Operation and Supply Chain Management
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Lecture - 10
Inventory - EOQ Model Graphs, with Backordering

In the previous lecture, we introduced the basic economic order quantity model, whose stock verses time relationship is shown in this figure.

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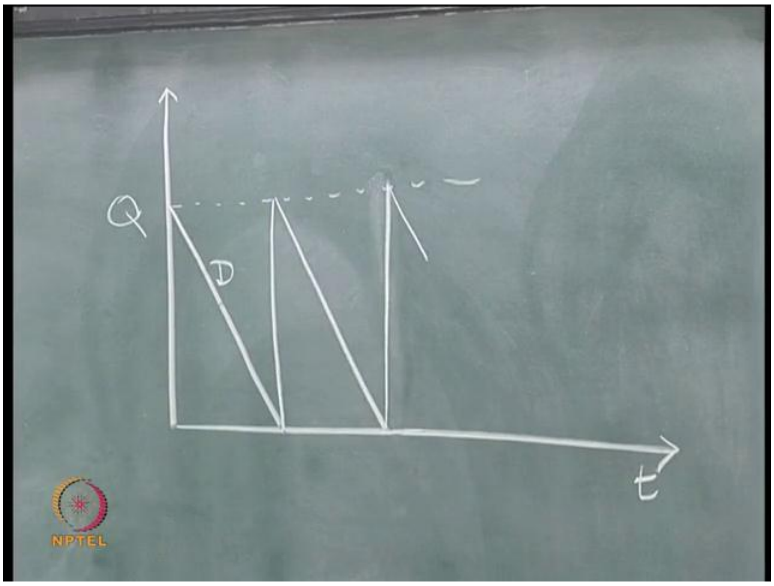

So, Q is the economic order quantity or the quantity that we order, every time we place an order. So, the model works as follows, let us assume that, we have just received Q units that has been ordered earlier. We also assume instantaneous replenishment which means, as soon as the order replaced, the items arrive. And then we consume these items at the rate of D till it reaches, the stock position reaches 0, and then we place another order and instantly get this Q and this process goes on forever.

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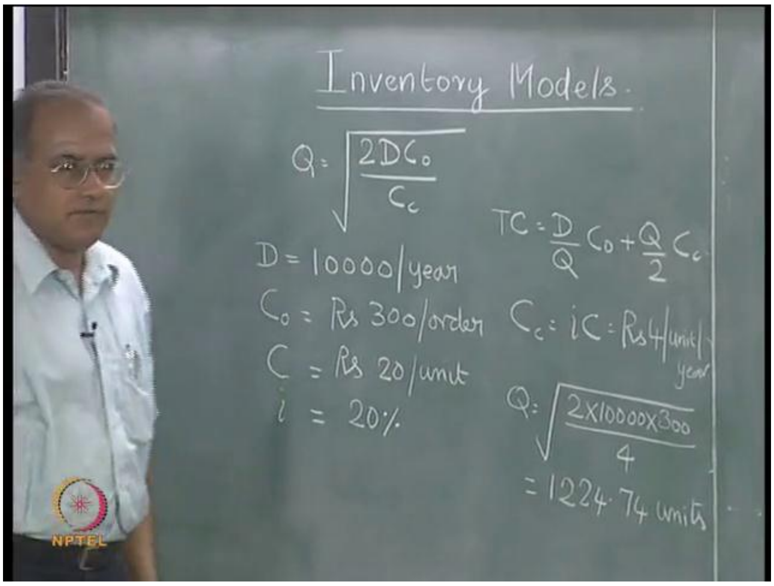

We also derived an expression for Q and we derived that Q is equal to root over 2 D C naught by C c, where D is the annual demand, C naught is the order cost or cost of placing an order and C c is the cost of holding or carrying inventor. This formula was derived when we tried to minimize this expression; total cost is equal to D by Q into C naught plus Q by 2 into C c. Now, this total cost is the total annual cost of inventory and in this model, the two relevant costs are the ordering cost and the inventory carrying cost.

We do not consider shortage cost, because one of the assumptions is, no shortage or back order is allowed which means that, this stock position will come right up to 0, it will not go beyond that. So, it will come only up to 0, so no shortage cost is allowed; we may or may not include the actual cost of the item. We saw in the previous lecture that, the actual cost of the item is D into C per year, this is per year or per given time period. So, it will be D into C if it is per year and it does not depend on Q, the ordering quantity.

Therefore, when we differentiate this, the term D into C does not contribute anything and it is customary, therefore to leave out that term, the term that involves the actual cost of the item and consider only the ordering cost and carrying cost and then optimize it to get this. Now, let us workout a numerical example to understand a few more things about this formula and the use of this formula.

Now, let us take an example, where D which is the annual demand of the item is say, 10000 per year, C naught which is the ordering cost is say, rupees 300 per order, C which is the unit price of the item let us say, is rupees 20 per unit and i is the interest rate, which let us say is 20 percent. So, C c will now be defined as, i into C will be 20 percent of rupees 20, which is rupees 4 per unit per year. Now, when we apply this formula, we are now given D, C naught and C c, 10000, 300 and 4.

