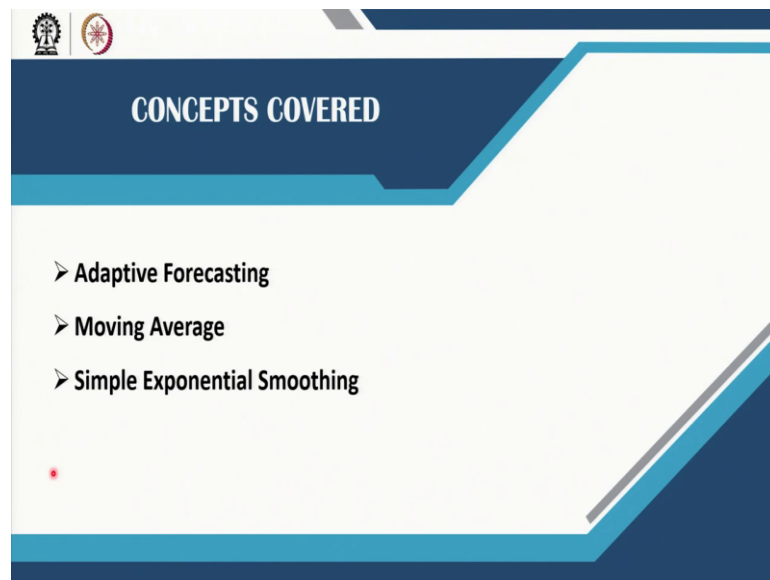


**Decision Support System for Managers**  
**Prof. Kunal Kanti Ghosh**  
**Vinod Gupta School of Management**  
**Indian Institute of Technology, Kharagpur**

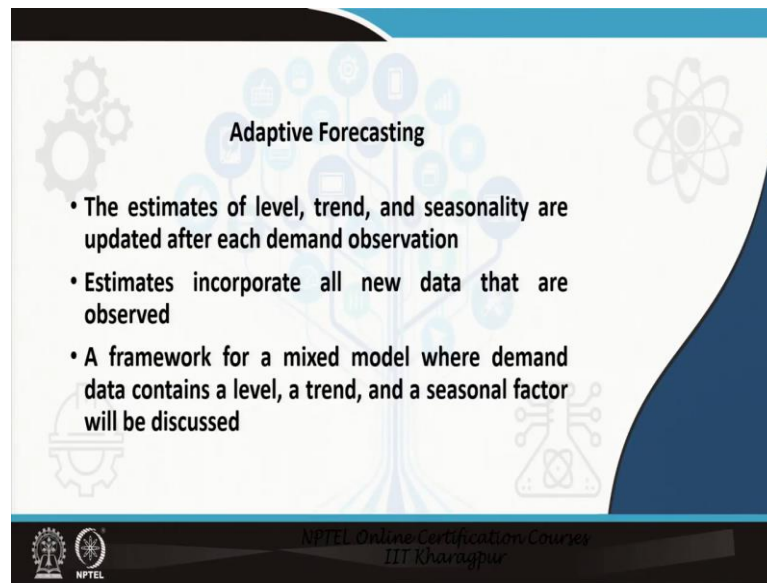
**Week - 03**  
**Module - 03**  
**Lecture - 13**  
**Decision Support Systems for Forecasting (Contd.)**

Hi, welcome to the 3rd module of week 3 on our course on “Decision Support Systems for Forecasting”.

