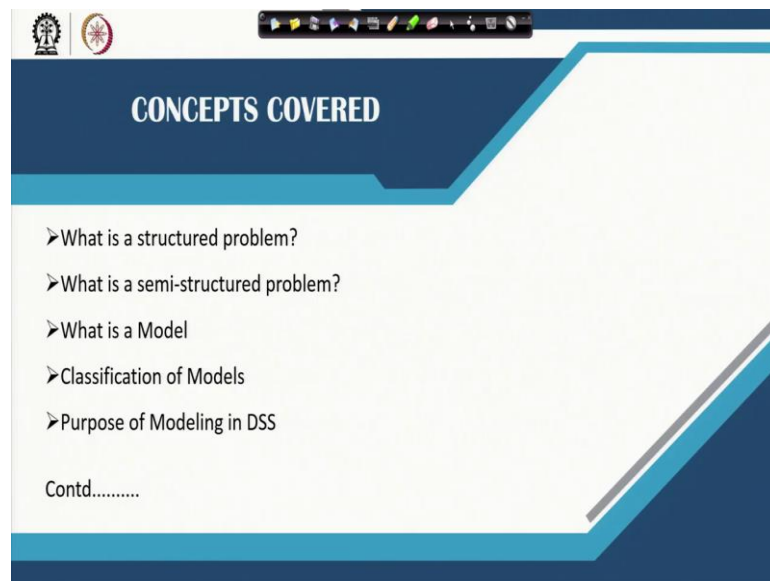


**Decision Support System for Managers**  
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**Vinod Gupta School of Management**  
**Indian Institute of Technology, Kharagpur**

**Module – 02**  
**Models in Decision Support System**  
**Lecture - 08**  
**Solution Techniques-Optimization: Linear Programming**

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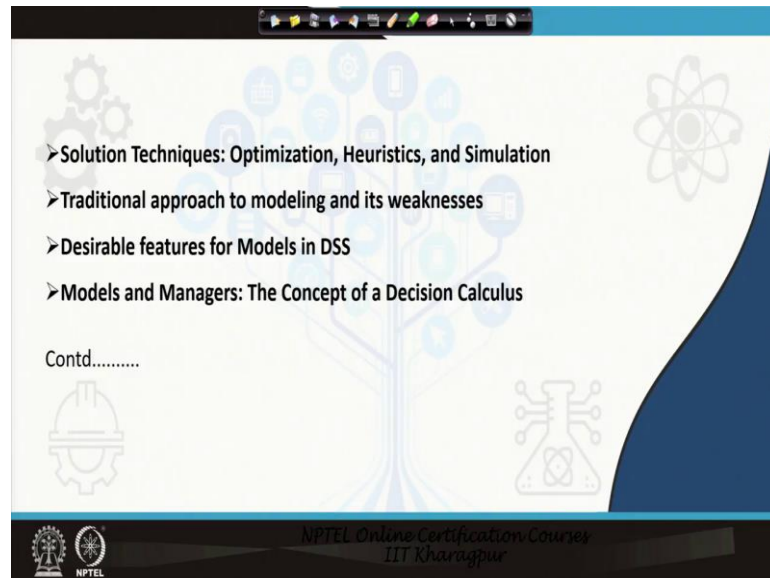
Hello and welcome to “Decision Support Systems for Managers”! We are into module 2, lecture 3 and this module deals with ‘models in decision support systems’ and today, we will deal with solution techniques, linear programming for optimization.

These are the concepts covered in this module, as we have mentioned earlier, what is a structured problem; what is a semi structured problem; what is a model; classification; purposes of modeling. All these we have covered and we have said, structured problem is very-very routine; we can put the problem into some predefined structure and get a mathematical model out of it.

