Econometric Modelling
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Module No. # 01

Lecture No. # 32

Simultaneous Equation Modelling

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Good afternoon, this is Doctor Pradhan here. Welcome to NPTEL project on Econometric Modelling. So, today we will discuss simultaneous equation modelling. In the last couple of lectures, we have discussed various aspects of modelling that like invariate modeling, bivariate modeling, multivariate modeling, and various issues related to econometric modelling like multicolinearity, heteroscedasticity, autocorrelation. Then, we have also discussed the structure of panel data modeling, that which is very interesting that you know, fixed effect model, random effect model and full data model.

So, anyway there are is one structure of econometric modelling, till whatever we have discussed till now. The fundamental issues till today, is with respect to single equation modelling. Today, we will discuss the issue called as a 'Simultaneous equation modelling'. So, before we proceed, the structure of simultaneous equation modelling, I would like to briefly highlight what is the exact difference between a single equation modelling and you know, simultaneous equation modelling.

First first of all, what is it all about the simultaneous equation modelling? Simultaneous equation means, it is you know, a structure of joint dependence of variables or joint dependence of equations. There are many ways in which we can represent. System describe joint dependence of variable is called as a simultaneous equation modelling. system describe joint dependence of variable is called as a simultaneous equation modelling

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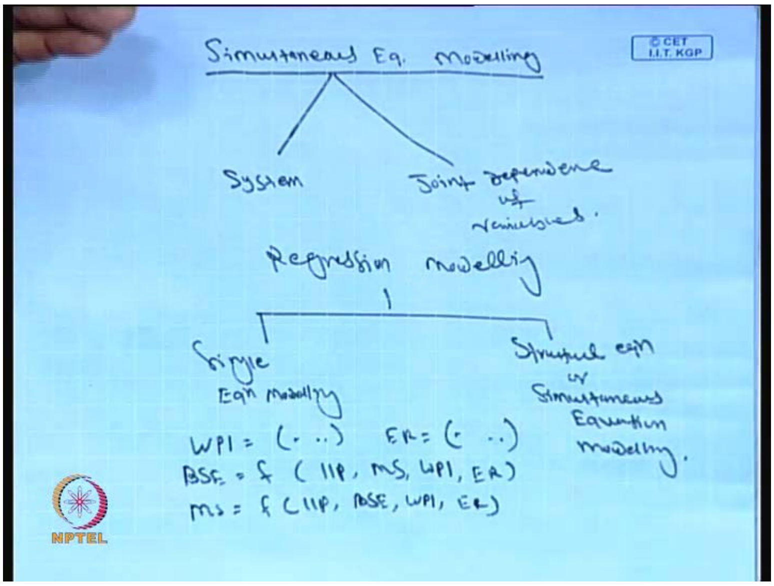

So, simultaneous equation means, you know, it is a system. First of all, it is a system describes the joint dependence of variable, joint dependence of variable. So, system consist of joints dependence dependence of variables. What exactly is this structure which is called as a joint dependence of variable? That means, before we proceed to joint dependence of variables, let us first know what is all about this single equation modelling so then we will move to simultaneous equation modelling.

So, whatever we have discussed you know, in the regression modelling, particularly in the regression modelling so we entirely or we can classify into two different agenda. One is called as a single equation modelling and second one is called as a structural structural equation or simultaneous equation modelling. simultaneous simultaneous equation modelling So, this is this is you know, either you can call its structural equation modelling or simultaneous equation modelling. They are more or less same but, there is little bit difference. So, we will discuss the structural equation modelling in details in a separate lecture. Today, we will specifically highlight the simultaneous equation systems. Let me first highlight the agenda of single equation methods and then we will go to simultaneous equation methods means single equation modelling then (( )) simultaneous equation modelling.

So, one in one case it is the single equation and in the case of simultaneous equation means it is the multiple equation situations. So, that means when generally till today, what we have discussed there is a problem. We have to transfer the entire problem into mathematical form of the model and then we will bring into the statistical form of the model. Then you have to go for collection of data, then estimation, reliability checking so many you know, fundamental issues related to that estimated model. Then use this models or we can call this model as the best fitted model. That model can be used for forecasting or policy use. But, whatever concepts we have discussed till now, so it is single equation we can describe the entire situation. For instance, here in this particular problem so we have taken BSE, IIP industrial index of industrial index of productions, money supply, wholesale price index, then exchange rate. So, we have discussed this particular problem in the case of various topics like you know multicolinearity issue and even water correlation issue. Everywhere you can side this problem.

But you know, here this particular system is five variable case and what what exactly is the single equation modelling is that, so, keeping one variable at a time, then rest of variable will be in the independent structure. For instance, I can write like this way BSE as a function of IIP, money supply, then wholesale price index, then exchange rate. Similarly, I can also write equation like you know, function of IIP, index of industrial productions, then BSE, then wholesale price index, then exchange rate. Similarly, I can write for WPI and similarly, I can write for you can say, exchange rate. Means, I will assume that all these variables are can be fitted into a model. For instance, once my issue is regarding prediction or forecasting of inflation then, I will I will assume that you know WPI as a function of rest of the other variables like you know, Bombay stock exchange index of industrial production, money supply and exchange rate.

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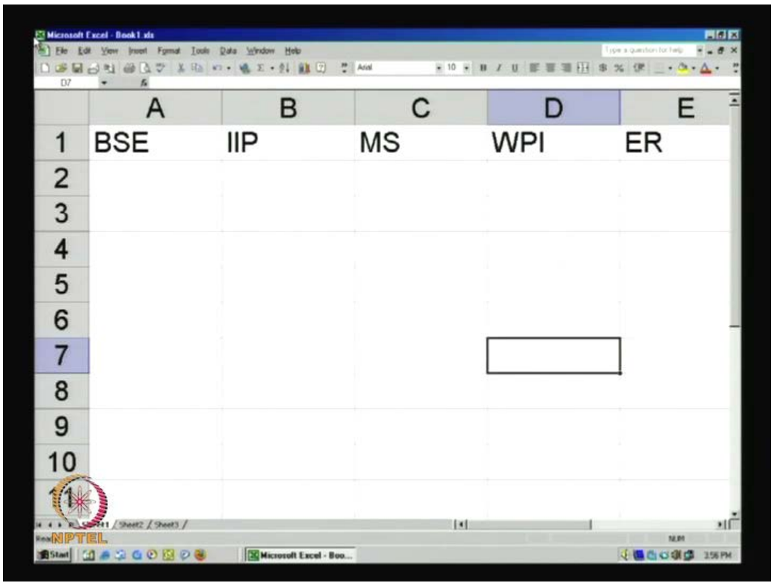

Similarly, suppose my objective is to you know, forecasting economic growth then, IIP is a function of BSE, money supply, wholesale price index and exchange rate. Similarly, if I have a problem that of exchange rate determination then, exchange rate as a function of stock price, economic growth, money supply, whole sale price. Like, many ways we have to represent the situations. So now, whatever your objective, whatever may be your objective, so you have to assume there is a dependent variable cluster and there is independent variable cluster. Then, we have to fit the model. Then, as usual you have to proceed for estimation, then reliability checking and its usefulness.

