

**Engineering Econometrics**  
**Prof. Rudra P. Pradhan**  
**Vinod Gupta School of Management**  
**Indian Institute of Technology, Kharagpur**

**Lecture - 14**  
**Descriptive Econometrics (Contd.)**

Hello everybody. This is Rudra Pradhan here. Welcome to Engineering Econometrics. Today, we will discuss with Descriptive Econometrics. And, that too we have already discussed descriptive econometrics, with you know univariate structure, and bivariate structure more specifically, we have discussed some of the econometrics, you know output with respect to a central tendency dispersions spreadness, and then the kind of you know bivariate econometrics like covariance and correlation.

Then ultimately, we may get some kind of you know intermediate output with the help of this you know variables and data, to report like you know mean, variance, standard deviation, correlations, covariance all these you know we can have, that will somehow you know address some extent the engineering problem.

But, ultimately the econometrics requirement is to test the particular outcomes and validate before you use for any kind of you know particle requirement, but ultimately for that we need to know the you know procedures through, which you can actually check the statistical you know reliability. And, for that few things are very much essential of course, the basic steps and structure which I have already highlighted in the last lecture.

And in order to understand better structures. So, means the kind of you know selection of the particular model and the selection of a particular you know technical tools. So, you should know more about something and that too the umbrella of you know inferential econometrics.

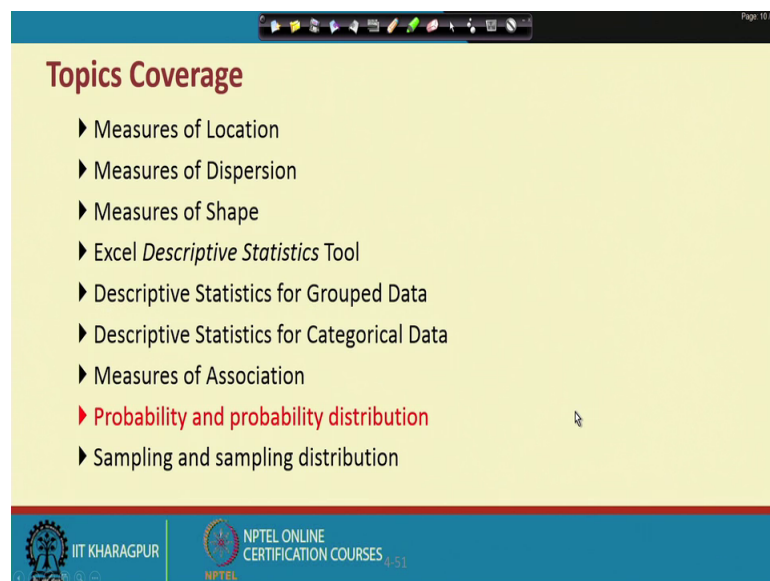
So, here we will be dealing with you know 2 things; one is the understanding of the probability and probability distributions, another one is the sampling and sampling distributions. Because, probability is one of the basic infrastructure in the engineering econometrics, any kind of you know testing empirical testing. So, you need to use the probability level of significance. So, knowing the details about the probability and the probability distributions will be very helpful to analyze the problem in a much attractive

way or you know much better way. So, let us see what is exactly the probability concept? And, what are these probability distributions?

Sometimes here you know regression modeling will be follow with here particular you know distribution only if the data is showing a particular distribution. So, then you know the model can be designed accordingly. Until unless you know the pattern of the data and the kind of you know structure so; that means, we need a complete visualization.

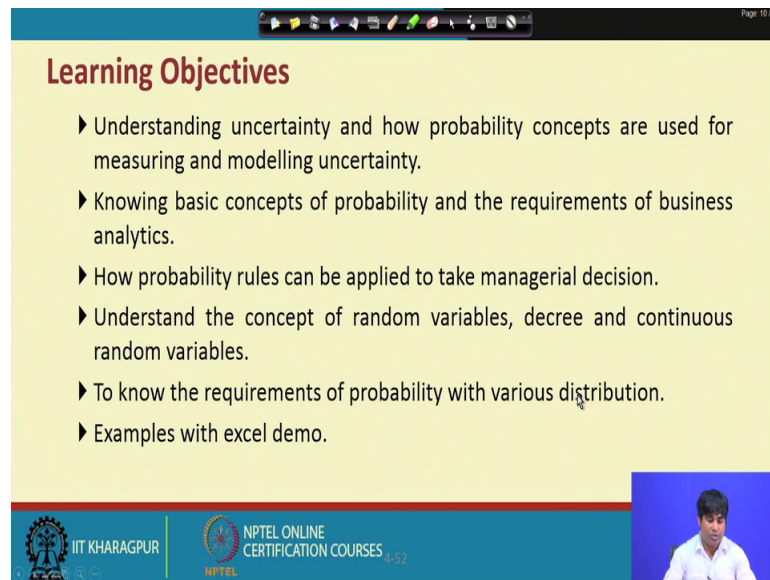
So, after knowing the visualizations you like to actually check how this data you know all about and then what kind of distribution they follow? So, according you can pick up that distribution or pick up that particular function for functional form to address or to analyze the problem. And; obviously, that will give you better inference and you know you know better insights as for here you know requirement.

(Refer Slide Time: 03:29)



So, ultimately so, we should like to know something about the all this structure.

(Refer Slide Time: 03:32)



The slide is titled "Learning Objectives" in a bold, dark red font. It contains a bulleted list of six objectives. The slide has a yellow background and is part of an NPTEL presentation, as indicated by the logos and text at the bottom. A small video inset in the bottom right corner shows a man speaking.

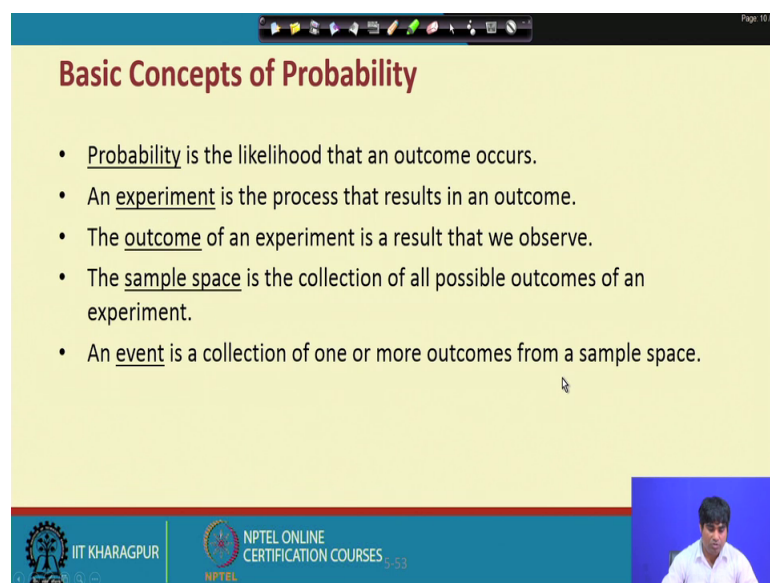
- ▶ Understanding uncertainty and how probability concepts are used for measuring and modelling uncertainty.
- ▶ Knowing basic concepts of probability and the requirements of business analytics.
- ▶ How probability rules can be applied to take managerial decision.
- ▶ Understand the concept of random variables, discrete and continuous random variables.
- ▶ To know the requirements of probability with various distribution.
- ▶ Examples with excel demo.

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 4:52

So, probability as usual very simple and the subject in the case of in an engineering econometrics, what I have already mentioned? This is actually basic infrastructure and it is up to you to understand and it is up to you to get to be acquainted. And what is exactly probability? What are the probability distributions? What are the probability rules? And how is the way you know, how you have to train interpret and the kind of way you know kind of way you know decision making process with respect to different kind of way you know situations. And some of the basic way you know items are way you know connected with the probability theory like way you know random variables, way you know discrete series continuous series discrete random variable continuous random variables.

So, these are things actually you are supposed to know of course, you have to just understand and you know these things and the suppression the calculation is concerned reporting is concerned, you can directly use the software. And again excel spreadsheet will help you a lot to get all these details. And then you know standard or advanced statistical software or economic software's further will add value to do your way you know processing.

(Refer Slide Time: 04:50)



**Basic Concepts of Probability**

- Probability is the likelihood that an outcome occurs.
- An experiment is the process that results in an outcome.
- The outcome of an experiment is a result that we observe.
- The sample space is the collection of all possible outcomes of an experiment.
- An event is a collection of one or more outcomes from a sample space.

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5:53

So, ultimately these are all your learning objectives. And probably simply actually the likelihood that an outcome occurs and it is a kind of you know experimental process, and we have a outcome, and that is with respect to a particular experiment.

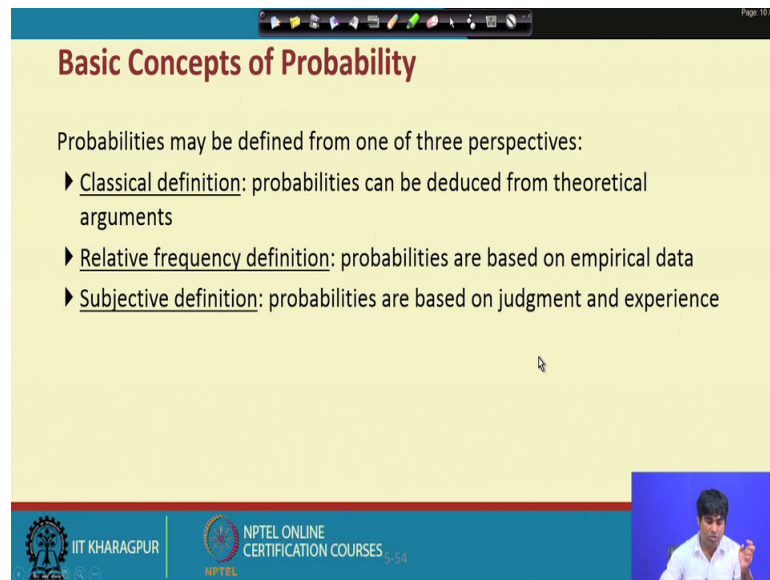
For instance you will toss a coin and what are the possible outcomes?. You know and what is the max total outcomes all together for instance if you if you are tossing a coin then it will be either head or tail so; that means, the a sample space will be either head or tail. So, so when you are tossing. So, it may appear the head or it may appear the tail. Ultimately that is the only alternatives so; that means, this is a simple examples so, so, the complex examples depends upon how is the game all about? For instance the same coin can be you know transferred into complex mode when you have it 2 different coins tossing simultaneously. And that times so, you will have a bigger sample space, like you know in the first case you have only 2 sample points head and tails.

Now, having 2 2 coin simultaneously so, your sample space will be again high. For instance it may be the appearance of a head head tail head head tails and tail tails so, for appearance will be there. Against so, a 3 coins can be toss simultaneously, then the sample space again we will be increased so; that means, it is a kind of you know a process of you know generalization. And, this is how you have to understand the system, understand the tress the statistical rules, econometrics rules, because you know their you know their basics actually until and unless you understand all these basics. So, you



cannot play the game in a you know better way. So, you must be acquainted. So, you have to understand and you have to connect as per the particular you know requirement. So, event is a you know particular item in the sample space.

(Refer Slide Time: 06:51)



**Basic Concepts of Probability**

Probabilities may be defined from one of three perspectives:

- ▶ Classical definition: probabilities can be deduced from theoretical arguments
- ▶ Relative frequency definition: probabilities are based on empirical data
- ▶ Subjective definition: probabilities are based on judgment and experience

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | 9:54

And so, the examples which I have already given is a kind of you know problems, which you can actually analyze through probability. For instance suppose one coin you are tossing. So, how is the kind of you know output and how you have to interpret the kind of you know output? The concept you know in the earlier lecture we have discussed the concept called as you know correlation. So, which is actually between minus one to one very standardized and as a result you have a very standard kind of you know interpretations as far as inference is concerned the kind of you know strategy application is concerned.

