**Decision Support System for Managers**
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**Module - 02**
**Models in Decision Support System**
**Lecture - 09**
**Solution Techniques - Optimization: Linear Programming ( Contd. )**

Hello and welcome to "Decision Support Systems for Managers"! We are into week 2 or module 2, the way you want to say and lecture 4 of this module. And we will continue with optimization, linear programming methods in this week or in this lecture; ok.

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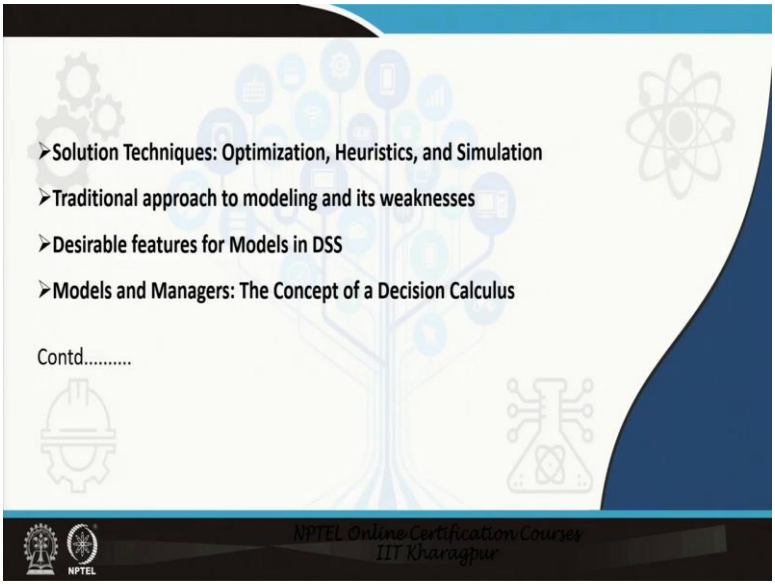

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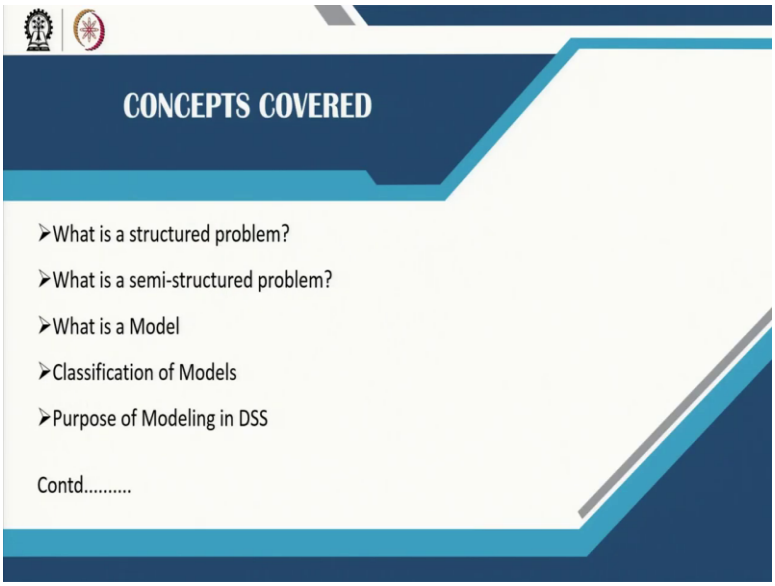


Now, in the previous week this was the contents for this week, as we have already mentioned concepts covered. And, we were into this part, this first part that is solution techniques optimization heuristics and simulation ok. Now, in optimization we had linear non-linear and integer programming; ok.

Now, we have started off with linear programming and we had solved one maximization problem. We had to manufacture two types of bags and the profits were given our objective function was to maximize the profit, for these bags right, our objective function was to maximize profit given the constraints. And, we drew one graph and then we found out how to or how many bags of ordinary and how many deluxe bags can be manufactured; ok.

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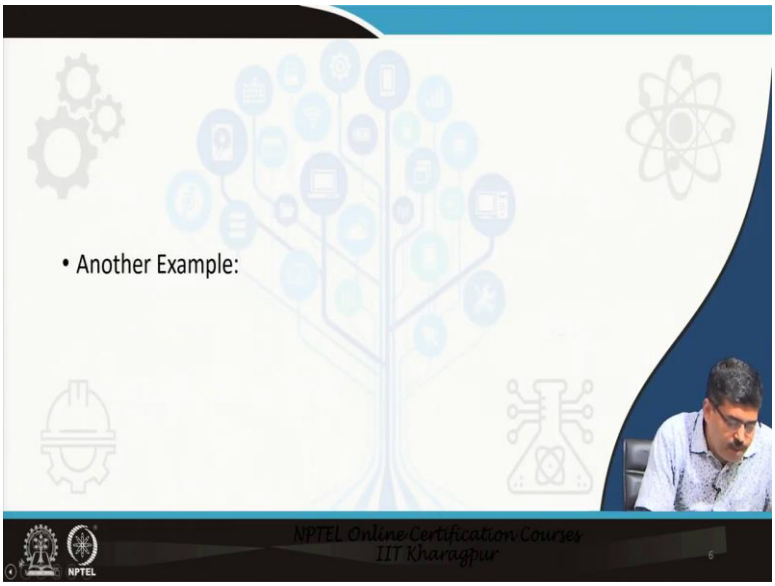


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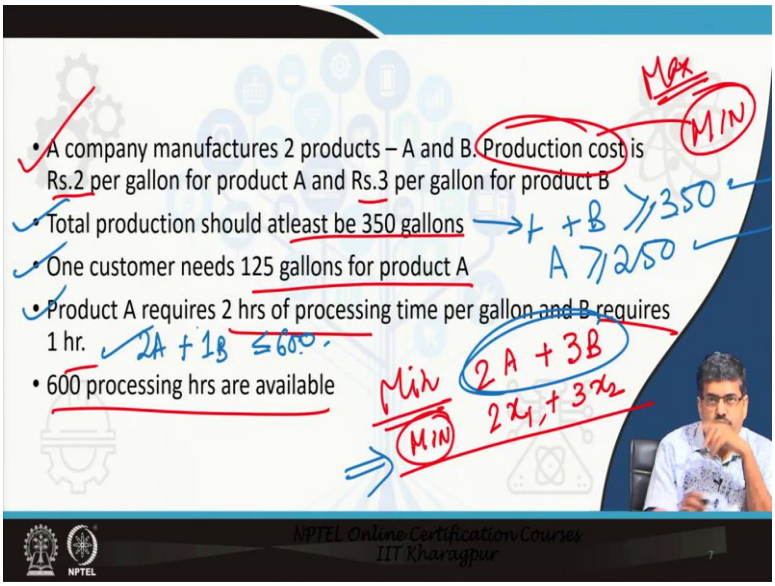


Today we will take another example; this is another dimension of optimization problem. Let us see a company manufactures 2 products A and B. Product cost is rupees 2 per gallon for product A and rupees 3 per gallon for product B. A company manufactures 2 products A and B production cost is rupees 2 per gallon for product A, and rupees 3 for product B right. 2 products are getting manufactured 3 for a 2 for B sorry 2 for a 3 for B.

