

Econometric Modeling
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Lecture No. # 23
Autocorrelation Problem (Contd.)

Good evening, this is Doctor Pradhan here, welcome to NPTEL project on econometric modelling. So, today we will continue the autocorrelation problem. So, in the last **last** lecture, we have discussed the basic framework of autocorrelation. So, let me highlight here, what is the basic issue of autocorrelations.

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Auto Correlation.

$$Y_t = \alpha + \beta X_t + U_t$$
$$U_t = \rho U_{t-1} + V_t$$
$$U_t = \sum_{s=0}^{\infty} \rho^s V_{t-s}$$

1. What is the problem Auto.
2. What are the causes of Auto.
3. Consequences
4. Detection
5. Removal measures.

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So, for autocorrelation we have the function like this, Y_t equal to α plus βX_t plus U_t so, where U_t is equal to $\rho U_{t-1} + V_t$. So, by the way, if we will expand, then you know U_t ultimately will be summation ρ to the power s V_{t-s} and s equal to 0 to n or infinity. When it will equal to infinity, then obviously the structure of autocorrelation is like this.

So, now the framework we have already discussed. So, right now we will be very specific. What is the exact issue? How you have to detect? What is its impact? And how

we are going to solve these particular problems? So, that means as far as autocorrelation is concerned so, we like to highlight here what is the exact problem? What is the problem of autocorrelations? Second is, what are the causes **what are the causes** of autocorrelations? Third: consequences four: detections, then fifth: a removal measures.

So, all together 5 components we are going to discuss. So, what is the basic framework of autocorrelations? What are the causes? Why autocorrelation happens? Then what are the consequences? Then detection criteria, then the procedural measures. Now, autocorrelation basically deals with correlation among the same variables, that too in error terms here. It is a deductive part of regression modelling or econometric modelling.

So, the moment which you see what is the actual procedure of econometric modelling? So, you have the mathematical problem model then, you get the statistical form of the model, then you estimate the model, you have the estimated model. So, now true model estimated model by the way we will get the error terms.

Now the moment you will get the error term, then the game will be very interesting. So, the way we will analyze the error terms, one of such problem we call as a autocorrelation problem. So, that means autocorrelation is the correlation or it is the linear association among the error terms in this particular econometric setup.

So, now autocorrelations like multicollinearity it is the degree of association, among the error terms. In the multicollinearity, we are discussing degree of association among the regressor that too linear in nature. But in the case of autocorrelation it may not be very specific linear issue it may be non linear in nature. So, that means it may be positive, it may be negative, it may be linear, it may be non-linear like this.

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$Y_t = \alpha + \beta X_t + U_t$
 $U_t = \rho U_{t-1} + V_t$
 $-1 \leq \rho \leq 1$
 ρ : Auto Correlation problem.
 $\rho = 0$ — No autocorrelation problem.
 $\rho = -1$ — Perfectly negatively autocorrelated.
 $\rho = 1$ — Perfectly positively autocorrelated.
 $\rho > 1$:
 $\rho < -1$:

So, now once you have Y_t equal to α plus βX_t plus U_t . Then U_t equal to $\rho U_{t-1} + V_t$. So, ρ generally lies between minus 1 to plus 1. So, you see here ρ is the signal of autocorrelation problem. Then, there are 3 extremes. So, ρ equal to 0, ρ equal to minus 1, ρ equal to 1, ρ greater than 1 and ρ less than 1.

So, these are the five. These are the 3 extremes besides there are two other cases. So, if ρ equal to 0 then, it is an indication of no autocorrelation. So, that means there is a no degree of association between the 2 error terms. So, that is called as a no autocorrelation problem. So, this is when ρ equal to 0. Then, this is called as a perfectly negatively auto correlated **negatively auto correlated** then, it is a perfectly positively auto correlated.