So, a probability is also similar kind of you know framework. So, the range of the probability will be always 0 to 1 and when it is you ones kind of you know perfectly there is no difference between predicted and the kind of you know happenings, but in the case of you know 0. So, there is no a kind of you know happening and in between you have to find out more occurrence and higher occurrence something like that. And so, the 2 extreme points are you know 0 and a 1 most of the instances, it will be appearing in between and the total probability will be exactly equal to 1 for instance if you are tossing a coin.

So, there are 2 sample points either head or tail so; obviously, it is the 2. So, that means, what will you do actually we have already discussed with you know, frequent you know variables information then you know frequency cumulative frequency. So, the cumulative frequency will give you the total sampling.

So, now with the total sampling if you will divide with individual sampling then that will give you the probability value. So, simple kind of you know a structure how you have to transfer the concept into a probability format? In fact, sometimes we can use the probability or normalization tool. So, to normalize the data and then use as per your you know modeling requirement. So, ultimately in this examples head and tail so, total is it 2. So, what will you do 1 by 2 for head and 1 by 2 for tail? So; that means, for individual event. So, the probability will be 1 by 2 that is 0.5 again, it is you know for tail it is 1 by 2 0.5, then if you sum these 2 again the total probability will be equal to 1.

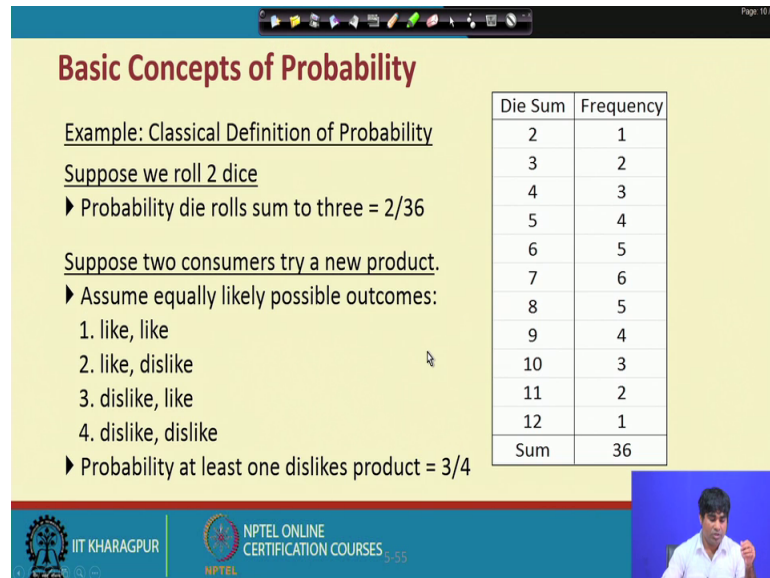
So, that means, the a so, for as a rule is concerned or the structure of probability is concerned it is a very standard, you know particular event the probability of a particular event will be in between 0 to 1. And, what are all probability if you will add, you know probability of all you know events if you add then it will give you actually one. So; that means, total probability is always equal to 1 and individual probability event specific is always between 0 to 1 that means, a compared to correlations. So, the probability value cannot be negative. So, it will be always in the positive sides, because it is a kind of an expectation nobody we will (Refer Time: 10:00) for you know negative kind of you know things.

So, this is the basics about the probability. And, there are various theories behind probability. So, we are not going in details. In fact, I have another subject you know NPTEL subject business and analytics for management decisions, where we have discussed all these details the first you know 2 3 chapters are you know more or less sense, because these are all basic requirements for any kind of you know econometric modeling and you know analytics.

So, still I am you know highlighting all these things, because it is the requirement of you know engineering econometrics. So, that means, ultimately for a hard core modeling. So, first one to you know 3 chapters are the basic inputs or the basic you know infrastructure

for you know hardcore, you know modeling. So, that is how we are actually in the processed you know all these details.

(Refer Slide Time: 10:54)



**Basic Concepts of Probability**

Example: Classical Definition of Probability  
Suppose we roll 2 dice  
► Probability die rolls sum to three =  $2/36$

Suppose two consumers try a new product.  
► Assume equally likely possible outcomes:

1. like, like
2. like, dislike
3. dislike, like
4. dislike, dislike

► Probability at least one dislikes product =  $3/4$

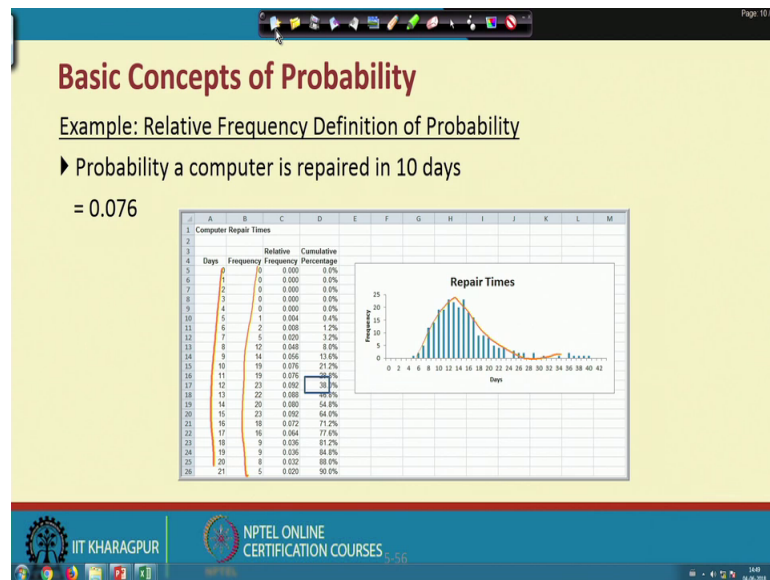
Die Sum	Frequency
2	1
3	2
4	3
5	4
6	5
7	6
8	5
9	4
10	3
11	2
12	1
Sum	36

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5:55

And, this is how the a kind of you know a another example. So, it is a kind of you know tossing a die and a tossing a dice a where you know we may have actually outcome like you know 1 2 3 4 5 6. And a similar like 2 coin if you did you know toss and 2 dice if you will toss then you will find plenty of you know outcomes. So; that means, in a real life situations you will be find you know different kind of you know you know interactive kind of you know issues. And when you are you know connecting each other then the sample space will be actually enlarged accordingly.

So, you should not know how is the particular you know structure and, how you can connect. And, how to get the sample space, and how to get the event specific output and then you can use this event specific output, and this kind of you know sample space for your you know problem requirement and the kind of you know investigation requirement.

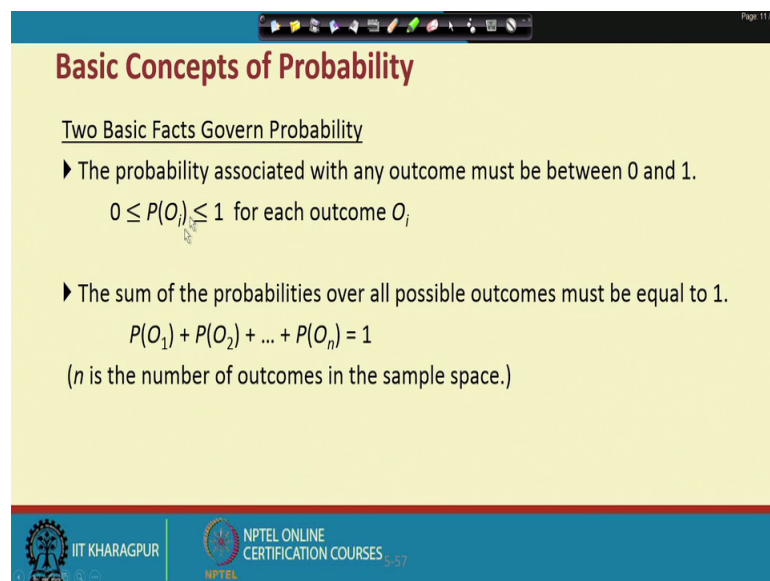
(Refer Slide Time: 11:56)



See here whatever I have already discussed. Let us say this is a kind of you know day wise informations and how you will transfer into you know probability format? So, this is a day wise informations. And, these are you know frequencies, what we will do actually you first calculate the cumulative frequency. And, then you know divide the total cumulative frequency with you know each individual then you will get you know relative frequency. And, by default that will be our you know probability. And, if we will add all these individuals individual probability, then ultimately at the end you will get you know total probability exactly equal to 1.

So, these are all actually even specific. And, we like to report all these probability and check how is the kind of you know distribution so; that means, the kind of you know approach of a variables with respect to frequency. So, that you will get to know that variable follows a means, what is the distribution this particular you know variable follows? So, accordingly you can develop the modeling, you know as for you know problem requirement until unless, you know the kind of you know distribution you may not in a position to pick up a particular model. So, that is how you have to know the concept of probability and the kind of you know probability distribution. So, this is how these simple examples.

(Refer Slide Time: 13:22)



### Basic Concepts of Probability

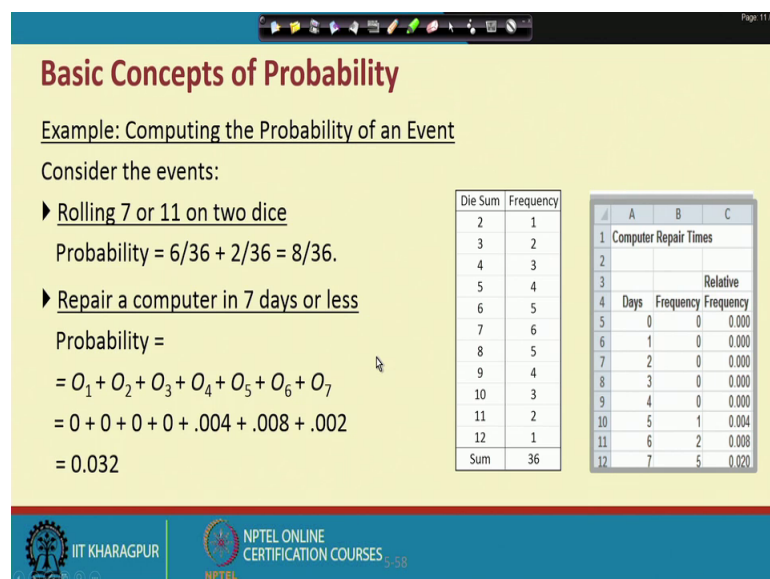
Two Basic Facts Govern Probability

- ▶ The probability associated with any outcome must be between 0 and 1.  
 $0 \leq P(O_i) \leq 1$  for each outcome  $O_i$
- ▶ The sum of the probabilities over all possible outcomes must be equal to 1.  
 $P(O_1) + P(O_2) + \dots + P(O_n) = 1$   
( $n$  is the number of outcomes in the sample space.)

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5:57

And, then similarly over that is how you know, what we have already a discussed. So, the probability range will be 0 to 1. And total probability exactly equal to 1.

(Refer Slide Time: 13:31)



### Basic Concepts of Probability

Example: Computing the Probability of an Event

Consider the events:

- ▶ Rolling 7 or 11 on two dice  
Probability =  $6/36 + 2/36 = 8/36$ .
- ▶ Repair a computer in 7 days or less  
Probability =  
 $= O_1 + O_2 + O_3 + O_4 + O_5 + O_6 + O_7$   
 $= 0 + 0 + 0 + 0 + .004 + .008 + .002$   
 $= 0.032$

Die Sum	Frequency
2	1
3	2
4	3
5	4
6	5
7	6
8	5
9	4
10	3
11	2
12	1
Sum	36

	A	B	C
1	Computer Repair Times		
2			
3			Relative
4	Days	Frequency	Frequency
5	0	0	0.000
6	1	0	0.000
7	2	0	0.000
8	3	0	0.000
9	4	0	0.000
10	5	1	0.004
11	6	2	0.008
12	7	5	0.020

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5:58

So, this is 2 basic you know principle of the probability and it is always true, whether the problem is a very simple one, you know very complex one. And this is standard examples here and you know dice sum because if, you are tossing 2 dice simultaneously. So, first happening second happening and their sum so, it will be start with you know 2 3 4. So; obviously, because dice is the minimum number is 1. So, if both will be having

minimum then the sum would be starts with you know 2 and then similarly 3 4 5 6 up to you know 12, because the maximum number of number in a dice a equal to 6. So, both will be happening same times then it will be total 12.

