

Decision Support System for Managers
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Week - 11
Module - 02
Lecture - 51

Decision Support Systems for Inventory Management (Contd.)

Hi, welcome to our 2nd module of the 11th week in our course on “Decision Support Systems”! And in this week we are deliberating on ‘Decision Support Systems for Inventory Management’. In our 1st module, we have talked about ‘economic order quantity’ and this ‘economic order quantity’ for inventory management is widely used by retailers.

And here there are some assumptions that we have already mentioned that the demand or the consumption takes place at a constant rate and the lead time is also known and constant or it can be even 0 that is instantaneous replacement. But, there are situations where production and consumption takes place at the same point in time particularly in a manufacturing environment this determination of economic batch size is a problem that operations manager often encounter with.

Here production and consumption they take place at the same point in time and whenever there is a change of one product type to another product type there may be some setup time involved in dismantling the dies for the previous product and rearranging the tools and dies for the product which is going to be manufactured all this takes time and during this time, there is no production.

So, there is an opportunity cost involved of not having production during the time over which the setup takes place. This is known as setup cost which is like the ordering cost that we discussed in our last module, ok. The only difference between economic order quantity and economic batch quantity is that in economic batch quantity model, there is production and consumption taking place at the same point in time.

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So, we will be deliberating on economic batch quantity determination and this model is built into the decision support systems for determination of economic manufacturing quantity also known as EMQ. Having done all the calculations through the input that is being provided from the database or by the managers, the batch sizes are reviewed by the operations managers and if necessary keeping an eye with the priority of the different products and the other dynamic environment that exists in a manufacturing shop.

This economic batch sizes may be revised based on their experience intuition and judgment; ok. And, the implications are also being flashed on the screen of a DSS and that gives the operations managers the feeling that to what extent changes in the computed batch size can be made and what are the consequences of that.

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The slide features a blue and white background with technical icons like gears and a network diagram. The title is 'SIMULTANEOUS PRODUCTION AND CONSUMPTION MODEL'. Below it are four bullet points, each with a square checkbox. A speaker's video feed is overlaid on the right side of the slide. At the bottom, there are logos for NPTEL and IIT Kharagpur.

SIMULTANEOUS PRODUCTION AND CONSUMPTION MODEL

- Economic order quantity works well with a wholesaler or retailer
- Large deliveries that instantaneously raise the stock level and series of smaller demands that slowly reduce it
- But consider the stock of finished goods at the end of a production line
- If the rate of production is greater than demand, goods will accumulate at a finite rate called as, finite replenishment rate

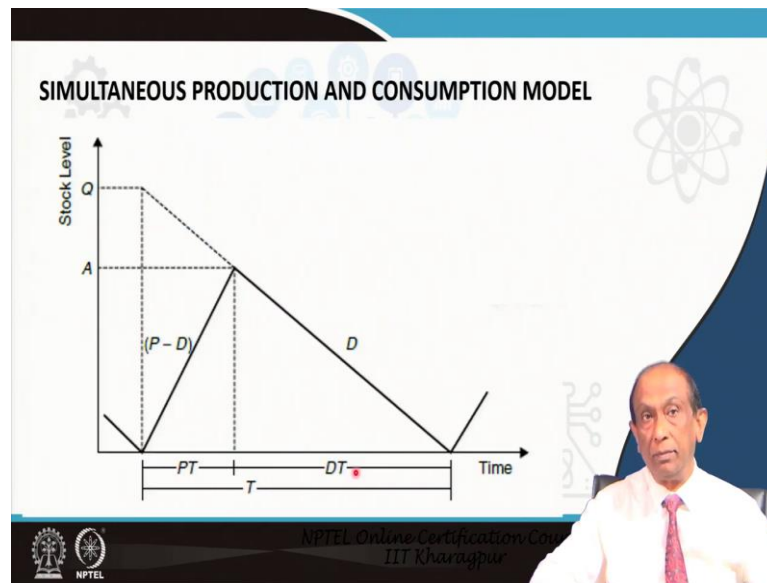
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So, in the last week when we discussed about economic order quantity, we had noted that this economic order quantity is widely used by wholesalers and retailers and that works fine with them. Large deliveries that instantaneously raise the stock level and series of smaller demands that slowly reduce it that is the environment where, this economic order quantity model is highly applicable.

