is of function of Q and therefore, C will change and D into C will also start influencing the calculation of the order quantity. The discount model that we have seen is called an all quantity discount. By which we mean that if you order a quantity of 2000, then for every item that is ordered, the cost is not rupees 20, but it is 98 percent of 20 which is rupees 19 and 60 paise. For each one of the 2000 units, the price is at 19.60 paise. Now we could also have something called marginal quantity discount. So, let us look at that. Let me write the marginal quantity discount.

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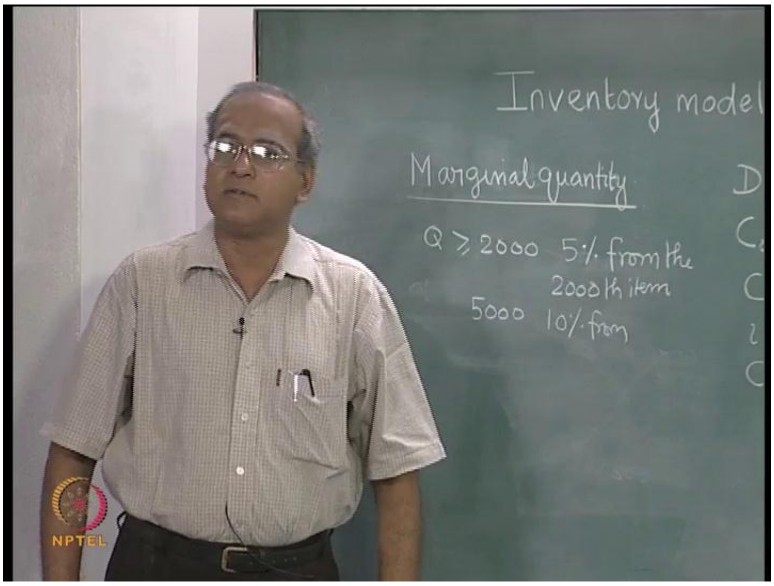

We could have something called marginal quantity discount. In marginal quantity discount the vendor may say that if you order for Q greater than or equal to 2000, let us say the vendor may give us, it is 5 percent from the 2000th item and say 10 percent from 5000th item. What is the difference? The first difference is this, if we ordered 2000 items in the all quantity discount, let us compare these two cases an all quantity discount of 2 percent versus a marginal quantity discount of 5 percent. If we order say 3000 items, then all the 3000 items will be priced at 19.60 paise. If we order 3000 items here, the first 2000 items will be priced at 20. The next 1000 which is after the 2000th item the next 1000 are now going to be priced at 95 percent of 20, which is 19 rupees.

So, at Q equal to 3000 only the item cost per year or at time of ordering, if we order 3000 the item cost will be 3000 into 19 rupees 60 paise. Here, it is 2000 into 20 plus 1000 into 90. The gain in the marginal quantity discount is slightly lesser than, the gain in the all quantity discount. For that reason you will see that the discount is higher in the marginal quantity, than in the all quantity discount. Let us work out this problem with marginal quantity discount of 5 percent from the 2000th item and 10 percent from the 5000th item.

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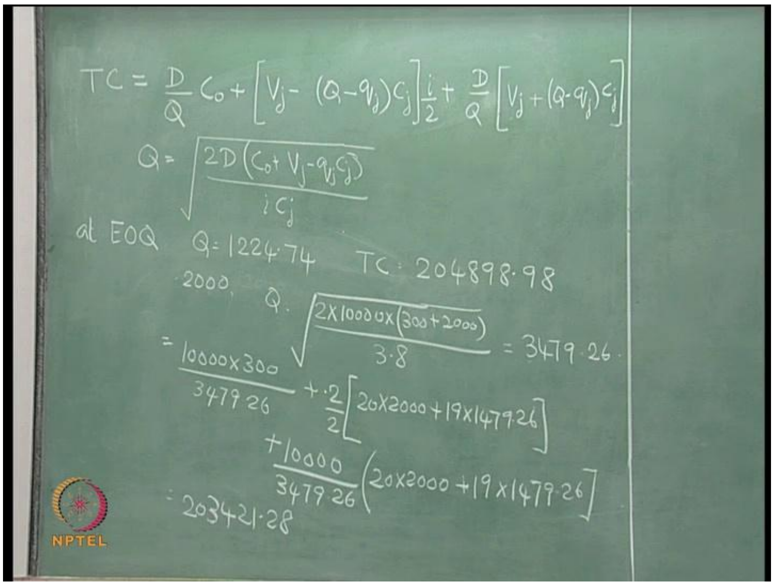

