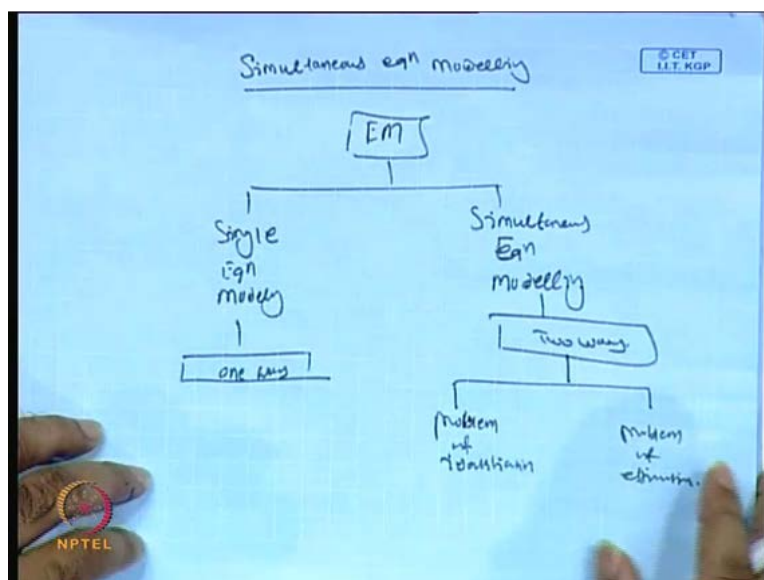


Econometric Modelling
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Module No. # 01
Lecture No. # 33
Simultaneous Equation Modelling (Contd.)

Good evening, this is doctor Pradhan here, welcome to NPTEL project on econometric modelling. So, today we will continue the simultaneous equation modelling. So, in the last lectures, we have highlighted very briefly the structure of simultaneous equation modelling and how it is different to you can say single equation modelling.

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So, in the econometric modelling altogether, econometric modelling can be classified can be classified into two groups, one is called as a single equation modelling single equation modelling and simultaneous equation modelling.

So, So, altogether the entire econometric modelling problem can be divided into two parts, single equation modelling and structural equation modelling, so now, now in the case of single equation modelling, there is a single equation, so our objective is to estimate the parameters, and you fit a models which can be considered as the best and can be used for forecasting and policy use, but in the other sides you can say simultaneous equation systems, there are several equations and we need to identify each

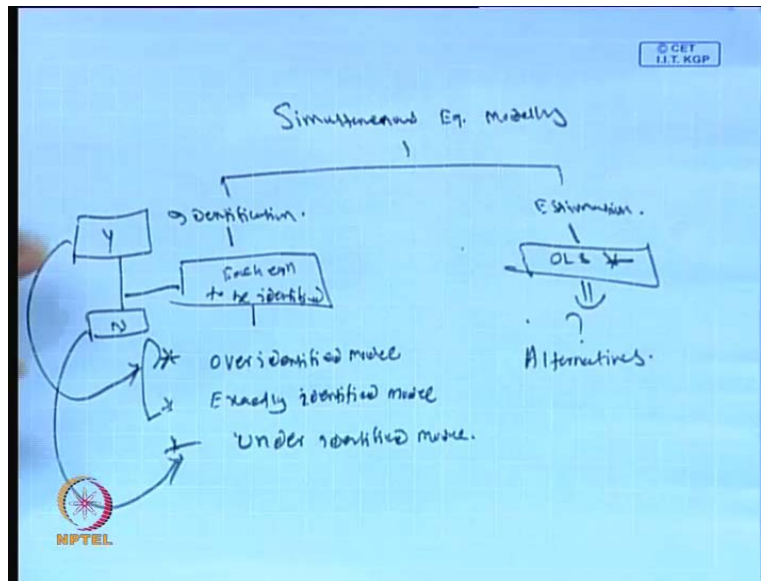
equation and in the same time, we need to estimate the parameters, and that model can be best models and **can be considered** can be used for prediction and policy use.

So, in the single equation modelling, there is a one the objective is **or** the structure is one way causality that means, that is the investigation is one way causality, so means it is only independent variables, which can influence on y, so that means there is only one dependent variable with several or single multiple independent variables. In the other side's there are several dependent variables and several independent variables these are called as a means dependent variable cluster is called as a endogenous variable and independent variable cluster is called as a exogenous variables **alright**.

So, now in these simultaneous equation systems, so there is a question of two ways causality, that means, there is a possibility of bidirectional causality. So, while in the case of single equation modelling there is a one way causality. So, when we handle the two way causality or you know simultaneous equation system, then obviously it is little bit complex because, various equations and various variables are go together, so means simultaneously. So, as a result, so there is a vary means it is a very much complex problem, so it **is this complexity** is a complexity issue is very high in the case of simultaneous equation modelling.

So, one such complexity means basically the entire complex means the complexity is divided into two parts, **for instance in the means** for instance single equation modelling this is one way causality. So, this is a one way causality and this is **two way causality** two way causality. So, once **once** you handle this particular issue say then obviously, there are two standard problems you have to face. So, one is called as a **problem of problem of identifications** problem of identifications and **problem of problem of estimations** problem of estimations, let me highlight once again here.

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So, now in the simultaneous simultaneous equation modelling in the simultaneous simultaneous equation modelling in the simultaneous equation modelling, so the problem is identifications and this is called as estimations so, estimations. So, that means here, we cannot directly the application of OLS is a question mark. And so far as identification is concerned, so then you know before you going to examine the identification or estimation, so there is a special issue here means, in the simultaneous equation modelling that means the model should be very consistent one.