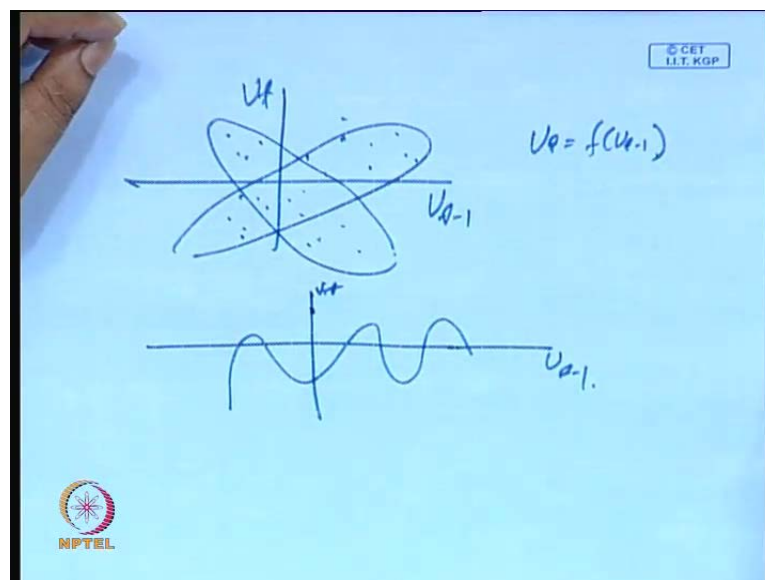
Then, so far as greater than 1 less than 1 it is very complex issue. How you have to properly interpret until unless you know the exact value of the term ρ . So, it greater than 1 means it may be 2, it may be 1.2, it may be 1.3, it may be 10, it may be 20, it may be 30 like this. So, similarly less than 1 means it may be 0.1, 0.2, 0.3, 0.4 like this. So, the range will be completely different until and unless you know the exact range then, you cannot interpret properly, because like multicollinearity, auto correlation cannot be removed completely.

So, autocorrelation most of the cases it will be always there. But, so problem is how much you have to bear it? So, what is the extent where the autocorrelation is not serious

problem? That will not affect the goodness fit of the model because ultimately so, our aim is to have the best fitted model which is free from all such problem.

So, now to have the best fitted models autocorrelation cannot be removed completely but autocorrelation there should be specific range. If the value of autocorrelation will be that specific range then, obviously it will give you the indication of best fitness of the model. So, otherwise it will be very problematic. So, that means like you see here graphically the picture will be coming like this.

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So far as autocorrelation problem is concerned let us say, I will take it here U_t and I will take to here U_{t-1} . So, with respect to U_{t-1} I have to plot U_t so that means U_t is function of U_{t-1} . So, one way I have taken it U_{t-1} . So, corresponding to U_{t-1} the picture will be like this. So, this may be another one way this will be another way so, this is how the linear relationship all about.

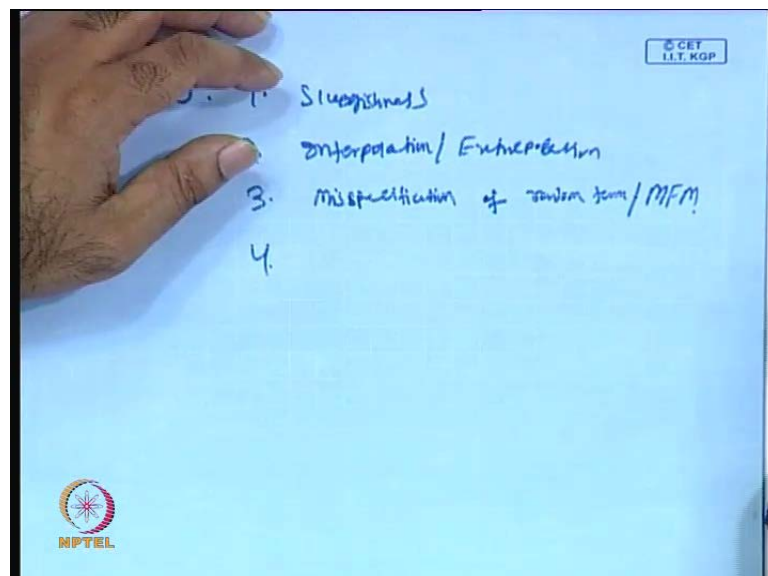
So, as far as non linearship is concerned then, the picture will be like this. So, this may be like this sometime half, sometime $(())$ sometime positive, sometime negative so, like this is U_t , this is U_{t-1} . So, that means here two things are very important. First thing is what is the degree of association among the error terms? This is the first issue if, degree is too high it is problem, if it is too low it is problem. So, it should be the optimum one. So, we will justify what should be the optimum one.