The derivation will be as follows. T C is equal to D by Q C naught plus Vj minus Q minus qj into Cj into i by 2 plus D by Q into Vj plus Q minus qj into Cj. This on differentiation would give us, Q is equal to root over 2 D into C naught plus Vj minus qj Cj divided by i into Cj. Let me explain this a little bit. It is also obvious that when we differentiate this with respect to Q, we are going to get this one.

As I said, there are 3 components. There is an order cost component, there is a inventory carrying cost component and then there is a cost of the item component. Order cost is very clear; D divided by Q into C naught, Q is Q what we are going to order. D by Q is the number of orders per year. D by Q into C naught is the order cost. Now, what is the carrying cost? Carrying cost originally is Q by 2 into C c. We can write it as Q by 2 into i into C. Can also be written as Q C by 2 into I; where C is the unit price of the item, Q is the order quantity and Q C is the money value of the order quantity.

Now let us look at order cost, as the money value of the order quantity by 2 which is the average into i. So, i by 2 is separated. This is the money value of the quantity order, by the very definition of a marginal quantity, if we are looking at this case 2000 and 5 percent for a quantity above 2000. For example, if we are calculating for 3000, we would do, 2000 into 20 plus 1000 into 19, which is given by this kind of thing. Vj is the value for the first 2000, Q minus Qj 3000 minus 2000 which is thousand into Cj which is the new value 19.

This is the expression for the money value for the item that is bought. So that into i by 2 is the inventory holding cost and this is the money value per order. This is the value the number of orders per year is D divided by Q. So, the total annual value will be D divided by Q into this. This on differentiation would give us this total value for the economic order quantity.

Let us work out the same problem for the marginal discount and if we do that now, the first one is at the economic order quantity at EOQ, Q is equal to 1224.74 and T C will be 204898.98. There is no discount. So, 4898.98 come from here. The cost of the item is 200000 so 204898.98 at Q equal to 2000. We need to substitute so you get 2 into 10000 into 300 plus 2000 is the gain.

For example, at 2000, 5 percent of 20 is 1 rupee. Vj minus qj Cj is 2000 into 1. For example, at 2000 if I had not availed the marginal discount, I would have paid 20 rupees. If I avail the marginal discount, I pay 19 rupees. Therefore, 2000 into balance 1, that is Vj minus qj Cj. This quantity at 2000, Q is equal to root over 2 into 10000 into 300 plus 2000 divided by 3.8 because there is a 5 percent discount, so 4 becomes 3.8. So this on computation gives us 3479.26.

If we really want to use this marginal discount of 5 percent from the 2000th item onwards, the best deal we will get is, if we order 3479.26 items, out of which the first 2000 items will be priced at 20, and the balance 1479 items will be priced at 19. The total cost associated with this will be 10000 into 300 divided by 3479.26 plus i by 2 into money value of 3479.26. So, 0.2 by 2, I by 2 into money value of this.

So, the first 2000 is at 20. So, 20 into 2000 plus 19 into 1479.26, that is the money value plus this money value, this will be 10000 divided by 3479.26 into 20 into 2000 plus 19 into 1479.26. All of these put together would give us 203421.28. This is the total cost that we will incur, if we want to avail this portion of 5 percent for a quantity of 2000.This is actually cheaper than 204898.98. We have to evaluate, what is the best deal we get, if we try and utilize this, and then we have to compare that with 203421.28 and choose the one which is cheaper. We will show those computations in the next lecture.