So, this is these possible all possible events and that is what the sample space of all about and then ultimately with the help of you know frequency, you can find out actually probability structures right. So, this is how means total actually coming 36 that is the cumulative frequency. If, you divide with you know individuals. So, then that will give you the event specific probability. So, and when the basis of that event specific probability you can check the distributions how is the kind of you know distribution.

Ultimately, most of the instances we will check whether the distribution is the normal one and not normal ome. So, if you know if not normal one, but what particular distribution other distribution it follows. Like you know, we have discussed you know very beginning and the functional relationship will be linear one. And, non-linear ones if it is a linear one the game is very restricted and very simples if it is in non-linear one then there are different kind of you know non-linear functions you may have. So, again the complexity we will start you know accordingly.

So, similarly the case of you know normal distribution and not normal distributions. So, so, but usually most of the things means in the case of normal distribution some of the things are very smooth and you know very streamlined, but in the case of you know other distribution it is not you know so, systematic.

So, ultimately a econometrics environment is like that, you know if you increase the sample size and structuring restructuring most of the distribution can actually converts to normal distribution. And, ultimately some I means most of the instances we do all these you know a iterations or the kind of you know structuring restructuring, ultimately try to bring in a kind of you know set ups maybe normal a completely normally distributed setup.

So, that you know the a the particular you know investigation process we will be very accurate very effective and will be very needful for the requirement. Otherwise you know it may be a kind of you know biasness and you will not ultimately get the right answer right output to address the problem. So, that is why we need to do some kind of you know structuring restructuring normalizations ok. So, increasing sample size means

there are variety of ways you can you know rotate the games as per your you know requirement. After, that if you are not getting; that means, the problem may not means may be dangerous or may not be handy to solve in the with the help of this particular you know engineering econometrics.

(Refer Slide Time: 16:54)

**Basic Concepts of Probability**

Example: Computing the Probability of the Complement of an Event

$$P(A^c) = 1 - P(A)$$

►  $A^c$ , the complement of  $A$ , consists of all outcomes in the sample space not in  $A$ .

Dice example:

►  $A = \{7, 11\}$   
 $P(A) = 8/36$

►  $A^c = \{2, 3, 4, 5, 6, 8, 9, 10, 12\}$   
 $P(A^c) = 1 - 8/36 = 28/36$

Die Sum	Frequency
2	1
3	2
4	3
5	4
6	5
7	6
8	5
9	4
10	3
11	2
12	1
Sum	36

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5:59

So, these are all basic you know concept and in addition to that to some of the basic ones like you know this is a compliment. So, total flow since total probability is equal to 1. So, probability of happening not happening so, if probability of a happening is available by default probability of not happening you can actually in get.

So; that means, it has a some kind of you know duty because you know some instance some, you know missing observations are there, but now with the help of this you know concept of probability or the rules of probability you can fill up this you know missing observations very easily.

(Refer Slide Time: 17:29)

**Basic Concepts of Probability**

Example: Computing the Probability of Mutually Exclusive Events

$$P(A \text{ or } B) = P(A) + P(B)$$

► Mutually exclusive events have no outcomes in common.

Dice Example:

►  $A = \{7, 11\}$

►  $B = \{2, 3, 12\}$

►  $P(A \text{ or } B) = \text{UNION of events } A \text{ and } B$

$$= P(A) + P(B)$$
$$= 8/36 + 4/36 = 12/36$$

Die Sum	Frequency
2	1
3	2
4	3
5	4
6	5
7	6
8	5
9	4
10	3
11	2
12	1
Sum	36

IIT KHARAGPUR NPTEL ONLINE CERTIFICATION COURSES 5:60

And, these are all various rules of the probability and I am not going in details.

(Refer Slide Time: 17:34)

**Basic Concepts of Probability**

Example: Computing the Probability of Non-Mutually Exclusive Events

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Dice Example:

►  $A = \{2, 3, 12\}$

►  $B = \{\text{even number}\}$

►  $P(A \text{ or } B) = \text{UNION of events } A \text{ and } B$

$$= P(A) + P(B) - P(A \text{ and } B)$$
$$= 4/36 + 18/36 - 2/36$$
$$= 20/36$$

Die Sum	Frequency
2	1
3	2
4	3
5	4
6	5
7	6
8	5
9	4
10	3
11	2
12	1
Sum	36

IIT KHARAGPUR NPTEL ONLINE CERTIFICATION COURSES 5:61

Against sometimes you know if there are 2 events a minimum in a in a particular you know set up. If it is only one event, then the game is very simple ones, then the game come game plan will be you know start come you know you know in a complex way. So, when your you know the events are you know more in numbers. And, the issue is that you know let us say there are 2 events and we that is the minimum from minimum number of complexity or minimum level of complexity. When and with is there so, and



we may be actually completely independent or there is a some kind of you know commonalities there.

So, that is why a you need to know some kind of you know rules of the probability how you have to bring the kind of you know situations for instance. If A and B is there then the probability says that you know what is the probability of where, B is not there what is the probability of B, where probably a is not there. And there is a chance that you know probability A and B simultaneously are there so; that means, both can actually very handy to represent the particular you know as specifications. So, all these details are there in the theory of you know probability and these are all the you know these are all the kind of you know requirements against for the a engineering econometrics, and that too inferential you know economics requirement.

(Refer Slide Time: 19:05)

**Basic Concepts of Probability**

Example: Conditional Probability in Marketing

- ▶ The Data shows the first and second purchases for a sample of 200 customers.
- ▶ Probability of purchasing an iPad given already purchased an iMac =  $2/13$

	A	B
1 Apple Purchase History		
2		
3 First Purchase		Second Purchase
4 iPod	iMac	
5 iPhone	MacBook	
6 iMac	iPhone	
7 iPhone	iPod	
8 iPod	iPhone	
9 MacBook	iPod	
10 iPhone	MacBook	

Count of Second Purchase	iMac	iPad	iPhone	iPod	MacBook	Grand Total
iMac	1	3	2	6		12
iPad	1	1	2	10		14
iPhone	3	4	14	21		42
iPod	3	12	12	30		57
MacBook	8	16	26	24		74
Grand Total	15	34	42	42	67	200

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5:62

So, the these are all various examples like whatever examples I have you know cited and this is a little bit more complex kind of you know example. Against like you know variance covariance matrix, we can create a probability matrix you know that that will give you a better pictures actually with respect to more number of features, more number of variables, means see the thing is that you know it is a very interesting kind of you know concept and very useful for modelling. Once your problem is the very much you know clear clean and you know understood properly, then you can actually process it as

per your you know requirement and probably help you lot to do the kind of you know needful.

(Refer Slide Time: 19:48)

### Basic Concepts of Probability

Example: Computing a Conditional Probability in a Cross-Tabulation

► Probability of preferring Brand 1 given that a respondent is male = 25/63

Respondent	Gender	Brand Preference
1	Male	Brand 3
2	Female	Brand 3
3	Male	Brand 3
4	Male	Brand 1
5	Male	Brand 1
6	Female	Brand 2
7	Male	Brand 2
8	Female	Brand 2
9	Male	Brand 1
10	Female	Brand 3

Count of Respondent	Brand 1	Brand 2	Brand 3	Grand Total
Male	9	6	22	37
Female	17	21	25	63
Grand Total	34	23	43	100

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5:63

So, this is another way of you know highlighting the concept of probability.

(Refer Slide Time: 19:52)

### Basic Concepts of Probability

Example: Using the Conditional Probability Formula

► Probability of A given B:

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

►  $P(B_1|M) = P(B_1 \text{ and } M) / P(M)$   
 $= (25/100) / (63/100)$   
 $= 25/63 = 0.397$

Summary of conditional probabilities:

P(Brand Gender)	Brand 1	Brand 2	Brand 3
Male	0.397	0.270	0.333
Female	0.243	0.162	0.595

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5:64

This is called as you know cross tabulation mechanism. And, this is what actually a you know another way to find out the a kind of (Refer Time: 20:01) address the problem. Sometimes what happens the a joint occurrence is there and a particular event outcome is there, the other one is missing. Now, with the help of this particular you know a concept,

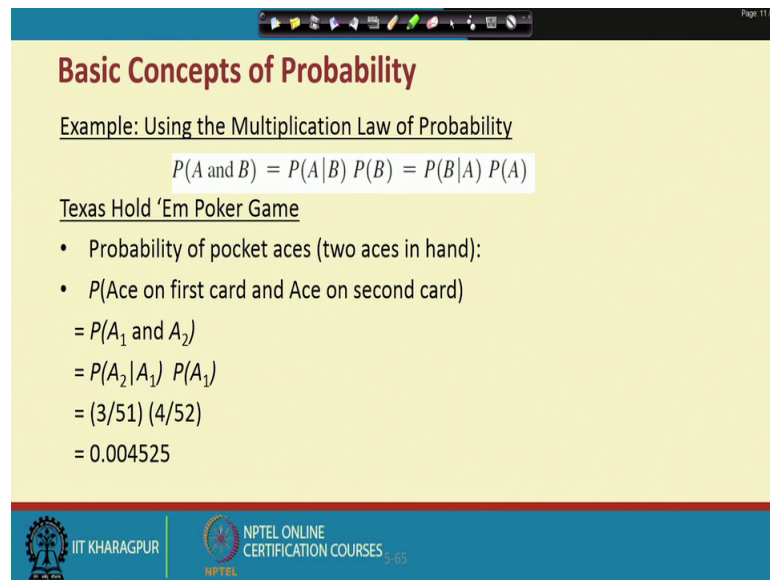
we can actually find a you know find out the missing observations for you know missing requirement.

So, the particular property is called as an you know the structure of you know base theorem and with the help of in this particular theorem the understanding is that you know, what is the probability of A given B. So, like what we have already discussed the concept in that case of you know partial correlations and the kind of you know multiple correlation.

So, in a real life scenario, it is the very you know very you know less instances, you will find the particular event or the particular variable is completely independent. So, most of the instances will be find the variables are actually behaved you know positively or negatively in presence of you know other variables. Or the other variables the behaviours or the impact may be a like you know pull or you know push, but ultimately it is the kind of you know requirement to connect, and check the kind of you know you know issues and the kind of you know solutions. Until unless you cannot check and you know address the problems you should not actually proceed for the hardcore modelling.

So, these are all basics you have to remove; that means, in every stage you need actually neat and clean environment before you go for the you know kind of you know hardcore modelling. Until unless have clean pictures so, you may not actually a estimate means a starting with you know choice of the models, estimation of the models, validation of the model, use of the models. So, some point of time it will you know you know give give you some kind of you know problem. So, that is why it is better to you know clean all these things and know all these things before you enter to the big better field.

(Refer Slide Time: 22:16)



**Basic Concepts of Probability**

Example: Using the Multiplication Law of Probability

$$P(A \text{ and } B) = P(A|B) P(B) = P(B|A) P(A)$$

Texas Hold 'Em Poker Game

- Probability of pocket aces (two aces in hand):
- $P(\text{Ace on first card and Ace on second card})$

$$\begin{aligned} &= P(A_1 \text{ and } A_2) \\ &= P(A_2|A_1) P(A_1) \\ &= (3/51) (4/52) \\ &= 0.004525 \end{aligned}$$

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5.65

So, likewise the is the base theorems can be actually simple and can be also in a complex a framework so; that means, ultimately what I have mentioned you start with a simple one then a add one after another then, you know the level of complexity we will start increasing. And like you we have already discussed in the case of you know (Refer Time: 22:35) setups. So, the concept of probability is also similar way.

So, they are not a different, but the structure is slightly a other way around, but ultimately the major goals or you know the kind of you know objectives are more or less concentrate towards the a investigation process that to or how to you know get some kind of you know empirical evidence to address the particular you know engineering problem.

(Refer Slide Time: 23:07)

### Basic Concepts of Probability

Example: Determining if Two Events are Independent

$$P(A|B) = P(A)$$

► Are Gender and Brand Preference Independent?