Semi-structured, we can do it partially, balance is left to the ingenuity and newer models can come in place and what is a model? Model is a mathematical expression of real life situations and we can replicate them in the real life and classification of the models, we

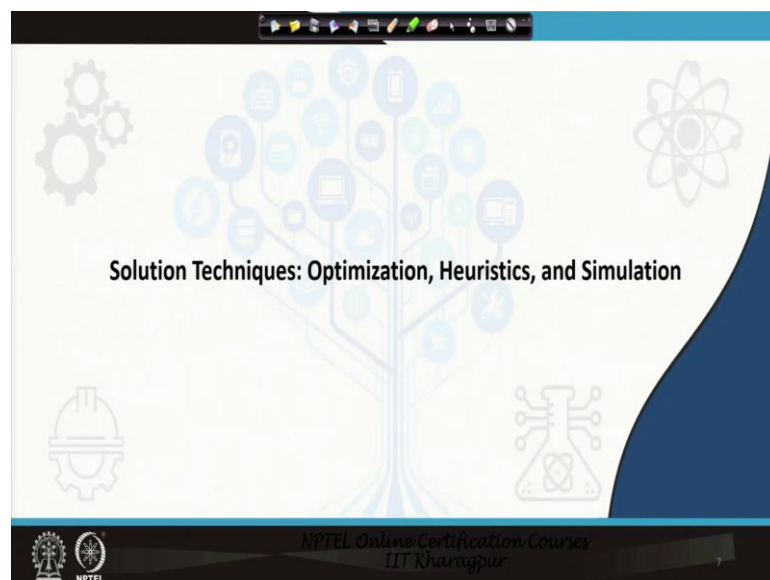
have done it; some choice models, sorting model, ranking model, etc.; purposes of modeling basically, it would enable us to take timely decisions.

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Now, what are the solution techniques possible; optimization heuristics and simulation; ok?

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Today, we will deal with, today we will deal with; this one we had covered. Today, we will deal with Solution Techniques, Optimization, Heuristics and Simulation. Of which today, we will deal with only the first part that is solution techniques optimization; ok.

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Now, optimization normally, we will take care of three; linear, integer and non-linear optimization; ok. Today, we will deal with linear optimization or linear programming which is commonly called linear programming optimizer of problem. Now, linear programming if you see it is I would say 95 percent structured; some people will say no, it is 100 percent structured. It is a very-very time-tested model; I agree 100 percent; fully agree it is a fully structured time-tested model, linear programming.

Why I am saying semi structured in the sense that partial little bit 5 percent is sensitivity analysis that within a range, this is the output possible. You can say that is also a part of fully structured models, I agree with it the way you want to look at it; it is up to you; ok. So, linear programming normally, we will call it as a fully structured model; ok.

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Example 1:

- A company manufactures 2 bags that go through the following processes:
- 1. Cutting and dyeing the material ✓
- 2. Sewing ✓
- 3. Finishing (inserting umbrella holder, club separators, etc.) ✓
- 4. Inspection and packaging ✓



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So, let us see what we mean by that. A company manufactures two bags that goes through the following processes. Company manufactures two types of bags rather two types of bags that goes to the following processes; cutting and dyeing the material, sewing, finishing ok, inserting the umbrella holder, the club separators inspection and packaging. So, cutting, sewing, finishing and packaging, these are the four things that go in; cutting, sewing, finishing and inspection; right.

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Department	Production Time (hours)	
	Standard Bag	Deluxe Bag
Cutting and Dyeing	$\frac{7}{10}$	1
Sewing	$\frac{3}{2}$	$\frac{5}{6}$
Finishing	1	$\frac{2}{3}$
Inspection and Packaging	$\frac{1}{10}$	$\frac{3}{4}$

- The profit for every standard bag is Rs.9 and every Delux bag is Rs.10. How many pieces of these these two types of bags should the company manufacture?

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Now, this is the time taken by each of these types of bags. What it is saying is that two types of bags are getting manufactured; one is standard bag and one is deluxe bag. Now, this there are four departments; cutting four processes; cutting, sewing, finishing and inspection; cutting, sewing, finishing and inspection.

Now, a standard bag takes seven-tenth of an hour of standard. If you see, I am moving the cursor and if you see here, a standard bag takes seven-tenth of a; sorry; standard bag takes seven-tenth of an hour to manufacture in the cutting and dyeing process. Deluxe bag takes 1 hour.

Sewing takes half an hour for standard bag and five-sixth of an hour in the deluxe bag. Finishing takes 1 hour for the standard bag and deluxe bag, it takes two-third of an hour; ok. An inspection takes one-tenth of an hour in the standard bag and one-fourth of an hour in the deluxe bag; ok.