So, we apply this formula to get Q is equal to root over 2 into 10000 into 300 by 4, which on simplification would give us 1224.74 units. We have already seen that, this Q when we derived it, Q is root over 2 D C naught by C c. We take only the positive quantity, we do not take the negative quantity, because Q represents an order quantity, so you get 1224.74 units as the economic order quantity in this case. Now, this means that, whenever we order, we will order for 1224.74 units to begin with, of course we will have questions like, how can I order for a fraction of a unit and so on.

We will answer those questions as we move along, but right now based on this formula, we say that, every time we place an order, the order for 1224.74 units. Now, let us also try and find out the cost associated with this, so we need to substitute Q equal to 1224.74 into this, in addition to D C naught and C c, to get this total cost. Now, instead of substituting 1224.74, let us try and substitute this value, which is the value of the economic order quantity. This equation for Q into this and compute this total cost as a function that is, that does not directly use Q, let us compute this in terms of D, C naught and C c, so let us go back and substitute.

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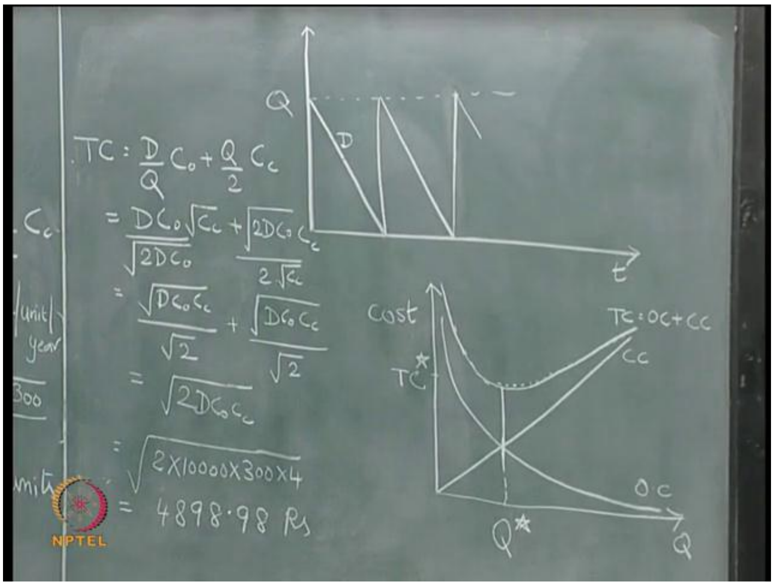

Now, total cost TC is equal to D by Q into C naught plus Q by 2 into C c, now Q is given by root of 2 D C naught by C c. So, on substitution, we will get D C naught by root over 2 D C naught C c plus root over 2 D C naught by 2 into C c. So, this on simplification would give us root over D C naught C c divided by root 2 plus root over D C naught Q by 2 into C c. So, Q is root over 2 D C naught C c, Q by 2, so this 2 comes here, this C c comes here.

So, on simplification, this root over C c and this C c will cancel to give another root over C c, so root over D C naught C c divided by root 2, because there is a root 2 here, there is a 2 here, so we get this expression. This on further simplification will give you 2 times root of D C naught C c by root 2, which will give us root over 2 D C naught C c. So, we can now write the total cost in terms of only the known parameters, which are D, C naught and C c and not in terms of the computed number, which is Q.

So, the total cost at the optimum, the minimum total cost that we will incur is root of 2 into D into C naught into C c. And in our example, when we substitute, we will get root over 2 into 10000 into 300 into 4, which on simplification will give us 4898.98 rupees per year. Now, this quantity is does not include the actual cost of the item, the actual cost of the item, if we take 1 year period will be 10000 is the demand per year, say 20 is the unit price, so 200000 will be the actual cost of the item.

So, if we say that, the total inventory cost is 4898.98 it means, we are only giving the order cost plus carrying cost for the year and we have not included the actual cost of the item. If we said 204898.98 it means, we have also added the actual cost of the item per year. Now, from this formula, we also observe an very interesting thing which is, at the optimum that is, when we substitute root over 2 D C naught by C c, at the optimum the component of the order cost and the component of the carrying cost are equal.

So, when we do this optimization to find out the value of the economic order quantity, we are essentially trying to find out that value of Q, for which the order cost is equal to carrying cost. Therefore, the total cost is 2 times the order cost and carrying cost, which comes to this expression. Now, let us also try and draw a graph that kind of depicts or shows the economic order quantity. So, if we look at, now cost versus order quantity, now the order cost is of the form D by Q into C naught, D and C naught are known, Q is the unknown, so it is constant divided by Q.

So, when we draw this, we will get, the curve will be like this, this will be the order cost curve, it will be a rectangular hyperbola, which will show this D by Q into C naught, this is the order cost, carrying cost is 2 by 2 into C c, which is constant into Q. So, it is a line, which passes through the origin, so the carrying cost will be like this. Now, we have already said that, the economic order quantity is the point, where the order cost is equal to the carrying cost, which we got from here, they are equal.

So, this is the economic order quantity, where the two curves intersect, this I call as Q star, which is the economic order quantity. You can write Q equal to or you can write Q star equal to, the best value of Q is called Q star and this is the place, where the total cost is also minimized, because we are substituting this, which is the minimum value into the total cost function. So, we will have, these two are equal, so this is the point, so the total cost curve will actually look like this, so this is how the total cost curve will look like.