Count of Respondent	Column Labels	Brand 1	Brand 2	Brand 3	Grand Total
Row Labels	Brand 1	Brand 2	Brand 3	Grand Total	
Female	9	6	22	37	
Male	25	17	21	63	
Grand Total	34	23	43	100	

P(Brand Gender)	Brand 1	Brand 2	Brand 3
Male	0.397	0.270	0.333
Female	0.243	0.162	0.595

► Is  $P(B_1|M) = P(B_1)$ ?

$$0.397 \neq .34$$

Gender and Brand Preference are Dependent.

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5:56

These are various examples against.

(Refer Slide Time: 23:08)

### Basic Concepts of Probability

Example: Using the Multiplication Law for Independent Events

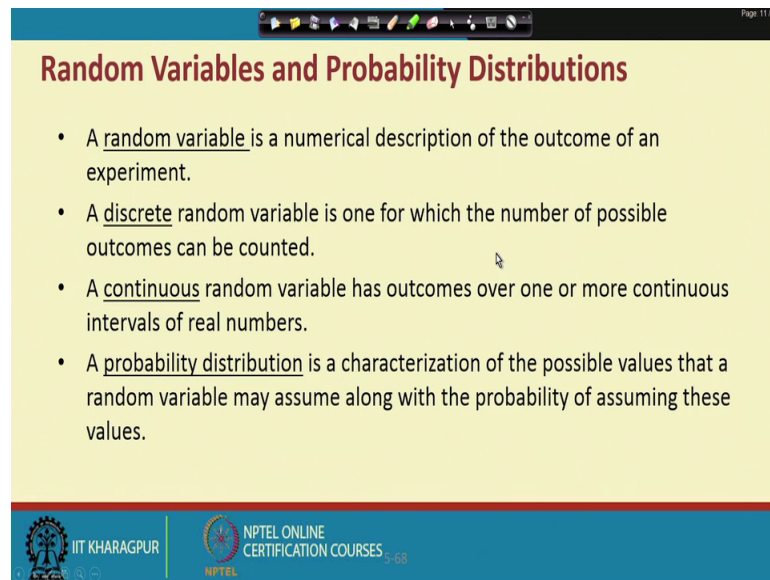
$$P(A \text{ and } B) = P(B) P(A) = P(A)P(B)$$

► Dice Roll Example:

- Rolling pairs of dice are independent events since they do not depend on the previous rolls.
- $A = \{\text{roll a sum of 6 on first pair die rolls}\}$
- $B = \{\text{roll a sum of 2, 3, or 12 on second pair rolls}\}$
- $P(A \text{ and } B) = P(A) P(B) = (5/36) (4/36) = 0.0154$

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5:57

(Refer Slide Time: 23:10)



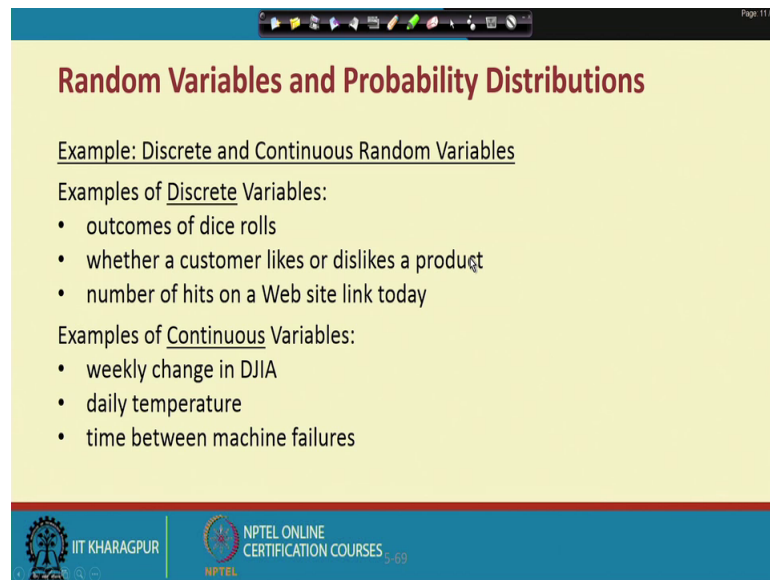
The slide is titled "Random Variables and Probability Distributions" in a bold, dark red font. It contains four bullet points, each starting with a small red dot. The text is in a black, sans-serif font. The slide has a yellow background and is framed by a blue border at the top and bottom. The top border contains a navigation bar with various icons and the text "Page 11/11". The bottom border contains the logos of IIT KHARAGPUR and NPTEL ONLINE CERTIFICATION COURSES, along with the number "5-68".

- A random variable is a numerical description of the outcome of an experiment.
- A discrete random variable is one for which the number of possible outcomes can be counted.
- A continuous random variable has outcomes over one or more continuous intervals of real numbers.
- A probability distribution is a characterization of the possible values that a random variable may assume along with the probability of assuming these values.

And I am just skipping these ones. And then ultimately a next item is actually the probability distributions and for that you need to know random variables. And, the random variables means variables, which will be you know randomly you know just you know means outcomes or you know some somehow, you know not actually very specific means there is you know biasness ultimately.

And the particular structure can be continuous can be a discrete and then that itself we will give you the understanding about the probability distributions, we have a different kind of you know probability distribution. So, let us see what are the distribution, and how is the particular you know functionality or functional forms, and how is the kind of you know requirement? Sometimes you know depending upon the particular problems you can you know apply a particular you know distribution. Sometimes you know once you understand and once you know the structure of you know particular distributions, you can easily actually connect and looking for the kind of you know inference.

(Refer Slide Time: 24:20)



**Random Variables and Probability Distributions**

Example: Discrete and Continuous Random Variables

Examples of Discrete Variables:

- outcomes of dice rolls
- whether a customer likes or dislikes a product
- number of hits on a Web site link today

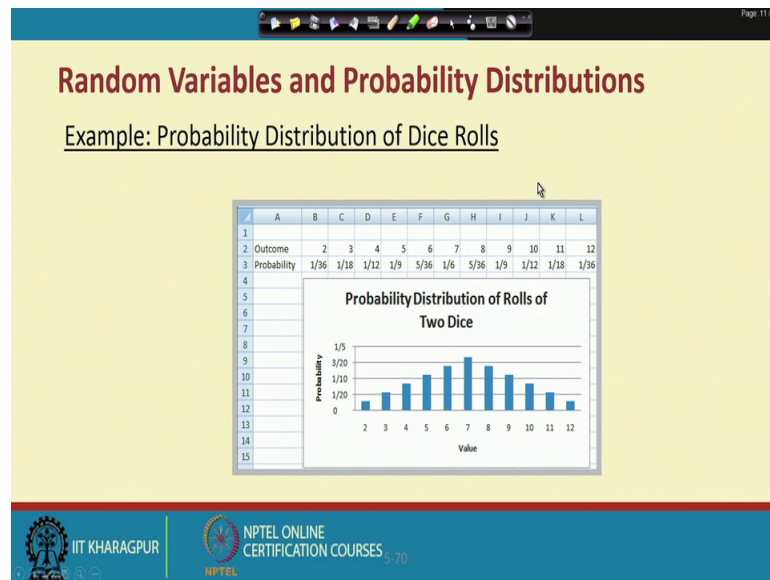
Examples of Continuous Variables:

- weekly change in DJIA
- daily temperature
- time between machine failures

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5:49

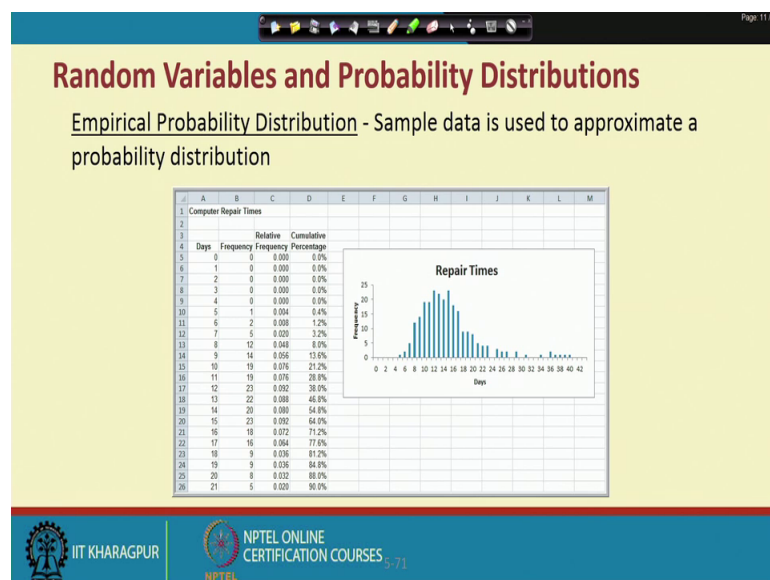
So, these are the classic examples of you know discrete variable case and continuous variable case for instance outcomes of you know dice rolls, whether a customer likes or dislike say product these are all called as you know discrete variables examples. And, the example of continuous variable is weekly change in D J I daily temperature time between machine failures. So, these are all continuous there is a need of you know continuity or some kind of you know consistency that is what you know and it is a very big link with you know time series data. And, that too with response to actually the kind of you know cross sectional kind of unilateral for the continuous variables, if you talk that this is very you know familiar with you know time series structure.

(Refer Slide Time: 25:11)



And, this is what actually another way of you know understanding. And once you know the a total outcomes then the kind of you know event issues. So, you can create here you know on probability structures and you can you know check the or you can design the probability distribution. Just you can visualize and ultimately you can get to know how is the kind of you know distributions? Whether it is your normal one or other way round.

(Refer Slide Time: 25:39)



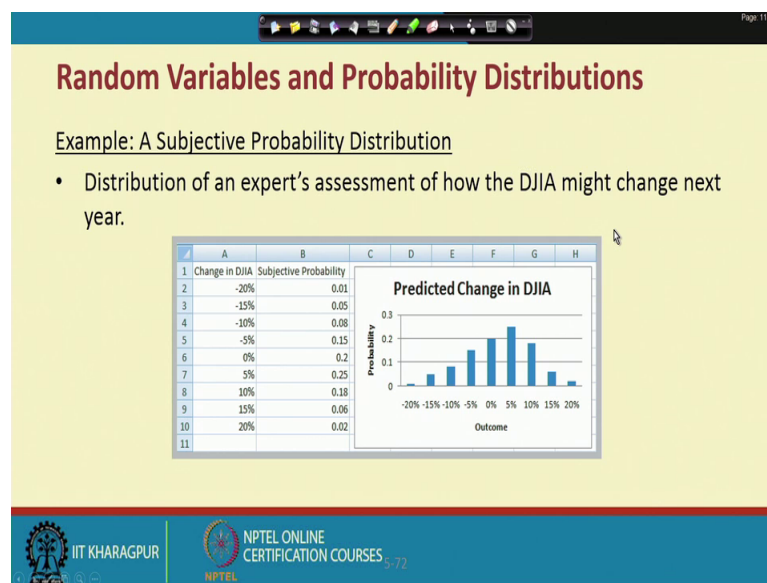
So, this is how is say yes this is how a kind of you know distributions? It is not exactly normal, but somehow close to normal, but ultimately this is how the shaping? And,



sometimes what happens some of the cases, you cannot just manipulate you know data or you may not have options to you know do the kind of you know transformation or some theory restructuring, but ultimately you need to have a balancing.

And, one way to you know means in such a situation one way to solve the situation, you know increase the sample size actually somehow if you increase the sample size and slightly you normalize. Then ultimately the distribution we will move from you know not you know means closing towards you know normal distributions. So, ultimately normal distribution, since normal discussion is the best ultimately we try to you know structure restructure. So, that the particular you know data structure or you know sample space living normally distributed and, as a result you can easily pick up a technique to address the problem.

(Refer Slide Time: 26:48)



This is another way of you know distributions.

(Refer Slide Time: 26:51)

**Discrete Probability Distributions**

Probability Mass Function  
a mathematical function  $f(x)$  specifying the probability of the random variable  $X$ .  
 $x_i$  represents the  $i^{\text{th}}$  value of  $X$ .