So, until unless you go to that particular structure we cannot analyze it, we cannot say right now. So, this is one aspect, and second aspect is the relationship, degree of association means it is otherwise known as degrees of relationship. Now, the relationship can be linear can be nonlinear, can be positive, can be negative. So, whether it is positive negative it is not at all issue. So, ultimately if it is highly positive or highly negative, it is lower positive, lower negative then, obviously it is a serious problem.

So, it should be in a range, and that range has to be perfectly optimum. So far as the best fitness of the model cannot be go other way around. So, that is how the entire system is all about. And, the relationship can be nonlinear also. Which has one of the short comes in the case of multicollinearity. **Multicollinearity** is the purely linear relationship, here there is no such linear relationship among the error terms, it can be linear, it can be non linear.

So, ultimately this is how the structure of the autocorrelation problem so, that means we get to know what is autocorrelation? How is it natures, and how we have to represent the situations? So, that is what the autocorrelation. Now, we like to know why autocorrelation usually happen in the case of econometric modelling.

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So, the question is why? Why there is an autocorrelation problem? So, there is term called as sluggishness. Most of the times this variable in the real life situation are very

interdependent. If they are very interdependent then, obviously there committing of error will also interdependent. So, that is why by default there will be autocorrelation problem.

Because, it is the association among the error terms, but in real life situations what we have observed that most of the time series variable are interdependent. If most of the time series variable are interdependent their committing error should be obviously by default will be interdependent. So, that is one of the most important factors why autocorrelation exist in the econometric modelling.

Second is the intrapolation and extrapolation. Intrapolation extrapolation is a mathematical technique which sometimes is very helpful for increasing data points that is increasing the size of the data points. So, that means sometimes you will extrapolate that (()). For instance, suppose there are 3 variables, one variable has 10, another variable is 15, another variable is say 12.

So, that means it is going against the econometric systems, because econometric system needs that all these 3 variables are uniform observations. But now here 10, 15, 12 so, obviously either you can go with 10 observations only without going for any intrapolation and extrapolation or you can say you have to extrapolate the data from 10 to 15 and 12 to 15. Then, you globe it or 10 to 12 then, 15 out of 15 you have to take 12, and already third variable 12 is there.

So, you can fit the model, that is another way. Third is you can extend both the variable to 15 then, obviously it will give you consistency samples size for all these 3 variables so, that you can you can go for estimations. This is one way how extrapolation will help you to increase the sample size. There is a very interesting technique, interpolation is one technique but under this technique there are several methods through which you can extrapolate the data.

Similarly, suppose there is some missing observation in between so, with the help of this interpolation and extrapolation you can also fulfill this missing observation. It may not be technically or mathematically correct but theoretically or you can say practically it is not so problem. In fact if we will go statistically, then this process is also not very positive process until unless you check something else.

Then, one such thing is called as a autocorrelation. That means the moment you will use interpolation and extrapolation to increase the sample size then, obviously one hand it will increase the sample size, and it will improve the degrees of freedom, and it will be very much helpful for significance of the parameters or you can say overall fitness of the model.

But in the other side it will add extra problem to this issue is called as a autocorrelation. You remember one thing if you have a model, and all these specification parameters are highly statistically significant then, overall fitness of the model denoted by r square is absolutely very high, and f is also very significant. Then, it can be considered as best fit models but in the same times if the autocorrelation value is extremely high or extremely low then, still the model cannot be considered as the best fitted model.

So, that means what we have discussed till now, the fitness of the model is not sufficient. Because, this is the necessary items which we have discussed in today(s) lecture or in future lectures we will discuss several such type of ratios, where the best fitness of the model is not a very small problem or small issue. It is a very complicated very compressed pictures, you will get to know when you have various aspects of knowledge on econometrics then, you can come exactly to a point that it is not so easy that you will get the best fitted models after so many clicks only.

So, it is very very complicated problem because so, many issues you have to discuss at a time. So, that means interpolation and extrapolation is another cause which can bring the autocorrelation issue. So similarly, so this first is sluggishness then, second is a interpolation or extrapolations, and third: misspecifications or random term.