Now, this is the TC star, this is the root of 2 D C naught into C c, so the total cost is 2 times this and it happens for the corresponding Q star. So, this is how the cost be, this is C c carrying cost, this is TC equal to order cost plus carrying cost. Now, let us try and answer a few simple questions, one is when we did this calculation, we have now found out that, the economic order quantity is 1224.74. So, the first question that we would ask is, are we justified in treating this Q, order quantity as a continuous variable?

When we differentiated this and then we set it to 0, when we set the first derivative to 0 to find the minimum, we assume that, the Q is continuous. Now, let us let us assume that, we took Q as a continuous variable, a assumption of Q being a continuous variable was made considering the ease of the mathematical derivation for it. If Q were to be defined as an integer variable then the methodology to get it would be different. Since for convenience, we define Q as a continuous variable, we could easily set the first derivative to 0 and get the value.

And show that, the second derivative is positive indicating that, this value is indeed a minimum for the plus value of Q. Now, because of that, we also assume to get a number like 1224.74, which poses the question, how can I order a fractional quantity of the item? So, the simplest thing to do perhaps is to try and round it off to the nearest number, which could be 1225, let us say. And let us see, what happens to the total cost, whether it is very, very different from the absolute minimum, if we look at 1225. So, let us try and do some quick computations.

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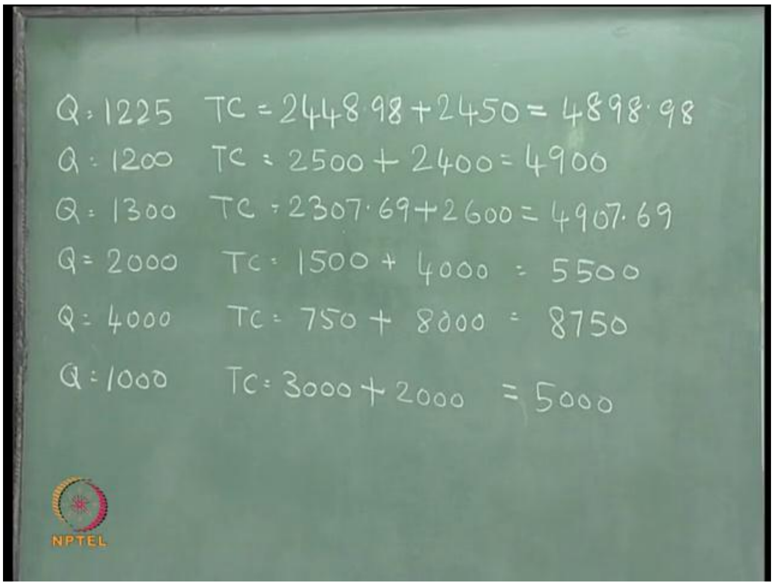

So, when Q is equal to 1225, TC is equal to - all we need to do is to substitute 1225 here. So, TC will become 10000 into 300 by 1225 plus 1225 divided by 2 into 4, which will become... Now, when we substitute Q equal to 1225, which is the upper integer value or the closest integer value to 1224.74 and substitute 1225 here as well as here, the total cost is, it remains at 4898.98, which is the absolute minimum. So, between 1224.74 and 1225, there is absolutely no loss, so instead of implementing 1224.74, we could implement 1225 if we have to keep Q as an integer.

Then, comes a next question, now can my vendor supply 1225, suppose my vendor is willing to supply only in multiples of 100 then I may have to consider 1200 or I may have to consider 1300, whichever is minimum. So, let us try and evaluate the TC for Q equal to 1200, now TC will become 10000 into 300 divided by 1200. So, 300 1200 is 4, so 10000 by 4 is 2500 plus here we have 1200 by 2 is 600, 600 into 4 is 2400, is equal to 4900.

Now, here I am specifically showing the order cost component and the carrying cost component. Now, if Q equal to 1300, TC is equal to 10000 into 300 divided by 1300 becomes 4907.69. Let us just look at one more calculation or two more calculations, let us say the vendor can give only 2000 and the vendor is giving at 4000. Whereas, the vendor is giving in multiples of 1000 let us say then we are looking at say three numbers, Q equal to 1000, Q equal to 2000, let us say Q equal to 4000.

Now, at Q equal to 1000 which means, the vendor is giving in multiples of 1000, here 10000 by 1000, which is 10, 10 into 300 which is 3000 plus, 1000 divided by 2 is 500, 500 into 4 is 2000, 5000. Whereas, the vendor will give us only in 2000 then this is 10000 divided by 2000, which is 5, 5 into 300 is 1500, 2000 divided by 2 is 1000, 1000 into 4 is 4000, which is 5500. Now, if the vendor would give us only 4000 then it becomes 4000 10000 into 300 divided by 4000. So, this is two and a half, two and a half into 300, which is 750, plus 4000 divided by 2 is 2000, 2000 into 4 is 8750.

Now, let us try and understand a few things that we have calculated here and let us try and show some of these into this graph and try to understand little bit from this graph. Now, the first thing we did was to round this off to the nearest numbers, we tried 1225, when we realized that, it just is the same, the total cost is the same. So, if the supplier can give in multiples of 1 or any integer quantity then ordering 1225 is indeed economical. So, first let us understand that, we need not really worry about the fact that, we have a fractional number or a decimal number coming here.

It can be a rounded off to the next highest integer and there is absolutely no change in the total cost. Then we start looking at multiples of 100 then we looked at the two closest numbers, which are 1200 and 1300. And we actually realize that, for 1200, the total cost was 4900, which is hardly a rupee more than the earlier one and it is absolutely negligible. The increase is absolutely negligible, not even of the order of 0.1 percent; it is much less than that.