Now, the profit is rupees 9 and rupees 10. Question is how many pieces of these two types of bags should the company manufacture? Now, if there was unlimited everything, let the company manufacture as many as they want; but that is not the case. There is a constraint or there are constraints. What are the constraints?

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• The Model:

$$\text{Max } 10S + 9D$$

subject to (s.t.)

$\frac{7}{10}S + 1D \leq 630$	Cutting and dyeing
$\frac{1}{2}S + \frac{5}{6}D \leq 600$	Sewing
$1S + \frac{2}{3}D \leq 708$	Finishing
$\frac{1}{10}S + \frac{1}{4}D \leq 135$	Inspection and packaging
$S, D \geq 0$	

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Let us take this example. A company manufactures two bags that goes through the following processes; cutting and dyeing the material; cutting and dyeing the material,

sewing, finishing and inspection and packaging; ok. Now, if you see cutting and dyeing takes up a seven-tenth of an hour, two types of bags are manufactured – standard bag and deluxe bag.

Cutting and dyeing takes seven-tenth sorry seven-tenth of an hour; deluxe pack takes 1 hour. Sewing takes half an hour; deluxe bag takes five-sixth hour. Finishing takes 1 hour, two-third hour, one-tenth hour this thing. So, if you see that normally, we are saying the deluxe bag takes a bit more time right; deluxe bag takes some bit more time now; ok.

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Department	Production Time (hours)	
	Standard Bag	Deluxe Bag
Cutting and Dyeing	$\frac{7}{10}$	1
Sewing	$\frac{1}{2}$	$\frac{5}{6}$
Finishing	1	$\frac{2}{3}$
Inspection and Packaging	$\frac{1}{10}$	$\frac{1}{4}$

• The profit for every standard bag is Rs.9 and every Deluxe bag is Rs.10.  
 How many pieces of these two types of bags should the company manufacture?

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So, the profit for every bag is rupees 9 and deluxe bag is rupees 10 right. How many pieces of these two types of bags should the company manufacture? The question is that this cutting process has a 630, 600, 708, 135; 708, 135; ok. Now, this cutting and dyeing, this is the time taken for 1 bag; but this cutting and dyeing process, total hours available is 630; total hours available is 630, total hours available for sewing is 600, finishing is 700, inspection and packaging is 135; right; ok.

So, what should be; what should be the; so, what should be the model; ok? So, this is the first, this is the first one that we are dealing with optimization techniques linear. What did we say? We said the company manufactures two types of bags, each bag goes through these four processes, these are the four processes, this is the time taken, profit is this, this and this ok. Now, so, let us go by it. So, profit is 9 and 10.

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Department	Production Time (hours)	
	Standard Bag	Deluxe Bag
Cutting and Dyeing	$\frac{7}{10}S$	$1D \leq 630$
Sewing	$\frac{1}{2}S$	$\frac{5}{6}D \leq 600$
Finishing	$1S$	$\frac{2}{3}D \leq 708$
Inspection and Packaging	$\frac{1}{10}S$	$\frac{1}{4}D \leq 135$

• The profit for every standard bag is Rs.9 and every Delux bag is Rs.10.  
 How many pieces of these two types of bags should the company manufacture?

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$\text{Max } 9S + 10D$

So, what is my objective? My objective is to maximize this profit right. How many standard bags should we manufacture? We do not know. So, let us keep it S. How many deluxe bags should we manufacture? We do not know.

So, let us keep it at D. What is the profit? Total profit then 9 per bag into S bags manufactured that is  $9 \times 1$  or  $9S$  and 10 per bag into D number of bags manufactures. So,  $10D$  or  $9 \times 1$  plus  $10 \times 2$  ok. What are the constraints? Constraints is cutting takes seven-tenth hour. So, seven-tenth of S plus 1 of D lesser than equal to how many hours were available? 630 right; ok. 630 cutting hours was available right.

Next one was if I am correct, 600 for sewing; 600 hours are available, half of S plus five-sixth of D; 1 of S plus two-third of D, I think this is 708, if I remember correctly; and 135 this is 135 ok. See is this clear? My model is maximize  $9S + 10D$ ; maximize  $9S + 10D$  subject to seven-tenth hour into S bag seven-tenth hour per bag into S number of bags manufactured. Seven-tenth hour per bag into S number of bags manufactured plus 1 hour per bag into D number of bags manufactured and so, the total hours available is 630; total hours available is 630.