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### Adaptive Forecasting

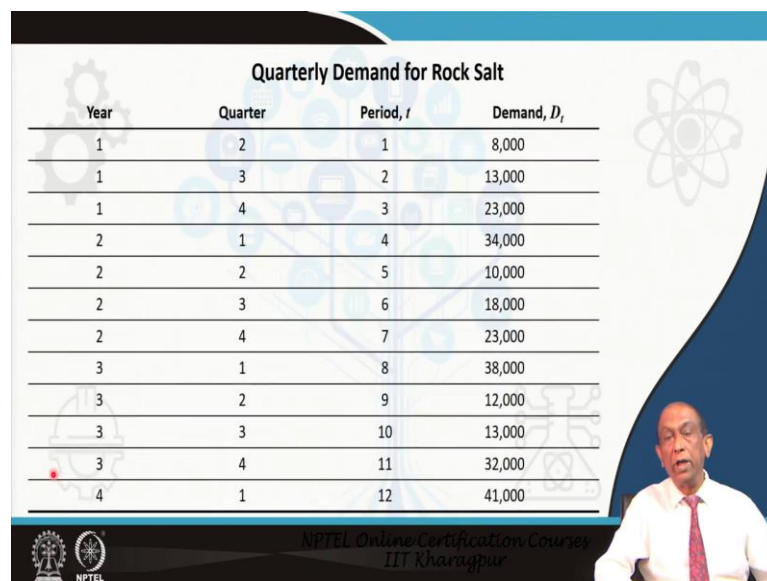
- The estimates of level, trend, and seasonality are updated after each demand observation
- Estimates incorporate all new data that are observed
- A framework for a mixed model where demand data contains a level, a trend, and a seasonal factor will be discussed

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Today, we will be talking about adaptive forecasting, moving average techniques and simple exponential smoothing. Unlike the static method of forecasting whatever we discussed in the previous module, in adaptive forecasting the estimates of level, trend, and seasonality are updated after each demand observation.

This estimates incorporate all new data that will be observed and we will be discussed being about a framework for a mixed model, where the demand data contains a level, a trend, and the seasonal factor.

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### Quarterly Demand for Rock Salt

Year	Quarter	Period, $t$	Demand, $D_t$
1	2	1	8,000
1	3	2	13,000
1	4	3	23,000
2	1	4	34,000
2	2	5	10,000
2	3	6	18,000
2	4	7	23,000
3	1	8	38,000
3	2	9	12,000
3	3	10	13,000
3	4	11	32,000
4	1	12	41,000

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We will be taking the same example, the quarterly demand for a rock salt, where there is seasonality along with trend as we had discussed earlier. You see, we start from the 2nd quarter of the 1st year, where the demand is 8000. Then, you see you come over here for the 2nd quarter of the 2nd year, the demand is 10,000 and then, you come to the 2nd quarter of the 3rd year a 12,000.

So, you will see these points the demand is the lowest and again, there is peak at a respective period. So, the periodicity is 4. We have 3 years of data and we want to discuss what is the forecast for the 4th year; 2nd quarter of the 4th year and things like that.

Because the 1st quarter already we know, 4th of the 4th year. So, we will be computing the forecast for the 2nd quarter of the 4th year. So, we have data for 3 successive years.

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The slide is titled "Adaptive Forecasting" and features a background with various icons including gears, a tree, a molecular structure, and a person. The main content consists of four bullet points:

- In here, we have a set of historical data for  $n$  periods
- The demand is seasonal with periodicity  $p$
- We have quarterly data, wherein the pattern repeats itself every year
- Periodicity  $p = 4$

In the bottom right corner, there is a video inset showing a man in a white shirt and red tie speaking. At the bottom of the slide, there are logos for NPTEL and IIT Kharagpur, along with the text "NPTEL Online Certification Course IIT Kharagpur".

So, in here, we have a set of historical data for  $n$  periods. The demand is seasonal with periodicity  $p$ . We have quarterly data, wherein the pattern repeats itself every year. So, the periodicity  $p$  equals 4.

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**Adaptive Forecasting**

$$F_{t+1} = (L_t + IT)S_{t+1}$$

❖  $L_t$  = estimate of level at the end of Period  $t$   
 $T_t$  = estimate of trend at the end of Period  $t$   
 $S_t$  = estimate of seasonal factor for Period  $t$   
 $F_t$  = forecast of demand for Period  $t$  (made Period  $t-1$  or earlier)  
 $D_t$  = actual demand observed in Period  $t$   
 $E_t = F_t - D_t$  = forecast error in Period  $t$

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So, in adaptive forecasting, we will be utilizing this particular expression, where the forecast of demand for the period  $t$  plus 1 is given by the expression  $L_t$  which is the estimate of level at the end of period  $t$  plus 1 times  $T$  into  $S_{t+1}$ . Here it will be actually  $T_t$ , the estimate of trend at the end of period  $t$  is  $T_t$  which is  $L$  into  $t$ .