So, one can use 1200 and even if we round it off to the higher 100, which is 1300, which is an increase of about 75. We still realize that, the total cost has just become 4907, which is somewhat like 8 rupees more than this. So, the increase is 8 rupees on 5000, which is 1.6 rupees on 1000 and 0.16 rupees on 100, so it is 0.16 percent increase. So, there is 0.16 percent increase if there is 75 by 1200 into 100, which is roughly about 6 percent increase.

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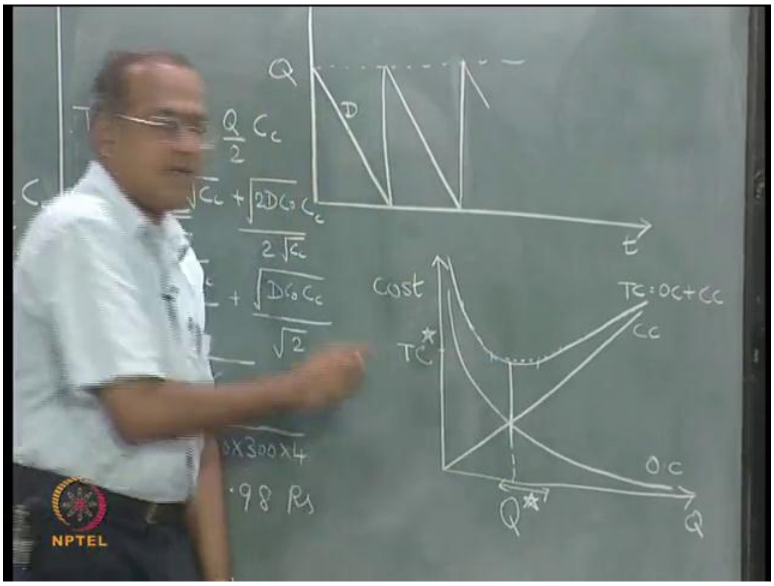

In the order quantity, is giving us 0.16 percent of increase in the total cost, which also shows somewhere that, the total cost curve is kind of flat near the optimum, which is what we have tried to show here, it is quite flat here at this zone. So, even if you kind of play around with Q roughly in this area, the TC, it does not increase very significantly. For example, even if we go to 1000 and 2000, which is the two nearest 1000s, if you want to approximate this, you realize that for 1000, the total cost is 5000, which is about 100 rupees increase.

But, for 2000, it shows about 600 rupees increase, which is a little significant, we are now talking of the order of 10 percent here, here you are still not talking of the order of 10 percent, you are still talking of the order of 2 percent here. And if you really increase it to 4000, this is where you are talking of a very, very large increase of nearly 70, 80 percent. So, as long as this Q is somewhere here, as long as any ordering quantity Q is near Q star, up to say 10 percent on either side of Q star, you realize that it is very flat near the minimum and therefore, the effect of increase in cost is not very high.

Only when you move the ordering quantity Q significantly away from this, either this side or this side, you realize that the total cost is very high. For example, if this is 1224.74, so let us say 4000 is somewhere here and therefore, we saw there it went up, total cost went up. Now, when it is 4000, you also realize that, total cost went up, the order cost is small, the carrying cost is large, so you see there the order cost is small carrying cost is large.

As you move to the right of Q star, the carrying cost is going to increase; the order cost is going to come down. For example, if you look at 1300, order cost is coming down carrying cost is going up, 2000, order cost is coming down carrying cost is going up, less than that 4000, order cost is coming down carrying cost is going up more. So, if the order quantity is less then Q star like 1000, see the order cost is going up and the carrying cost is coming down.

So, to the left of it if you realize, so somewhere here if we are then you realize that the order cost part is higher the carrying cost part is lower, but the total cost will always be higher, because this is the minimum. So, the one of the important lessons from the economic order quantity formula is the realization, that it is very flat at the minimum and therefore, one need not worry about the decimal or the fraction coming in here. One can conveniently round it off to the nearest 100 and still not incur a significantly high cost. Next thing that we need to look at is, if we are ordering 1224.74 units, every time we place an order. Now, in one year, we will be ordering 10000 divided by 1224.74.

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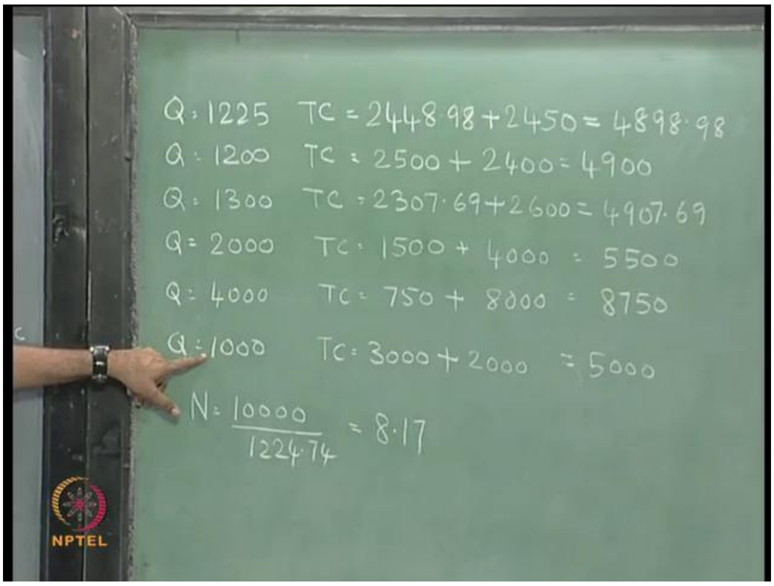