Total production should at least be 350 gallons. One customer needs 125 gallons for product A. Product A requires 2 hours of processing time per gallon and B requires 1

hour. 600 processing hours are available, very simple problem. 2 products right A and B; right; ok. Production cost is given I do not know the selling price. So, do I cannot maximize it right, selling price is maximized not production cost. So, production cost is minimized; ok. So, production cost is minimized; ok.

So, let us see minimize what? Rupees 2 per gallon for product A, and rupees 3 per gallon for product B. How much product A is manufactured? 2 per gallon. How much product A is being manufactured? Let us take it as A or you can take x 1, how much B is being manufactured? 3 per gallon. Let us take B or you can take it as x 2. So, 2 x 1 plus 3 x 2; ok.

So, how many A is manufactured? I am repeating again x 1 quantity of A is being manufactured x 2 quantity of B is being manufactured. Cost for x 1 is 2 per piece and 3 for x 2. So, this is basically the I need to minimize this cost, I need to minimize this cost 2 x 1 plus 3 x 2; ok; now clear to now done.

Now, let us see to so this is the first point is taken care of a company manufactures 2 products A and B, production cost is rupees 2 per gallon for product A and 3 per gallons I have to minimize this 2 x 1 plus 3 x 2.

Let us take the next one total production should at least be 350 gallons, this point total production should at least be 350 gallons; ok. So, I can take A, I can take x 1 let us take A. So, A; I am sorry; so, A plus B should at least be 350 means this should be greater than 350, A plus B should be greater than 350 gallons. One customer needs 250 gallons for product A.

So, product A definitely has to be manufactured 250 it can be more, if we produce more we will give it to somebody else sell it to somebody else, but it should be definitely 250 it can be more.

So, A greater than 250 are you understanding it is not equal, one customer needs 125 gallons of product A another customer may need. So, I should produce minimum of 250. So, it should be greater than, equal to 250; ok. And product A requires 2 hours of processing time; this one.

So, product A requires 2 hours of processing time, per gallon and B requires 1 hour of processing time and 600 processing hours are available. So, this total thing can be should be completed within less than 600 hours agreed? So this is my problem. Let us go to the next slide and see. So, just look at it min 2 x 1 plus 3 x 2 what are the constraints A plus B greater than 350 this is my production constraint, one supplier wants 250 and my production hours available is 600 hours; ok.

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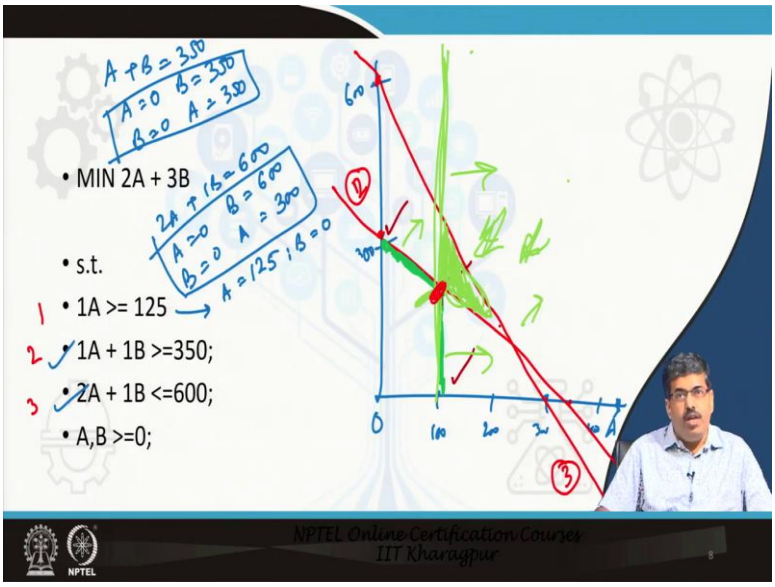


So, let us; this is the problem; minimize 2 A plus 3 B go back 2 A plus 3 B here 2 A plus 3 B; ok; subject to A is 125 one customer wanted 125 total production has to be 350 A plus B has to be 350. And the hours 600 processing hours are there, 2 is required by A and 1 hour is required for B. So, 2 A plus 1 B 600 processing hours are required; right.

Now, how do we; how do we prepare the graph very simple, let us again prepare the graph; erase all; ok, A plus B is 350. So, A plus B is equal to 350 this one I am doing, if A is 0 B is 350 B 0 A is 350, this is one. Second is 2 A plus 1 B is 600 2 A plus 1 B is equal to 600 A is 0. So, B is 600 B is 0 so, A is 300 and the other 2 constraints are A is 125. So, A is 125 this one B is 0.

Let us draw the graph; A values are 350 and 300; ok. So, this is 0 this is A 100, 200, 300, 400 100 200 300 400 ok and B values are maximum 600. So, let us 300 let us put this at 600 A 0 B 350 sorry A 0 A 0 B 350, B 0 A 350 A 0 B 350, B 0 A 350 equation 2 ok. This is equation 1 2 3 to A 0 B 600, A 0 B 600, B 0 A 300 equation 3 and A is 125.

Let us use a different color A is 125. So, it should be somewhere here and B is 0; ok. So, what is the minimal? The minimal point is either it this is the range right this is the range, minimal point is either this one or this one or this one ok. So, we will have to solve now the intersection of these two lines intersection of this point is anyway easy intersection of this point, we need to calculate and see how much we can produce right. And this is the A 125 line; ok.