So, what is mean by consistent one? So, that means, each equation can be identify, each equations means not can be should be identify, each parameters parameters of each equation should be estimated, and the system should be consistent very very much consistent, that means number of equations should be equal to number of parameters, that means, so the system simultaneous equation modelling will be in unique form if the model is a statistically in unique form each equation can be identify, and parameters of the means system should be estimated, and number of equation should be exactly equal to number of parameter, then the system will be very consistent and systematic, if it is other way around then obviously, it will give you lots of baseness or you can say the system will itself will give you inconsistent setup.

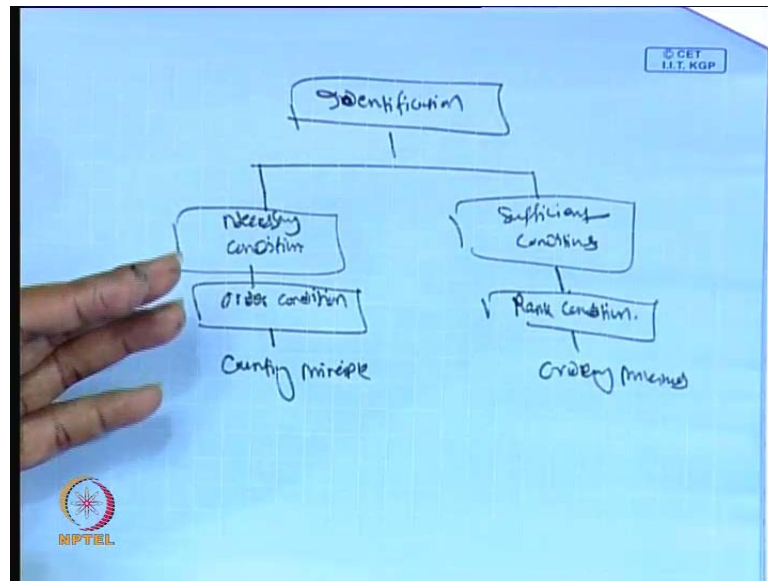
So, now so if that is the case after all these issues, so when we will go for generally whatever discussion we have tilled now, so we start with first OLS techniques then

ultimately, we have to redesign restructure till you get the better fitted model. So, now here the starting point of application of OLS gives you the biased result, so that means it will give you serious estimation problem. So, the moment you will go for serious estimation problem, so you need to have alternative, so that means we look for **alternatives** alternatives, so these objectives, so now in the case of identification so here, so each equation to be identify in that particular systems.

So, that means here is each equation **each equation to be identified to be identify each equation** to be identify in the systems. So, now, how do you identify, so when will you go for identification, then obviously there are three **three** different setup you will find, one is called as a **over identify**, over identified model **over identified model** then, **exactly identified models exactly identified models** exactly identified model then, you know this is called as a **under identified models under identified models under identified** under identified model. So, this is **this is;** that means, when there is question of identification, so you have two answers yes and no. So, if is no then it will come to here, if it is yes then it will come to here.

So, that means, so if equation is identified then obviously, question is a whether it is exactly identified or whether it is over identified. So, **when it is** when a **question is** answer is no then obviously it is question of under identified model. So, now how will you go for this identification? So, we start with first identification then will we move into the estimation sides.

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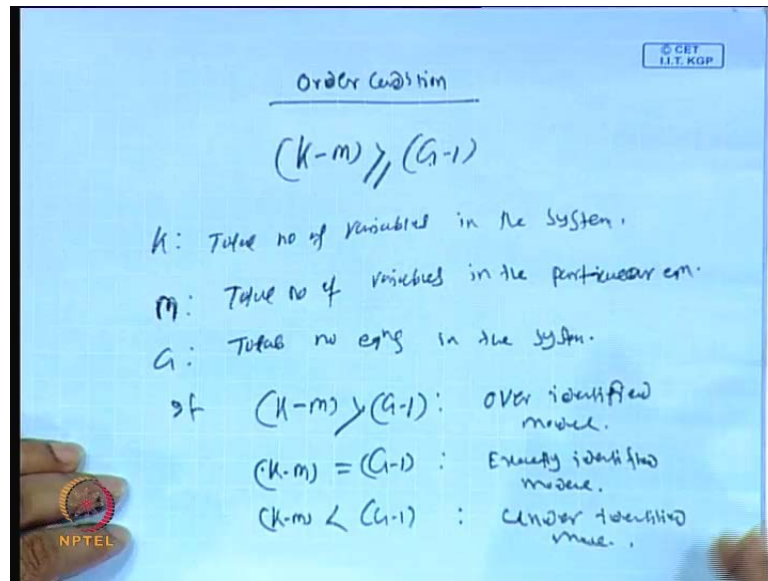
So, what is this identification problem altogether? So, now so far as identification problem is concerned, under simultaneous equation modelling identification problem. So, the identification problem basically we use two different criteria to identify the particular equation in a particular system means simultaneous equation system.

So, what is this identification? So, the first $(())$ are called as a necessary conditions. So, the rule is to examine necessary conditions. So, there are two condition necessary conditions and sufficient conditions. So, there are two conditions to identify the model in a identify equation in a particular system. So, necessary condition and sufficient condition this is called as a order conditions, this is called as a order condition this is called as a this is called as a this is called as a rank conditions this is called as a rank conditions order condition and this is called as a rank condition, this is sometimes called as a counting principles counting principles and this is sometimes called as a ordering principles ordering principles ordering principles.

So, now we like to highlight what what the setup is of you know, this other condition and you know, these rank conditions. So, order condition means what is necessary condition that means, it is the counting rule is the variable included in a particular equation, the difference between the total number of variable in the systems, and the total number of variables in a particular equation should be greater than equal to number of equations minus 1. So, this is the structure of the necessary conditions.