Number of orders n is equal to 10000 divided by 1224.74, this 10000 is the annual demand, which is shown here. So, I have a demand of 10000, every time I place an order for 1224.74, therefore I have 10000 divided by 1224.74, that many orders per year. Now, this on simplification would give us something like an 8.17 orders per year. Now, this rises another question, whether the number of orders per year should it be an integer or should it not be an integer.

The easier thing to do is to assume that, let it be an integer, so I order 8 times in a year, so roughly I say, I order every one and a half months. So, one and a half into 8 is 12. So, I order roughly every one and a half months and it is also convenient that, if we make this as 8 orders per year, the order quantity would become 10000 divided by 8, which is 1250, instead of 1224.74, 1250 is a much more convenient number. And we have already seen here that, whether it is 1200 or 1300, we are still in the border of 4900 to 4907, which is hardly 10 rupees on 4800 or 5 rupees.

So, it is always possible to try and have an integer number here, more for the sake of convenience. And when we have an integer number here and if this is a very comfortable number like 10000 then the order quantity also becomes a very comfortable number or even if it does not become a comfortable number or it is a fraction, it can always be rounded off to the nearest 100 on either side and so on. As such there is no sanctity about the fact that, this has to be an integer, need not be an integer.

It simply means that, if it is 8.17 orders per year and if the year comprises of 365 then it is like 47 days I place an order. So, every 47 days or every 46 days I place an order, plus minus 1 day does not matter at all, the order quantity can be adjusted suitably. But, what we learned from this is that, even though the actual results of the economic order quantity like 1224, 4898, sometimes or many times they may not be implementable exactly as they are.

But, the important learning is that, we need to fix our order quantities as close to the economic order quantity as possible, subject to other constraints and considerations so that, the actual cost that we incur is still around this 4898.98. It could be this, it could be (Refer Slide Time: 30:14) this, sometimes it could be this, sometimes it could even be this, but it should not be this. So, if right now we are having a vendor, who is giving us only 4000, because of which we are incurring an 8750.

If we do the economic order quantity and understand that, it is somewhere near 5000, the next thing we need to do is to see, whether our vendor can give this 1000 so that, we get a cost of 5000. Otherwise, we should try another vendor, who can give 1000 and spend an additional 5000 per year on this item, rather than by 4000 at a time and spend 8750 on that. This is the most important learning from the economic order quantity formula; now let us move to the next model.

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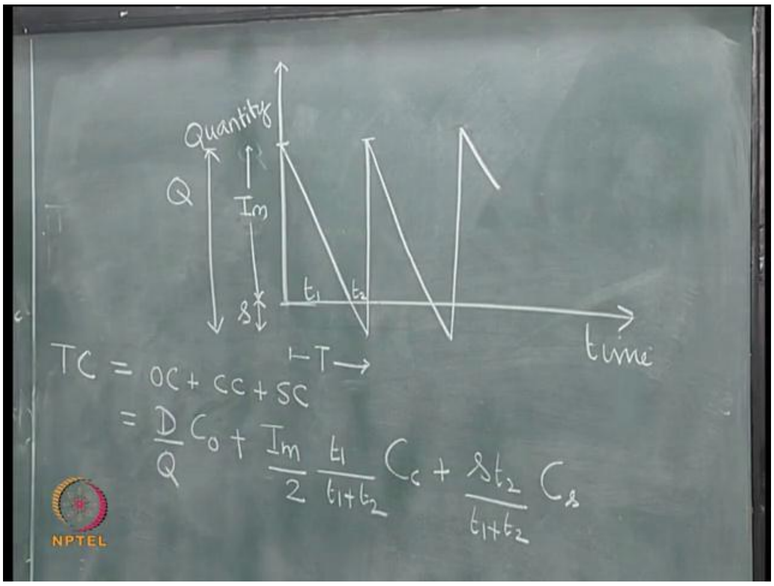

The second model is very similar to the first model, except for relaxing one assumption, in the earlier model or the first model; we did not allow shortages or backordering. Now, we are trying to see, at least to begin with mathematically, what happens when we allow the backordering. We defined four costs, we used only three of them, we also showed that amongst the three, one of them does not affect the decision. The actual cost of the item does not affect the decision; we left out the backorder stroke shortage cost, because of the assumption, that there is no backorder.

Now, let us try and allow some backordering at an additional cost and see, how this model behaves. So, what we do now is, let us assume that, we start somewhere here, so we consume, we reach the point where the stock position is 0. We also assume instantaneous replenishment which means, if I place an order here, I am going to get it immediately, which is what we did in the earlier model. Now, we are going to allow a little bit of backordering, so we do not place an order here.

We allow backordering, we build up some backorders then we place an order for Q, for an order quantity Q, which is instantly replenished. So, it is replenished and let us say, after the replenishment, it reaches this point, the point where we started. So, once again we consume at the rate of D, like what we did, once again build some backorder and at this point, you place the order for Q and get it replenished instantly and then it proceeds. So now, this is time, this is quantity, now we are going to use some additional notation, now this part, this part the quantity that we order here.