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**Adaptive Forecasting**

- The forecast for Period  $(t+1)$  standing at Period  $t$  will use the estimate of level and trend in Period  $t$
- ( $L_t$  and  $T_t$  respectively) and is given as,

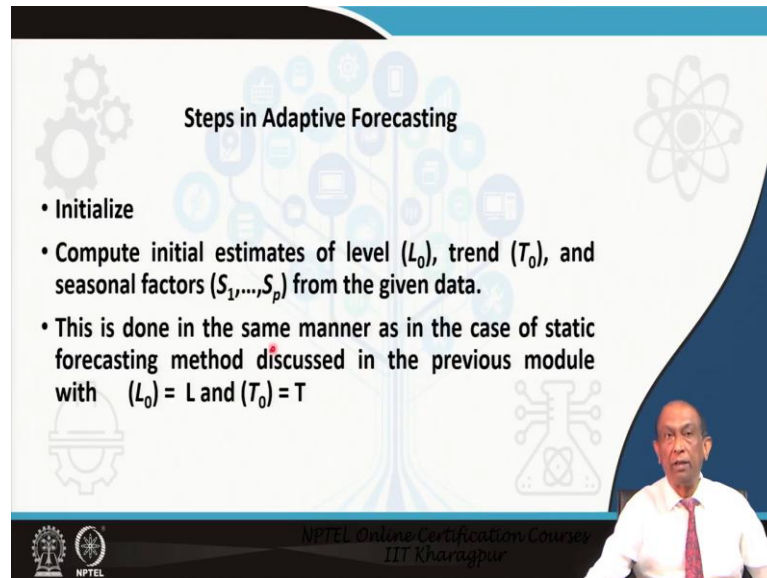
$$F_{t+1} = (L_t + IT)S_{t+1}$$

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So, the forecast for period  $t$  plus 1, we are interested in and we are making this period at this forecast standing at the point in time at period  $t$ . So, we will be using the estimate  $L_t$

and  $T_t$  which we had estimated at the point or at period  $t$  and then, we will substitute in this expression to compute the forecast for the period  $t + 1$ .

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**Steps in Adaptive Forecasting**

- Initialize
- Compute initial estimates of level ( $L_0$ ), trend ( $T_0$ ), and seasonal factors ( $S_1, \dots, S_p$ ) from the given data.
- This is done in the same manner as in the case of static forecasting method discussed in the previous module with  $(L_0) = L$  and  $(T_0) = T$

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So, the steps involved in adaptive forecasting are first one is initialize. In the initialized phase which is the first step, we will compute the initial estimates of level and trend given by  $L_0$  and  $T_0$  and the initial seasonal factors  $S_1$  to  $S_p$  from the given data. And, this will be done in the same manner as in the case of static forecasting method, which has been discussed in the previous module and once, we compute  $L_0$  that will be equal to  $L$  and  $T_0$  will be  $T$ .

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**Steps in Adaptive Forecasting**

- Forecast
- Given the estimates in Period  $t$ , forecast demand for Period  $(t + 1)$  using the equation  $F_{t+1} = (L_t + IT)S_{t+1}$
- Our first forecast is for period 1 and is made with the estimates of level, trend, and seasonal factor at Period 0

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Then, given the estimates in period  $t$ , the forecast demand for period  $t$  plus 1 or say  $t$  plus 1 uses the equation  $F_{t+1} = (L_t + IT)S_{t+1}$ . This is actually  $L_t + T_t + S_{t+1}$ . Since our first forecast is for period 1 made with the estimates of level, trend and seasonal factor at period 0.

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**Steps in Adaptive Forecasting**

- Estimate error
  - Compute error  $E_{t+1} = F_{t+1} - D_{t+1}$
- Modify estimates
  - Modify the estimates of level ( $L_{t+1}$ ), trend ( $T_{t+1}$ ), and seasonal factor ( $S_{t+p+1}$ ), given the error  $E_{t+1}$

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We will be computing the error for the first period which is the difference between the forecast that we compute for the first period and the difference between that forecast and the demand that we observe in the period  $t$  plus 1. After doing that we will be modifying

the estimates of level, trend and seasonal factors. Once we get the error, the entire thing will be clear when we take an example.