Properties:  $0 \leq f(x_i) \leq 1$  for all  $i$   
 $\sum_i f(x_i) = 1$

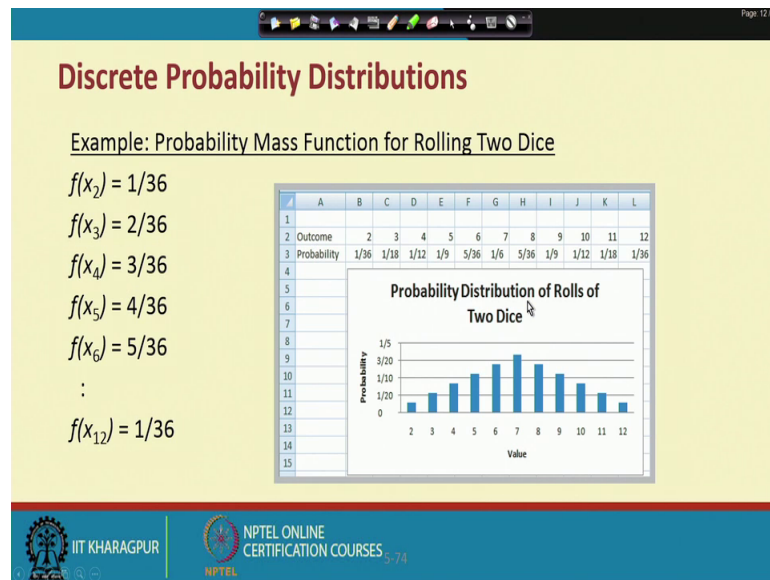
Cumulative distribution function:  
 $F(x) = P(X \leq x)$

Handwritten notes:  $X: x_1 x_2 x_3$   
 $P: P_1 P_2 P_3$

And, usually probability distribution if you talk about probability distribution so; that means, there is a random variables corresponding it has some kind of you know event specific. And, then corresponding to each events we must have a probability for instance. The simple structure of understanding is  $X$ ;  $X$  is a random variable which carries you know events like  $x_1 x_2 x_3$  and so on. And, the corresponding probability will be  $P_1 P_2 P_3$ .

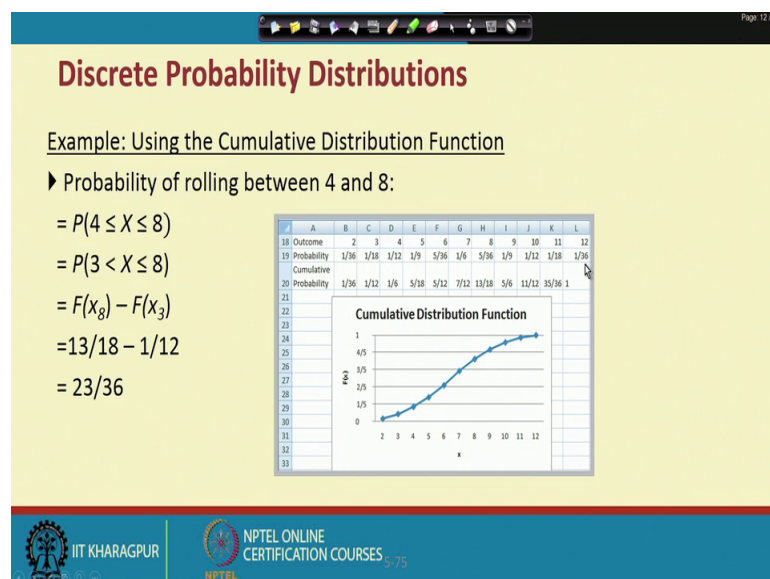
And so, now, the question is you know how you are getting the probability? So, that depends upon actually a particular you know distribution. So, it depends you know how is the kind of you know (Refer Time: 27:29) structure, but whatever ways you can you know address. So, ultimately the probability value will be in between 0 to 1 and total probability exactly equal to 1. That is that is why we have written here. And, this is like you know same case like you know correlation, whatever ways you can handle simple partial multiple. Then, ultimately the range is will be between minus 2 1 and you know means it is a kind of you know structured framework or what we can call as a standardized framework. So, that you know it is very easy to address easy to understand and easy to actually use for the kind of you know requirement. So, that is how the simple structure open up probability distribution.

(Refer Slide Time: 28:16)



And, then what is happening actually you can just design the probability requirement then you plot after plotting you will get a get to know the kind of you know distribution.

(Refer Slide Time: 28:24)



This is what actually cumulative distributions functions.

(Refer Slide Time: 28:29)

Page 12/12


## Discrete Probability Distributions


Example: Computing the Expected Value  
of the sum of values on 2 die rolls

$$E[X] = \sum_{i=1}^{\infty} x_i f(x_i)$$

$E[X] = 2(1/36) + 3(1/18) + \dots$   
 $12(1/36) = 7$

	A	B	C
1	Expected Value Calculations		
2			
3	Outcome, x	Probability, f(x)	x*f(x)
4	2	1/36	1/18
5	3	1/18	1/6
6	4	1/12	1/3
7	5	1/9	5/9
8	6	5/36	5/6
9	7	1/6	1 1/6
10	8	5/36	1 1/9
11	9	1/9	1
12	10	1/12	5/6
13	11	1/18	11/18
14	12	1/36	1/3
15	Expected value		7

 IIT KHARAGPUR

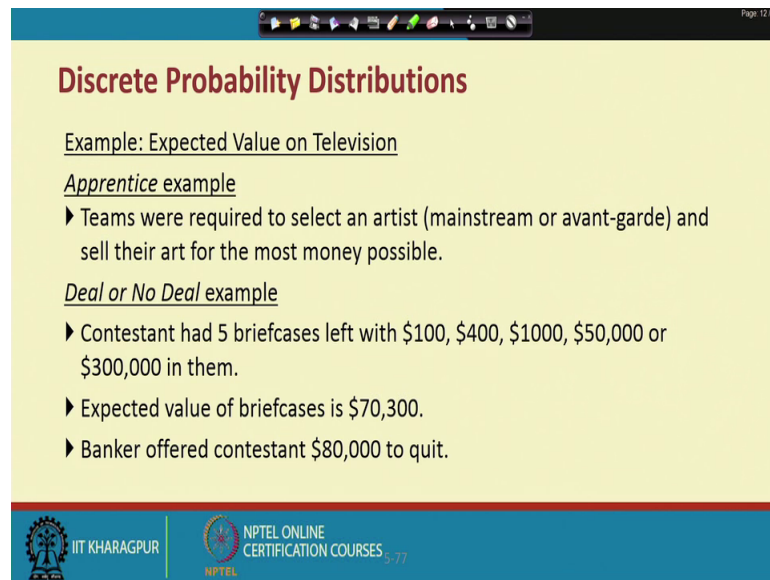
 NPTEL ONLINE  
CERTIFICATION COURSES 5.76

And, this another example called as you know means say we have actually discrete econometric say, you know where we have discussed centrals tendency dispersions and a kind of you know bivariate econometrics like covariance and correlations. Ultimately, here is also case having actually you know variables and the corresponding probability. So, you can also touch upon all these components again and get to know how is the kind of you know series all about.

Ultimately the difference between the these two is a sometimes when when you have a variable information say x 1 and x 2 x 3 x 4 like this. And, they have you know connection with you know probability. And, sometimes they have a connection with you probably depends actually most of the future predictions where you know, the probability attachment is always there, but when as something already happened and you have collected something else.

So, there is no need to actually connect with you know probability, but whatever may be the ways whether it is a simple structure or probability structure, ultimately the game is a exactly as per the particular you know requirement. So, you can plan accordingly and then use as per your you know or you know, engineering problems you know need or you know applications.

(Refer Slide Time: 29:51)



**Discrete Probability Distributions**

Example: Expected Value on Television

Apprentice example

- ▶ Teams were required to select an artist (mainstream or avant-garde) and sell their art for the most money possible.

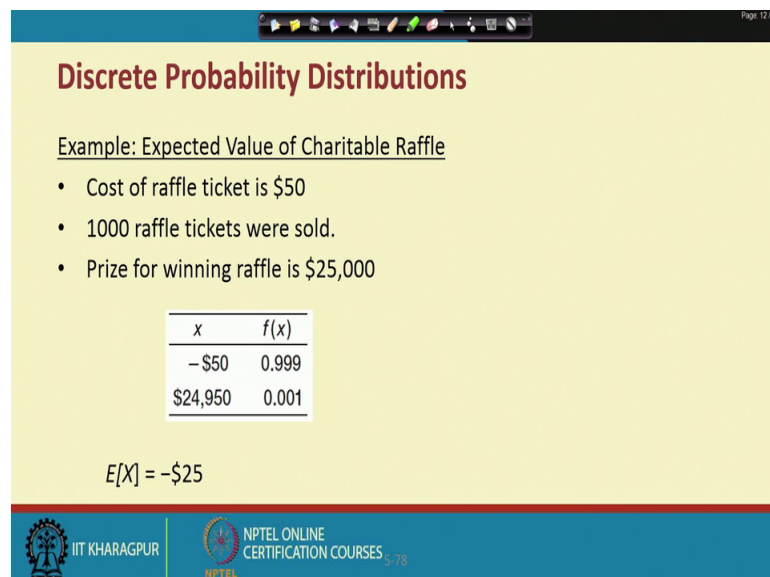
Deal or No Deal example

- ▶ Contestant had 5 briefcases left with \$100, \$400, \$1000, \$50,000 or \$300,000 in them.
- ▶ Expected value of briefcases is \$70,300.
- ▶ Banker offered contestant \$80,000 to quit.

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5.77

So, these are all you know various examples and this called as you know discrete probability distributions.

(Refer Slide Time: 29:59)



**Discrete Probability Distributions**

Example: Expected Value of Charitable Raffle

- Cost of raffle ticket is \$50
- 1000 raffle tickets were sold.
- Prize for winning raffle is \$25,000

$x$	$f(x)$
-\$50	0.999
\$24,950	0.001

$E[X] = -\$25$

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5.78



And, under the discrete probability distributions, we have a couple of distributions.

(Refer Slide Time: 30:02)

## Discrete Probability Distributions

Example: Airline Revenue Management

- Full and discount airfares are available for a flight.
- Full-fare ticket costs \$560
- Discount ticket costs \$400
- $X$  = ticket price paid
- $p = 0.75$  (the probability of selling a full-fare ticket)
- $E[X] = 0.75(\$560) + 0.25(0) = \$420$
- The airline should not discount full-fare tickets because the expected value of a full-fare ticket is greater than the cost of a discount ticket.
- Break-even point:  $\$400 = p(\$560)$  or  $p = 0.714$

 IIT KHARAGPUR
  NPTEL ONLINE CERTIFICATION COURSES 5:79

And, this is the simple example of you know airline revenue management. And, here we have actually experience and corresponding probability, then you can you know you can get to know what is the main clause and variance clause all these details you can here and then you can address the problem.



