Decision Support System for Managers Prof. Kunal Kanti Ghosh Vinod Gupta School of Management Indian Institute of Technology, Kharagpur

# Week - 11 Module - 03 Lecture - 52 Decision Support Systems for Inventory Management (Contd.)

Hi, welcome to our 3rd module on "Decision Support Systems for Inventory Management" corresponding to the 11th week; ok. Today we are going to discuss about the concepts related to safety stock or what is also popularly known as – buffer stock and the reorder level.

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So, the concepts to be covered is: what is safety stock, and how to compute the reorder level.

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So, safety stock is the additional amount of stock which is kept on hand to safeguard against the fluctuations in lead time, or against the fluctuation in demand, or against the fluctuations in both of these variables that is the lead time and demand, both of them if they fluctuate at the same point in time.

Once again, safety stock is the additional amount of stock kept on hand to guard against the variation or fluctuation in either the lead time, or fluctuation in demand, or to safeguard against fluctuations in both lead time and demand at the same point in time. And, this additional stock, which is also known as the buffer stock, helps to reduce the chances, which statistically is known as the probability of stock-out.

So, in order to determine the safety stock, the operations manager, or the materials manager, or the supply chain manager, has to specify the service level, which is a policy measure that help determine the level of safety stock that needs to be kept, to protect the managers from stock-out situations. Higher the service level, higher is the level of safety stock that needs to be maintained.

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The service level of 95 percentage means that there is a chance of stock-out during this replenishment cycle and that chance is nothing but 5 out of 100, 5 percent. Two types of service level, that are commonly used for management of inventory are number 1 is a measure based on the proportion of order cycles, in which no stock-outs occur, that is what we I just discussed.

In a second concept of service level, that it is a measure based on the proportion of customer demands that are satisfied from the on hand inventory, which is also referred to as the fill rate.

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There are several methods for determination of safety stock, but each one of them require thorough analysis of the historical lead time and the demand data.

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When adequate data is available, then we can fit a statistical distribution to describe the demand during lead time. Usually, normal distribution is assumed, although other continuous distributions may be used.

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The following three situations may be encountered, while determining safety stock. Number 1, the demand is variable and the lead time is constant, which is normally the case. Demand is constant, the lead time is variable that is the second case, and the third case is both demand and lead times are variable.

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This is what; this is the extra stock that I am keeping on hand. The demand during lead time is assumed to be normal, this is the level which basically depicts the average

demand during lead time as long as the actual demand is less than that no safety stock is required.

As soon as the actual demand is more than this expected average demand during lead time, we should have some safety stock on hand to guard against that fluctuation. And, how much additional stock you will keep will depend on the service level that you specify.

This picture shows a cycle service level of 85 percent, corresponding to which from a standard normal distribution curve, you can find out the value of Z. So, this portion is the safety stock and the probability of stock-out is given by 0.15 that is shown with a red color.

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So, now we will consider some very simple cases to show you, how the safety stocks are computed using a model in any decision support system. Let us consider the case, when demand is variable and lead time is constant.

In finding out the reorder level, reorder level is the level of stock, at when the ordering takes place. If the stock will get consumed and if the level of stock reaches that reorder point, a fresh order is released on to the supplier. And, we have discussed this concept of reorder level in the first module.

The assumptions here are for the examples that I have discussed, that our stock level is reviewed constantly on a continuous basis. That means, whenever there is an issue of stock from the stores, that withdrawal quantity will get subtracted from the level of stock.

Whenever there is a receipt, that receipt quantity will get added to the level of stock. And, this kind of inventory system can be easily implemented through a computer. This inventory system is reviewed on a continuous basis that is the assumption we have made, typically, the corresponding system is known as continuous review inventory systems or queue system, which we will discuss in one of the modules.

The inventory system involves a single item and demand for the item is variable that is random. Under these assumptions how to compute the safety stock we are going to discuss. There are various assumptions; there are various situations, in which lot of models can be derived and embedded into any decision support systems for computing the buffer stock. But we are going to discuss in this course some very simple models.

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The underlying statistical distribution of demand can be estimated. Normally, we said that we will assume that it is a normal distribution. There can be complex derivations, if you assume other type of distributions. Even, distribution free approaches are also there. In here we have assumed that the lead time is constant and known. So, in this continuous review system a fixed cost is incurred every time an order is placed, that is the ordering cost. Which we have denoted by S, S equals the ordering cost. If, D the annual demand for the item and C is the unit cost for the item, with annual interest rate of i then, the economic order quantity Q equals root over of twice DS by i C, which is the order size and this order quantity will be released onto the suppliers whenever our stock level will reach the reorder point or the reorder level.