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**Moving Average**

- ❖ Used when demand has no observable trend or seasonality  
Systematic component of demand = level
- ❖ The level in period  $t$  is the average demand over the last  $N$  periods
$$L_t = (D_t + D_{t-1} + \dots + D_{t-N+1}) / N$$
$$F_{t+1} = L_t \quad \text{and} \quad F_{t+n} = L_t$$
- ❖ After observing the demand for period  $t + 1$ , revise the estimates
$$L_{t+1} = (D_{t+1} + D_t + \dots + D_{t-N+2}) / N, \quad F_{t+2} = L_{t+1}$$

Among these adaptive forecasting methods, the most simple is the moving average method. This moving average method is used when demand has no observable trend or seasonality. Herein, the systematic component of demand is just the average level and the level in period  $t$  is the average demand over the last  $N$  periods that is the expression for  $L_t$  is this divided by  $N$ . Once we compute that, then the forecast for the next period  $t$  plus 1 is the estimate of  $L_t$ .

So,  $F_{t+1}$  is  $L_t$  and that estimate will remain same because you know there is no growth, there is no seasonality. So,  $F_{t+N}$  will also be  $L_t$ . After observing the demand for period  $t$  plus 1, we can revise the estimates and in that case  $L_{t+1}$  will be given by this expression and having computed  $L_{t+1}$ , we forecast for the period  $t$  plus 2 is the estimate of level computed at the period  $t$  plus 1.

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**Moving Average Example**

- A supermarket has experienced weekly demand of milk of  $D_1 = 120$ ,  $D_2 = 127$ ,  $D_3 = 114$ , and  $D_4 = 122$  gallons over the past four weeks
- Forecast demand for Period 5 using a four-period moving average
- What is the forecast error if demand in Period 5 turns out to be 125 gallons?

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We will take an example. See do not get afraid with all these expressions and all. Moving average technique is very simple. If there is a series of data that is given and suppose, you are computing 3 months moving average.

Then, for the first 3 periods 1 2 3, you just compute the average. That will be the estimate of the level, that estimate will become the forecast for the fourth period or fourth month. Then, having observed the demand for the fourth month, again you recompute the estimate of level. How will you do that?

You discard the value corresponding to the first month's demand and then, compute the average considering the period second month, third month and fourth month because in this case you know the actual demand for the fourth month. So, the average value of demand for the second, third and fourth period will be the estimate of level that you compute at this point in time and that will be the forecast for the fifth month.

Again, you will be observing the demand for the fifth month. In that case, while recomputing the estimate of level, you will discard the demand data corresponding to second month and compute the average considering the demand for third, fourth and fifth month and that estimate will be the forecast for the sixth month. That is rolling period.

So, in this example, a supermarket has experienced weekly demand of milk. For the first period, it is 120; for the second period it is 127; for the third period, it is 114 and for the



fourth period, it is 122 gallon a weekly demand. Problem is to forecast the demand for period 5 using a four-period moving average and the next portion is what is the forecast error if demand in period 5 turns out to be 125 gallons? It is very simple.