(Refer Slide Time: 30:20)

## Discrete Probability Distributions

Example: Computing the Variance of a Random Variable

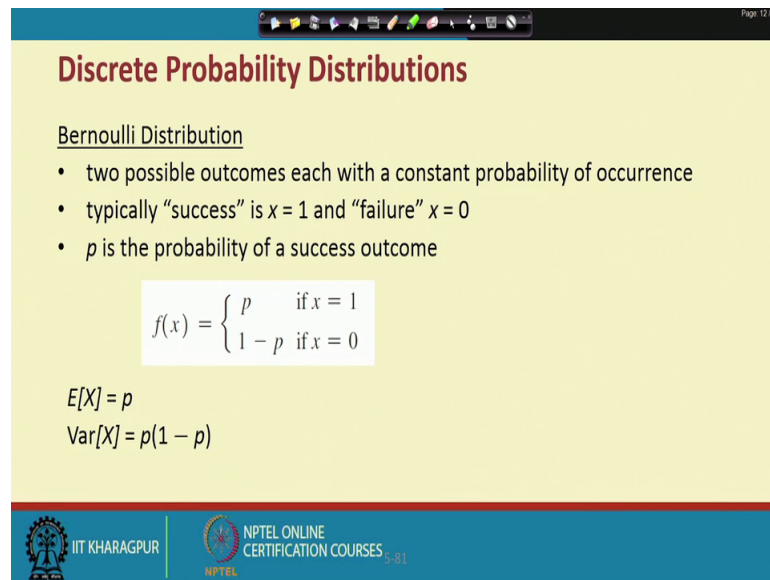
$$\text{Var}[X] = \sum_{j=1}^{\infty} (x_j - E[X])^2 f(x_j)$$

	A	B	C	D	E	F
1	Variance Calculations					
2						
3	Outcome, $x$	Probability, $f(x)$	$x \cdot f(x)$	$(x - E[X])$	$(x - E[X])^2$	$(x - E[X])^2 \cdot f(x)$
4	2	1/36	1/18	-5	25	25/36
5	3	1/18	1/6	-4	16	8/9
6	4	1/12	1/3	-3	9	3/4
7	5	1/9	5/9	-2	4	4/9
8	6	5/36	5/6	-1	1	5/36
9	7	1/6	7/6	0	0	0
10	8	5/36	10/9	1	1	5/36
11	9	1/9	1	2	4	4/9
12	10	1/12	5/6	3	9	3/4
13	11	1/18	11/18	4	16	8/9
14	12	1/36	1/3	5	25	25/36
15	Expected value		7	Variance		5 5/6

 IIT KHARAGPUR
  NPTEL ONLINE CERTIFICATION COURSES 5:80

And this is how the variance can be all these are you know same like whatever we have discussed in the case of you know.

(Refer Slide Time: 30:24)



The slide is titled "Discrete Probability Distributions" in a bold, dark red font. Below the title, the text "Bernoulli Distribution" is underlined. A bulleted list contains three points: "two possible outcomes each with a constant probability of occurrence", "typically 'success' is  $x = 1$  and 'failure'  $x = 0$ ", and " $p$  is the probability of a success outcome". A white box contains the probability mass function: 
$$f(x) = \begin{cases} p & \text{if } x = 1 \\ 1 - p & \text{if } x = 0 \end{cases}$$
 Below this, the expected value and variance are given as  $E[X] = p$  and  $\text{Var}[X] = p(1 - p)$ . The slide footer includes the IIT Kharagpur logo, the NPTEL logo, and the text "NPTEL ONLINE CERTIFICATION COURSES 5.81".

**Discrete Probability Distributions**

Bernoulli Distribution

- two possible outcomes each with a constant probability of occurrence
- typically "success" is  $x = 1$  and "failure"  $x = 0$
- $p$  is the probability of a success outcome

$$f(x) = \begin{cases} p & \text{if } x = 1 \\ 1 - p & \text{if } x = 0 \end{cases}$$

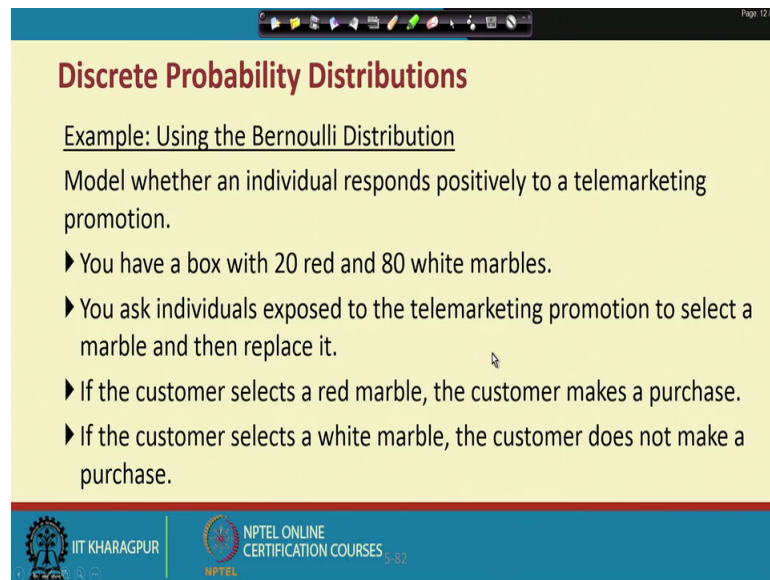
$E[X] = p$   
 $\text{Var}[X] = p(1 - p)$

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5.81

First 2 lectures and that too the discrete econometrics with you know as 1 variable case and you know 2 variable case. And, one of the discrete world distribution is called as a by binomial distributions, where the structure is right you know  $P$  is the probability of success. And, its counterpart is called as you know failure which is nothing, but  $1 - p$ . And that is how we transfer? And, this is very you know very much requires in the case of you know as econometric engineering econometrics, where will we where will we use some of the you know typical or you know structure kind of you know modelling. So, like you know logit modelling, probit modelling, in that case we will be use we will need this particular you know distributions.



(Refer Slide Time: 31:18)



**Discrete Probability Distributions**

Example: Using the Bernoulli Distribution

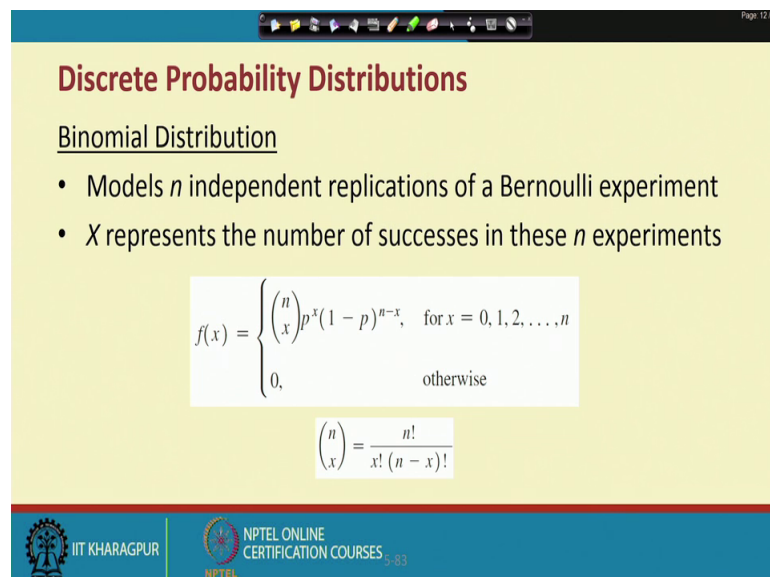
Model whether an individual responds positively to a telemarketing promotion.

- ▶ You have a box with 20 red and 80 white marbles.
- ▶ You ask individuals exposed to the telemarketing promotion to select a marble and then replace it.
- ▶ If the customer selects a red marble, the customer makes a purchase.
- ▶ If the customer selects a white marble, the customer does not make a purchase.

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5:52

So, these are you know specific examples of you know you know Bernoulli or binomial distributions. And, here you need to have some kind of you know specifications.

(Refer Slide Time: 31:33)



**Discrete Probability Distributions**

Binomial Distribution

- Models  $n$  independent replications of a Bernoulli experiment
- $X$  represents the number of successes in these  $n$  experiments

$$f(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & \text{for } x = 0, 1, 2, \dots, n \\ 0, & \text{otherwise} \end{cases}$$
$$\binom{n}{x} = \frac{n!}{x! (n-x)!}$$

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5:53

And, the distribution it should be like this you know  $n C x p$  to the power  $x$  1 minus  $p$  to the power  $n$  minus  $x$ , this is what the functional for either the particular variable we will follow that functions or not you see this is the kind of you know examples. Ultimately all kind of you know distribution, we have a parameters and the kind of the random variable. So, we just specify ultimately it will give you the indications. Ultimately, we



can use the excel spreadsheet to get all these numbers excel spreadsheet has this kind of you know functions, you can easily get all these things, just you know generate the kind of you know requirement. And, then ask this software to or you know spreadsheet to report the particular you know items ultimately we get it. So, this is actually one needs job, but ultimately you need actually understanding.

(Refer Slide Time: 32:25)

### Discrete Probability Distributions

Example: Computing Binomial Probabilities

- Suppose 10 individuals receive the telemarketing promotion.
- Each individual has a 0.2 probability of making a purchase.
- Find the probability that exactly 3 of the 10 individuals make a purchase.

$$f(3) = \binom{10}{3} (0.2)^3 (0.8)^{10-3}$$

$$= 10! / (3! 7!) (0.008) (0.2097152)$$

$$= 120 (0.008) (0.2097152) = 0.20133$$

IIT KHARAGPUR NPTEL ONLINE CERTIFICATION COURSES 5-84

So, this is one kind of you know classic example.

(Refer Slide Time: 32:27)

### Discrete Probability Distributions

Example: Using Excel's Binomial Distribution Function

True:  $F(x)$   
False:  $f(x)$

$\text{BINOM.DIST}(\text{number\_s}, \text{trials}, \text{probability\_s}, \text{cumulative})$

$P(x = 3) = 0.20133$   
 $= f(3)$   
 $= \text{BINOM.DIST}(3, 10, 0.2, \text{true})$

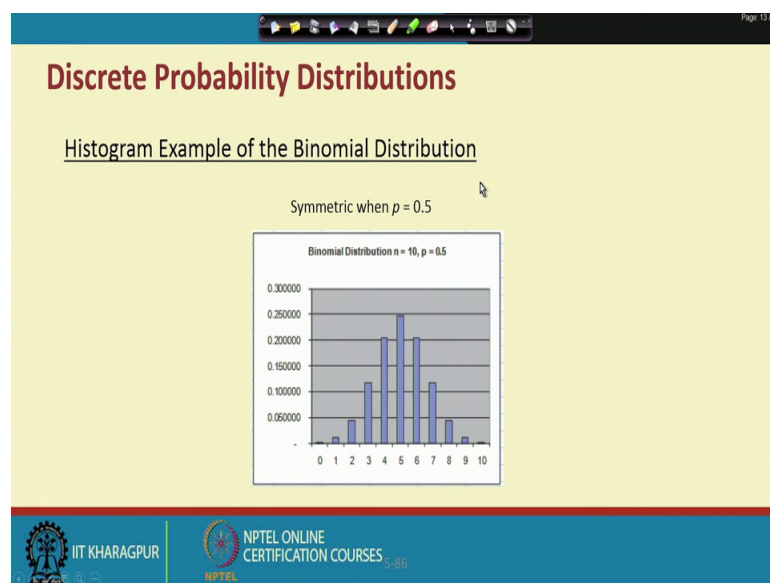
$P(x \leq 3) = 0.87913$   
 $= F(3)$   
 $= \text{BINOM.DIST}(3, 10, 0.2, \text{false})$

x	f(x)	F(x)
0	0.10737	0.10737
1	0.26844	0.37581
2	0.30199	0.67780
3	0.20133	0.87913
4	0.08808	0.96721
5	0.02642	0.99363
6	0.00551	0.99914
7	0.00079	0.99992
8	0.00007	1.00000
9	0.00000	1.00000
10	0.00000	1.00000

IIT KHARAGPUR NPTEL ONLINE CERTIFICATION COURSES 5-85

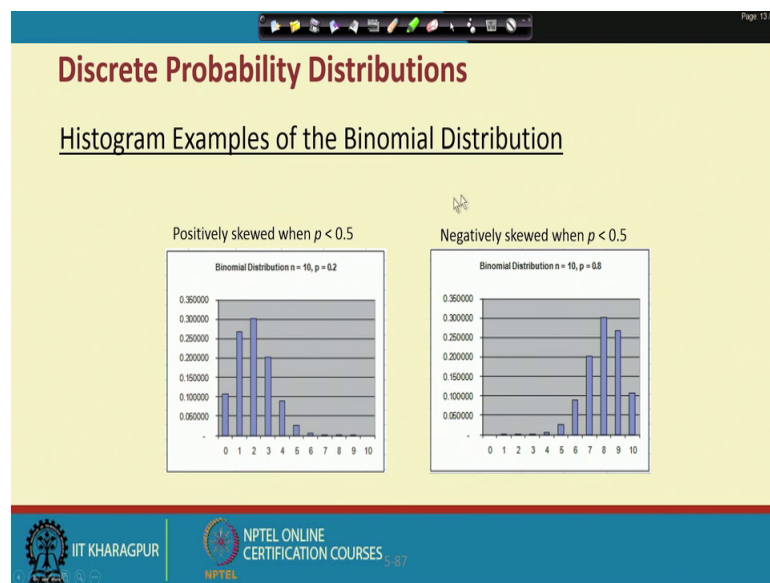
Similarly, we have a different kind of so, this is how the procedure through which actually you work in the excel sheet. So, for a binomial distribution the following command you have to give only, then ultimately for a particular you know random variable  $x$  equal to 3. And you are actually the requirement  $n$  and  $p$  you have to specify then ultimately, you will get actually this is  $p$  requirement, and this is the  $n$  requirement, and this is the trials, then ultimately you will get to know what is the probability value? This is ultimately this will be coming under you know, same 0 to 1, but the functional forms are you know different. So, this is how the typical examples.