So, the minimum point is this block because, this one is coming down this one is equation 2 is greater than so, it is going up equation 3 is coming down and this one anyway is greater than. So, this one also is going this way. So, this is the minimum point the minimum point is this particular block, the minimum point is this particular point ok, did you understand I am repeating again.

Let me clear away; this eraser let me clear away this part. What is happening; see equation 1, 1 A greater than equal to 125 equation 1, 1 A greater than equal to 125.

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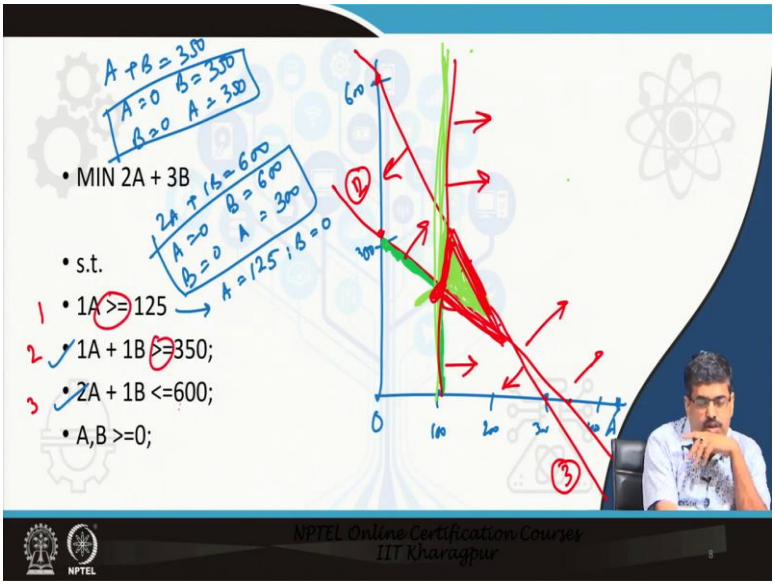


This is your equation one line greater than means it is moving in this direction ok. Equation 2 is also greater than, this is what? This is equation 2. So, this one is also moving in this direction and equation 3 is less than. So, this is equation 3 equation 3 is this line is equation 3 line it is less than.

So, this is the common space this is the common space as in green this is the common space right. This is the common space agreed? So, I am sorry I missed this 125 line in the earlier thing.

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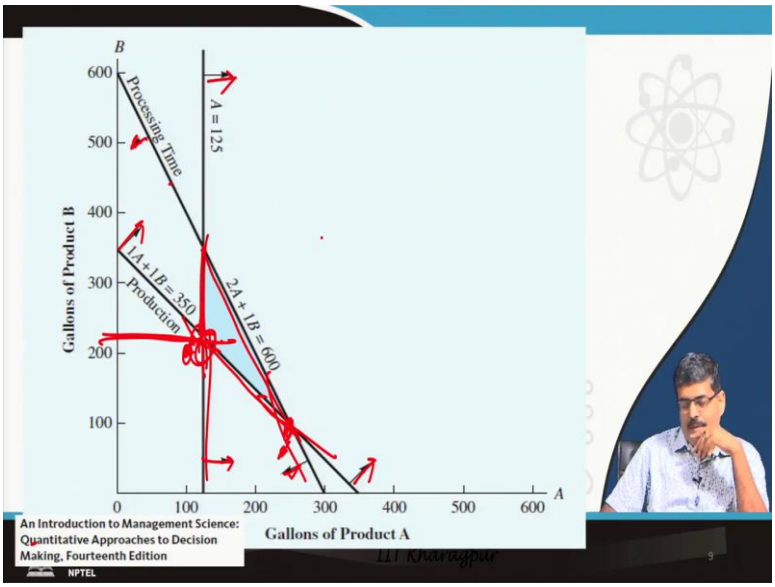


So, if you go now. So, you see as we mentioned this was this way this was going here greater than equation 1; ok, this was my equation 2 and this was my equation 3 it was coming down. So, the minimum point is this one; ok. This is the minimum point we have to find out the minimum point here.

Definitely this cannot be the minimum point, this has to be the minimum point or this has to be the minimum point. We have to solve these two we have to solve these two equations to get this point and these 2 equations to get the value at this point. And then put it in the min main equation and see how much the value is; ok.

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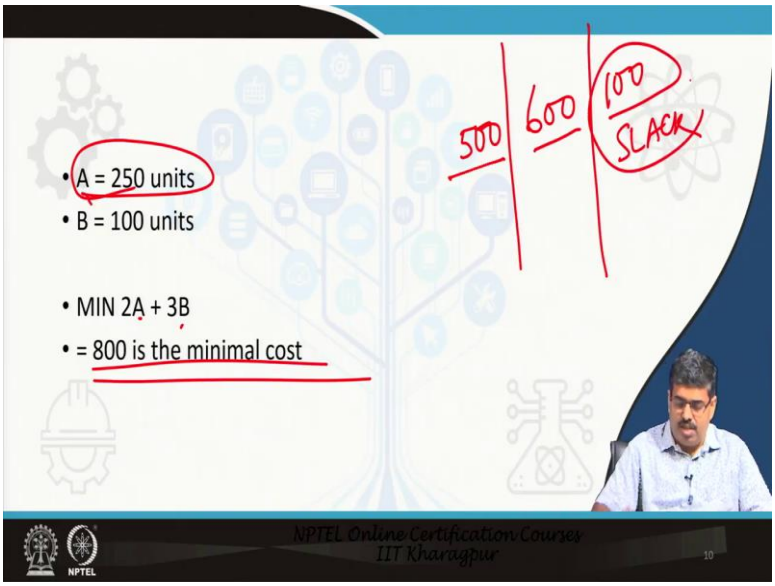


So, 800 will be the minimal cost 250 and 100 is the price. So, this is the minimal point 100 and 250 this is the minimal point right; ok. So, and if you put 2 into 250 is 500 3 into 100 is 300. So, 800 is the minimal cost; ok.