And in the case of rank condition, we like to construct a matrix which is called as a reduce form of a matrix, and that matrix should have a significant value that means, the determinant of that reduce form of matrix should be have a positive value. So, to examine that situation we will highlight **a** in a brief way here. So, let us start with this you know counting principle first, so what is this order condition altogether.

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So, now order condition is that, what is order condition is perfectly. So, there is a since it is a counting rule or you can say necessary condition, so the principle is here. So, the apply principle is K minus M should be greater than equal to G minus 1.

So, K is here is **total number of total number of variables** total number of variables in the systems **in the systems**. You see here, so we have a system just like you know we have last class we have discuss the cancel model system, and there is a system simultaneous equation modelling having two equations. So, that means it is a system where there should be **there should be** at least two equation together otherwise, it cannot be called as a simultaneous equation modelling. In that systems there should not be one equation there should be multiple equations, so the starting of the game on the simultaneous equation modelling is that there should be at least two equations in a particular system. So, if it is two equation then it is called as a simultaneous equation modelling, if it is more than two then obviously, we will continue that, if it is only one equation in the system then it is called as a single just single equation modelling, so that means, in this

simultaneous equation modelling, you must have multiple equations and your objective is to identify each equation and to estimate the parameters of each equation. So, that is the two major problems of simultaneous equation modelling.

So, now so far as an ordering orders condition is concerned. So, we like to know, what is the total means, total number of variable included in the system, including whatever number whatever equations are there? So, total number of variable in the systems minus number of variables included in a particular equation should be greater than to greater than equal to number of equations minus 1. So, now so I will I will mathematically I will write $K - M$ should be greater than equal to $G - 1$.

So, K represents total number of variables in the systems, and M represents total number of total number of total number of variables total number of variables in the particular equation, in the particular equation which we have identified for identification in a in a particular equation in a particular equation, so this is a called as M principles. So, now G equal to total number of total number of equations in the systems in the system total number of equation in the in the systems, so you see here you see here So, if $K - M$ greater than to $G - 1$ then it is called as a over identified models is called as a over identified identified over identified models.

Then if, $K - M$ is exactly equal to $G - 1$, then it is called as exactly identified models exactly identified models exactly identified models exactly identified models. So, similarly if $K - M$ less than to $G - 1$ it is called as an it is called as a under identified models. it is called as a under identified models under identified models. Let us take an example here, so means basically here the agenda is to means to examine the identification of that particular simultaneous equation modelling.

So, that means first thing is the model must be in a unique form means statistically in unique form each equation can be identified then, parameter of the equation should be estimated, and the means the system should have a number of equation should be number equal to parameters then, we we will we will say that this is the unique systems. So, now once it is unique system, so the starting point of the system is that, we have to identify each equation whether it is and over identified, exactly identified, or you can say under identified. And to know the particular structure, so we usually apply this strict rule is

called as a counting rule. So, that is necessary condition or you can say you know order conditions.

So, now in the order condition the formula is **K minus** K minus M should be greater than equal to G minus 1. K is total number of variables in the systems, M is total number of variables in a particular equation, which we like to identify, and G is total number of equations in the systems, so that means, the system says that the identification rule says that, the difference between total number of variables in the system, and total number of variables that the equation to be identified should be greater than equal to number of equations minus 1. So, if it is so then it is this consistent, otherwise this system will be inconsistent.

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$$Y_1 = 3Y_2 - 2X_1 + X_2 + U_1$$

$$Y_2 = Y_3 + X_3 + U_2$$

$$Y_3 = Y_1 - Y_2 - 2X_3 + U_3$$

$\{Y_1, Y_2, Y_3\}$: Endogenous variables.
 $\{X_1, X_2, X_3\}$: Exogenous variables
 U_1, U_2, U_3 : Stochastic disturbance term

$K = 6, \quad G = 3$

Eq 1: $m = 4 \quad (K - m) \gg (G - 1):$
 $(6 - 4) \gg (3 - 1) \Rightarrow 2 = 2 : \text{Eq 1.}$

Let us see here, so now so we will take an example here let us start with a simple problem here, so **y 1** Y_1 equal to 3 Y_2 **3 y 2** minus 2 X_1 plus X_2 plus U_1 . So, then **then** Y_2 equal to Y_3 plus X_3 plus U_2 then, Y_3 equal to Y_1 minus Y_2 minus 2 X_3 plus U_3 . So, now what **what** are the variables here, so now the variables are here, so Y_1, Y_2, Y_3 are one cluster and X_1, X_2, X_3 are another cluster. So, now Y_1, Y_2, Y_3 are called as a means they are called as a endogenous variables and X_1 and X_2, X_3 are called as a exogenous variables **alright.**

So that means, here Y_1, Y_2, Y_3 are called as a endogenous variables, **it is called as an endogenous variables,** then X_1, X_2, X_3 are called as a **exogenous variables** exogenous

variables, and U_1, U_2, U_3 are stochastic disturbance term **stochastic disturbance term** or error term disturbance term. So, 3, 2, 1 then 1, 1, 1, 1 these are coefficients. So, which we will discuss details in the case of rank condition because, for the **for the** time being in the case of order condition, the coefficient of this particular variables are not essential however, the essential is just you have to count that is **that is** why it is known as a counting rule.

So, you have to count the total number of variables in the systems, and total number of **total number of** variables for equations then, number of equation in the system. So, you need three items only. So, total number of variables in the system, total number of variables for each equation and total number of equations in the systems, once we have these three information we can able to detect or you can able to identify, each and every equation in the simultaneous equation system alright. So, now for first equation let us see we like to target first equation.