But, consider the stock of finished goods at the end of a production line. Now, and there you are changing from one product type to another, and in such a situation when one product is getting manufactured how the batch size is accumulated that is the thing to be noted. In here, if the rate of production is greater than the rate at which the items are consumed; that means, rate of production is greater than the rate of demand.

This products or goods will get accumulated at a finite rate which is called the finite replenishment rate. If P is the rate of production and D is the rate of demand then their finite replenishment rate equals P minus D , where P is greater than D . And, till the point in time the production is carried on the batch will get accumulated at the rate of P minus D per unit time.

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Look at this figure P is the rate of production and D is the rate of consumption ok. So, the slope of this straight line is P minus D and you see PT is the time over which the production takes place; ok. So, the maximum inventory or stock that gets accumulated over the period production time PT is P minus D into the production time PT; ok.

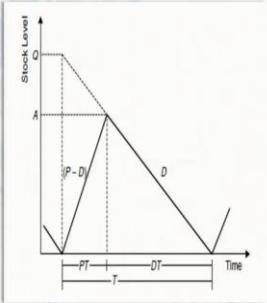
So, A is the level which depicts the maximum stock that can get accumulated and once the production stops, the item gets consumed at the rate capital D over a period of DT I have depicted in the figure.

So, the sum of PT and DT equals T that is the total cycle time. The stock level never reaches Q. The Q is the batch size, but since there is consumption going on, at the same time when product is getting produced, the maximum inventory that can get accumulated is P minus D into PT.

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SIMULTANEOUS PRODUCTION AND CONSUMPTION MODEL

- ❖ The adjoining figure depicts the inventory versus time for the EBQ model.
- The item is produced in lot size of Q , at the production rate of P and consumed at the demand rate of D .
- Hence, inventory will build up at the rate of $(P - D)$ during the production period PT , if $P > D$



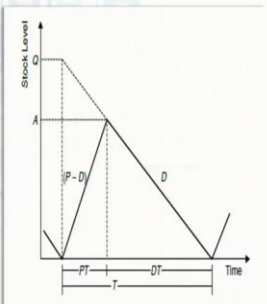
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So, the adjoining figure depicts stock versus time for the economic batch quantity model. As I have already said, the item is produced in lot size of Q at the production rate of P and consumed at the demand rate of capital D . Hence inventory will build up at the rate P minus D during the production period PT , if the rate of production is greater than the rate of demand.

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SIMULTANEOUS PRODUCTION AND CONSUMPTION MODEL

- ✓ At the end of the production period (PT), inventory begins to deplete at the demand rate of D until it is exhausted at the end of the inventory cycle, $T [= (PT + DT)]$
- ✓ With a finite replenishment rate, stock never reaches Q , rather reaches at a maximum level A
- ✓ The maximum stock level, A , is lower than Q and occurs at the point where production stops



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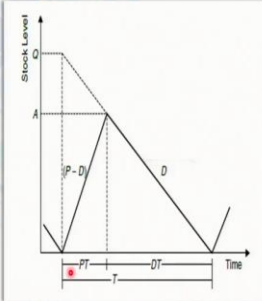
At the end of the production period PT , inventory begins to deplete at the demand rate of D until the total stock is exhausted at the end of the inventory cycle T which I have

already said is the sum of PT plus DT. With a finite replenishment rate of P minus D the stock never reaches the batch size Q but rather the maximum inventory level is specified by A which is: P minus D multiplied by the production time, PT. So, you note that the maximum stock level A is lower than Q and occurs at the point where production stops.

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SIMULTANEOUS PRODUCTION AND CONSUMPTION MODEL

- Looking at the productive part of the cycle we have,
 $A = (P - D) \times PT$
- Also total production during the period is,
 $Q = P \times PT$ or $PT = Q/P$
- Substituting for PT into the equation for A gives,
 $A = Q \times (P - D)/P$



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So, you said that looking at the productive part of the cycle we have A equals P minus D into PT. Also, the total production during the period is Q which is the lot size which is equal to P the rate of production into PT, the time over which the production takes place. Therefore, PT production time equals lot size Q divided by capital P the rate of production.