This is Q, so Q is starts from here. This is what we order. So this is our Q, the order quantity. Now, this part, which is the backorder part is called small s, which is the backorder and the level to which the inventory gets build up, it is called I m maximum inventory, which is here. So obviously, Q is equal to I m plus s. Let us say this is a cycle, which we call as capital T and the cycle repeats. Now, this cycle now has two parts, this part is called T 1, where there is inventory and there is this part called T 2, where there is back order and T 1 plus T 2 is equal to T.

So now, there are four costs that we will have, so let the economic order quantity be Q or the quantity that we order is Q. So, the order cost component, so the total cost has, TC has all the four costs, order cost plus carrying cost plus shortage cost plus item cost. Now, Q is the variable which is the order quantity, I m is a variable, s is a variable, but they are related, Q equal to I m plus s. Now, if we are going to compute this total cost for a year then the item cost is going to be D into C.

So, the item cost will not contain any of the variables Q, I m or s, so like we did in the earlier model, we leave out the item cost. So, we are going to have three costs in this model, we had two costs in the earlier model. So, if D is the annual demand and Q is the quantity that we order, the number of orders per year will be D by Q, like we did in the previous model. The order cost or cost for order is C naught, so D by Q into C naught is our total order cost per year.

Now, we need to look at the total carrying cost per year, the broad or general definition is average inventory into the inventory carrying cost. Earlier it was Q by 2 into C c, now we have to compute the value of the average inventory. Then if we take any cycle, say cycle like this, the total inventory that we hold in that cycle is area under this curve, which is half into I m into t 1. So, this much inventory is held for a period t 1 in the cycle and this is the backorder part of the cycle, so there is no inventory held.

So, zero inventory is held for a period t 2, the average inventory of I m by 2 just as we got Q by 2 in the earlier, the average inventory of I m by 2 is held for a period t 1. An average inventory of 0 is held for a period t 2, therefore the average inventory as such, at any point will be I m by 2 into t 1 by t 1 plus t 2. Let me repeat again, total inventory held in a cycle is area of this triangle, half into I m into t 1. So, that much total inventory is held for a total period of t 1 and 0 is held for a period of t 2.

So, average inventory is I m by 2 into t 1 plus 0 into t 2 by t 1 plus t 2, which simplifies to I m by 2 into t 1 by t 1 plus t 2, this is the average inventory, this into the inventory holding cost, which is C c. Similar manner, the average shortage or backorder is, backorder is 0 for a period t 1, average backorder of s by 2 for a period t 2. So, 0 into t 1 plus s by 2 into t 2 divided by t 1 plus t 2, which on simplification would give s into t 2 by t 1 plus t 2 into the backorder cost, which is defined as C s.

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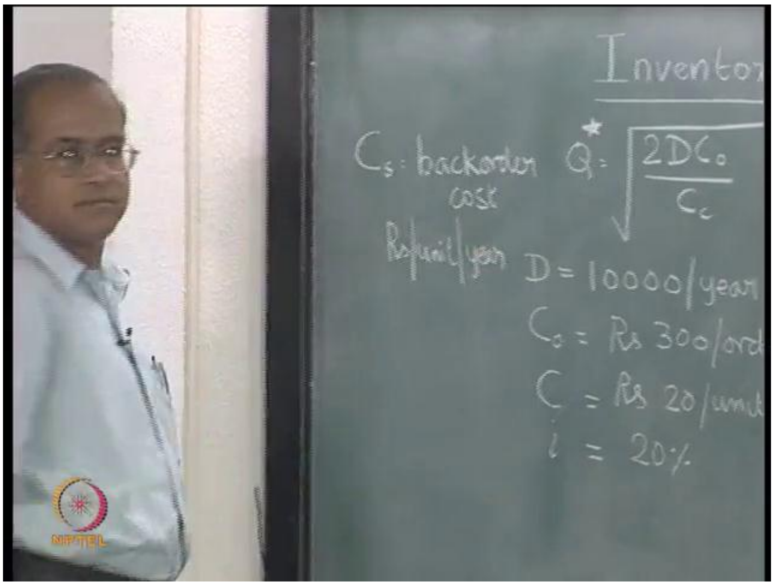

So, let us now define C s, which is the back order cost and this is right now defined with the same units as C c. So, this will be rupees per unit per year, C s and C c will have the same units, rupees per unit per year.

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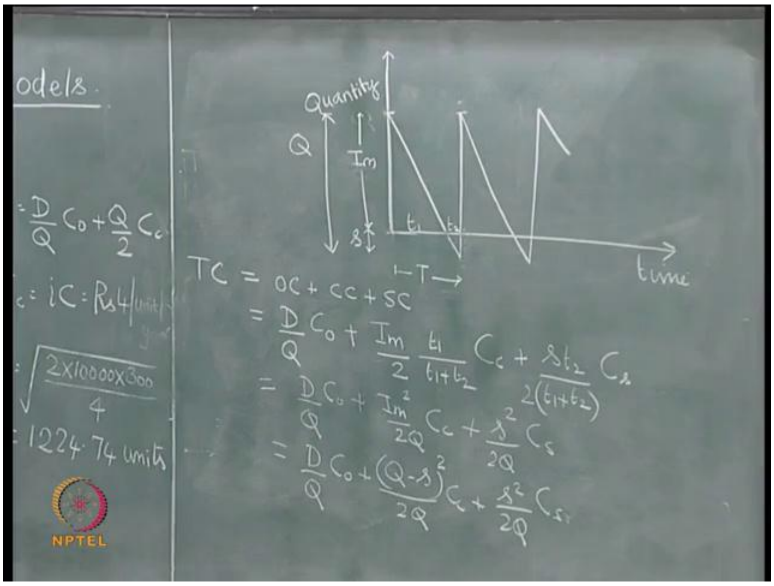