(Refer Slide Time: 33:10)



And likewise so, we have similar kind of you know distribution you can check after plotting.

(Refer Slide Time: 33:11)



(Refer Slide Time: 33:17)

Discrete Probability Distributions

Poisson Distribution

- Models the number of occurrences in some unit of measure (often time or distance).
- There is no limit on the number of occurrences.
- The average number of occurrence per unit is a constant denoted as  $\lambda$ .

$$f(x) = \begin{cases} \frac{e^{-\lambda} \lambda^x}{x!}, & \text{for } x = 0, 1, 2, \dots \\ 0, & \text{otherwise} \end{cases}$$

IIT KHARAGPUR NPTEL ONLINE CERTIFICATION COURSES 5:88

And, this is another kind of you know discrete probability distributions, where you know we have actually a functional from  $e$  to the power minus  $\lambda$   $\lambda$  to the power  $x$  by  $x$  this exponential functions. And, ultimately again some of the you know engineering econometrics models; we need these distributions to solve some of the engineering problems. Ultimately same so, you need parameters value and the random variable specification and again ask this software to report these variables.

(Refer Slide Time: 33:49)

## Discrete Probability Distributions

Example: Computing Poisson Probabilities

- Suppose the average number of customers arriving at a Subway restaurant during lunch hour is  $\lambda = 12$  per hour.
- The probability that exactly  $x$  customers arrive during the hour is given by the Poisson distribution.
- Find the probability that exactly 5 arrive during lunch hour:

$$f(5) = e^{-12}(12^5)/5!$$
$$= (0.000006144)(248,832)/120$$
$$= 0.1274$$

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5.89

So, these are the, you know instances you can use actually a poisson and distributions.

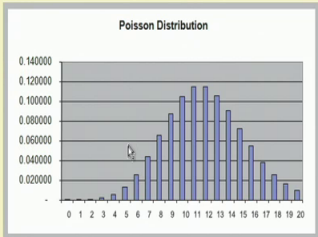
(Refer Slide Time: 33:55)

## Discrete Probability Distributions

Example: Using Excel's Poisson Distribution Function

POISSON.DIST( $x$ , mean, cumulative)

True:  $F(x)$   
False:  $f(x)$



x	f(x)	F(x)
0	0.00001	0.00001
1	0.00007	0.00008
2	0.00044	0.00052
3	0.00177	0.00229
4	0.00531	0.00760
5	0.01274	0.02034
6	0.02488	0.04522
7	0.04368	0.08890
8	0.06552	0.15442
9	0.08736	0.24178
10	0.10484	0.34662
11	0.11437	0.46100
12	0.11437	0.57537
13	0.10557	0.68094
14	0.09049	0.77143
15	0.07239	0.84382
16	0.05429	0.89811
17	0.03832	0.93643
18	0.02555	0.96200
19	0.01614	0.97814
20	0.00968	0.98782

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5.90

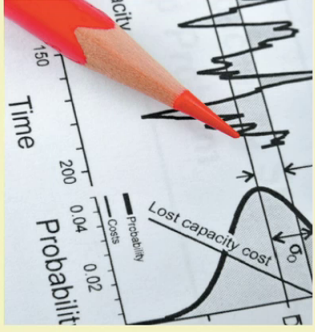
And, ultimately he say same you know the excel function will be like this and you get to know whether it is actually normally distributed or not normally distributed.


(Refer Slide Time: 34:04)


## Discrete Probability Distributions

Analytics in Practice: Using the Poisson Distribution for Modeling Bids on Priceline

- ▶ Pricing strategies for Kimpton hotels on Priceline is modeled using a Poisson distribution.
- ▶ The number of bids placed per day 3 days before arrival is  $f(x) = e^{-6.3}(6.3^x)/x!$ .
- ▶ Using the model increased sales 11% in one year.







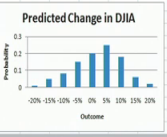
NPTEL ONLINE  
CERTIFICATION COURSES 5:91

(Refer Slide Time: 34:07)

## Continuous Probability Distributions


	A	B	C	D	E	F	G	H
1	Change in DJIA Subjective Probability							
2	-20%	0.01						
3	-15%	0.05						
4	-10%	0.08						
5	-5%	0.15						
6	0%	0.2						
7	5%	0.25						
8	10%	0.18						
9	15%	0.06						
10	20%	0.02						

Change in DJIA using 5% increments





	A	B	C	D	E	F	G	H
11	Change in DJIA Subjective Probability							
12	-20%	0.0050						
13	-17.5%	0.0100						
14	-15%	0.0200						
15	-12.5%	0.0314						
16	-10%	0.0400						
17	-7.5%	0.0504						
18	-5%	0.0750						
19	-2.5%	0.0884						
20	0%	0.1000						
21	2.5%	0.1114						
22	5%	0.1250						
23	7.5%	0.1084						
24	10%	0.0900						
25	12.5%	0.0750						
26	15%	0.0600						
27	17.5%	0.0400						
28	20%	0.0200						

Change in DJIA using 2.5% increments



Approaching a smooth curve





NPTEL ONLINE  
CERTIFICATION COURSES 5:92

And likewise so, these are all various cases and the other sides the continuous probability distribution that is called as you know normal distribution basically.

(Refer Slide Time: 34:16)

**Continuous Probability Distributions**

Probability density function

- ▶ A curve described by a mathematical function that characterizes a continuous random variable

Properties of a probability density function

- ▶  $f(x) \geq 0$  for all values of  $x$
- ▶ Total area under the density function equals 1.
- ▶  $P(X = x) = 0$
- ▶ Probabilities are only defined over an interval.
- ▶  $P(a \leq X \leq b)$  is the area under the density function between  $a$  and  $b$ .

$$P(a \leq X \leq b) = P(X \leq b) - P(X \leq a) = F(b) - F(a)$$

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5.93

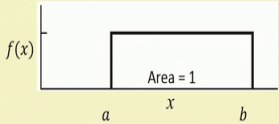
So, under normal you know means continuous case, the probability density you know here you need a kind of you know confidence intervals. Through which you can actually a project the particular you know outcomes.

(Refer Slide Time: 34:31)

**Continuous Probability Distributions**

Uniform Distribution

- All outcomes between a minimum ( $a$ ) and a maximum ( $b$ ) are equally likely.

$$f(x) = \begin{cases} \frac{1}{b-a}, & \text{for } a \leq x \leq b \\ 0, & \text{otherwise} \end{cases}$$
$$F(x) = \begin{cases} 0, & \text{if } x < a \\ \frac{x-a}{b-a}, & \text{if } a \leq x \leq b \\ 1, & \text{if } b < x \end{cases}$$


IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5.94

So, starting with you know uniform distributions.

(Refer Slide Time: 34:34)

**Continuous Probability Distributions**

Uniform Distribution

- Expected Value =  $EV[X] = \frac{(a + b)}{2}$
- Variance =  $Var[X] = \frac{(b - a)^2}{12}$

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5.95

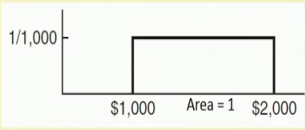
Then assembly mean reporting, the variance reporting.

(Refer Slide Time: 34:38)

**Continuous Probability Distributions**

Example: Computing Uniform Probabilities

- ▶ Sales revenue for a product varies uniformly each week between \$1000 and \$2000.
- ▶  $f(x) = 1/(2000-1000)$   
= 1/1000



IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5.96

Then you know these are the classic examples of you know uniform distributions.




(Refer Slide Time: 34:42)

**Continuous Probability Distributions**

Example: (continued)  
Computing Uniform Probabilities

- Find the probability sales revenue will be less than \$1,300.
- $P(X < 1300) = (1300 - 1000)(1/1000) = 0.30$



IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5.97

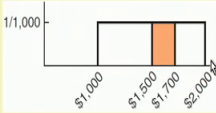
And, ok.

(Refer Slide Time: 34:43)

**Continuous Probability Distributions**

Example(continued): Uniform Probabilities

- Find the probability that revenue will be between \$1,500 and \$1,700.



- $$\begin{aligned} P(1500 \leq X \leq 1700) &= P(X \leq 1700) - P(X \leq 1500) \\ &= F(1700) - F(1500) \\ &= 300/1000 - 500/1000 \\ &= 0.20 \end{aligned}$$

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES 5.98

So I am just skipping because ultimately these are all not our requirement.



(Refer Slide Time: 34:46)

**Continuous Probability Distributions**

Normal Distribution

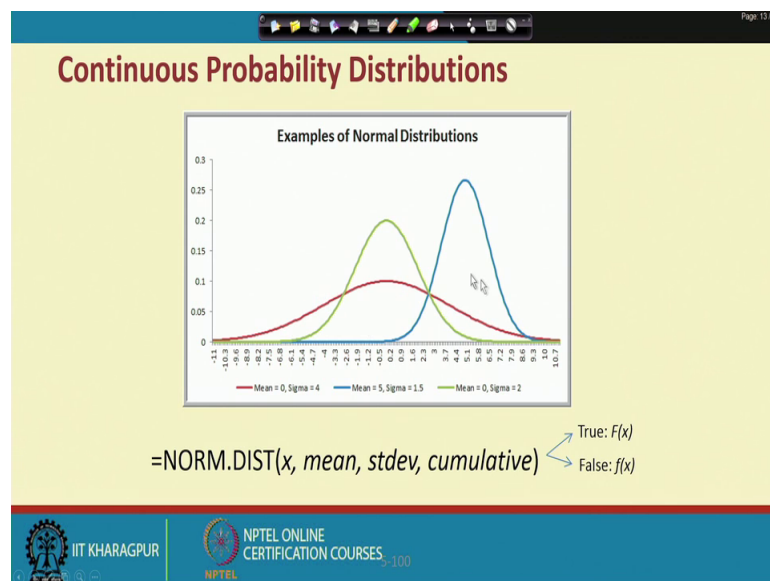
- $f(x)$  is a bell-shaped curve
- Characterized by 2 parameters
  - $\mu$  (mean)
  - $\sigma^2$  (variance)
- Properties
  1. Symmetric
  2. Mean = Median = Mode
  3. Unbounded
  4. Empirical rules apply

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES

Just I will get to you know and say I will like to highlight you know these are the various distributions are available in the probability theory, which is actually one of the requirements for our, you know engineering econometrics a modeling. And in the normal distribution this is one of the highest you know distributions or you know distribution at the highest levels where you know, most of the engineering econometrics problems can be solved through this particular you know distributions.

And, yes the parameters are you know  $\mu$  and standard deviations and by default  $x$  is the random variables. And, the variable we will give you snap shot very simple you know kind of you know conclusion symmetric or non-symmetric. And, where you know I mean few properties are you know mean median mode will be coincide and it is a completely unbounded against and empirical rules you know easily can be applied, how you have to work out with this particular you know so; that means, when you know in infinite kind of you know dense. So, it is very difficult to you know work out. So, in that context normal distribution will be very handy to solve these particular problems

(Refer Slide Time: 35:59)



Like, when this is the typical structure of you know normal distributions and like you know excel spreadsheet you can just put normal distribution. And give the random variable indications and the parameter wells mean standard deviation that is the one of the means, these 2 are you know major indicators of this in our normal distribution then ultimately you get to know the probability value.