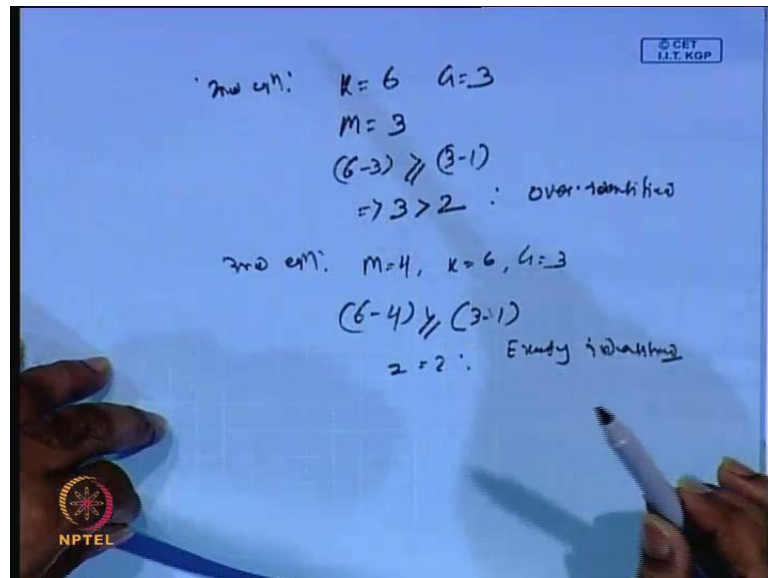
So, now obviously in this particular setup, so your K equal to 6, so K equal to 6 and what is the G here so G is equal to here 3. So, now you see here, so when a simultaneous equation structure is with you then obviously, K your objective is go for identification then obviously, your objective is to identify each equation in the system. So, now when you target one by one equation then obviously, every **every** equation means every identification case, so K and G will be always there. So, that is why for every equation K will be remains same and G will be always remain same only **only** variability factor is that M component.

So, M should very difference upon equation to equation that means, is the M is the potential element, which can which can exactly identify the structure of means exactly identify the nature of the particular equations. So, now in this particular problem, so K equal to 6 and G equal to 3. So, now we will we target for equation 1. So, now for equation 1, so **1 2 3 4** 1 2 3 4 so that means it is M equal to 4 here, so now what is the balance sheet, so balance sheet is K minus M into G greater than into G minus 1, so that is our formula.

So, means here K is 6 here is M equal to 4 here, so greater than equal to G minus 1, 3 minus 1. So, that means, it is 2 equal to 2, so that means, it is exactly identified the system is it is called as an exactly identified. So, this particular equation that means, in

this particular simultaneous equation system, first equation is a exactly identified. So, then we will go for second equation. in the second equation in the second equation let us take this one first in the second equation in the second equations.

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So, for second equation second equation 1 2 3. So, K is already 6 then G already equal to 3 then obviously, this is this is 3. So, second equation M equal to 3 M equal to 3.

So, that means 6 minus 3 6 minus 3 greater than equal to 3 minus 1, so that means, the balance sheet is, this is 3 greater than to 2, so that means this is identification second equation is over identified this second equation is over identified over identified. So, then you come to third equation third equation. So, third equation 1 2 3 4, so this is 4, so that means M equal to 4 here, so K is already 6, G is already 3, so that means 6 minus 4 is greater than equal to 3 minus 1 so that means 2 equal to 2 that is exactly exactly identified exactly identified.

So, that means that means, in this particular system, so we have a there we have three different an equations, and altogether there are 6 6 variables. So, including three endogenous and three exogenous, So, so far as a counting counting identification rule is concerned, so we are not serious how many are endogenous, and how many are exogenous. So, we are very much concerned, what are the total number of variables is in the system, what are the total number of variables in a particular equations, and what are

the number of equations. So, these three conditions are required to justify the other condition for identification of problem.

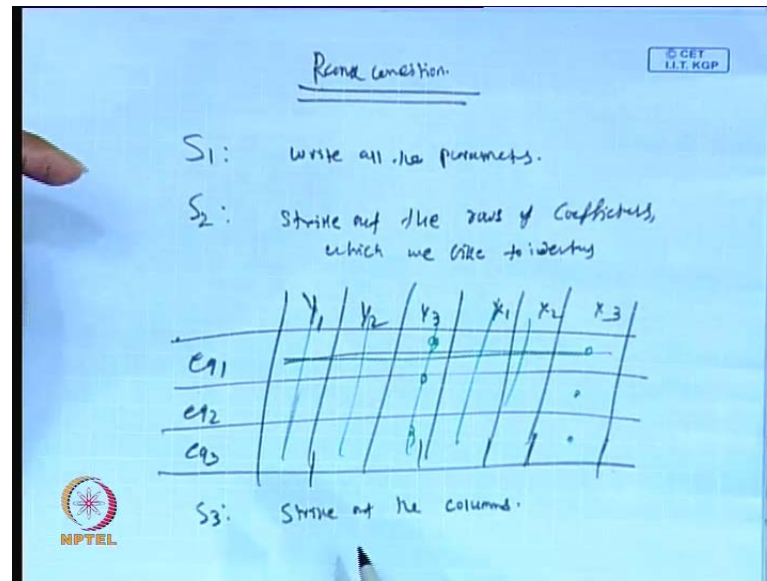
So, in this particular problem, so we have three equations, so consist of six variables that too three endogenous variables, three exogenous variables. So, now by having such informations, obviously there are three equations. So, first equation is identified as exact, second equation is over identified, and third equation is exactly identified. So, now this is not in fact sufficient. So, other condition will give you signal whether the system is a means identified or not, but the it has to be verified through sufficient condition that is rank condition.