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**Moving Average Example**

$$L_4 = (D_4 + D_3 + D_2 + D_1)/4$$
$$= (122 + 114 + 127 + 120)/4 = 120.75$$

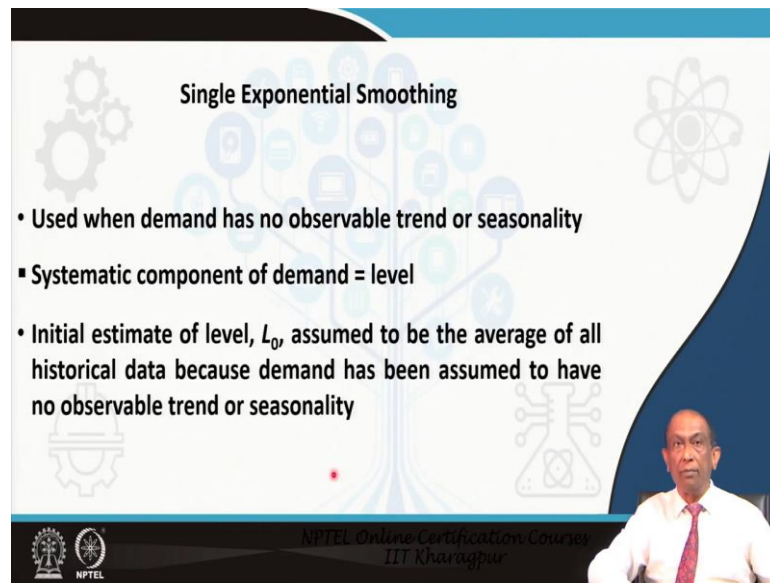
- Forecast demand for Period 5  
 $F_5 = L_4 = 120.75$  gallons
- Error if demand in Period 5 = 125 gallons  
 $E_5 = F_5 - D_5 = 120.75 - 125 = -4.25$
- Revised demand  
 $L_5 = (D_5 + D_4 + D_3 + D_2)/4$   
 $= (125 + 122 + 114 + 127)/4 = 122$

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What do you do? First, you compute the estimates of level for the fourth period that is  $L_4$  because it is a four period, four-month, four week moving average. So, you compute  $L_4$  as 120.75. This estimate will be the forecast for the period 5, 5th period. So, forecast for the 5th period standing at the end of fourth period is 120.75 gallons and the error can be only noted after observing the demand for the period 5.

So, error in forecast is the difference between the forecast value and the demand which is equal to minus 4.25. Then, you basically revise that estimate. So, in that case, you will be neglecting or discarding the demand for the first period and then, you the revised estimate value will be 122 which will be the forecast for the sixth period on a rolling basis as if the average is moving as simple as that.

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**Single Exponential Smoothing**

- Used when demand has no observable trend or seasonality
- Systematic component of demand = level
- Initial estimate of level,  $L_0$ , assumed to be the average of all historical data because demand has been assumed to have no observable trend or seasonality

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The slide features a background with various icons including gears, a tree, and a molecular structure. A small red dot is visible on the slide. A presenter is visible in the bottom right corner of the slide frame.

Now, we will talk about single exponential smoothing model. This model is used when there is no observable trend or seasonality. So, in this case, the systematic component of demand is only the level, estimate of level. So, initial estimate of level which we denote by  $L_0$  is assumed to be the average of all historical data and that assumption is based on the fact that the demand has been assumed to have no observable trend or seasonality.

So, whatever demand data you have got, you just take the average demand considering all the periods over which the demand data is given that will be the initial estimate of level which we denote by  $L_0$ .

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**Single Exponential Smoothing**

Given data for Periods 1 to  $n$

$$L_0 = \frac{1}{n} \sum_{i=1}^n D_i$$

Current forecast

$$F_{t+1} = L_t \quad \text{and} \quad F_{t+n} = L_t$$

Thus  $F_1 = L_0$

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And that initial estimate of level computed at period 0 will be the forecast for the period F 1, the first period. So, thus F 1 equals L 0. So, the generalized expression for the current forecast is F t plus 1 is L t.

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**Single Exponential Smoothing**

- After observing the demand,  $D_{t+1}$ , for Period (t +1), we revise the estimate of the level as follows
- $L_{t+1} = \alpha * D_{t+1} + (1 - \alpha)L_t$
- $\alpha$  =Smoothing constant for the level and lies between 0 and 1

Thus  $F_2 = L_1$

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After observing the demand for the period t plus 1, we revise the estimate of the level and the revised estimate of the level for the period t plus 1 is given by the expression L t plus 1 is alpha times the demand for the period t plus 1 plus 1 minus alpha times the estimate that was computed initially that is standing at for the period t which is L 0.

So, in this case, we compute the revised estimate L 1 which becomes the basis for the forecast, for the period for the second period that is F 2 equals L 1.