So, this is my linear programming model; ok; let us go. So, you see this is the model in the proper form  $\text{max } 9S + 10D$  subject to seven-tenth of S plus one-tenth of D; half of S plus five-sixth of D; 1 of S plus two-third of D and one-tenth of S plus one-fourth of D less than equal to 135. So, this is the model. Now, the question is now the question is

how to solve this? How do we get the quantity of S and the quantity of D? How to solve this; ok.

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The Graph =

1.  $\frac{7}{10}S + 1D \leq 630$   
 when  $S=0$ ;  $D = 630$   
 when  $D=0$ ;  $S = \frac{6300}{7} = 900$

2.  $\frac{1}{2}S + \frac{5}{6}D \leq 600$   
 when  $S=0$ ;  $D = \frac{3600}{5} = 720$   
 when  $D=0$ ;  $S = 1200$

S	D
0	630
900	0

S	D
0	720
1200	0

0.1062  
708,0

0.540  
1350,0

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So, let us now go through that ok. The graph. Now, what was my equation? Let us take one, seven-tenth of S plus 1 D is less than equal to 630; seven-tenth S plus 1 D is less than equal to 630 right, when s is equal to 0, D should be equal to 630 right. For simplicity purpose, we will make this as equal right when D is equal to 0, s is equal to what? 6300 by 7 that is equal to 900; 7 9's are 63.

So, for first constraint, constraint number 1, what did we get? S and D points; s is 0, D is 630; D is 0, S is 900 right. These are the two points that we get. Understood? First equation was this, seven-tenth S plus 1 D is equal to 630 when s is equal to 0, D is equal to 630; when D is 0, s is equal to 6300 divided by 7 that is 900. So, what are the two points? s 0, D 630; D 0, s 900.

This is the first equation point. Sorry; let us go to the second point, let us go to the second point, what was the previous equation half S plus five-six D less than equal to 600; half S plus five-six D less than equal to 600 ok. Let us go sorry, half S plus five-sixth D less than equal to 600 right; ok.

So, when s is equal to 0, D is equal to what? 3600 by 7; sorry 3600. So, sorry, I am so sorry; this is 3600 by 5 right 36 sorry; anyway 3600 by 5 that is equal to 3600 by 5 is



equal to 720. When D is equal to 0, s is equal to 1200; ok. So, my next equation becomes s and D right. Half S plus five-six D is less than equal to 600; s 0, so D becomes 3600 by 5 is 720 and D is this; ok.

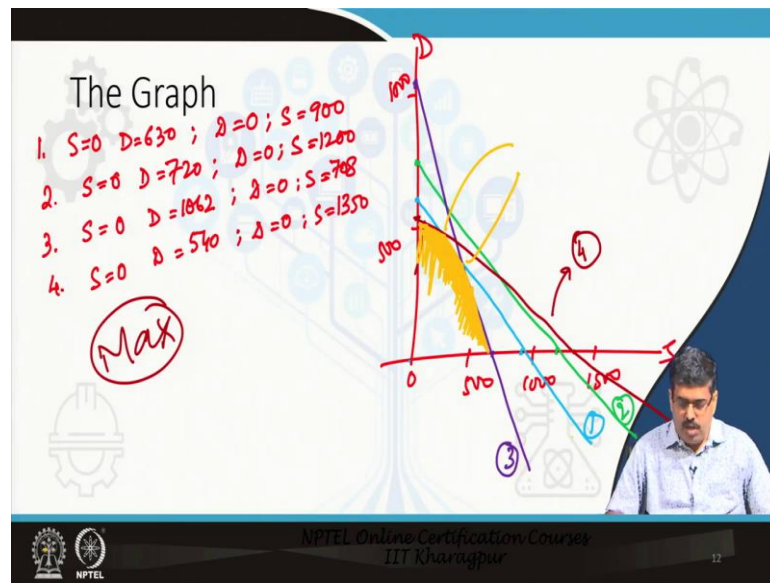
So, in this way, we can get the list of all equations clear. So, for equation 3, I will just for equation 3, we will get 0, 1052 and sorry 1062 and 708, 0 and for equation number 4, we will get 0, 540 1350 and 0 clear. So, this is the; ok.