So, what is rank condition? Rank condition is a means in a system of G equation any particular equation can be identified, if a and only if possible to construct at least one non zero determinant of order G minus 1, on the coefficients of variable included, excluded **excluded** from the particular equations, but included in other equation models that means, it is a just derived from that counting principles so that means, what is rank condition, in a rank condition it is you know we have to we have to find out the rank which is from the reduced form of the matrix.

So, that means we apply the counting rules, so by the way we have to reduce the entire system into order of G minus 1 coefficients then, you have to apply the matrix the value of the matrix should be be positive, if the value of at least one of the value of the matrix is positive, then obviously the condition is a satisfied. Otherwise, **even if I** even if the order condition is a identifying a particular equation then, obviously we cannot conclude that this model is a exactly identified or over identified.

That means, for any equation to be identified it has to be **it has to be** done through order condition, and also rank condition. If one condition satisfied, and another condition are not satisfied then obviously, this equation cannot be possible that is that will be treated as a under identified.

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So, now let us see what this structure of rank condition? is. So, now rank conditions in the rank conditions rank conditions. So, it is this in a in a system of G equation any particular equation is to be identified if it is possible to construct at least one non zero determinant of order G minus 1 of the coefficient of the variable included from the particular equation, but included from the other equation of the model. So, this is how the definition of rank condition.

So, there are a certain guidelines to apply the rank condition, but that guidelines is very perfect, very accurate, very systematic. So, there is no confusion at all. So, you just follow me. So, what is rank condition here? So, there are altogether there are five steps, step one step one, just you write all the parameters you write all the parameters in a separate tables, write all the parameters in the separate tables including all these you know, including all these error term also including means, take all these error terms in the other side's then you take the all the parameters in that particular sides.

Step two, then strike out strike out the rows of the coefficient rows of the coefficients which we like to which we like to identify which we like to identify, for instance suppose as per the previous problem. So, we have Y 1, Y 2, Y 3, X 1, X 2, X 3 in that case this is equation 1, this is equation 2, this is equation 3 this is equation 3. So, we need to have a prepare a table like this. So, write all these parameters in a you know tabular forms. So,

this is how you have to represent all these things. So, this is how you have to prepare these particular tables.

So, now second **second** step is strike out all the rows of the coefficient which you are you like to identify for instance suppose we like to first we fill up all these gaps, **I will** I will highlight all these things with respect to a particular example. So, once you fill up all these means this **this** particular matrix is the coefficient of all these variables. So, which we have not use in the case of counting rule. So, in the order condition the coefficient of all these variables is very important to test the rank condition for identification problem. So, now there are three equations here equation one equation two equation three.

So, if it is first equation to be identified then you have to close this particular row, you have to close this particular row like this. So, it is better you put here. So, you have to cross this particular row then again in the third step **in the third step in the step step three** strike out all the columns **strike out all the strike out the column columns columns strike out the columns** where the variable is already there in the system. For instance, for first equations we have Y 1, Y 2, X 1 and X 2 so that means, it has to be remembered all this sorry Y 1, Y 2, X 1 and X 2. So, this **this** will be remain there, and **this will be remain there** this will be remain there.

So, now in the step four, so you prepare a reduce form of the matrix this is structural matrix. So, you prepare a reduce form of a matrix after you know deleting all the zeroes, and deleting all this columns. So, whatever left out items, so we have we we will call it as a reduce form of the matrix. So, then once you have reduce form of the matrix, so you design a you design a matrix or determinant whose values should means you have to design a square matrix of order say 1 1 or 2 2 or 3 3 like this. So, then if the value of a if an means there may be several such square matrix can be obtained.

For instance it may be 1 it may be obviously, it cannot be one. So, it may be 2 2 **2** 3 3 4 4 like this way. So, the value of that matrix should be at least one of such value should be you know positive, then you know the system order sorry rank condition can be satisfied. Otherwise, even a order condition satisfy this equation cannot be identified. So, let me take an example here, this same example we will highlight here. So, you see here. So, what is that example?

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$Y_1 = 3Y_2 - 2X_1 + X_2 + U_1$
 $Y_2 = Y_3 + X_3 + U_2$
 $Y_3 = Y_1 - Y_2 - 2X_3 + U_3$

$-Y_1 + 3Y_2 - 2X_1 + X_2 + U_1 = 0$
 $-Y_2 + Y_3 + X_3 + U_2 = 0$
 $-Y_3 + Y_1 - Y_2 - 2X_3 + U_3 = 0$

Step 1

	Y_1	Y_2	Y_3	X_1	X_2	X_3
eqn ¹	1	3	0	-2	1	0
eqn ²	0	-1	1	0	0	1
eqn ³	1	-1	-1	0	0	-2

$\Delta = 1$
 $\Delta = -2 - 1 \neq 0$

So, the example is Y_1 equal to here y_1 equal to here $3Y_2$ minus $2X_1$ plus X_2 plus U_1 . Then, similarly Y_2 equal to Y_3 plus X_3 plus U_2 . plus x_3 plus u_2 So, then Y_3 equal to Y_1 minus Y_2 minus $2X_3$ plus U_3 . So, what you have to do to take all these items in the left side. So, then you prepare the matrix here. So, now this matrix is **step one process** step 1 process, so $Y_1, Y_2, Y_3, X_1, X_2, X_3$. So, Y_1 so you bring all this things here, so that means if this, if you will bring all this things then means minus Y_1 plus $3Y_2$ minus **minus** $2X_1$ **minus** $1X_1$ plus X_2 plus U_1 equal to 0. So, this is first equation this is we are just simplifying this equation.