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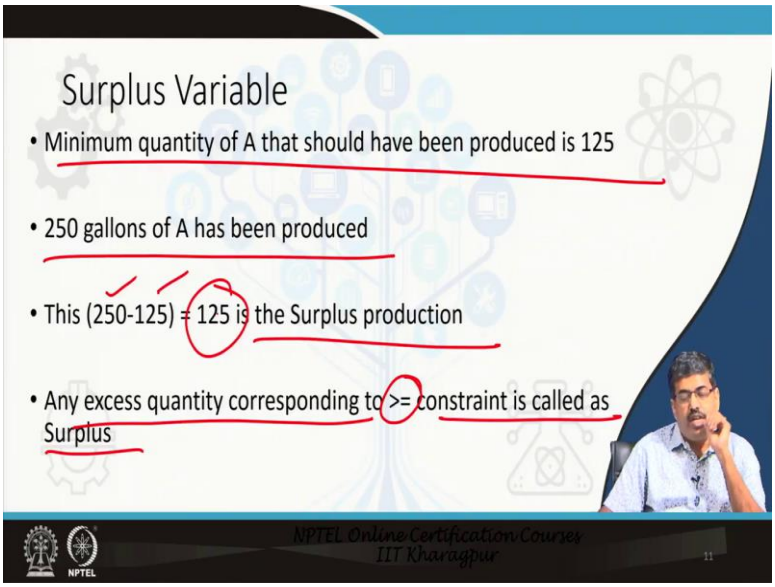


Now, in the previous class we had mentioned about what is a surplus variable or a slack variable, which one slack variable or a surplus variable slack variable. Now, in the previous class we have mentioned what is a slack variable. Slack variable is what when

you have used 500 units of your resources, and resources available to you was 600. So, how much is left over? 100 is left over, this 100 was called as slack variable; ok.

Today, we will explain what a surplus variable is; ok. Let us go now minimum quantity of A that should have been produced is 125 in the given problem, how much are we producing of A? As per solution we are producing 250 units right. So, 250 gallons has been produced.

So, this 250 minus 125, 125 is the surplus production. And, excess any excess quantity corresponding to a greater than equal to constraint is called as surplus. Any excess quantity corresponding to greater than equal to constraint is called as a surplus.

Let us see what do we mean by that let us go by the equation, any greater than quantity this one this one; ok. Any excess greater than any excess in greater than is surplus here we got 250 125, we have used we had to we had produced 250. So, that is surplus any excess quantity corresponding to greater than is called as surplus; ok.

This gives us an indication how much more we can produce given the constraints, this is the learning for the manager. This is the learning outcome for the manager, how much more we can produce given a surplus? Sorry, given a particular output or target; ok.

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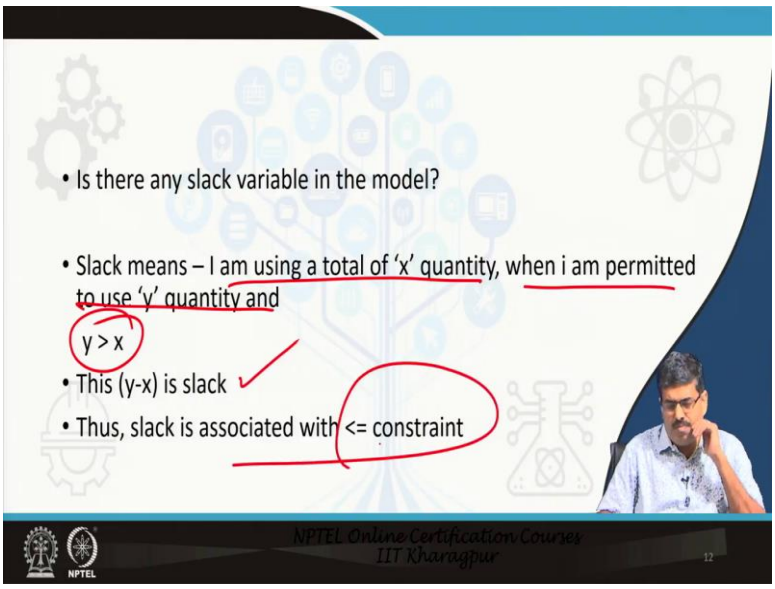


Is there any slack variable in the model, what is the slack variable remember? You have used 500 you had 600. So, 100 is the slack is there any slack variable. Slack means I am

using a total of x when I am permitted to use y, and y is greater than x. I have permitted to use 600 I have used only 500, this balance is the slack; slack, is associated with less than constraint you do not need to remember this.

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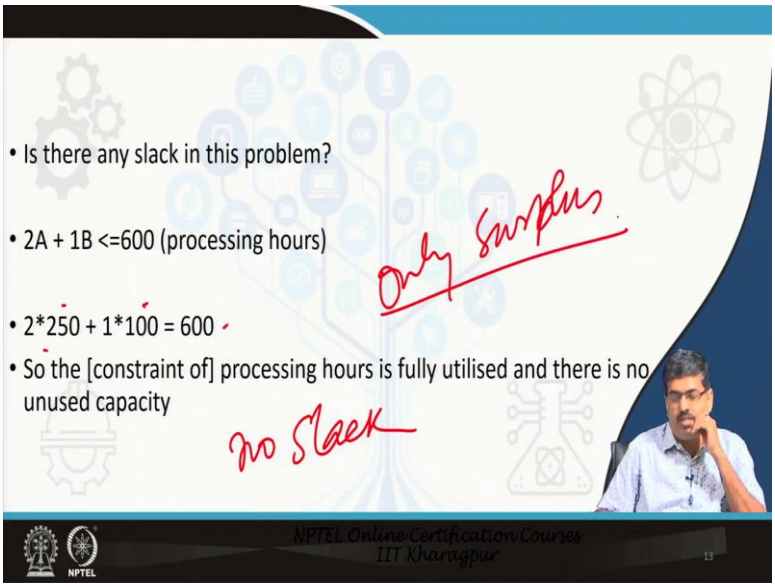


Is there any slack 2 A plus 1 B less than 600 2 into 250 A 100 is B 600. So, the constraint of processing hours is fully utilized and there is no unused capacity. So, this model has no slack; ok, only surplus; ok.

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Now, let us come to another term called sensitivity analysis.