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- Final Solution:
- S = 540
- D = 252

So, what I will do is, I will erase this and is it clear? First if you solve these four constrained equations, we get these values right. So, now, we will do a graph of these, we will do a graph of this. Let us see; thank you!

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Here let us see. So, we first equation, we got  $S = 0$ ,  $D = 630$ ;  $D = 0$ ,  $S = 900$  right. Second equation, we got  $S = 0$ ,  $D = 720$ ;  $D = 0$ ,  $S = 1200$ .

These two we solved; ok. Now, for the third constraint, we get  $S = 0$ ,  $D = 1062$ ;  $D = 0$ ,  $S = 708$ . For 4 we get,  $S = 0$ ,  $D = 540$ ;  $D = 0$ ,  $S = 1350$  right. Now, let us draw a graph ok. Let this be  $S$  and let this be  $D$ . Now, what is the maximum value of  $S$ ? Is 1350? So, let us go at 0, 500, 1000, 1500.

What is the maximum value of  $D$ ?  $D$  gets me at 1062. So, let us go for 500, 1000. Let us take the first equation  $S = 0$ ,  $D = 630$ . I will use different color.  $S = 0$ ,  $D = 630$ ;  $S = 0$ ,  $D = 900$  ok;  $D = 0$ ,  $S = 900$ ;  $D = 0$ ,  $S = 900$ . This is my equation 1; ok. Then,  $S = 0$  let us use another color then, let us use a green;  $S = 0$ ,  $D = 720$ ; ok;  $D = 0$ ,  $S = 200$ ;  $D = 0$ ,  $S = 1200$ . Let us take the third one  $S = 0$ ,  $D = 1062$ , somewhere here  $D = 0$ ,  $S = 708$ . This is equation 3. Then,  $S = 0$  equation 4, let us use a fourth color; let us use this.  $S = 0$ ,  $D = 540$  and  $D = 0$ ,  $S = 1350$ ; so this one is my equation 4; right.

So, if you see what my objective was; sorry; what was my objective? My objective was maximization right. So, my common space; so, let us go back again; if I can, I do not know whether this color will work; my; sorry, my common space is this one right. So, I

have two points ok. So, I have these two feasible points right of which whichever is maximum, I will get it.

Now, what are the; these two are the intersection point of these two lines; line 4, line 1 and line 3 and line 1 ok. So, if you can solve these two simultaneous equations, you will get a value for this, for both these and the value for S and D whichever one gives me the maximum number that is the maximum profit; ok. So, if you solve this, you will get S as 540 and D as 252; ok. So, this is the way by which you solve a graphical solution; ok.

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**Slack Variable**

• 'Unused' is 'Slack'

Constraint	Hours Required for $S = 540$ and $D = 252$	Hours Available	Unused Hours
Cutting and dyeing	$\frac{7}{10}(540) + 1(252) = 630$	630	0
Sewing	$\frac{1}{2}(540) + \frac{5}{6}(252) = 480$	600	120
Finishing	$1(540) + \frac{2}{3}(252) = 708$	708	0
Inspection and packaging	$\frac{1}{4}(540) + \frac{1}{4}(252) = 117$	135	18

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Now, I have a question, if you solve this you will get; ok. You will get 540 S we have manufactured and 540 S we have manufactured and 252 D we have manufactured right. So, seven-tenth of hour per bag into 540 bags, 1 hour per bag into 252 that is equal to 630 hours right. Similarly, half hour per bag into 540 bags plus five-six of per hour into 252; so, this is basically the number of hours required.