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**Single Exponential Smoothing**

Given data for Periods 1 to  $n$

$$L_0 = \frac{1}{n} \sum_{i=1}^n D_i$$

Current forecast

$$F_{t+1} = L_t \quad \text{and} \quad F_{t+n} = L_t$$

Revised forecast using smoothing constant ( $0 < \alpha < 1$ )

$$L_{t+1} = \alpha D_{t+1} + (1-\alpha)L_t$$

Thus

$$L_{t+1} = \sum_{n=0}^{t-1} \alpha(1-\alpha)^n D_{t+1-n} + (1-\alpha)^t D_1$$

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So, this is the general expression, if you note this particular expression.

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**Single Exponential Smoothing**

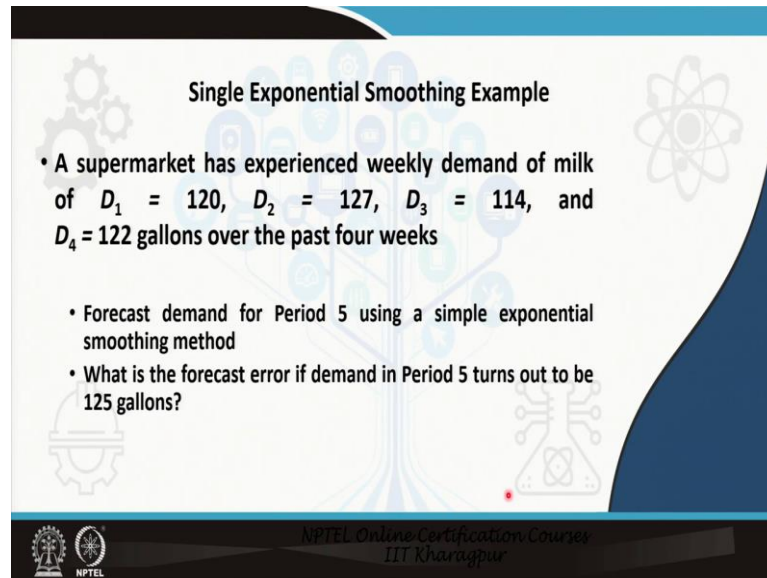
- The current estimate of the level is a weighted average of all the past observations of demand, with recent observations weighted higher than older observations
- A higher value of  $\alpha$  corresponds to a forecast that is more responsive to recent observations, whereas a lower value of  $\alpha$  represents a more stable forecast that is less responsive to recent observations

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Then, what you find that the current estimate of the level is nothing but a weighted average of all the past observations of demand, with recent observations weighted higher than the older observations. So, the significance of which it means or implies that a higher value of this moving constant alpha corresponds to a forecast that is more

responsive to recent observations; whereas, a lower value of the smoothing constant represents a more stable forecast which is less responsive to recent observations.

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**Single Exponential Smoothing Example**

- A supermarket has experienced weekly demand of milk of  $D_1 = 120$ ,  $D_2 = 127$ ,  $D_3 = 114$ , and  $D_4 = 122$  gallons over the past four weeks
- Forecast demand for Period 5 using a simple exponential smoothing method
- What is the forecast error if demand in Period 5 turns out to be 125 gallons?

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So, again here do not get you know swayed away with all these expressions and all. Simple exponential smoothing or single exponential smoothing whatever you might call it, is extremely simple; suppose, you are trying to make a forecast for the same month of October.

And you are standing at the end of September, standing at the end of September, if you want to make a forecast for the demand of that item in the month of October, it is very simple. Forecast for the month of October is nothing but whatever you had forecasted for the month of September plus the smoothing constant times the difference between the forecast and the actual demand that was observed in the month of September.

As simple as that; forecast for the month of October is whatever we had forecasted for the month of September plus smoothing constant times the difference between the forecast and the actual demand that was observed in the month of September.

That is this. And if we give a very high value of alpha that is the smoothing constant, we are giving more importance or weightage to the recent observations compared to whatever has happened in the past. But if the manager or the decision maker feels that ok, due to some particular reason which may not hold good in future.