So, then minus Y_2 plus Y_3 plus X_3 plus U_2 equal to 0. Then minus Y_3 **minus** plus Y_1 minus Y_2 minus $2X_3$ plus U_3 is equal to 0. So, this is how the transformation. So, now it is equation 1, **equation one this is** equation 2, then equation **equation 3 equation equation three** alright. So, now you have to just enter the **value** value. So, **Y** for first equations this is Y_1 is minus 1, then Y_2 equal to 3, then Y_3 0, then X_1 minus 2, then X_2 1, then **then then** this is minus 2, **minus 2** 1, then it is 0 **then 0** alright. Then come to second equation, **in the second equation** in the second equation Y_1 **y 1** is not available. So, 0 then this is minus 1, this is 1, then 0, 0, this is 1.

Then for equation 3, so Y_1 is there so 1, then Y_2 is there minus 1, Y_3 is there minus 1, then X_1 0, X_2 0, then X_3 is minus 2. That means, if some variables are there then obviously there will be some coefficient, if variable is not there then you assume that **that**

coefficient is 0 coefficient is zero. So, as a result you prepare this you prepare this you know structural matrix. So, this is called as a structural matrix this is called as a structural matrix of this particular simultaneous equation modelling.

So, now suppose our target is to identify equation 1, our target is to test whether equation 1 is means, we have already got that equation 1 is exactly identified by the counting rule. So, we have to test through rank condition. So, now if will you apply rank condition then by this principle of rank condition. So, equation 1 row of each element in that case has to be you can say removed. So, that means this particular line has to be removed alright. So, now then all columns where the coefficients are not zero has to be a removed.

So, now Y 1 has to be removed, then Y 2 Y 2 has to be removed then then then Y 3 is Y 3 is not there, so it will be remain there, X 1 will be removed, then X 2 will be removed. So, now we prepare the left out matrix, so that is reduced form of the matrix. So, the reduced form of matrix is nothing but Y 3, X 3 then equation 2 equation 3. So, that is a reduced form of the matrix. So, Y 3 so it is 1, this is minus 1, this is 1 this is one minute, x 1 x 2 sorry this is Y 1, Y 2, X 1 and X 2 alright this is first equation.

Ah. So, 1 minus 1, minus 1 1 minus 2 1 minus 2 this is. So, now this is the matrix of square matrix, so we can prepare this matrix. So, now minus 2 into this one minus 1 is equal to say you know minus 2 this is minus 2 minus 2 minus 1 which is not positive which is not coming positive. So, it is not equal to 0. So, it is not equal to 0 it is not equal to 0. So, it is not equal to 0. So, that means it is the this condition is satisfy, if the value of the matrix is equal to 0 then; obviously, the coefficient cannot be a means that equation cannot be identified.

So, similarly let us see you know this this structural reduce form matrix is very simple because, we are getting only a single matrix one because here you see if I will go one by one then delta 1 is you can say only 1, so delta 2 is this is delta two is delta 2 is minus 2 minus 1 this is order of 2 into 2, this is 1 into 1. So, now we have determinant means it should be you know sum square form. So, that has to be not equal to 0 not equal to zero if it is equal to zero, then that equation has to be declared as a under identified.

So, similarly we can go for suppose our target is to identify the second equation, let us see, in fact second equation may be little bit complex, because it is coming simple, so we

will see the second equation structure. So, as usual we have to prefer these particular structure particular equations.

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The image shows a handwritten derivation on a blue grid background. At the top right, there is a small logo for 'CET I.I.T. KGP'. The main part of the image contains a system of equations in matrix form:

$$\begin{array}{c|ccc|ccc} & Y_1 & Y_2 & Y_3 & X_1 & X_2 & X_3 \\ \hline e_{11} & -1 & 3 & 0 & -2 & 1 & 0 \\ e_{12} & 0 & -1 & 1 & 0 & 0 & 1 \\ e_{13} & 1 & -1 & -1 & 0 & 0 & -2 \end{array}$$

Below this, there is a smaller matrix:

$$\begin{array}{c|cc} & Y_1 & X_1 & X_2 \\ \hline e_{11} & -1 & -2 & 1 \\ e_{13} & 1 & 0 & 0 \end{array}$$

At the bottom, there is a calculation of a determinant:

$$\Delta = \begin{vmatrix} -1 & -2 \\ 1 & 0 \end{vmatrix} = 2 \quad \Delta = \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix} = -1 \quad \Delta = \begin{vmatrix} -2 & 1 \\ 0 & 0 \end{vmatrix} = 0$$

There are also some handwritten notes like 'e_{12}?' and 'x' near the matrices.

So, this is what Y 1, so this is Y 2, this is Y 3, **this is y three** this is X 1, this is X 2, this is X 3, then equation 1, equation 2, equation 3. So, this is how we have to prepare, so as a result you have to draw these line, so X 1, X 2, X 3, then equation 2, then equation 3. So, then this is equation 1.

So, now we have to **we we have to** pinpoint all these items. So, now for Y 1, so equation 1 **this is this is this is minus 1** this is minus 1, then this is 3, this is minus 2 **minus 2** this is 1, so other items is to be 0 **other item has to be 0** then, **this is** this is minus 1, this is **minus** 1, then this is 1, **rest item will be 0** rest item will be 0. Then third, this is **minus 3** is minus 1, this is 1, this is minus 1, so then this **is minus 2** x 3 is minus 2, **x 3 is minus 2** then x 1 is 0, x 2 is 0. So, this is **this is** what the structural matrix is all about. So, now let **us** say equation 2, **equation 2 is 2** equation 2 has to be identified. **this is has to be identified**

So, now what you have to do. So, now first **first** step is a to remove this **row** second row then, you have to see this column has to be removed where there is an elements. So, now what is the reduced form equation? So, reduced form is this equation 1, then equation 3, equations 2 will be removed automatically. So, now this is Y 1, then X 1, then X 2. So, this is minus 1 **this minus** 1, then **then** minus 2 0, 1 0. **So, 1 0.** So, now so this is very, in

fact little bit complex because this particular matrix with this form of matrix is not **in a** in a square form.