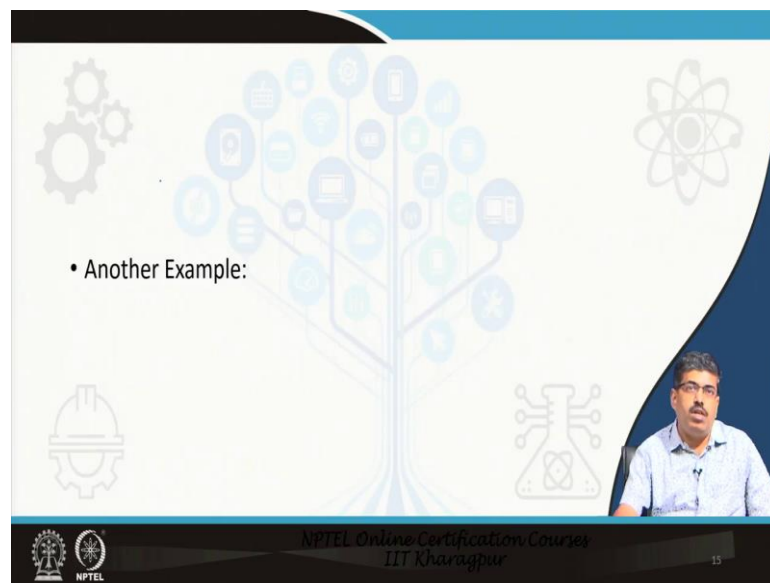
What was the hours available? 630, 600, 708, 135. So, here hour required 630, we had 630. So, unused is 0. 480 hours were used, we had 600 hours; so, 120 is unused. 708, 708 is 0. 117 hours were used, we had 135 hours; so, eighteen hours were unused. These unused hours; unused; these unused hours is called as slack variable.

These unused hours is called slack variable; ok. Just this, nothing else. Unused constraint, unused quantity in a constraint is slack; ok. Just remember this thing; unused

is slack; ok. That means, we do not need this. This is one managerial decision. You can find out wastage, you can find out where to reduce cost.

You can find out where to reduce cost because this is unused, you can negotiate with the suppliers give me less; negotiate with the finance department, all along we have been purchasing more; a new machine has come, this is not required; reduce the number of may be working hours; ok. So, this is the use of slack variable; ok.

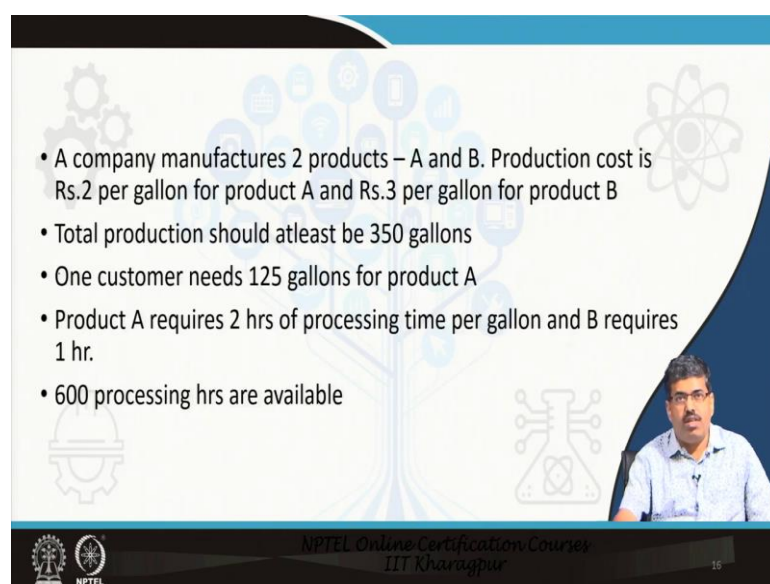
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• Another Example:

The slide features a background with a stylized tree of icons representing various technologies and industries. The icons include gears, a lightbulb, a smartphone, a laptop, a server, a network diagram, a microscope, and a chemical flask. The slide is part of an NPTEL Online Certification Course from IIT Kharagpur, as indicated by the logos and text at the bottom. A small inset video shows a man speaking.

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- A company manufactures 2 products – A and B. Production cost is Rs.2 per gallon for product A and Rs.3 per gallon for product B
- Total production should atleast be 350 gallons
- One customer needs 125 gallons for product A
- Product A requires 2 hrs of processing time per gallon and B requires 1 hr.
- 600 processing hrs are available

The slide features the same background as slide 15, with a stylized tree of icons. It is part of an NPTEL Online Certification Course from IIT Kharagpur, as indicated by the logos and text at the bottom. A small inset video shows a man speaking.

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• References:

1. Mathematical Modeling for Business Analytics; William P. Fox, CRC Press
2. An Introduction to Management Science; Anderson, Sweeney, Williams; Cengage Learning

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We will take on with another example in the next class; ok. These are the references.

Thank you!