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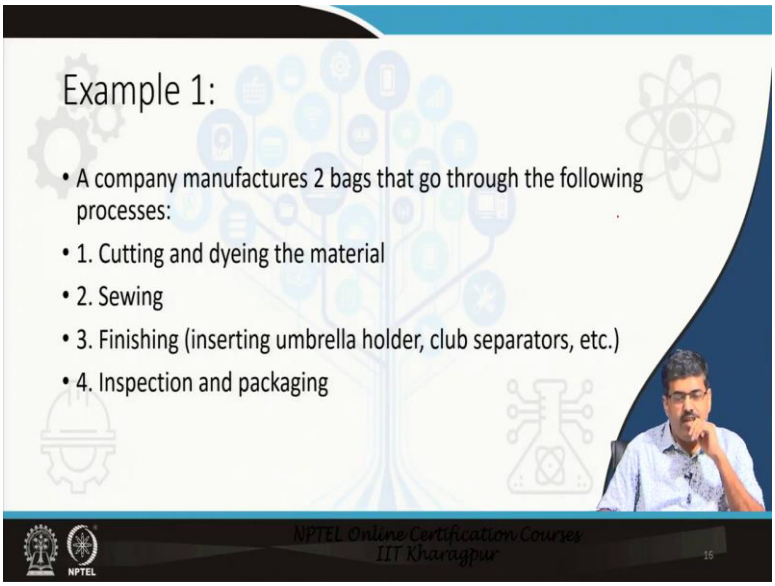


What is sensitivity analysis? Recall our example in the previous lecture, which is replicated here it was 2 bags cutting, dyeing, sewing finishing; ok.

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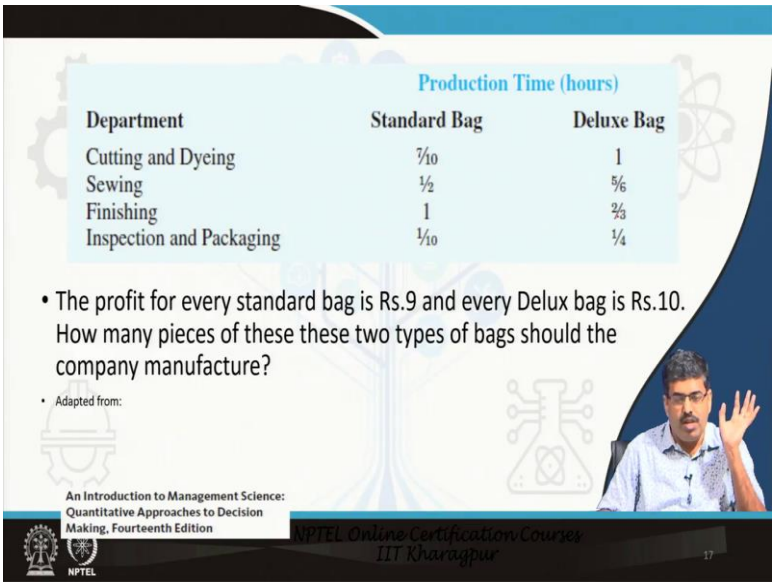


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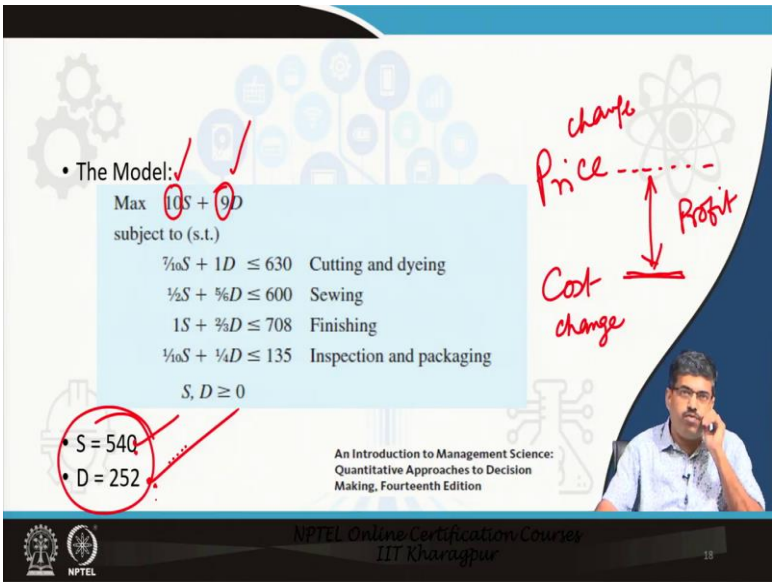


Standard bag this was it and ultimately, we got the solution as S is equal to 540 bags and deluxe standard is 540 bags and deluxe is 252 bags; ok. What was my profit per bag? 10 and 9 right S was 540 and deluxe was 252 bags.

Sensitivity analysis says that tomorrow. What is it assuming this profit? This profit is assuming that my cost is this and my market price is this. So, this balance is my profit right agreed. Tomorrow my costs may change my price may also change, agreed try to

understand it very simple right. Now, the model says that 10 rupees is the profit for every standard bag and 9 rupees is the profit for every deluxe bag right.

So, if 10 rupees is the profit per standard bag and 9 rupees is the profit per deluxe bag. You should produce 540 of standard bags and 252 of deluxe bags 252 of deluxe bags. But, tomorrow market price may change, price change; cost may also change. So, this profit may shrink or expand question is that.

Then I should this my production quantity should also change, my production quantity should also change, issue is this is called as sensitivity it changes; ok. Question is what is the range of output? Now, absolute output is 540 and 252 question is what is the range of output, if I produce within that range my profits will not change right. So, let us move to understanding this.

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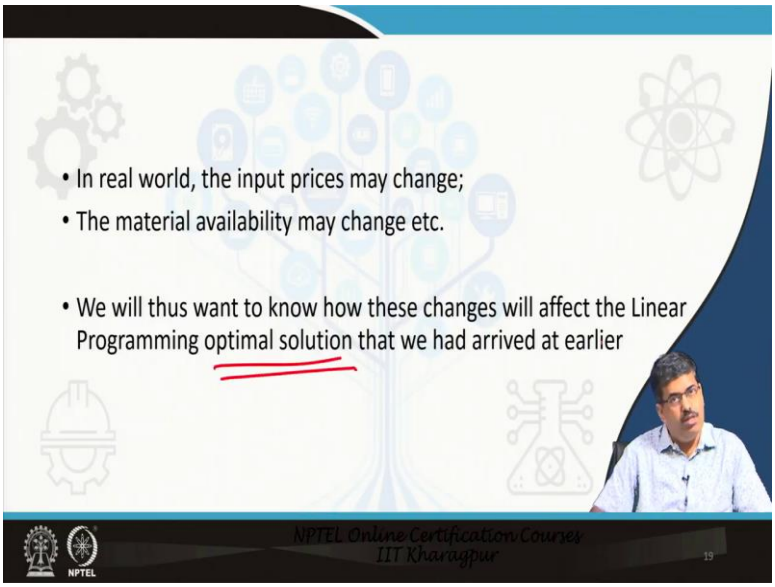


In real world the input prices may change, the material availability may also change. We will just want to know, how these changes will affect the linear programming optimal solution that we had arrived at earlier; ok.