Generally, we use error terms or random terms just to adjust the models. In a particular problem, in a particular setup is very very difficult to identify all such variable which can influence Y . There are certain factors which cannot be measure, which cannot be easily quantify, which cannot be easily represent in a or introduce in the particular systems like drought, famine, earthquake, terrorist attack, log factors these are the things it cannot be easily measurable.

So, for these factors the proxy is used as a error component. In fact, there is another type of proxy called as a dummy variable technique. So, we will discuss little bit later but in the case of these variables or for these things we are introducing the error term. That

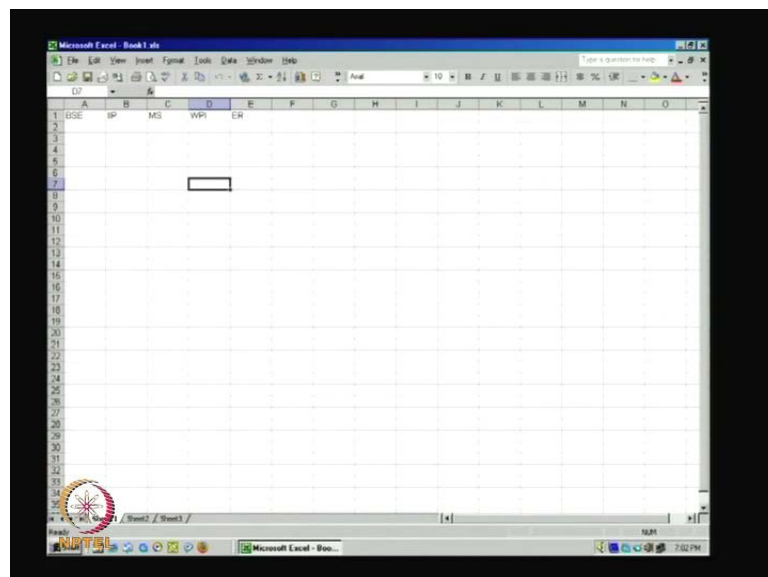
means we are observing that the variables which we are not incorporate that can be influenced by U term only.

But there are certain cases it is not that every time whatever variable you could not identify that you have to put it in error term. So, there should not be situation that the percentage of error should overtake the percentage of x plane variable. So, if the x plane variable percentage will be very high than error percentage then, obviously you are in the right track. But if the other way around then, you are in the wrong track, and that is one of the major issues, and that may lead to the serious autocorrelation problem.

Then, remember autocorrelation is a negative aspect of best fitness of the model. So, the values should be optimum, and it should be as per the accuracy of best fitness of the model. Otherwise you have to redesign, formulate and estimate the model till you get the better fitted model.

So, misspecification of random term or error term is also one of the factors which can influence. For instance, let us say last class I have discussed the stock price that is B S E as a function of index of industrial productions, money supply, whole sale price index and exchange rate.

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So, now as per my knowledge, that the stock price as a function of economic growth, money supply, inflation and exchange rate. So, obviously when we will fit this model

there will be some error term here. So, that the error term which can capture some of the other variables like government stability or you can say governmental policy, market news that may influence the stock price.

But, since we are not representing all these items at a time here, and it cannot be fitted in the model directly. So, we are assuming that these U will capture all these weightage to this stock price. But you remember I I P G measureable, quantified, and it is readily available. Money supply is quantified, measurable and readily available. Wholesale price index, indicator of inflation is readily measurable and it is also available, exchange rate is already readily available.

So, now by any chance artificially what we will do? We will drop inflation factors or let say we will influence; you will drop the money supply factors. Then, obviously if you drop all these various variable artificially then, it may be miss specified it is not a question of maybe it is definitely the case of misspecification. So, that is one way the misspecification order and similarly, that leads to in fact a autocorrelation problem.

Similarly, there is mathematical imperfection of the model. For instance, we are discussing about this particular setup. In that particular setup, we are representing that the model will be linear in nature so, the way we have discussed $Y_i = \alpha + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \dots + \beta_k X_{ik}$.

So, in that particular context, we are putting all these things in a linear set. But most of the things cannot be in a linear set so, there are certain things it can be nonlinear in nature but ultimately you have to transfer into linear by using lots of transformation rule which we will discuss sometimes later.