(Refer Slide Time: 36:19)

Continuous Probability Distributions

Example: Using NORM.DIST to Compute Normal Probabilities

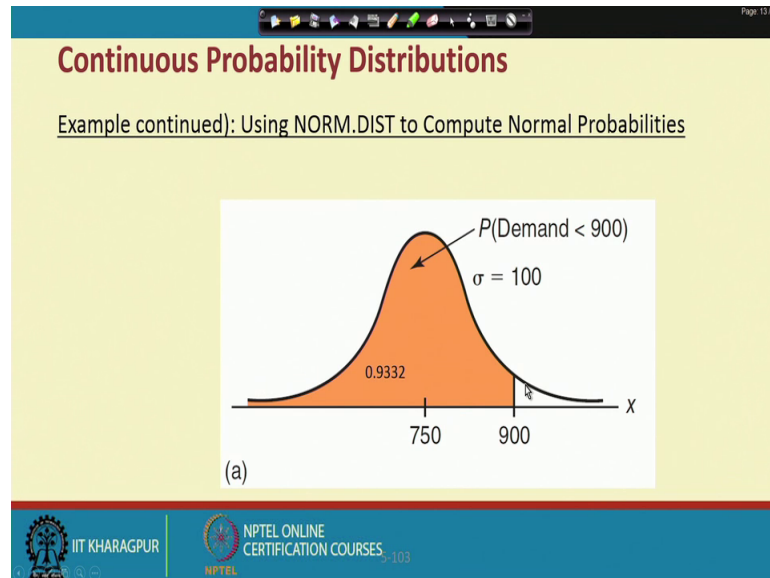
- ▶ The distribution for customer demand (units per month) is normal with:  
mean = 750  
stdev. = 100
- ▶ Find the probability that demand will be:
  - a) at most 900 units/month
  - b) exceed 700 units/month
  - c) be between 700 and 900 units/month

IIT KHARAGPUR NPTEL ONLINE CERTIFICATION COURSES 101

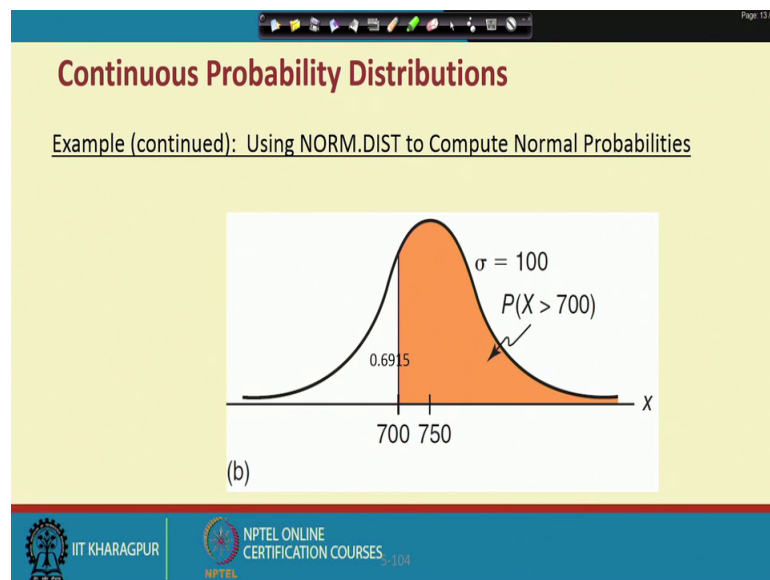
So, again whether it is a binomial distribution or poisson distribution or by you know a normal distributions the range of the probability will be 0 to 1. And the total property

will be ones depending upon the you know sample space and the kind of you know event specifications.

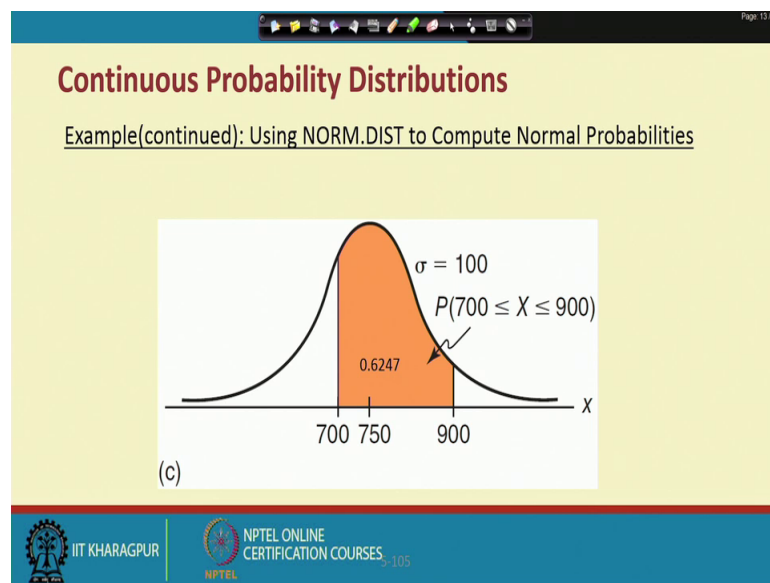
(Refer Slide Time: 36:40)



(Refer Slide Time: 36:40)

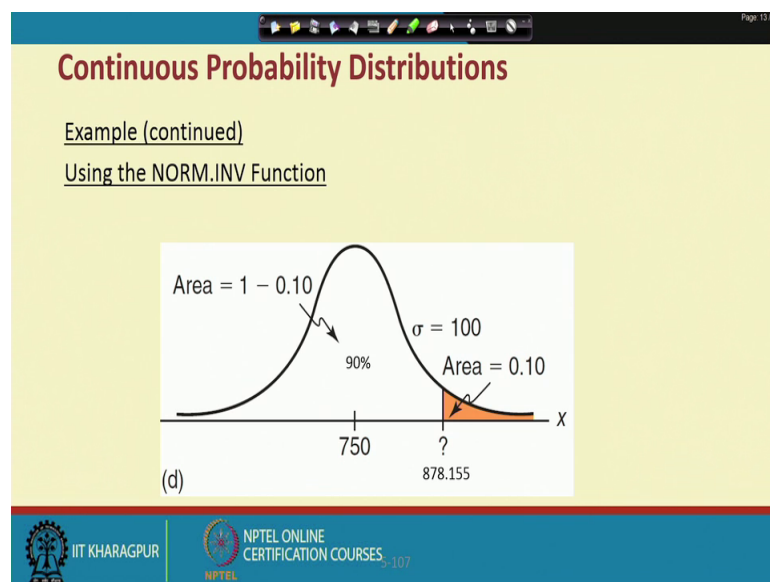


(Refer Slide Time: 36:42)

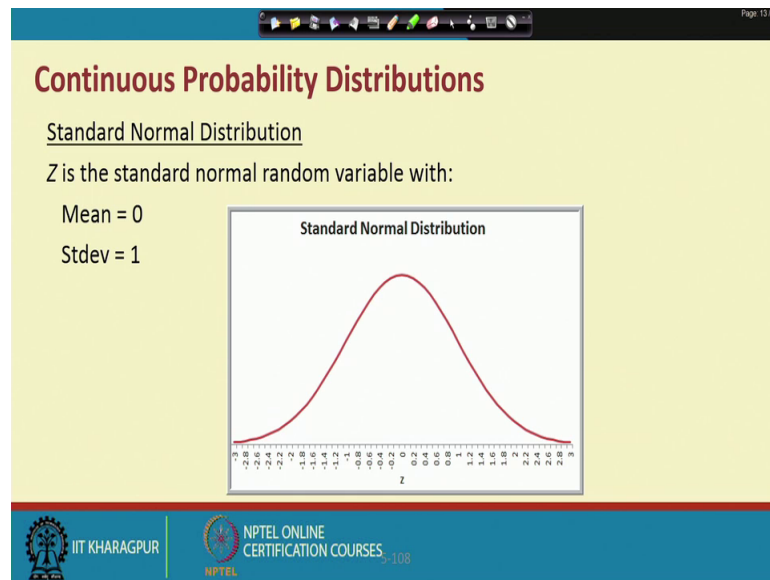


So, likewise there are you know different instances all together depending upon you know left skewed, right skewed, left attack, right attack kind of you know things.

(Refer Slide Time: 36:50)

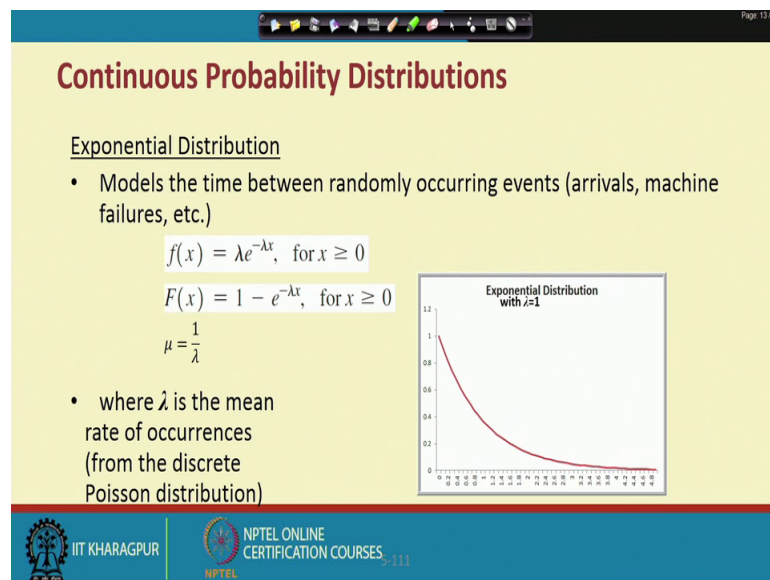


(Refer Slide Time: 36:51)



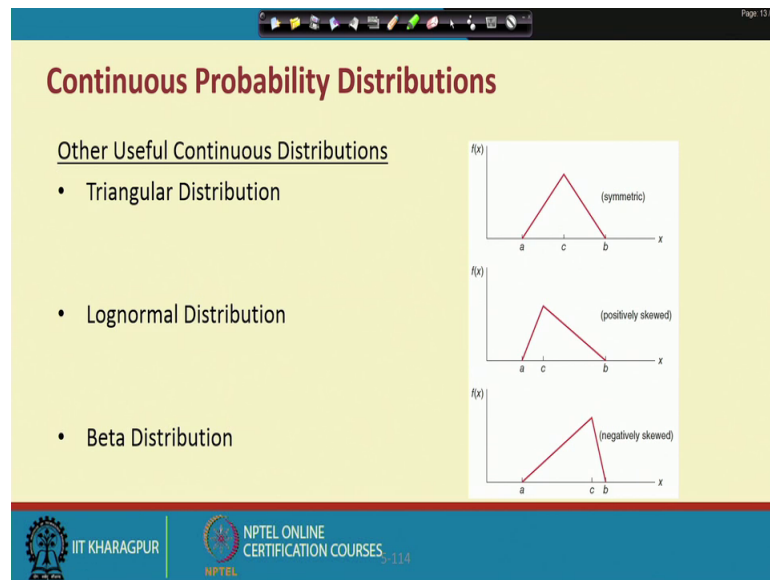
And so these are all ultimately means, this is another kind of you know exponential distributions.

(Refer Slide Time: 36:56)



And again it is one of the basic requirements of you know engineering econometric modelling which will be a use in the later stage.

(Refer Slide Time: 37:13)



And so; that means, technically a so, again these are all additional you know different distributions, log normal distribution, beta distribution, triangular distribution. Ultimately the point is that you know before you start for a you know solving any engineering problems through engineering econometrics. So, again this what I you know every times you know representing that these are all basic infrastructure actually for this you know engineering econometrics investigation.

So, you must be acquainted with the probability and various distributions associated with the probability, with you know a simple structure and complex structures. And depending upon your you know engineering problems, you have to check whether the particular you know problem is connected with your particular distributions. And then you have to develop the models and look for the solution as per the particular you know distribution only.

So, with this we will stop here. And, in the next class we will discuss the details about the sampling and sampling distributions have a nice time and.

Thank you very much.