Now, if we substitute this value of PT in the expression for A we get A equals Q into P minus D divided by P which is nothing, but Q into within bracket 1 minus D by P that is the expression for maximum inventory.

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SIMULTANEOUS PRODUCTION AND CONSUMPTION MODEL

- Now the average inventory level = $A/2$
= $[Q/2] \times (1 - D/P)$.
- The total annual inventory cost can be state as:
- Total Annual Inventory Cost = Annual product cost + Annual holding cost + Annual setup cost
- $TAIC = APC + AHC + ASC$ where

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Now, the average inventory level is half of that just like what we discussed in the last module. So, the average inventory level becomes $A/2$ which is nothing, but $Q/2 \times (1 - D/P)$. So, now we have to find out the total annual inventory cost. The total annual inventory cost is the sum of annual product cost, annual holding cost and annual setup cost.

So, if we designate total annual inventory cost by TAIC then this becomes equal to APC that is the annual product cost plus AHC that is the annual holding cost plus ASC that is the annual setup cost. So, that is what we have written in the next slide.

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SIMULTANEOUS PRODUCTION AND CONSUMPTION MODEL

- TAIC = total annual inventory cost
- APC = annual product cost
- AHC = annual holding cost
- ASC = annual setup cost

The graph shows Stock Level on the y-axis and Time on the x-axis. The inventory level starts at Q, decreases linearly to 0 over a period T. The production rate is P and the demand rate is D. The setup cost is represented by a vertical line at the start of each cycle. The holding cost is represented by a triangle with base PT and height A. The production cost is represented by a triangle with base PT and height P-D. The demand cost is represented by a triangle with base DT and height D. The total annual inventory cost is represented by the area under the inventory level curve.

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TAIC is the total annual inventory cost, APC is the annual product cost, AHC is the annual holding cost and ASC is the annual setup cost. So, the idea is to find out the batch size Q such that this total annual inventory cost can be minimized.

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SIMULTANEOUS PRODUCTION AND CONSUMPTION MODEL

- Let R = annual requirement or demand
- C = total cost of one unit of the finished product
- S = cost of setting up the equipment to process one batch of the product
- k = holding rate, where annual holding cost per unit = $k \times C$
- Q = Economic Batch Quantity

The graph shows Stock Level on the y-axis and Time on the x-axis. The inventory level starts at Q, decreases linearly to 0 over a period T. The production rate is P and the demand rate is D. The setup cost is represented by a vertical line at the start of each cycle. The holding cost is represented by a triangle with base PT and height A. The production cost is represented by a triangle with base PT and height P-D. The demand cost is represented by a triangle with base DT and height D. The total annual inventory cost is represented by the area under the inventory level curve.

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So, first let us write down the expression for the total annual inventory cost. Let capital R be the annual requirement or demand for that product and C is the total cost of one unit of the finished product. S is the cost of setting up the equipment to process one batch of a particular product and this is equivalent to ordering cost that we have said already

because whenever we are changing from one product type to another product type, the tools and dies for the previous product needs to be kept aside they have to be dismantled.

And, the tools and dies for the product that is going to be manufactured has to be fixed up properly all this things they take time and during that time no production can takes place. So, there is an opportunity cost for not having production during this time over which the setup is taking place and hence it is called a setup cost.

Let small k is the holding rate like a you know interest charge or you can replace k by the cost of capital of that particular company and hence annual holding cost per unit equals k multiplied by C that is the total cost of one unit of the finished product.

Of course, you have to be very careful about the way this k is specified whether this k is on an ELD basis or a monthly basis. So, accordingly you have to take care of that unit and let Q be the economic batch quantity. It can be also known as economic manufacturing quantity, sometimes it is known as economic batch size so many equivalent adminologies.

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SIMULTANEOUS PRODUCTION AND CONSUMPTION MODEL

- $TAIC = APC + AHC + ASC$
- $TAIC = (R \times C) + [Q/2 (1 - D/P) \times k \times C] + [(R/Q) \times S]$
- The optimum batch size (EBQ) can be determined by taking the first derivative of TAIC with respect to Q and then setting it equal to zero.