But, if you keep something needed transformation, and if you are not doing and you are going with the original setup. Then, obviously it will lead to autocorrelation by default, mathematical identification of the model is not correct so, there is a mathematical imperfection. So, that is how there is way to autocorrelation problem. So, this is misspecification of the random term or mathematical form of the models there may be two reasons.

Then, I have just identified here, that a stock price as a function of index of industrial production, money supply, wholesale price index and exchange rate. So, I will add

another items say C P I consumer price index sometimes, whole sale index and consumer price index they may have different effect.

But sometimes they look like same. Now, if I will put B S E function of I I P, M S, wholesale price index, exchange rate then, obviously, and also function of consumer price index. Then, obviously this is called as a overidentified models, means unnecessarily we are introducing consumer price index where you have already use the inflation factor W P I. Now, in that particular case it will lead to one of such problem called as say autocorrelation problem.

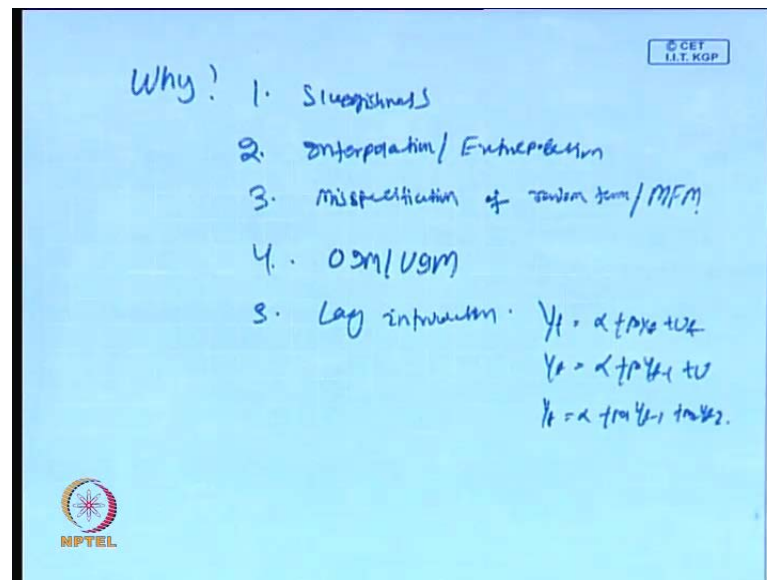
Similarly, suppose this is the optimum case or you just increase these terms so, that it can be visible properly. Now, we have taken B S E as a function I I P, money supply, wholesale price index and exchange rate. Now, if we add another variable say consumer price index into the systems then, there is enough possibility that the model is called as over identifying model. If there is identifying model then, definitely there will be some kind of autocorrelation means high presence of autocorrelation.

Similarly, what I will say this may be the optimum case but, artificially, I will drop variables say money supply. So, I will just now integrate B S E with I I P, W P I and E R. In that particular case it is called as a underidentified models. Underidentified models are also have a autocorrelation problem that means, you need to have a optimum model.

So, optimum specification is very important that is how very beginning I have very clearly mentioned that your theoretical knowledge must be very perfect, and before you go for econometric modelling your problem formulation or setting up of the problem should be perfectly ok.

If it is not perfectly ok then, obviously there may be chance of misspecification of the random term, and there may be chance of misspecification of the mathematical form of the (()) model. There may be chance of you can say overidentified model, and underidentified model. In all these cases the ultimate result is the high and high autocorrelation, and if this is the case then, obviously this model cannot be considered as a best fitted model. So, we must be very careful about that.

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So, that is how it is called as a over identify model identify or under identify model so, this is another reasons. Since, sometimes there is question of lag introduction. We are just discussing $Y_t = \alpha + \beta X_t + U_t$ but I will take models like this. Lag introduction means instead of taking $Y_t = \alpha + \beta X_t + U_t$ I can take Y_t as a function of $\alpha + \beta Y_{t-1} + U_t$ or I can take $Y_t = \alpha + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + U_t$.

So, this is how I can also extend. If I will follow this particular structure then, it is called as a lag introduction. If there is lag introduction then, there is enough chance that multicollinearity will increase and in the enough chance there is high and high autocorrelation. So, lag introduction will lead to autocorrelation program but you cannot avoid it. These are all sometimes very much required for different point of **(())** B O M for different objectives, and different interesting structure of econometric modeling.