So now, so we briefly highlight the details setup of single equation modelling and how we will go to or how we will enter into this simultaneous equation modelling. Because in a real world situation there are certain, there are variables which are more or less interdependent in nature. It is very difficult to find out a situation single variable which has you know, means which is, which can be influenced by only several independent factor variables. That same independent variable can be as again can be considered as a also dependent variable. So, there are many ways it can be analyzed or you can say propose. But, how means what is the specialty of this single equation modelling? And how you have to converge? Means, we have to integrate with the simultaneous equation modelling. So, let me highlight what is the single equation modelling system.

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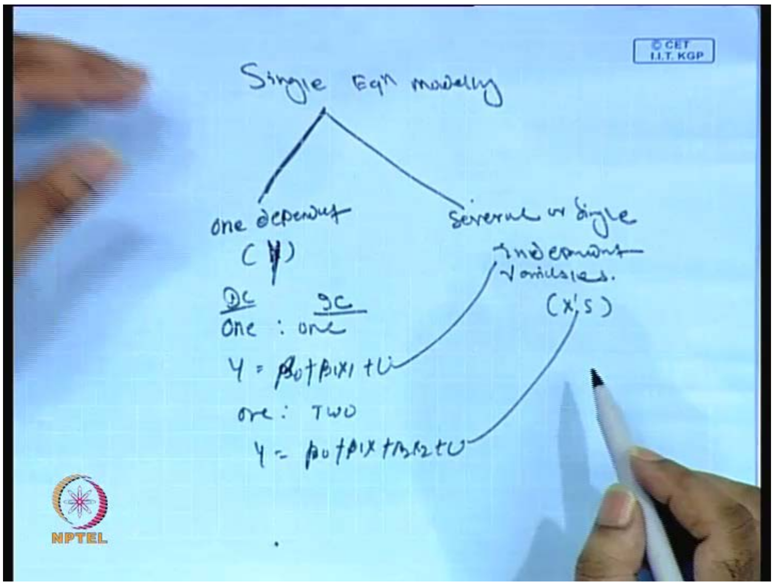

This is first single equation modelling. Then, we will go to simultaneous equation modeling. Single equation modelling, so, in the single equation modelling so what is the structure? So, structure is in every case there is one dependent one dependent. So, say x, then several several or you can say single independent variables independent variables. So, there is single or several independence variables.

So, now single equation modelling; so in the single equation modelling, your setup, is means it is very much interesting with respect to the variable classification, that dependent classification and independent classification. So, whatever may be the situation, so far as a single equation modelling is concerned, there is one variable which is called as a dependent variable and rest of the variables either it is one or multiple, it is are there in the dependent clusters. That means, every time there is one dependent variable with one or multiple independent variables.

So, that means if one dependent variables, so it is one dependent variable, means we can call it Y here. Then we will call it is x here. So, suppose the cluster is one one, one one then obviously it is Y. You can say, alpha plus or beta 0 plus beta 1 x 1 plus u. Suppose there is cluster of 1 2; so this is independent cluster and this is dependent cluster, this is independent cluster and this is dependent cluster. So, if it is 1 2, then obviously the structure will be beta 0 plus beta 1 x 1 plus beta 2 x 2 plus U. So similarly, it will continue like that way. So now you know, this means, it has a connection with this, you know, this particular independent side. So, we have two different setup, one dependent structure, and independent structure. Now in the single equation modelling, so every time there will be single dependent variables and one or several independent variables.

But, when we will go for estimation? Generally there are many ways. We have discussed the issues like the so far as estimation concerned, so we usually apply OLS technique. If OLS technique will not provide you know, efficient results then we have to apply GLS technique and WLS techniques. So, what we have discussed? means up to today's discussion, we have highlighted OLS technique, G L S technique and W L S techniques. But, out of these techniques, OLS is the standard technique and everywhere first we like to use. Then, if the problem is not in our boundary, then we have to go for alternatives by the use of OLS technique and WLS technique. Means, specifically we have discussed that issue in the heteroscedasticity problem and you know panel data model.

So now, so in this particular structure, so the classification is dependent classification and independent classification. Dependent is every time there will be one dependent and there will be 1 independent or several independent. So, if it is 1 to 1, then it is simple bivariate modelling. If 1 to 2, then it is trivariate modelling and 1 to many then it is called as a multivariate modelling multivariate modelling that to single equation systems. So, when there is single equation system, there may be several variables but, the variable classification is only one dependent variable and several independent variables. However, like this.

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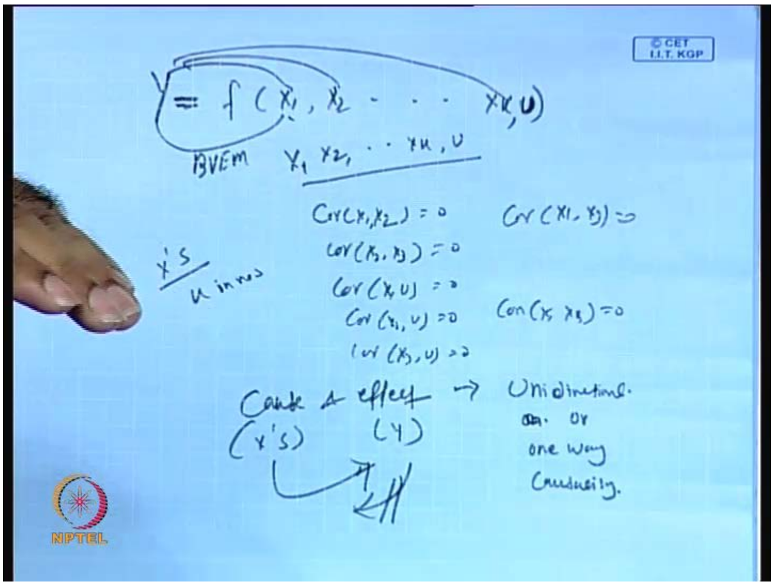