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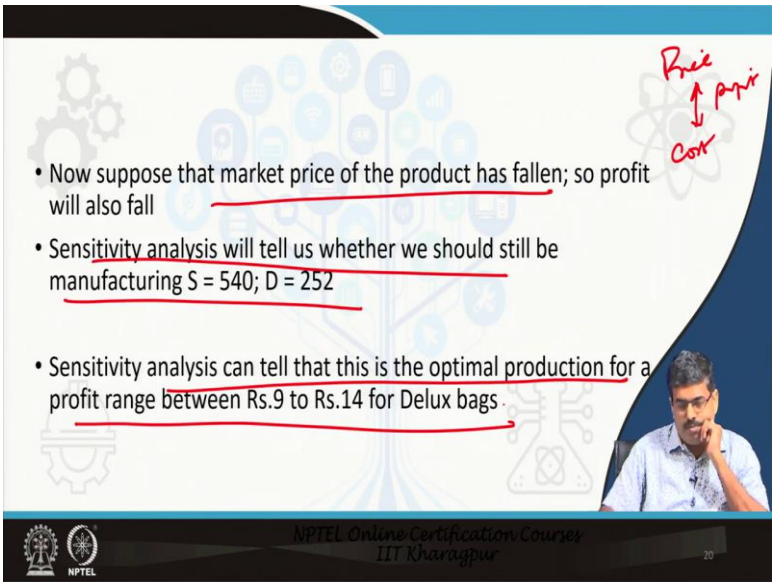


Now, suppose the market price of the product has fallen remember, you are just saying price and here is the cost so, balance is the profit. Now, suppose the market price of the product has fallen so, profit will also fall.

Sensitivity analysis will tell us whether we should still be manufacturing S 540 and D 252. Sensitivity analysis can tell that this is the optimal production for a profit range between 9 to 14 for deluxe bags if such a thing is there; ok. Sensitivity analysis can tell us that if you produce, this your profit range will be between 9 to 14 for deluxe bags that is what sensitivity analysis will tell us; ok.

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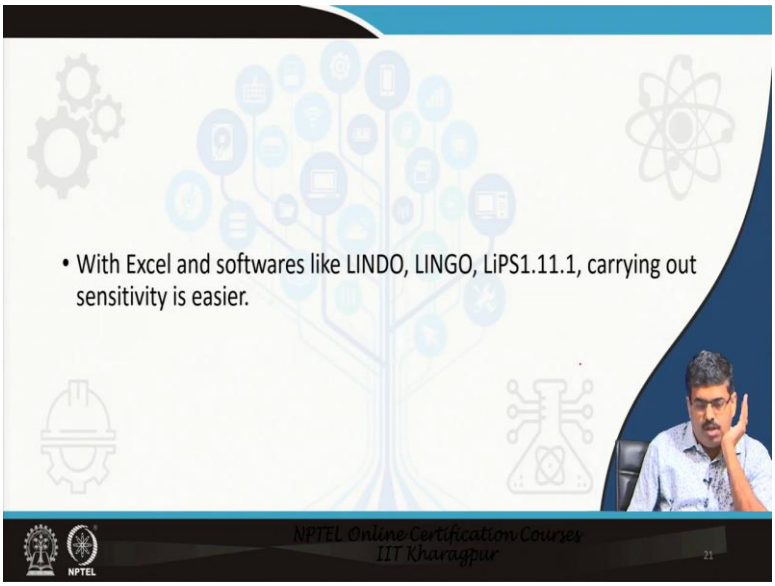


With Excel and software's like LINGO, LINDO, LINGO, LiPS carrying out and sensitivity analysis is very easy; ok.

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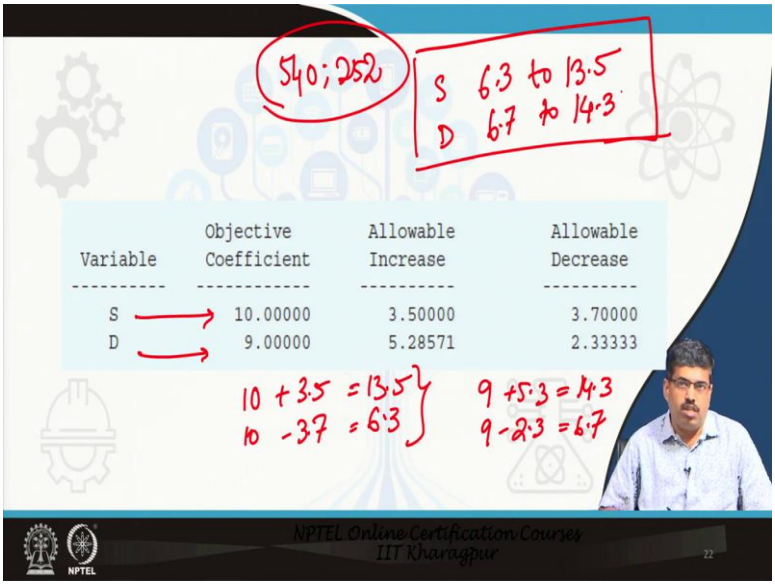


Let us take this table which is which shows you, if you see my bags was S 10 rupees was the profit per bag D deluxe bag 9 rupees was the profit. Allowable increase 3.5 and D is 5.28 allowable decrease 3.7 D is 2.33 what does that mean.

That means that if you keep the production at 540 and 2 540 and 252, if you keep the production at 540 and 252. Because, changing production is a bit difficult you have

employed workers you have brought in raw materials. So, you do not you have calibrated your machines. So, you do not want to change production you want to see, if I fix this production how will my profits change. Because, constraints are changing I do not want to change the production quantity because, I already set my machines.

So, how will my profit change? This says that allowable increase is 3.5; that means, 10 plus 3.5 is equal to 13.5. Allowable decrease 3.7 so, 10 minus 3.7 is equal to 6.3. So, if I keep on producing this quantity my profit for S will range from 6.3 to 13.5.