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**REORDER LEVEL - VARIABLE DEMAND, CONSTANT LEAD TIME** When demand is normally distributed with a known mean and a standard deviation, the reorder level, 'R' is given by,  $R = dL + SS = dL + Z^* \sigma_d$  where,  $\mathbf{*}\overline{d}$  = Average Demand L = Lead Time SS = Safety Stock \*Z = Number of Standard Deviations for a Specified Cycle Service Level  $alpha \sigma_d$  = Standard Deviation of Lead Time Demand

So, when demand is normally distributed with a known mean, known mean is average demand, which we have expressed by the symbol d bar, L is the lead time, which is the time that elapses between the release of an order onto the supplier, and the corresponding receipt of the lot from the supplier. So, this average lead time is L. So, d bar multiplied by L, this expression is the average demand over the average lead time or over the fixed lead time.

So, average demand during the lead time is expressed by d bar plus d bar into L. And, the reorder level is nothing but the average demand during lead time plus the safety stock. Now, in this case we have said that the lead time is constant.

So, the safety stock which is denoted by SS equals Z times sigma d, where sigma d is the standard deviation of lead time demand. And, Z is the number of standard deviations for a specified cycle service level. So, this Z value will be dependent on the service level

that I choose, sigma d is the standard deviation of lead time demand, that we need to compute.

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So, in this case the units for average demand and the lead time must be consistent. That is if the demand period is specified in days, then the lead time must also be in days. If, in a manufacturing situation or in a given problem, suppose we know the standard deviation of demand per day, that is standard deviation of daily demand.

And, lead time may be expressed as 7 days, 10 days, 12 days, daily demand we know; lead time can be 15 days. So, in that case the expression for the reorder level R equals d bar into L plus Z into sigma d into root over of L, where L is the lead time in days.

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So, demand period is the time span over which the demand has been estimated. So, that must be consistent with the units for lead time.

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If, the demand period is less than the lead time, what will happen? Consider the following example, where the standard deviation of demand is 4 units per day and lead time is 3 days. Assuming the demand for each day is independent, the standard deviation of lead time demand is equal to the square root of the sum of the variances of daily demand.

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That is where in the expression that root L thing come, because there will be 3 such distributions of daily demand over the lead time period.

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Here is a numerical example. The daily demand for an item is normally distributed with a mean of 100 units and a standard deviation of 3 units. The procurement lead time is 6 days. The question to be answered is, compute the standard deviation of lead time demand, very simple, 7.35 units.

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NUMERICAL EXAMI	PLES ON REORDER LEVEL AND SAFETY STOCK
The daily demand f time for the item is time demand is 1 situation to satisf during lead time.	for an item is 20 units. The procurement lead a 10 days. The standard deviation of the lead 2 units. Determine the order level for this y an 85% probability of not stocking out
	· ;;
(Source: Problems and Solutio	ns in Inventory Management by Shenoy and Rosas, 2018)
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Look at another problem, the daily demand for an item is 20 units. The procurement lead time for the item is 10 days. The standard deviation of the lead time demand is 12 units. Determine the order level for this situation to satisfy an 85 percent probability of not stocking out during the lead time.

So, here you see slight variation. Demand is 20 units per day, procurement lead time on an average is 10 days but the lead time is variable, lead time is varying with an average of 10 days and the standard deviation of 12 units.

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So, again what we do? That we first go and try to find out the value of the standard normal random variate Z, which is equal to 1.04 corresponding to a probability level of 85 percent, the other values are given. So, the reorder level R is equal to average demand during the average lead time, which is 20 multiplied by 10 plus Z into corresponding value of 12; ok.

Standard deviation of demand during lead time is directly specified in the value of units. So, this is 213 units. Otherwise, you have to compute that and find out the value of sigma d and substitute in this expression. So, the reorder level is 213 units and the corresponding safety stock is this extra portion, which is equal to 13 units.

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Look at another example. Demand for an item in a month is normally distributed with a mean of 100 units and a standard deviation of 3 units. If, the lead time is 1 week, compute the standard deviation of the lead time demand, is very absolutely simple.

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N	UMERICAL EXAMPLES ON SAFETY STOCK
	In this problem, the demand period is 1 month and lead time is 1 week.
>	Since demand period is greater than the lead time, we use
4	$[\sigma_{dl} = \sigma_d / \ln n]$ to obtain the standard deviation of lead time demand.
۶	Here n =4
>	Therefore, $\sigma_{dl}$ = (3 / $\sqrt{4}$ ) = 1.5 units
4	The standard deviation of lead time demand is 1.5 units
A	(Source: Problems and Solutions in Inventory Management by Shenoy and Rosas, 2018)
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Here, you have applied that central limit theorem concept. In this problem the demand period is 1 month and lead time is 1 week. Since, demand period is greater than the lead time we use standard deviation of demand during lead time equals sigma d by root n.