So, because in the earlier case we get the matrix in a square form, so your job is little bit simpler, but here the job is little bit complex because, once you have such type of you know uneven numbers for means if it is not square matrix then, you cannot get the value of determinants. So, what you have to do, you have to now prepare a square matrix here. **So, So, it is.** So, there are three matrix **you can** you can prepare here. So, Δ_1 , so minus 1 minus 2 1 0 this is one case, then **$\Delta_2 \Delta_2$** Δ_2 equal to minus 1 1 1 0, so this is Δ_2 , then Δ_3 is equal to minus 2 1 0 0. **So, this is equal to 0** this is equal to 0, **so this case is rejected** this case is rejected.

So, this is nothing but **this is nothing but 2**, **this is 2** so which is not equal to 0. So, this is **this is** minus 1, so not equal to 0. So, one determinant is coming positive and not equal to 0, so that means that means the system second equation is a order condition is satisfied. So, now for **which particular equation has to be identified** which particular equation has to be identified for that, you need to **you need to** again go to order condition so that means, **in the case in the** in the case of identification problem, so once the means starting point is you **have** to go by counting rules, then once counting rule is **((then))** you have to tested through rank condition.

But you know, but **if will you** if this is you know double way process its better you start with a first rank condition, then ultimately, **if it is a** if it is you know the equation is not satisfying rank condition then obviously no point **no point** to go order conditions, but if will you go by order condition then ultimately it is to be tested through rank conditions so that means, sufficient condition must has to be satisfied. So, first my suggestion is first you check the order sorry rank condition if rank condition says **((then))** you has to go to order condition.

What is the need of order condition here, the need of order condition is that, so it will give you whether the equation is exactly identified or over identified. **So, by the way.** So, you **you** will be come to a conclusion that, which particular whether the equation is exactly identified or you know over identified or you can say under identified. So, this is how this is all about the structure of identification problems. **this is all about the structure**

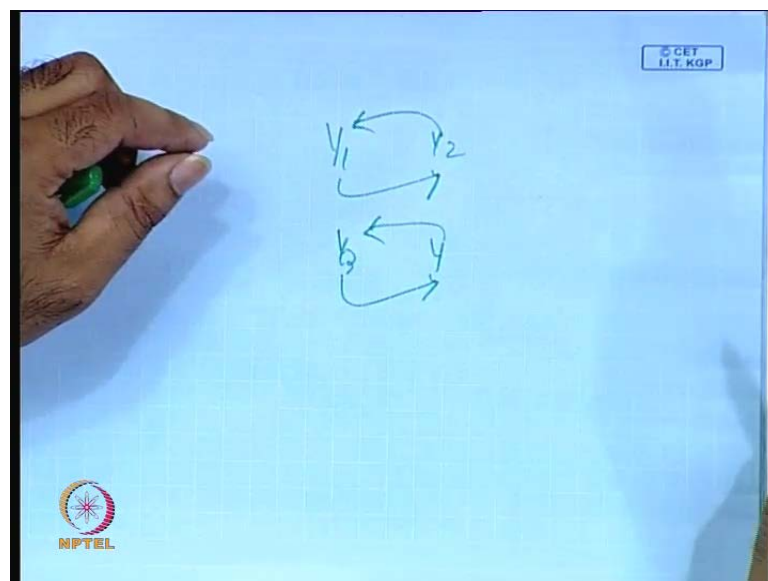
of identification problem So, that means so next part of this problem is called as an estimation problem.

So, now so means basically what we what we are highlighting here. So, once you have a system of simultaneous equation then obviously, it it is something different to single equation modelling. So, in the single equation modelling, where there are you know means there is always dependent variable, one dependent variable and one or multiple independent variables and the condition is that this there is only one way causality that means, every time only independent variable has an influence on dependent variables.

But in the case of, but in the case of you know simultaneous equation modelling it is the it is not a structure of single equation system where there is a one way causality, but here there is a you know two way causality that means, the there is a means there are interdependent. So, that means there is possibility of bidirectional causality. So, a exactly not bidirectional there is a two way causality that means, the unidirectional and bidirectional there is something, there is inside hidden factor, so that we will discuss all details in the case of pure time series modelling.

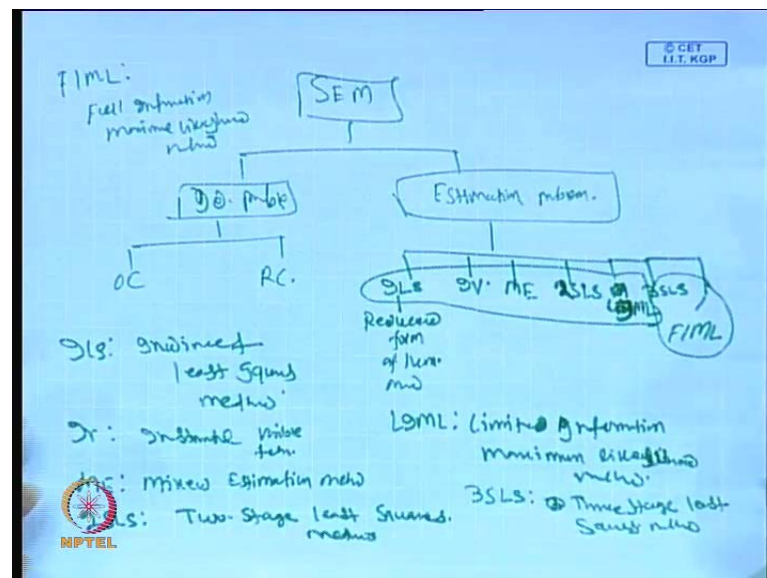
So, times times means in if will you apply the time series modelling the concept of bidirectional means, every means every variable has an (()) for instance if there are like this.