Let us take 9 plus 5 let us take its 5.3. So, it comes to as 14.3 and 9 minus 2.3 is 6.7. So, my D will be 6.7 to 14.3. So, what does this say that if I keep the optimal production quantity same. If I keep the optimal production quantity same, my profit for S will vary between 6.3 to 13.5 and my profit for D will vary between 6.7 to 14.3 if the constraints keep on changing; ok.

So, I if I do not change the quantity if; however, I do not want this profit I need to redraw, then we will have to again go back to the linear programming problems and solve it again. This is without altering the solution, without altering the optimal production, without altering the optimal production, if it if I want to see how much the profit is varying this is the solution.

If I am acceptable with this profit I will continue with this production, if I am not acceptable I will redraw and I will go for a newer set of linear programming problems; ok.

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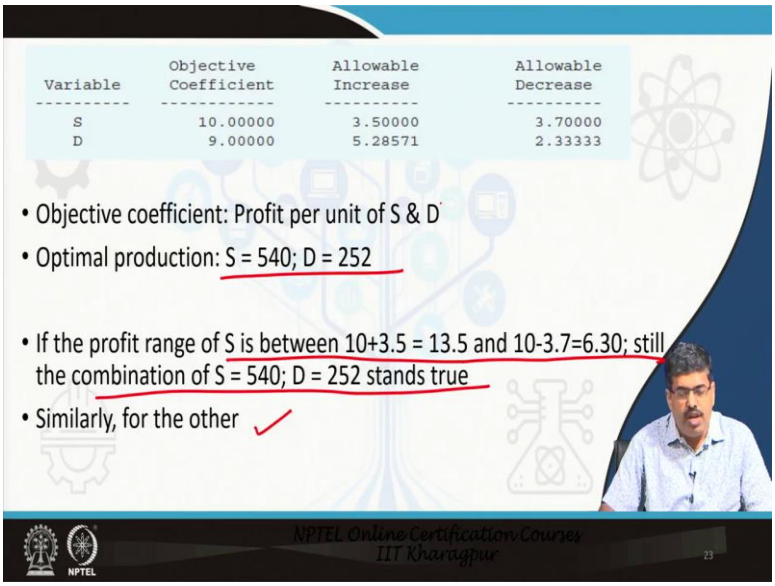


So, this is what it means objective coefficient profit per unit of S and D optimal production S 540 D 252. If the profit range of S is between 13.5 and 6.3 still the combination of S 540 D 252 stands true, similarly for the other; ok. So, this is what is called as sensitivity analysis.

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Now, I just want to show you something before I end today's lecture. That is modeling in LiPS; LiPS is a very-very simple, easy-to-use software you can just try and take a

permission from the developer and I think it is easily available. I think they do not charge also if I am correct; it is available in the internet also.

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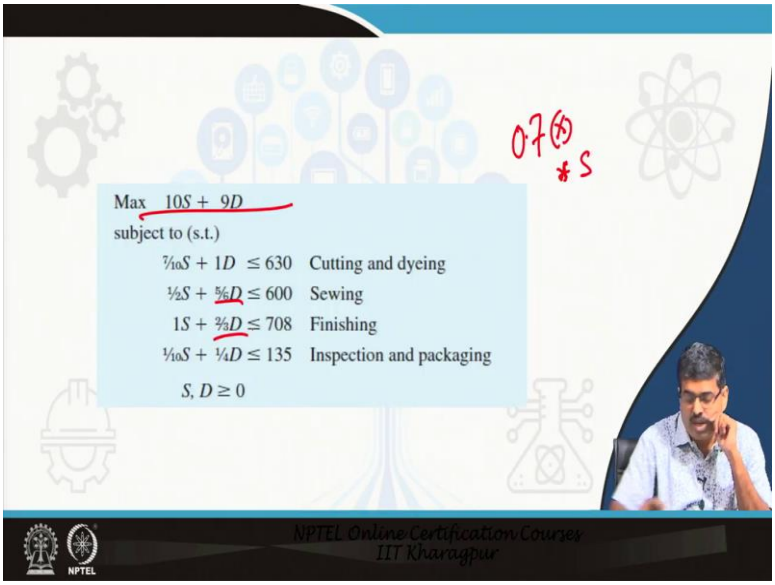


So, you see this was the original model that we had, what we just did is we just simply wrote it down in play an English, see what was it max 10 S plus 9 D.

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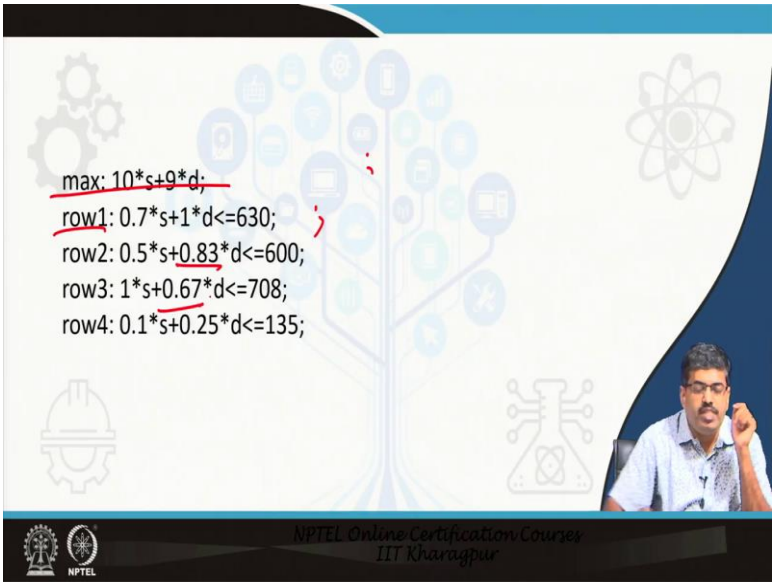


What did we write max 10 into s plus 9 into d. Let us see go back max 10 S plus 9 D, we are writing max 10 into s plus 9 into d that multiplication sign. We are putting and we are putting a semicolon that is it and we are putting a semicolon.