And, hence sigma dl becomes 1.5 units. So, standard deviation of lead time demand is 1.5 units.

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Now, let us consider the case, where the replenishment lead time is variable and the demand is constant. The reorder level may be determined under the following assumptions, inventory system is again reviewed on a constant basis, the inventory system involves a single item, demand for the item is known and constant, lead time is random, but the distribution governing the lead time is known that is normally distributed.

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Very simple; this is the expression we have already discussed; just substitute the values; for example, here is a problem.

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Hospital SSKM performs 10 heart surgeries each day with one stent for each surgery. The hospital procured stents from England. The procurement lead time is normally distributed to the mean of 10 days and the standard deviation of 3 days. If the hospital management wants a 95 percent probability of not stocking out during lead time compute the safety stock and the reorder level for stents.

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You know all the data have been specified, d is 10 stents, L is 10 days, the Z value corresponding to a 95 percent service level is 1.64, sigma L is 3 days. Just substitute all these values in the expression for the reorder level, which gives 49s 149 stents, the safety stock being this portion, is 49 units.

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REORDER LEVEL	- VARIABLE DEMAND, AND VARIABLE LEAD TIME
We will now co are variable. normally distri	onsider the case where both demand and lead times Assuming that the demand and lead times are buted, the reorder level, R, is given by
$ ightarrow \mathbf{R} = \overline{d}^* \overline{L} + \mathbf{Z}^* \sqrt{d}$	$\overline{L}\sigma_d^2 + \overline{d}^2 \sigma_{LT}^2$
$\succ \overline{d}$ = Average d	emand per period
$\succ \overline{L}$ = Average re	plenishment time
σd = Standard	deviation of demand per period
> σιτ = Standard	deviation of lead time
(Source: Problems and	Solutions in Inventory Management by Shenoy and Rosas, 2018)
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Let us see another case where both demand and lead times are variable. Assuming that the demand and lead times are normally distributed, in this case the reorder level is given by this expression. Wherein this portion is average demand over average lead time plus Z

times root over of L bar into sigma d square plus d bar square into sigma LT square. This is the resultant variance, resultant standard deviation taking care of the fluctuations in both lead time and demand.

Sigma d is the standard deviation of demand per period. So, sigma d square is the variance of that, that we have multiplied by L to take care of the period of lead time, if it is more compared to the demand per period, which is given.

Here, you see sigma LT square is the variance of lead time that we have multiplied by d bar square to get the expression for quantity, the sigma LT is in days. So, that has to be multiplied by d bar average demand per period to get the corresponding quantity.

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	KAMPLES ON	REORDER LE	VEL AND SAFETY	r STOCK	
□ The daily de assembler is a standard d RAM for the o lead time fo distributed w day. If the ass stocking out o (ii) Safety stoo	emand experim normally distri eviation of 6 computer from or supply of ith a mean of 3 embler desires during lead tim ck, SS	enced by sm buted with a r units. The as a supplier in f the RAM chi 3 days and a st s to ensure a 9 he, compute (i)	all home compu- nean of 20 units a sembler sources the local market. T ps is also norma andard deviation o 0% probability of Reorder level, R a	ter ind the The ally of 1 and	
÷	,			6	
(Source: Problems and	Solutions in Invento	ory Management by	Shenoy and Rosas, 2018)		

So, this is here is one example. The daily demand experienced by a small home computer assembler is normally distributed with a mean of this, 20 units, standard deviation of 6 units. The assembler sources the RAM from the computer for the computer from a supplier in the local market. The lead time for supply of the RAM chips is also normally distributed with a mean of 3 days and the standard deviation of 1 day.

If, the assembler desires to ensure a 90 percent probability of not stocking out during lead time, compute the reorder level, and the safety stock. So, look at the data carefully.

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Here, both the demand and lead time are variable; the value of z for a probability of 0.90 is 1.28. So, if you substitute these values in the expression for R, which we have just discussed, you find that the reorder level comes out to be 88.8 units that is rounded off to 29 units.

With 29 units being the measure of safety stock, to maintain a desired surface level cycle service level of 95 percent and corresponding to 90 percent, the standard normal random variate value is 1.28 from the table.

So, these are the kinds of, you know simple problems that are encountered in real life but in a factory environment, even things are even simpler. You have to just understand how to apply this kind of models and build decision support systems. So, we will discuss one such case.

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This is the reference I have used for preparing this lecture. Any book on production and operations management, they give you plenty of such models and the underlying principles are the same. Next module, we will discuss about a simple decision support systems wherein this concept has been applied.

Thank you all!!