So, what what we will see here? so Let say, will take a case of multiple multivariate regression modelling. So, Y as a function of x1, x2 up to xk. So This is how the structure is all about. Now, what we have to do you know, this is multivariate model. So, if we will regress this to this, this this 1 to 1 situation and that too called as a bivariate econometric modelling and if we will you know, like this, then it is trivariate econometric modelling, then if we will club all these things, it is called as a multivariate equation modelling. But, when we will go for this issue of single equation modelling, we have different game plans with respect to right hand side that is, with respect to independent cluster. But, in the why because there are several items in this right hand side, in fact you have not, we have not introduced here. So, that is random term or error term.

So, that means, when we will apply OLS, then our standard assumption is that, the this you know, x1 and x2; x1, x2 up to xk and U, though all are should be you know, you know they should not be correlated. That means, if there correlated, then there is multicolinearity problem. That means, covariance of x1, x2 should be equal to 0, then covariance of x2, x3 should be equal to 0, then covariance of x1, U should be equal to 0. Covariance of x2 upon U should be equal to 0. Covariance of x3 upon U should be equal to 0. Then, covariance of x1, x3 should be equal to 0. Similarly, covariance of x1, xk should be equal to 0. So, like this and variance of variance you know variance of U should be constant everywhere. This is how we have to set the structure. Means, it is a single equation modelling. Every time Y is the dependent classification and others are independence classification. And, within that system there are several things we have to discuss. And that too right side of the pictures, where there are multiple variables are there, independent variables are there, and U. is U is a random term that is a which represents the error component. That means, which which is not already captured in the system. So, we will take care of that the U issue. Anyway, so this is the structure of you know what we will call single equation system. So, the variable classification is dependent classification and independent classification. Here, Y and x s, x s means it is k, k in numbers k in numbers.

But, what is the interesting thing here is that, so once you will go for handling the single equation modelling, then that times one picture is very much accurate. That means very much essential that is one dependent variable, with one or several independent variable. This is most importance feature of single equation modelling. And, second feature of the single equation modelling is that, it is the its its means it is the question of cause and effective relationship. So now, this cause and effective relationship is nothing but, you know, it is purely unidirectional. For instance, the way we are discussing here, so this is nothing but, cause and effective relationship. It is the cause and effective relationship, so cause, we will generally put it in x s and effect will consider it is in Y. So, the regression will be like this, so Y to x now, sorry x to Y. So, this is the cause and effective relationship.

So now, so this particular cause, that means the reverse causality does not allow here. So, reverse causality that is this specialty of single equation modelling or you can say that it may be limitation of the single equation modelling. so That means, it is one way a causality. so That means, it is the independent to dependent clusters. While dependent to independent cluster is not allowed, so that is the structure of single equation modelling. So, that means it is the x which can causes Y but, vice versa is not true. Even, may be there, but, our standard assumption is that, there is no such relations. Reverse causality, so that means in the single equation system, so one dependent variable, several independent variables and the causality issue are very much unidirectional. So that means, it is the independent variables which usually or which can explain or which can cause the dependent variables. so That means, dependent variable is effect side and the causes are you know independent variables.

Cause and effective relationship so, that too is unidirectional in nature. That too, it is unidirectional in nature. Unidirectional unidirectional or sometimes it is called as a one way causality. It is otherwise called as a one way causality it is otherwise called as either it is unidirectional causality or sometimes it is called as one way causality.

So now, with this particular setup, we will move to simultaneous equation modelling. So that means, in these particular system, there is only single equations and where independent variable and error term is influencing the dependent variables. so That means, Y has a function of several x s and error terms but, reverse is not allowed here. So, this is the system can be called as single equation modeling.

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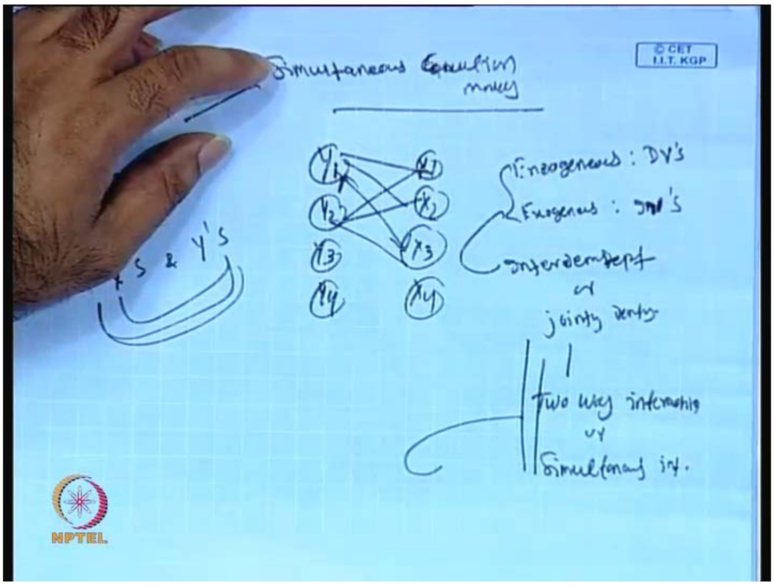

So now, we will come down to you know, simultaneous equation modelling. Now, how simultaneous equation modelling simultaneous equation modelling can be you can say generated? so simultaneous equation modelling simultaneous equation modelling

So now, in the simultaneous equation systems, it is called as the system, where there is joint dependent of variable. So here, so there is no one means; one there is not a concept of you know, one dependent variable and several independent variable. In this particular simultaneous equation, instead of using independent cluster and dependent cluster, we usually represent the endogenous cluster and exogenous clusters.

So that means in this system, so the structure will be like this, Y1 Y2 Y3 Y4, then similarly, X1 X2 X3 X4. Like this way. So, we will means our structure is like this. so It is Y1 influence Y2 Y1 X3 and similarly, that means it may be causes like this way so X3 may be causes Y2; Y2 may be causes X3. So, like this. There is are several several that is How it is jointly dependents and inter dependents among the variables.