What are the constraints 7 10th S plus 1 D less than equal to 630 what are we writing 7 10 the software cannot take we will have to put in decimals. So, 0.7 into s plus 1 into d less than equal to 630. And again put a semicolon and use a word called row 1; it is very-very simple. Whatever, you are writing by hand just put a multiplication sign and in computers the multiplication sign looks like this. So, just put a multiplication sign; ok.

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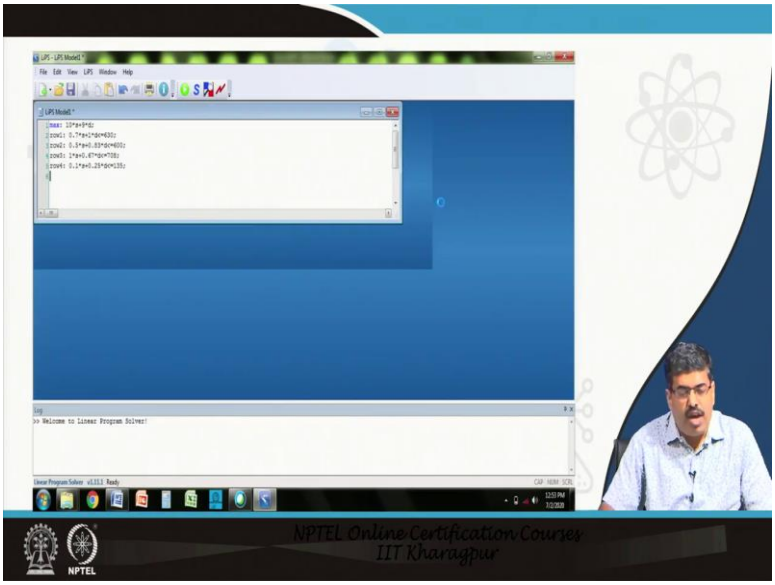


So, do this is the LiPS screenshot go into lips, go into text model because we are writing right. So, it is a text model go into text model write this down or you may write it in MS Word and just copy paste, it go to LiPS again on the top click on that you will find something called solve model; ok.

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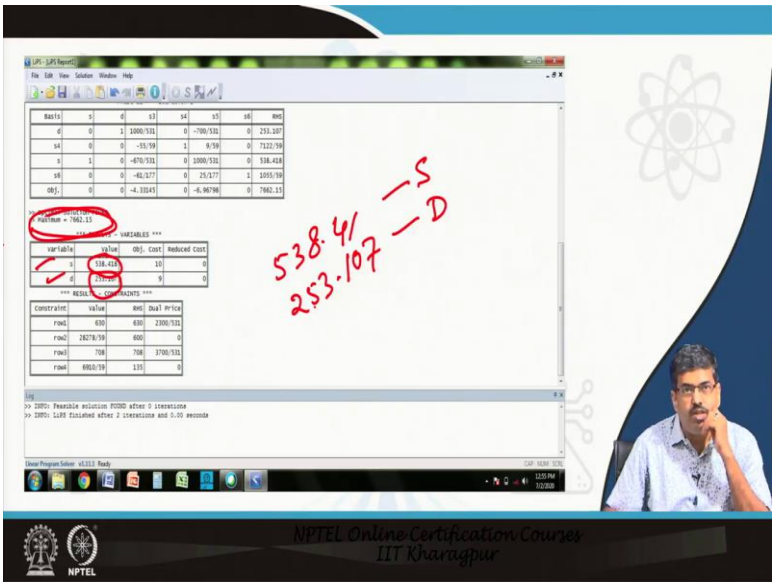


Moment you click on solve model this result will come. What it is saying? If you look at it this is a screenshot, it is saying 538.41 and 253.107. This is for S and this is for D.

We had said 540 remember we are said 540 and 252, why has this solution changed 540 and 252? This is because for some fractions we had rounded it off for LiPS; ok. We had taken 0.83 0.67, these are all rounded off for five sixth and two third that is why there is a slight difference; ok, otherwise no, and ok. So, this is telling us how much we should manufacture and what will be the total profit? 7662.15; ok. This is the screenshot; right; ok.

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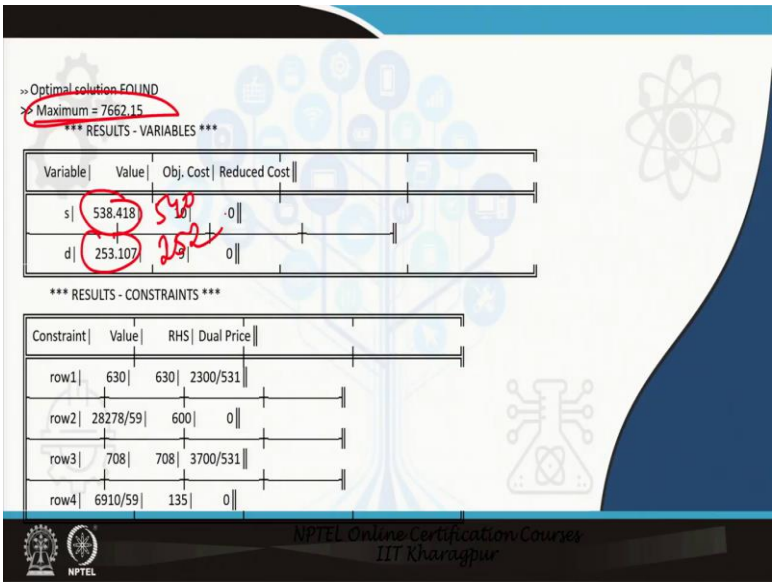


So, this is how it looks I am magnified it 7662.15 and this is just the way the version is done; ok. 538 we have got as 540 253, I think we got as 252; ok. So, this is it I think this much pretty much sums up your linear programming problems.

In the next lecture that is lecture 5, we will deal with integer programming problems; ok. And I hope you just go back to the reference material the books that we have told and, I think if you practice some problems at home, you will be able to solve them at ease remember there is nothing difficult in this world. Just again relook at the video how I framed a problem, I drew lines, I underlined it and immediately, I noted down the equations then and there only.

So, similarly you look at these problems try to structure it and immediately underline and note down the equations. This is the way you go about in solving a structured problem; ok. And, it is already there you just need to arrange it, rearrange it, and put it in some structure; ok.

And then you can solve it using any method, any software; linear program is one method; use any software, and you will get a proper solution. So, you practice it in the next class; we will use integer programming; ok.

Thank you!