Note the deference, in the earlier model this was Q by 2 into C c, now it is I m into t 1 by 2 into divided by t 1 plus t 2 and so on. Basic motivation is that, if you take a cycle, part of the cycle which is t 1, is the time where we hold inventory and the remaining part of cycle which is t 2, is where the backorder is there. So, the inventory portion will contain the area under the inventory curve I m by 2 into t1 for a period t 1 plus t 2, s by 2 into t 2 for a period t 1 plus t2.

So now, this expression for TC has several variables now, it has Q which is unknown, it has I m which is unknown, it has t 1 that is unknown, t 2 that is unknown and s that is unknown, but the unknown variables have some relationships. We have already saw that, I m plus s is equal to Q and t 1 plus t 2 is equal to some T, that we have not used. So, we try and simplify this a little bit using some similar triangle property. Now, t 1 by t 1 plus t 2 from similar triangles, is equal to I m by I m plus s, t 1 by t 1 plus t2 will be I m divided by I m plus s.

But, I m plus s is Q, so this will become D by Q C naught plus I m by 2 into t 1 by t1 plus t 2 is I m by I m plus s, which is I m by Q. So, this will become I m square by 2 Q into C c. Similarly, t2 by t 1 plus t2 is equal to s by s plus I m, this will give t 2 by t 1 plus Tt 2 is equal to s by Q. So, on simplification, this will give s square by there is a 2 that I have missed, here s by 2, so s square by 2 Q into C s.

Let me explain it again, when we calculated this portion, which is the backorder cost portion, average backorder in a cycle is, for t 1 part of the cycle, there is no backorder, for this part of the cycle, the total backorder is area under the curve half into s into t 2. So, the average back order at any point in time is half into s into t 2 divided by t 1 plus t 2. So, half into s into t 2 divided by t 1 plus t 2, this multiplied by the backorder cost is C s.

So, on simplification, t 2 by t 1 plus t 2, t 2 by t 1 plus t2 is equal to s by I m plus s, which is s by Q. So, this portion becomes s by Q, so this becomes s square by 2 Q into C s. So now, we are eliminated the t 1 and t 2 from this, because they are also dependent on, they are related to I m and s, so we have eliminated these. Now, we have only three unknowns, which are Q, I m and s and we also know that they are related, because Q is equal to ii m plus s.

So, what we do now is, we eliminate I m by substituting I m is equal to Q minus s to get D by Q into C naught plus Q minus s the whole square by 2 Q into C c plus s square by 2 Q into C s. So, this way we have now eliminated I m also by substituting I m is equal to Q minus s, so we have only two unknowns, which are Q and s. The two unknowns are, how much to order, which is the order quantity and what is the level of backorder, which is the back order level, so Q and s are the two unknowns.

And now, this expression does not have any dependency, all the dependencies have been taken care of by the substitution using similar triangles as well as using Q is equal to I m plus s. So now, we can differentiate this partially with respect to the two variables Q and s, to try and get the optimal values of Q and s. So, let us do that, so first let us differentiate this with respect to Q and then we differentiate this with respect to s.

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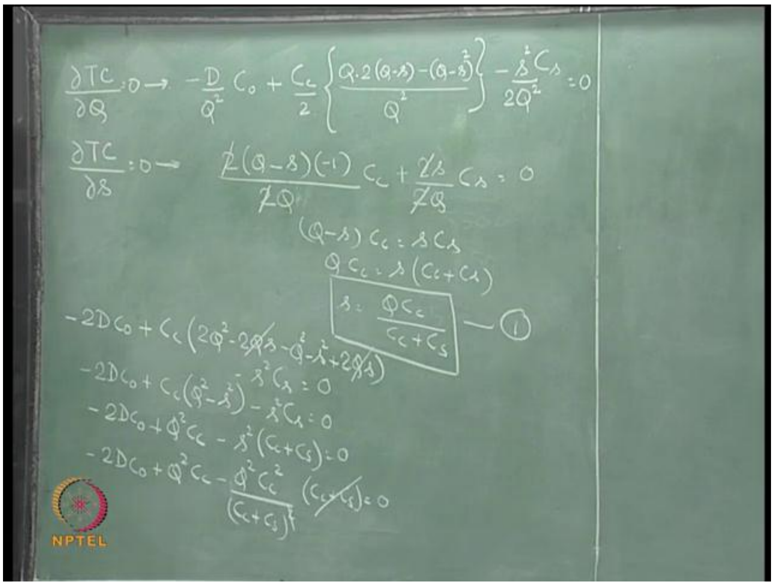

So, dou TC by dou Q equal to 0 gives partially differentiating total cost with respect to Q would give us, this would give us the term minus D by Q square into C naught, which is a same term that we got in the earlier model. So, minus D by Q square into C naught plus, now this is a term where Q appears in the numerator as well as in the denominator. So, here we need to use the u by v differentiation, so v square will come in the denominator v d u minus u d v by v square.