The slide includes a graph of Stock Level vs Time. The y-axis is labeled 'Stock Level' and the x-axis is 'Time'. The graph shows a sawtooth pattern where inventory increases linearly during production (rate P) and decreases linearly during consumption (rate D). The peak inventory level is A . The time to produce a batch of size Q is $P^{-1} \times Q$, and the time to consume it is $D^{-1} \times Q$. The total cycle time is T . The average inventory level is $A/2$.

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So, the total annual inventory cost equals APC plus AHC plus ASC. Now, annual product cost is nothing, but annual demand into unit cost of the item that is R into C plus annual holding cost equal to Q by 2 into 1 minus D by P that is the average inventory multiplied by k into C . So, that gives the value, plus ASC annual setup cost.

So, if R is the annual demand and Q is the batch size, then R by Q is a total number of setups over the year multiplied by the setup cost S . The optimum batch size EBQ can be determined by taking the first derivative of $TAIC$ with respect to Q and then setting it equal to zero; very much like what we did in the last module.

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SIMULTANEOUS PRODUCTION AND CONSUMPTION MODEL

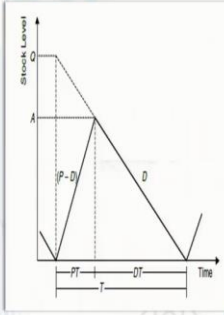
➤ A second derivative of $TAIC$ can also be taken with respect to Q to prove that the $TAIC$ is a concave function, and thus $\frac{d(TAIC)}{dQ} = 0$ is at the lowest point of the cost curve.

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A second derivative of $TAIC$ can also be taken with respect to Q to prove that the $TAIC$ total annual inventory cost, the expression we have already derived is a concave function, and hence d dQ of the total annual inventory cost if we equate it to 0, the cost is at the lowest point of the cost curve, the corresponding Q value you get.

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SIMULTANEOUS PRODUCTION AND CONSUMPTION MODEL



$$\begin{aligned} \frac{d(TAIC)}{dQ} &= 0 + \left[\frac{1}{2} (1 - D/P) \times k \times C \right] + \\ &\quad \left[-1 \times R \times S \times 1 / Q^2 \right] \\ &= \left[k \times C / 2 (1 - D/P) \right] - RS / Q^2 \end{aligned}$$

➤ Now, $\left[k \times C / 2 (1 - D/P) - RS / Q^2 \right] = 0$ gives us

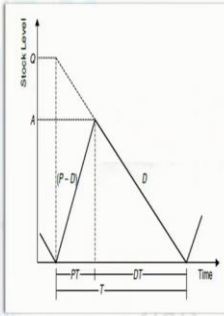
➤ $\left[k \times C / 2 (1 - D/P) \right] = RS / Q^2$

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So, what we did here? We have differentiated the expression for total annual inventory cost with respect to Q and then equated it to 0.

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SIMULTANEOUS PRODUCTION AND CONSUMPTION MODEL



➤ $\left[k \times C / 2 (1 - D/P) \right] = RS / Q^2$

➤ $Q^2 = (2RS / kC) \times (P / (P - D))$

➤ $Q = \sqrt{(2RS / kC) \times (P / (P - D))}$

➤ The second derivative of TAIC is ≥ 0

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If we do that then what we get? We get the expression for Q equals root over of 2RS by kC into P divided by P minus D and the second derivative of TAIC is greater than equal to 0.

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SIMULTANEOUS PRODUCTION AND CONSUMPTION MODEL

➤ Similar to the EOQ model, the annual product cost drops off from the expression after the first derivative is taken, indicating that annual product cost does not affect the batch size decision if the unit cost of each product produced is constant; thus the annual product cost is also ignored in the EBQ model.

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So, similar to the EOQ model, here you will find this annual product cost does not really affect our derivation because this product cost drops off from the expression after the first derivative is taken, indicating that annual product cost does not affect the batch size decision if the unit cost of each product produced is constant. Now, that is the condition. Thus the annual product cost may also be ignored in the EBQ model like the way we had ignored it in the EOQ model.