The demand for this recent observation has become very high, but actually whatever we have observed in the past that is the actual thing or pattern. Then, you give a low value to the smoothing constant. Let us take this example. Same example what we had discussed in case of this moving average method, same supermarket example.

A supermarket has experienced weekly demand of milk and the data is given. We have to forecast the demand for period 5, using a simple exponential smoothing method and what is the forecast error if demand in period 5 turns out to be 125 gallon. So, what we will do? We will take first the average of all these 4 weeks to compute the initial level of the estimate that is  $L_0$ .

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Simple Exponential Smoothing

- Supermarket data

$$L_0 = \sum_{i=1}^4 D_i / 4 = 120.75$$
$$F_1 = L_0 = 120.75$$
$$E_1 = F_1 - D_1 = 120.75 - 120 = 0.75$$
$$L_1 = \alpha D_1 + (1 - \alpha)L_0$$
$$= 0.1 \times 120 + 0.9 \times 120.75 = 120.68$$

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So,  $L_0$  is nothing but the sum of the past 4 weeks of demand which if you look at which is  $D_1$  plus  $D_2$  plus  $D_3$  plus  $D_4$  divided by 4 which is 120.75 and that initial level initial estimate of the level that is  $L_0$ , we will serve as the forecast for the first period.

So, with this method the forecast for the first period  $F_1$  is  $L_0$  is 120.75 and the error corresponding to the forecast that was made for the first period is  $E_1$  given by the difference between  $F_1$  and  $D_1$  which is in this case 120.75 minus the demand for the first period over 120. So, what you see? This is 0.75.

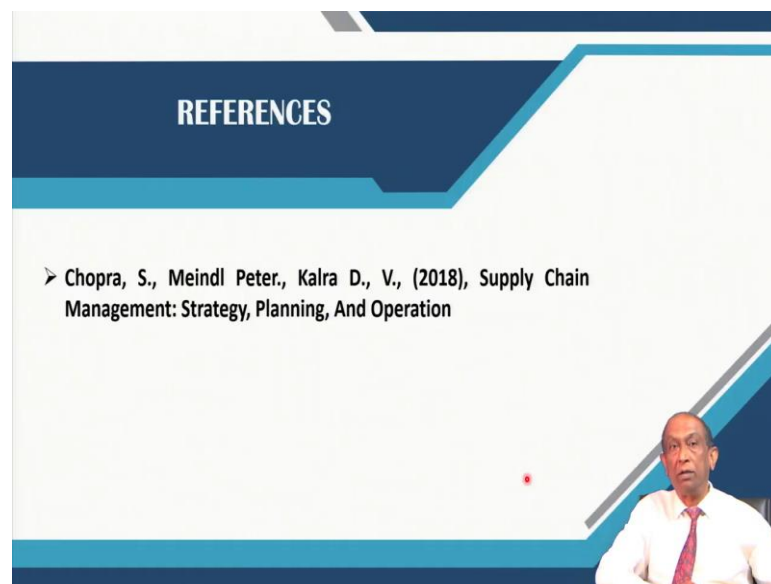
This error you can only compute, when you observe the actual demand for the first period. Then, at the end of the first period, you revise the estimate for the level and that

is given by  $L_1$ . This revised estimate for the level standing at the end of period 1 is given by the expression  $L_1 = \alpha D_1 + (1 - \alpha)L_0$ , where  $L_0$  is the estimate of level computed at the initial starting period which we have denoted by  $L_0$ .

The smoothing constant varies between 0 to 1 and in here, we have give chosen a smoothing constant value of 0.1,  $\alpha = 0.1$ ; that means, we are not giving much more importance to the recent periods data because there is no growth, there is no seasonality, it is a stable demand pattern.

So, there is no reason why should we give more weight or importance to the demand data that we have observed in the first period. Having done that, we compute  $L_1$ . In this case, it turns out to be 120.68. Now, this estimate  $L_1$  will serve as the forecast for the second period because  $F_2$  is nothing but  $L_1$ .

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This is the reference that we have used for computing, for discussing single exponential smoothing method, moving average and adaptive forecasting. This book gives all the details.

Thank you!