So, that means here in this particular simultaneous equation system so there are more more x and more Y. More Y so that means it is x s and the cluster means it is the game between multiple X and multiple y. So, that means it is the, it is not 1 1 1 or 1 many case. It is many many case all together. so that means That is this specialty of simultaneous equation system. So, simultaneous, in the simultaneous equation system, so there are several dependent variable and several independent variables.

The dependent variable is otherwise called as endogenous cluster and independent variable otherwise called as a exogenous cluster. So, that means here, so there is are two groups of data, so one is endogenous endogenous clusters and this is called as a dependent variables dependent variables and then there is exogenous exogenous clusters, that is, independent variables clusters that is independent independent variables clusters. And, they are very much interdependent. So, these two are interdependent interdependent. These two are very much interdependent interdependent or you can say jointly dependent jointly dependent jointly dependent.

So now, the so in one case, it is means in the single equation modelling, so it is only 1 Y with several X s and here there are multiple Y with several X. Now, this is somewhat little bit simple. And this one is somewhat little means it is somewhat complex in nature. Because, there is lots of cost casualty. is there So, that means it is not question of one way causality, rather, it is two way causality. So, that means there is bidirectional causality. Between the two, that means we are assuming that for instance if Y1 causes Y2 then, there may be chance Y2 may be cause Y1.

So, this is how it is. It is called as a two way causality which is not possible in the case of you know, single equation modelling. In the single equation modelling, it is only Y1 causes Y2 while Y2 does not cause Y1. So, this is how the single equation structure and in the simultaneous equation structure it is the reverse causality allowed. So, that means it is it is the system, where we where our objective is to study two way interactions or two way causality. That is you know because, it is the there is question of joint dependence of variables. So, there is possibility of two way causality or bidirectional causality.

So, it is two way intersection, two way interactions or simultaneous interactions. So, either it is called as a two way interactions two way interactions interactions or it is called as a simultaneous simultaneous interactions simultaneous interactions. So this is how we have to check.

The classification depend means here In this particular structure, the classification of dependent and independent is very confusing or you can say it is doubtful because it is difficult to describe the situation because it is a joint dependence of variables in the system. So, it is very very difficult to say that, this is purely independent variable and that is purely dependent variable. Of course, in a particular system we can classify. Still, you know, it is very difficult to describe the situation in the case of simultaneous equation system because every equation has an error term and every equation depends upon other equation. So, that is why the system itself is a very complex process. So now, with this particular setup, so we have to we have to describe this simultaneous equation modeling.

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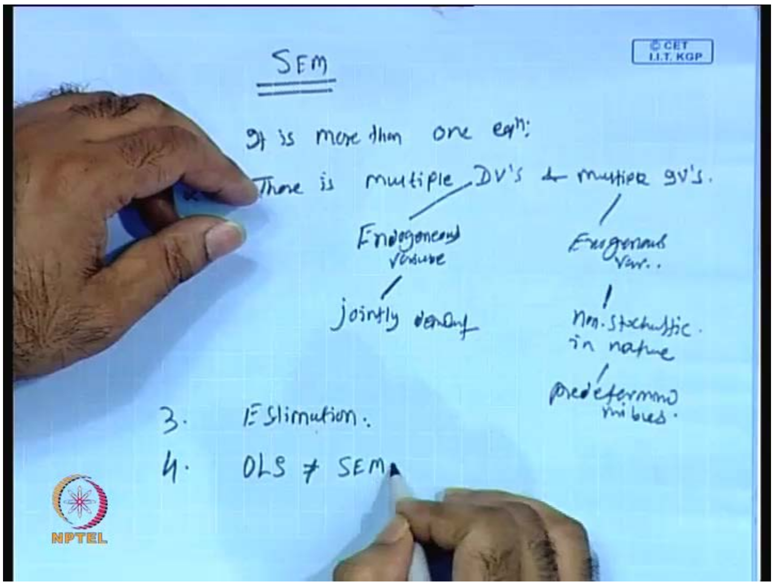

So now, what is the structure of simultaneous equation modelling? now simultaneous equation modelling so First of all, first of all before we go for this typical issues about simultaneous equation modelling, so I would like to highlight a special feature of simultaneous equation modelling. We will then go for its problem aspect or estimation aspect.

In the structural equation modelling, so first condition is, it is more than 1 equation. It is more than more than 1 equation. So, it is obviously more than 1 equation and second you know the variables are, there is there is there is multiple x, multiple dependent variables and multiple multiple independence variables.

So now, this dependent variable cluster is called as an endogenous variable it is called as an endogenous variable. This is called as an exogenous variable exogenous variables variables it is called as a exogenous variables. And you know, these endogenous variables, it is jointly dependent it is jointly dependent. So, this is this is you know, non-stochastic in nature this is non-stochastic in nature stochastic non stochastic in nature and it is otherwise called as a predetermined variables predetermined predetermined variables predetermined variables.

In the simultaneous equation, one cannot estimate the parameter of single equation. That means in the simultaneous equation there are several equations. So actually, what is the object of econometric modelling objective? of There are may be several objectives, but one of interesting objective is to estimate the value of parameters in a in a particular equation. Whether, it is single equation system and simultaneous equation system when your objective is to estimate the parameter. Typically, for a single equation model it is not a problem.

So, we have to every time, we have to go by systematic way. Then, you have to test, retest. Then it finally, will get come to a conclusion. that are Come to a point where you will get the best fitted model. But, in the case of simultaneous equation modelling, there are several equations. So, there are parameters involved in each and every equation. So, to estimate the parameters of your single equation is somewhat very difficult in the case of simultaneous equation system. Until, unless you know the information about the other equation in the systems.

So in the third case, simultaneous equation, the estimation problem is there. This is nothing but, estimation problem estimation problem in the one cannot estimate the parameter of single equation without taking into account the information provided by other equation in the system. So, this is how the structure about the simultaneous equation modelling. In the simultaneous equation modelling, the parameter of one equation cannot be properly estimated. Yes, it can be estimated, but that estimated value cannot go with blue properties. So, it is very difficult to estimate the parameters which are satisfying with the blue theorem until unless you know the information provided by other equation in the system.