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So, let us say Y 1 and Y 2, so Y 1 (()) from this influence and Y 2 this this upon influence, so like you knows Y 3 Y 4. So, this is influence like that way this is influence like that way, but you know here in the simultaneous equation modelling. So, we cannot directly call bidirectional causality we can call two way causality, Sometimes what happens all this dependent variables may not be similarly influence exactly on other variable. For instance, Y 2 may be influence by Y 1, but not exactly Y 1 influence by Y 2, it may be Y 1 again influence by Y 3, so like that there is different cluster altogether. So, this is the specialty of you know structural equation modelling, so the exact bidirectional causality problem. So, we will discuss in the case of time series modelling which we will discuss after few lectures only. So, now here here the major issue is that you know identification problem. So, which we have already highlighted and the next issue is the estimation process, so what is all about this estimation tricks.

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So, what is this estimation problem here. Put it in other way here is simultaneous equation modelling, simultaneous equation modelling which we have highlighted here identification problem identification problem, so which we have which which we can settle through order condition and rank conditions, but when we will go for you know estimation problems when we will go for estimation problem then obviously, the estimation problem, so that means, what is the problem that means, OLS the application of OLS to a particular equation without having without having substantial information about the other equation then the system will be biased inconsistent, inefficient or cannot

be means, **it cannot** altogether it cannot satisfy the blue theorem. So, ultimately we have to look for alternative techniques, there are series of alternative techniques which are available for simultaneous equation modelling, let me highlight here. The starting procedure is called as a **indirect least square methods** indirect least square methods ILS or its otherwise called as a reduced form of the models. **reduced form of the form of the models reduced form of the model** ILS stands for **indirect least square methods indirect least squares least squares methods** indirect least squares methods.

So, then next is instrumental variable technique, instrumentals I V, **instrumentals instrumental variable technique instrumental variable technique** then next is mixed estimation methods, so M E stands for mixed estimation methods, **mixed estimation mixed estimation methods** then there is called as a two staged least square methods **two staged two SLS two staged least square** methods 2SLS. **So, that is nothing but two staged two staged least squares least squares methods T staged least squares methods. So, this is this is two SLS.**

Then **then** maximum likelihood information method, **limited** its better put limited **maximum limited** information maximum likelihood methods **limited I I M I limited** L I M L, **limited limited information of maximum limited informations limited informations maximum likelihood estimators maximum likelihood methods likely likely likelihood likelihood methods likelihood methods limited information limited information maximum likelihood methods** then there is a called as a three staged least square methods and another is called as a full information maximum likelihood methods F I M L. **ok**

So, one is called as a three stage least square methods 3 S L S, **that is three stage that is three stage three stage least square methods three stage least squares methods** and another is called as a **F I M L** F I M L, it is called as a **full information** full informations maximum likelihood **maximum likelihood likelihood methods maximum likelihood methods**. So, now you see here this first technique, **the** you know there are altogether one, two, three, four, five, six, sevens, so there are seven techniques. **So, these are all one groups.** So, these are all one groups and three stage **three stage** least square method and full information maximum likelihood method is another another groups.

So, the standard trick is that, so this is called as a this particular you know instrumental variable technique, indirect least square method, maximum likelihood estimation method, two stage least square method, then limited information maximum likelihood method these are all called as a single equation method technique, and three stage least square methods, and full information maximum likelihood methods are called as a simultaneous system methods. So, this is single equation method and system methods.

So, that means so far as a estimation problem is concerned. So, direct application of OLS to an individual equation in the simultaneous equation system will give you bias and inconsistent result. So, ultimately we have to apply we have to apply a we have to apply, you can say an alternative methods, So, so far as alternative methods are concerned. So, we have a we have series of techniques starting from reduce form of the methods, that is indirect least square method, instrumental variables, mixed estimation method, two staged least square method, limited information maximum likelihood methods, limited information maximum likelihood methods then three stage least square methods, then full information maximum likelihood method.

So, except except three stage least square method and full information maximum likelihood methods all other methods are treated as a single equation method, and three stages least square method and full information maximum likelihood methods are called as a system methods. So, these are the basic structures of you know model estimations means, the estimation problem of simultaneous equation modelling. So, I am not going to discuss the detail about all this you know techniques that means, that is related to indirect least square method, in a instrumental variable technique, mixed estimation techniques, two stage least square methods, limited information maximum likelihood method, three stage least square method and full information maximum likelihood.

These methods are you know very not too much complex except full information maximum likelihood methods, other very simple like (()) technique, but there is a little bit different structure and setup. So, we have no time to discuss each and every technique separately, so that is how I am restricting this particular component here only. So, it requires more number of lecture to describes all this details with respect to all these detail derivation of this technique, but our problem is not to highlight details about all these techniques, our problem is to a specify how is the estimation problem and how quickly we can solve this type of estimation problem through various techniques. ok

So, with this **we can** we can conclude this simultaneous equation modelling system here in the next times related to simultaneous equation modelling there is a problem called as a structural equation modelling. So, we will discuss structural equation modelling in the next class with this we can conclude this particular session thank you very much have a nice day.