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NUMERICAL EXAMPLE ON ECONOMIC BATCH QUANTITY (EBQ)

□ A company manufactures a crucial component internally using an advanced technology. The supply chain manager wants to determine the economic batch size to ensure that the total inventory cost is minimized. The daily production rate (P) for the component is 200 units, annual demand (R) is 18,000 units, setup cost (S) is \$100 per setup and the annual holding rate is 25 percent. The manager estimates that the total cost (C) of a finished component is \$120. Assume that the plant operates 360 days per year.

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Let us look at one example that will make things absolutely clear. A company manufactures a crucial component internally using an advanced technology. The supply

chain manager wants to determine the economic batch size to ensure that the total inventory cost is minimized.

Daily production rate for the component is 200 units, annual demand is 1800 units, setup cost is 100 dollars per setup and the annual holding rate is 25 percent. The manager estimates that the total cost of a finished component is 120 dollars and assume that the plant operates 360 days per year.

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NUMERICAL EXAMPLE ON ECONOMIC BATCH QUANTITY (EBQ)

□ Solution

✓ The daily demand rate, $D = 18,000 / 360 = 50$ units per day

✓ Economic Batch Size = $EBQ = \sqrt{\frac{2 \cdot R \cdot S}{kC}} \times \sqrt{\frac{P}{P-D}}$

✓ $EBQ = \sqrt{\frac{2 \cdot 18000 \cdot 100}{0.25 \times 120}} \times \sqrt{\frac{200}{200-50}} = 400$ Units

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So, in here what we get the daily demand rate is 18000 by 360 which is 50 units per day economic batch size equals root over twice RS by kC is similar to this portion is similar to that EOQ model multiplied by a factor of root over P by P minus D.

So, we substitute the values 2 times 18000 times 100 divided by 0.25 in the rate of k given into 120 into 200 by 200 minus 50 which is equal to 400 units. If you look at it see here the annual holding rate is 25 percent. So, it is given that k equal to 0.25 and the rest of the data can be obtained; ok.

So, this is the economic batch size. Now, this economic batch size will be reviewed by the operations managers and may be depending on the dynamics of the environment under which the product is getting manufactured the operations manager, might change this value and the corresponding cost consequences will be computed and shown to him and then we will take a decision whether to stick to it or change it.

The simplest model for determination of economic manufacturing quantity and very widely used in medium size industry as well as large scale industry even in automotive shops where they are operating large size spaces, determination of batch size is a regular decision problem and the DSS can be built using this kind of models.

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NUMERICAL EXAMPLE ON ECONOMIC BATCH QUANTITY (EBQ)

□ Solution

- ✓ The maximum inventory level $A = Q(1 - D/P) = 400(1 - 50/200) = 300$
- ✓ The annual product cost = $R \times C = 18000 \times \$120 = \$2,160,000$
- ✓ The annual holding cost = $[(1/2) \times A] \times k \times C = \$ (300 / 2) \times 0.25 \times 120 = \4500

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And, the maximum inventory level corresponding to that batch size what we have computed comes out to be 300 from the expression of the maximum inventory that we have already derived. The annual product cost will be this much from the given data we have just shown because in the on in the DSS screen all these values will be computed and shown. The annual holding cost is this much. So, this will also be reflected on the screen.

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NUMERICAL EXAMPLE ON ECONOMIC BATCH QUANTITY (EBQ)

□ Solution

- ✓ The annual setup cost = $[R/Q] \times S = (18000 / 400) \times \$100 = \$4500$
- ✓ The total annual inventory cost = $\$2,160,000 + \$4500 + \$4500 = \$2,169,000$
- ✓ The length of a production run, $PT = [EBQ / P] = [Q/P] = 400 / 200 = 2$ days
- ✓ The length of each inventory cycle = $T = [EBQ/D] = 400/50 = 8$ days
- ✓ The number of inventory cycles per year = $360 \text{ days} / 8 \text{ days} = 45$ cycles

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Annual setup cost; total annual inventory cost; the length of the production run (it is the time over which the production will take place), this is also computed and shown. In this case, it is 2 days.

The length of each inventory cycle that is PT plus DT is nothing, but EBQ by D which is nothing, but 8 days and in that case the number of inventory cycles per year will be 360 days working days divided by 8 which is 45 cycles. The simplest model is given in any operations management book.

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You can refer to these books which I have consulted for preparing this lecture.

Thank you!