So, there are two problems associated with this particular structural equation modelling. One is estimation problem and second is the identification problem. so Until and unless you identify the proper structure, then obviously you cannot go for estimations. So, obviously, estimation depends upon identification. so Until, unless you know the proper identification, then you cannot go for its estimation. So that, again when there is estimation, so which particular technique you have to apply, because, only OLS in a simultaneous equation system cannot be feasible or cannot be fit.

So, there are several methods which we usually handle to solve the simultaneous equation system. We will discuss in details subsequently. So, in the mean time, this is estimation problem. for so that means Why it is estimation problem because, OLS cannot be applied to this simultaneous equation modelling directly because, it there are several problems associated with this OLS techniques to the simultaneous equation system. So, one one of the major problem associated with the introduction of OLS is the estimated parameters will not be as far as the blue theorems.

So that means, the parameters are most probably it cannot be unbiased and it cannot be efficient or you can say it cannot be consistent. So, to so when there is simultaneous equation modelling, then the application of OLS, the use of OLS is very limited. so Because, it will give you some biased result and you know, inconsistent results. So, to get consistent result, then unbiased results, then obviously so we have to go for different estimation technique to simplify the simultaneous simultaneous equation setup.

So this these are the conditions through which simultaneous equation modelling usually operates. So, let me take an example; simple example. Then, I will highlight the structure of simultaneous equation modelling.

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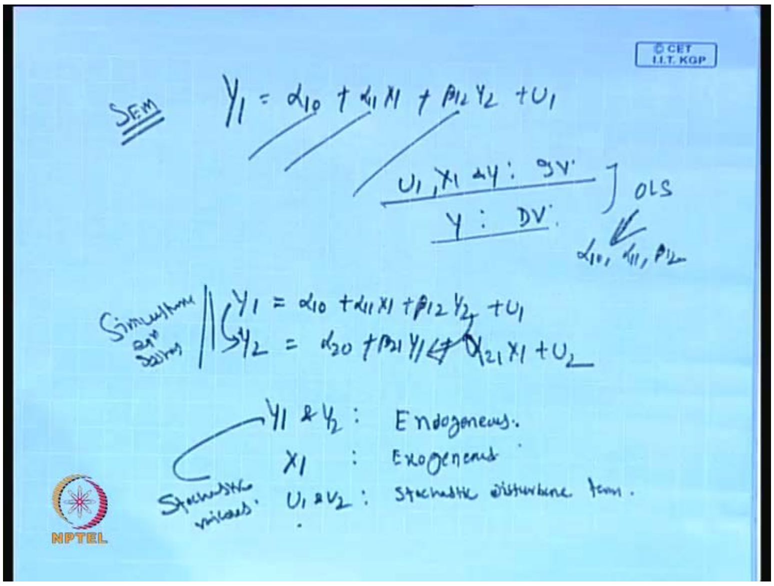

So, I will take two equations here. Y1 equal to alpha 1 0, plus alpha 1 1, X 1 plus beta 1 2, Y 2 plus U 1. So, this is one equation. so When there is single equation model, then obviously this equation is there. So, when we will go for single equation modelling single equation modelling then our objective is to get alpha 1 0, alpha 1 1 and beta 1 beta 1 2. Because, here once you will go for single equation modelling then X1 and Y2 is treated as an independent variable and Y1 treated as a dependent variable.

So, obviously this is U1. Also, these are in the independent cluster. And these are all dependent clusters and since it is a single equation modelling, then obviously you can apply the OLS and get the estimated parameters like you know, alpha 1 0, alpha 1 1 and beta 1 2. So, this is how after getting, applying the OLS you will get all these results. But, when we will go for simultaneous equation modelling then system will be like this. So, Y1 equal to alpha 1 0 plus alpha 1 1 X 1 plus beta 1 2 Y 2 plus U 1. So this is first equation and second equation second equation is Y 2 equal to alpha 2 0 plus beta 2 1 Y 1 plus you know, I will put rather yes alpha, this better, put alpha alpha 2 1 X 1 plus U 2. So, this is this is a this is simultaneous equation system simultaneous simultaneous equation system you know, so far as a simultaneous equation system is concerned. So, the system will be consistent. Mathematically, the system will be consistent if number of equations exactly equal to number of variables. Otherwise, the system will be inconsistent.

Similarly, this can be also applied here, in the case of statistical modelling. Now, before we go ing that is how there is a need of proper identification, whether the system so far as identification is concerned. So, there are 3 different steps of identification one is called as exactly identify, then over identify, then under identify. That means, once you have identification problem, then obviously that should be in a simultaneous equation system and each equation has to be identified.

So, when we will go for identifying a particular equation, then there are 3, 2 different ways you have to classify. Either the equation is identified or not identified. If it is identified, then obviously whether it is exactly identified or its over identify, generally exact identification is the best. But, still we have to see what is the step of the means exact identification. Then, we have to until, unless you know, the identification situation you are not sure which particular technique has to be applied to solve this particular system.

So now, in this particular setup, there are several variables. So, Y1 and Y2 are mutually interdependent. So, that is how it is called as an endogenous variable. So, these are all Y1 Y2 are mutually dependent variables because, Y1 depends upon Y2 and even Y2 depends upon also Y1. So, this is how they are mutually interdependent to each other.

So, Y1 Y2 are endogenous variables and X1 is only one variable. So, that is exogenous variable. This is exogenous variable. so In other words, in the by as far as the single equation system, so this is called as a independent variable, these are called as a dependent variables.

So U1 and U2 are non-stochastic sorry stochastic disturbance term. These are stochastic disturbance term. Stochastic disturbance terms and Y1 and these are called as stochastic variables these are called as a stochastic variables these are called as a stochastic variables.

So that means, this is a simultaneous equation system where, there are 2 equations and the number of equations that means number of equations are 2, number of endogenous variables are 2, number of exogenous variables are 1 and 2 error terms, that is, U1 and U2. That is the complete description of this particular model.

So, this is how we have to represent various system simultaneous equation modelling. Because, overall in the broad levels so you will find that everywhere there will be model and that model cannot be single equation that too several equations or multiple equations. Now, our objective is to know how they are interdependent to each other and how quickly we have to get the estimated parameters and you will get the better fitted models that can be used for forecasting and policy use.

So, better fitted model means, it is not single equation for every equation has to be identified properly and that too you have to get the estimated parameters which should be very reliable, very consistent, very feasible and should be totally unbiased. This is the motto of this simultaneous equation modelling. So, with this setup, so I will give you some examples here. so, This is this is one way example. So, I will put another equation.