So, we take this C c by 2 outside, so plus C c by 2, u by v will give us v d u minus u d v by v square. So, we put a Q square here, v d u, so this is Q into differentiation of this. So, Q into 2 times Q minus s is what we get, 2 times Q minus s into 1, we differentiate this Q you get 1, so v d u minus u d v, Q minus s the whole square into differentiation of this Q which is 1. So, minus Q minus s the whole square is what we have here, this term Q appears only in the denominator, so it is easy to differentiate, minus s by 2 Q square into C s equal to 0.

This Q will give minus 1 by Q square, so minus 2 Q square s square C s v equal to 0, partially differentiating, so s square will remain as s square, s square C s is 0. Now, partially differentiating with respect to s dou TC by dou s equal to 0 would give us, this term will not contribute anything, because there is no s term here, so this is a constant as particular as partial derivative is concerned.

So, this term will not contribute anything, this term has s only in the numerator, so this will become 2 times Q minus s into minus 1 divided by 2 Q square C c plus 2 s square by 2 Q, s square on differentiation would give us 2 s. So, 2 s by 2 Q into C s is equal to 0, it may repeat, s appears in the numerator, so 2 times Q minus s into minus 1 divided by 2 Q into C c, 2 s by 2 Q into C s equal to 0. So, this 2 and this 2 will get cancelled, when you take this Q to the other side it goes, so we have minus Q plus s into C c plus s C s equal to 0 or let me put it or let me simplify it differently.

I will simply take this term to the other side, so Q minus s into C c is equal to s C s, so Q C c is equal to s into C c plus C s, from which s is equal to Q C c by C c plus C s. So, this is the first thing that we derive, we have this, now we go back to this one and then we have to use s is equal to Q C c by C c plus C s on to this one, to try and get the value for Q. We will do that now, now I multiply this by 2 so that, the 2 Q square gets canceled, so I will have minus 2 D C naught plus C c into, this is 2 Q square minus 2 Q s minus Q square minus s square plus 2 Q s minus s square C s equal to 0.

I just multiplied this by 2 so that, I have this expression, now here I can cancel this plus 2 Q s and minus 2 Q s, plus 2 Q square minus Q square will give me Q square, so minus 2 D C naught plus C c into Q square minus s square, minus s square C s equal to 0. Now, this will become minus 2 D C naught plus Q square C c minus s square into C c plus C s equal to 0.

Now, we substitute for s square from here, minus 2 D C naught plus Q square C c minus s square is Q square C c square divided by C c plus C s the whole square into C c plus C s equal to 0. Now, this will go and this will remain, now this will simplify to minus 2 D C naught.

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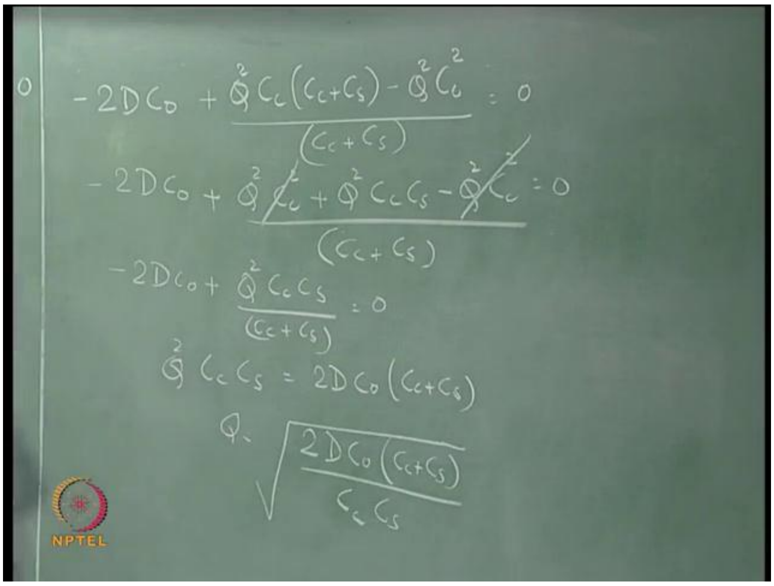

Take this here, only here plus Q square C c into C c plus C s minus Q square C c square by C c plus C s is equal to 0, I am leaving this as it is, I am taking this only up to this. Now, minus 2 D C naught plus Q square C c square plus Q square C c C s minus Q square C c square divided by C c plus C s equal to 0. Now, these 2 terms will get canceled, so minus 2 D C naught plus Q square C c C s by C c plus C s equal to 0, from which Q square C c C s by or Q square C c C s is equal to 2 D C naught into C c plus C s, from which Q is equal to root over 2 D C naught into C c plus C s divided by C c into C s.

So, when we use this model, where backorder is allowed and then we derive, we get the expression s, the backorder quantity is Q C c by C c plus C s, the order quantity Q is 2 D C naught by C c C s into C c plus C s. Now, we can substitute this s and Q back into the total cost function, which is here to try and find out, what is the total cost at the optimum, that and comparison of the model with backordering. Comparing a model with back ordering to a model without back ordering which means, the similarities and differences between models 1 and 2, we will see in the next lecture.