So, these are all required you we cannot avoid all this thing but within that problems you have to find out the optimum solution. So, the another cause is called as a manipulation of data, sometimes lots of people with respect to lack of data availability so, they sometimes manipulate the data. The moment you will manipulate the data, then obviously one of the testing problem will be the autocorrelation problem.

So, you must be very careful about that. So, another is manipulation of data then similarly, there is a term called as stationarity. So, stationarity is another issue through

which autocorrelation problem can be detected. If the variable is not stationary, that means what is mean by stationary? Stationary means a series having a mean variance are equal with respect to different time (()) then, it is called as stationary.

If there is any kind of volatility in between then, it is called as non stationary or that is usually called as unit root problem. We have a separate chapters to discuss all these unit root problems. So, right now we are now discussing, but you remember one thing. So, a non stationarity issue is also sort of violation problems. So similarly, we have just now discussed the transformation rule, if the proper transformation rule is not ok then, obviously it will lead to autocorrelation problem.

These are the specification and through which we can say, you have to represent the various reasons so, that autocorrelation can occur. So, it is by default there will be autocorrelation because, there are many factors which cannot be near control which can **forcely** lead to autocorrelation problem Yes, we need to what is the alternative? We need to have some solution means you have to make lots of compromise so, to get a best fitted model. So now, this is the cause behind this autocorrelation then, next is detection criteria.

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Detection Criteria

DW Statistics: 'D'

$$Y_t = \alpha + \beta_1 Y_{t-1} + u_t$$

$$u_t = \rho u_{t-1} + v_t$$

$$D = \frac{\sum_{t=2}^n (e_t - e_{t-1})^2}{\sum_{t=1}^n e_t^2} \quad f = \frac{\text{cov}(u_t, u_{t-1})}{\text{var}(u_t)}$$

$$= \frac{\sum_{t=2}^n e_t^2 + \sum_{t=2}^n e_{t-1}^2 - 2 \sum_{t=2}^n e_t e_{t-1}}{\sum_{t=1}^n e_t^2}$$

$$= \frac{2 \sum_{t=2}^n e_t^2 - 2 \sum_{t=2}^n e_t e_{t-1}}{\sum_{t=1}^n e_t^2}$$

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You remember one thing, there are numbers of techniques are available to measure the autocorrelation. The basic objective is to know what is the degree? That is more important degree of association among the error terms. So, what is the amount of degree

of association? that is more important to project or to connect or to integrate the best fitness of the model.

So, one of the standard measures to know the level of autocorrelation is called as Durbin Watson d statistic. It is otherwise called as a D W statistics. What is D W statistics? It is sometimes denoted as small d so, d is calculated as a summation $e_t - e_{t-1}$ whole square $t = 2$ to n by summation e_t^2 $t = 1$ to n. Why it is $t = 2$ here because, if it is 1 then it becomes 0. So, there is an associative term so, obviously if it is 2 then, it will be $t - 1$ then it will be 0. So, obviously it is **unsignificant** also, because of if we will start with $t = 1$ so, obviously the starting point is equal to $t - 1$.

So, now you see we like to know what is the Durbin Watson d statistic. D value because will influence the degree of autocorrelation present in the system. So, how do we calculate for that? So, let us we simplify this one so, if we will simplify then, it will be coming summation e_t^2 plus summation e_{t-1}^2 minus 2 summation $e_t e_{t-1}$ into $e_t - e_{t-1}$. So, $t = 2$ to n then, this is summation $t = 2$ to n then, summation $t = 1$ to n, divide by of course, $t = 2$ to n here. So, summation e_t^2 $t = 1$ to n. So, this is how the simplification is all about.

So, now you see here. So, what I have to do? But you remember summation e_t^2 is equal to summation e_{t-1}^2 , when $t = 2$ to infinity. When n stands for infinity then, the difference between e_t and e_{t-1} will be equal. So, that means when n is large and large and large then, obviously in the lag will be minimum, the lag difference will be very minimum.

So, that is why summation e_t^2 is equal to summation e_{t-1}^2 . So, that means so, we can call it $2 \sum e_t^2$ $t = 2$ to n minus 2 summation $e_t e_{t-1}$ divide by summation e_t^2 $t = 1$ to n, $t = 1$ to n. So, then this is $t = 1$ by summation e_t^2 , $t = 1$ to n.