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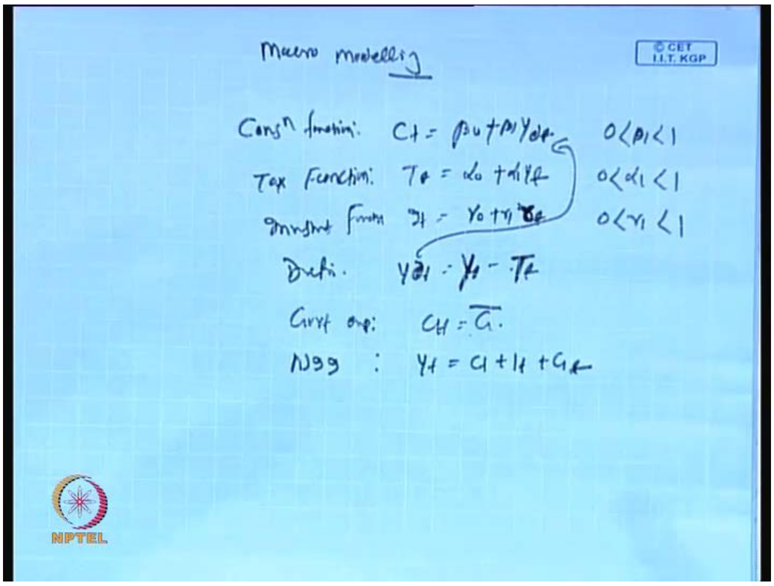

So, take a case of macroeconomic modelling. One such example is called as a macroeconomic modelling. So, in the macroeconomic modelling, so there is a standard concerned model. So, this is called as a conception function. This is called as a conception function where C t equal to beta 0 plus beta 1 beta 1 Y t Y d t disposable income where 0 less than beta 1 less than 1. So, this is the condition. Then, we have tax functions. We have tax functions, tax functions p t equal to alpha 0 plus alpha 1 Y t so 0 less than alpha 1 less than 1, then investment functions investment functions. This is investment functions. So i t equal to gamma 0 plus gamma 1 then, then rate of interest r t r t 0 less than gamma 1 less than 1.

Then definitions, there is another definition is available. So, Y d t equal to Y minus Y t minus t t. so This, is national income and this is total amount of tax collected.So, the difference will give you the disposable income. This disposable income is used here. So, then, there is government expenditure government expenditure so which is nothing but, G t equal to G. Now, there is national income identity, so which is nothing but, Y t equal to C t plus i t plus G t. So, G t is G bar is given here.

So now, this is the complete, means in the last last examples I have just sited 2 simultaneous equations. 2 equations in a simultaneous equation modelling where there are 2 dependent 2 endogenous variables Y1 Y2 and 1 exogenous variable X1 and there are 2 error terms Y1 Y2.

So now, here we have a system here there is a macroeconomic modelling, Cancian macroeconomic modelling. so In the Cancian macroeconomic modelling, the major component is Y t, that is national income which is influenced by consumptions. Consumption expenditure time period t, investment expenditure time period t, and government expenditure time period t. So, which is where you know C t means if you will go by single equation modelling, then Y t equal to C t plus i t plus G t plus U. But, in reality C t is not a independent variable. C t depends upon here, income disposable income and further disposable income depends upon national income and tax structure.

So similarly, you know, so tax structure we will get it here. So similarly, investment function depends upon investment function, investment amount is not constant. It depends upon the level of rate of interest in the economy. So, as a result the system is very you know, inconsistent. So, you have to make the system very consistent. Means, the model is very much consistent. So, what you have to do so it looks like inconsistent? But, you have to go for its proper identification. The moment you will go for proper identification, it will give you a signal. What is the step of this? Or what is the structure of this particular simultaneous equation modelling.

So, with this background, we will highlight this particular simultaneous equation problem. Now, when we have a simultaneous equation problem so, for instance, in the simultaneous equation problem, so we have series of equation. So, while in the case of single equation modelling, there is one single equation. Here, there are series of equations. So, when we will apply OLS technique then, obviously the estimated parameter cannot be, you know, cannot be consistent or cannot be unbiased.

So, there is bias arising in the process of estimations if, you apply the direct OLS techniques. so That means, so there is are several techniques which we will take care of the issue of biasness inconsistence result. So that means, if I initially I have mentioned very many times.

So, when you have a particular problem, particular in the single equation modelling, so you may not get initially the best fitted model. so But, if you will go by different methods like you know, if your applying instead of OLS, if you will apply GLS or maximum likelihood estimator or would weighted least square method then, obviously there is small change of result. Sometimes, the insignificant variable may be significant or that is how you are changing the different techniques. So, that means ultimately, aim is to get the best fitted model. Sometimes, the best fitted models you can get easily by different different process. so You have to apply different techniques and you have to go for its estimation and testing. Then finally, come to a conclusion that this model is best fitted and can be used for forecasting.

So that means, where there is no such problem of direct bias, so that has to be tested and it can be cured properly. But, in the case of simultaneous equation modelling, so when we will apply OLS, then there will definitely be bias, because the equation is not independent equation. It depends upon several other equations. So that means, the systematic bias is always there in the case of simultaneous equation modelling if you directly apply the OLS technique.

So, the standard trick is that, you first identify the exact exact equation in a particular system. Then, you have to go for its technique choice. So, once you apply the different techniques, reliable technique then, obviously the system will be give you the systematic result. That is, you know, consistent. It should be practically feasible and it also should give you unbiasness.

So now, so in the case of single equation modelling, so the pure technique is OLS and if you're not getting the best models through the OLS, then you have to go for mathematical transformation, statistical transformation, data transformations. Then, increase of sample size, decrease of sample size, dropping of variables, inclusion of variables, like so many ways you will redesign, restructure till you get the best model which can be which can be essentially called as a too much exclusively best and can be used for predicting or forecasting or it can be used for policy use.

But, in the simultaneous equation modelling so before we go through all such condition. Restructuring, retransformation, etc etc. So, one one first agenda is, to identify that particular systems. Whether that system is simultaneous or how how how much they are interrelated to that. means You can directly apply OLS in a particular equation, in a particular system where a particular equation has a linkage with other equation in the model. That means, it depends upon the information of the other equation until, unless you know the information about other equation model. Then you cannot apply the OLS technique to the standard single equation model in a particular structural equation system.

Now, I will highlight here, so how the systematic bias can be arise in a particular simultaneous equation modelling if you apply directly with OLS technique to a simple equation means one particular equation in the system.

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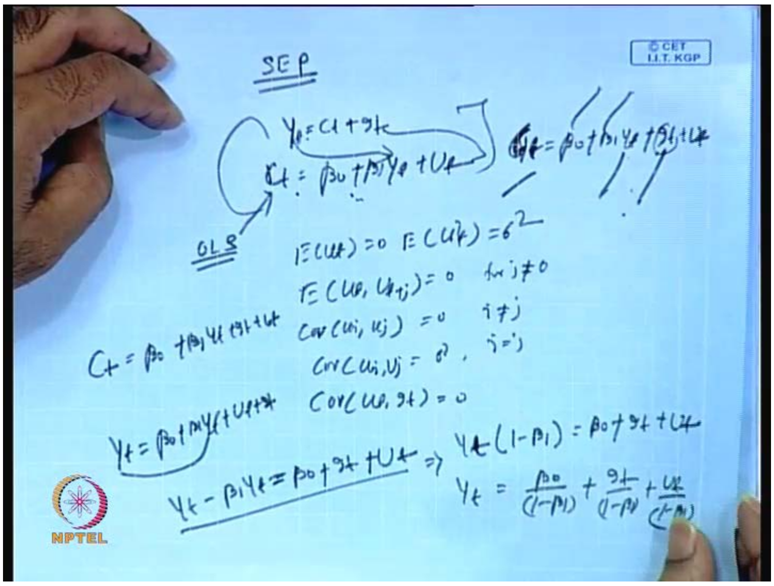

Let us start with, like this, so we know Y t Y t equal to C t plus i t. So, just now we have highlighted here, this is Cancian cancian macroeconomic model. So, when there is in the case of Cancian macroeconomic model. so We just are you tell to remove this G component so that, means it can be included G, it cannot be included G, so G depends upon comment expenditure involvement.

So for simplicity, we are just taking Y t Y t equal to C t plus i t. Now so here, Y t equal to Y t means in the Y t equal to C t plus i t while C t equal to beta 0 plus beta 1 plus beta 1 Y t plus U t. So, this is how the condition. so This is called as a Cancian macroeconomic model and that too consumption determination of consumption functions.

So now, what you have to do so, if we will apply OLS technique in this particular system. Suppose, you're interested to apply OLS technique to the consumption functions, now what is the essential condition we have to choose here? Means assumption? So first, is E U t must be equal to 0. Then E U square t must be variance of that error term should be constant sigma square. Then covariance of error terms U t U t plus j should be equal to 0 for j not equal to 0 for j not equal to 0. That means, covariance of covariance of U i U j equal to 0 for I not equal to j. Or else, covariance of U I, so what we have written here, U i U j equal to sigma square provided, I equal to j. This is how the condition all about.

So similarly, in the same time, in this particular equation, covariance of U t i t must be U t i t must be equal to 0. U t i t must be equal to 0. That means what is U t i t? Now I will integrate this 2 equations, then I will get Y t equal to Y t equal to beta 0 plus beta 1 Y t plus plus i t plus U t plus U t.

So that means, this is one variable, this is another variable, this is another variable, another variable. Now, no in fact this is C t so C t. So, I just what I have done, so I put C t Y t into this particular system. So, put Y t in this particular system because we are now discussing the consumption determination in a particular Cancian economic setup.

So now, C t, the putting equation 1 in equation 2 so, C t equal to beta 0 beta 1 Y t plus i t plus U t. So, this is this is the models which we like to estimate. So, that means we like to estimate beta 0 here, beta 1 here and here, i t is there is no coefficient at all.

So anyway, so we will introduce this concept but, in the meantime we are just clubbing. so What we will what we can do here, now C t equal to C t equal to beta 0. C t equal to beta 0 beta 0 plus beta beta 1 Y t plus i t plus U t.

So now, what I have to do so I will I will take like this, Y t say, Y t equal to put C t here so C t equal to beta 0 plus beta 1 Y t plus U t plus i t. Now, these and these so Y t minus beta 1 Y t is equal to beta 0 beta 0 plus plus i t plus i t plus U t plus i t plus U t.

So now, take Y t common so, that implies if you will take Y t common Y t common then it is 1 minus beta 1 equal to beta 0 plus i t plus U t. So, that means Y t equal to beta 0 by 1 minus beta 1 plus i t by 1 minus beta 1 plus U t by 1 minus beta 1. So, this is how we have derived.

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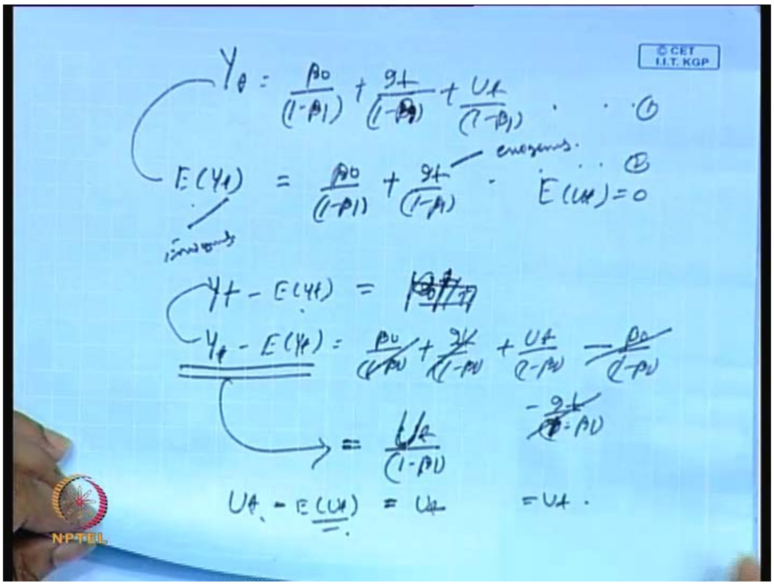

So now, so if we will further simplify, then, you see here, so what is this simplification here? The latest equation is Y t equal to beta 0 beta 0 1 minus beta 1 1 minus beta 1 plus i t by 1 minus beta 2 no sorry 1 minus beta 1, so write 1 minus beta 1 1 minus beta 1 plus U t by 1 minus beta 1 1 minus beta 1.

So now, we we we have E Y t mean of Y t that is nothing but, beta 0 by 1 minus beta 1 1 minus beta 1 plus i t by 1 minus beta 1 1 minus beta 1 because because E of U t E of U t. We have assumed as a 0, so this means U i t is this much. so Then similarly, similarly, i t being a exogenous variables. so This is exogenous variable exogenous variable and this is endogenous variables this is endogenous variables.

So now, what you have to do, so we will make a difference this and this. So, Y t minus E Y t Y t minus Y t put it equation number 1. So, this is equation number 2. Now, subtracting equation 1 to equation 2, so we will have Y t minus E Y t equal to simply U t. means You see here, beta 0 beta 0 by 1 plus sorry I will write once again.

So, Y t minus E Y t is nothing but, beta 0 by 1 minus beta 1 plus i t by 1 minus beta 1 plus U t by 1 minus beta 1 minus beta 0 by 1 minus beta 1 minus i t by 1 minus 1 minus beta 1 1 minus beta 1. So, this cancels, this cancels, so that means it is nothing but, U t by U t by U t by 1 minus beta 1 U t by 1 minus beta 1.

So, Y t Y t is nothing but, this is U t by 1 minus beta 1. So similarly, we can calculate U t minus E U t. So, that means U t U t is already there. So, minus E U t E U t, so which is nothing but, U t because we just like to it put it in a deviation format. This is U t equal to E minus U t. So, this is so that means U t equal to 0. So this is only U t. So now, so you you have Y t minus U i t. You have U t minus U i t.

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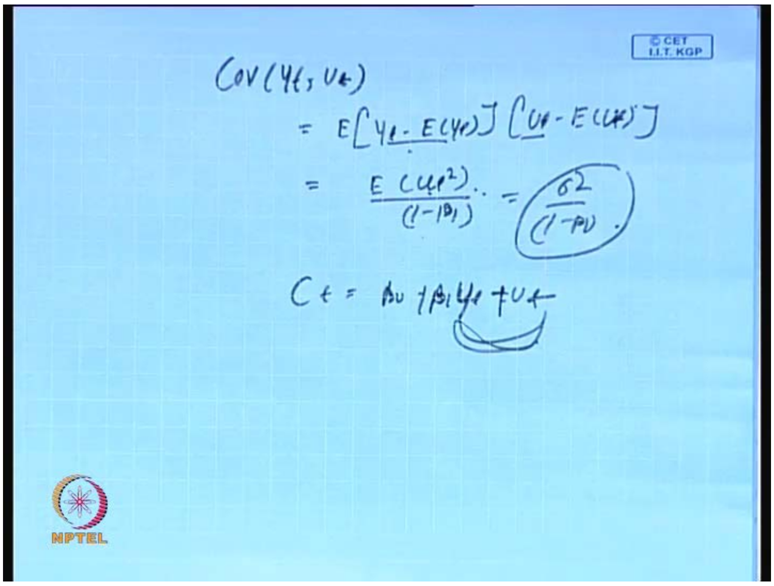

So, what is the next? so We like to know, covariance of covariance of covariance of U t and Y t and U t covariance U t Y t. So, this is nothing but, E upon E upon Y t minus E upon Y t then U t minus E upon U t. So, this is how the cluster is all about.

So now, which is nothing but, see E U t squares U t squares by 1 minus beta 1 1 minus beta 1, because this is U t and this is U t. So, the difference we have received, U t by 1 minus beta 1 so 1 minus beta 1 into U t into E t so E U t square. This will give you sigma square 1 minus beta 1 sigma square 1 minus beta 1. You see, here sigma square is positive. So, covariance of Y t and U t cannot be 0 because we start with C t, we start with C t equal to beta 0 plus beta 1 U t plus U t beta 1 beta 1 Y t beta 1 Y t plus U t.

So, that means this should be equal to 0, by the standard assumption of OLS. Just like you know, when I am discussing about the multivariate regression modelling. So, I just highlighted few minutes back that, all the variables should not have any linear relationship. So, that that leads to multicolinearity problem and in the several in the same time, the independent variable should not have any linkage with or any linear relationship with the error terms; if it is there, then it is called bias.

So now, in the simultaneous equation system, if we will apply OLS directly then obviously the systematic bias will be always there. So, sigma, since sigma square cannot be cannot be 0, so that is why covariance of Y t and U t U t cannot be equal to 0. So, as a result, it goes against the OLS property.

So, once it goes against the OLS properties, so the so the standard assumption, standard interpretation is that or you can say conclusion is that, so there is always systematic bias when we will apply OLS, direct OLS technique to you know, a single equation in a simultaneous equation system. So, that means, what is these means the ultimate conclusion is that this ultimate conclusion is that that means OLS technique cannot be directly used in the case of simultaneous equation system.

So, there are several techniques, other alternative techniques are available so to handle the simultaneous equation modelling. Now, in a means in this particular system, since one equation depends upon other equation and parameters are interdependent, variables are interdependent, then, obviously it is very difficult to handle this type of this type of problem by O L S technique. So, there are standard, several other techniques. are there so With the help of all these other techniques, we can solve the simultaneous equation system.

But, before we apply the simultaneous equations, means other techniques, to solve the simultaneous equation system, so we like to first highlight the issue of identification that means, specifically in the simultaneous equation modeling. Generally, we face two different problems, one is called as a identification problem, another is called as a estimation problem. So that means, since it is a system where there are several equations, are there so each equation has to be identified, to know each equation. means Once, your objective is to identify each equation in the system, then this leads to identification problem and say in the same time, each equation has to be estimated, so in that case then you know, there is estimation problem.

So, as a result that means because we have we have just proved that OLS techniques will give you biased and inconsistent results. So, as a result so in the simultaneous equation system, so there is always identification problem and there is always estimation problem. So, obviously our issue is how to identify a particular equation in a particular system and in the same time, we like to know, what are the techniques we usually apply in the simultaneous equation system so that, so the estimated model will be efficient, reliable and you can say, very much unbiased and can be used for you can say forecasting and policy use.

So, in the next class, we start with the discussion of identification problem and also, we will discuss the estimation problems. So, with this we will conclude this particular session. Thank you. Have a nice day.