So, is already there so, $e_t - e_{t-1}$ square $e_t - e_{t-1}$ square equal to 1 so, you will get the result $2 - 2$ this particular item is (C) . Remember, we start with $Y_t = \alpha + \beta X_t + U_t$ and $U_t = \rho U_{t-1} + b_t$ so, that means a rho means this rho is the covariance of U_t , U_{t-1} by U_t variance of U_t . So, this is how the rho component and this is how the rho is all about.

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The image shows a whiteboard with handwritten notes. At the top, the formula $d = 2(1 - \rho)$ is boxed, with the condition $-1 \leq \rho \leq 1$ written below it. Below this, three cases are listed:

- For $\rho = 0$, $d = 2(1 - 0) = 2(1 - 0) = 2$. Below this, it says $d = 2$: no autocorrelation.
- For $\rho = -1$, $d = 2(1 - (-1)) = 2 \cdot 2 = 4$, perfectly negative autocorrelation.
- For $\rho = 1$, $d = \frac{2(1 - 1)}{0} = \frac{0}{0}$, perfectly positive autocorrelation.

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So, that means we have the point here So that means if we will simplify further then, d will be coming like this d equal to 2 into 1 minus rho hat. So, this is the Durbin Watson d statistic. So, Durbin Watson d statistic d is represented by 2 into 1 minus rho. So you see here rho we have highlighted there which is equal to minus 1 less than equal to 1 corresponding to this value d value will so indicate the degree of association.

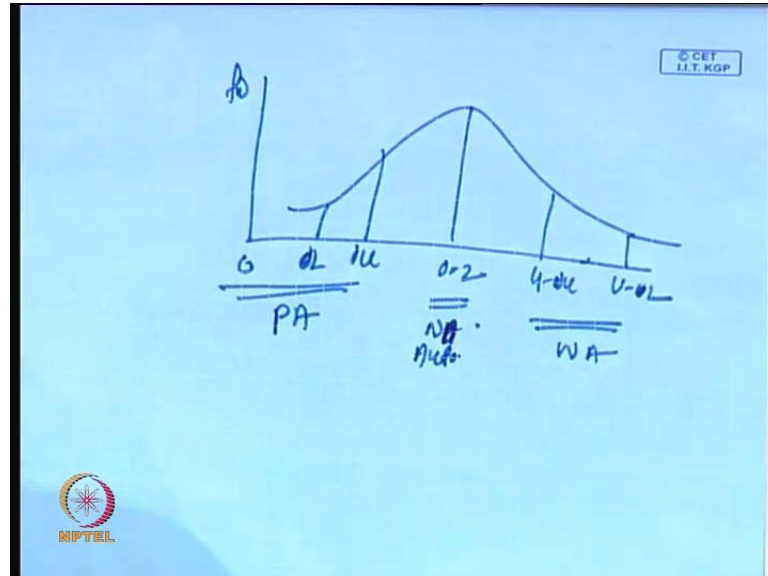
Because without having the Durbin Watson d statistic theoretically we justify that, if the rho value is diverge towards the minus 1 or you can say diverge towards plus 1 then, obviously this will lead to autocorrelation problem at the higher degree. So, if it is a converse to 0 then, obviously the degree of autocorrelation is very low. Now, putting all these value then we like to know what is the d structure, you see here.

So, we have the one case, different case, extreme case is rho is equal to 0. If rho equal to 0, then d equal to 2 into 1 minus rho; so that means 2 into 1 minus 0 so, it will be ultimately equal to 2. So, that means if rho equal to 0 then, d equals to simply 2. Now, since rho equal to 0 no autocorrelation now, when d value is there then, it will indicate that there is no autocorrelation.

So, similarly if rho equal to minus 1, then obviously d equal to 2 into 1 minus into minus one; so it will be equal to 2 into 1 plus 1 so 2 so this will be equal to 4 only; that means when rho equal to minus 1, d equal to 4 so, that means it is perfectly negative autocorrelation. So, now if we will put rho equal to 1 then, d equal to 2 into 1 minus 1 so

d equal to $2 - 1$ so it is nothing but 0 so, $2 - 0$ it will be equal to 0 then, it is called as a perfectly negative autocorrelation.

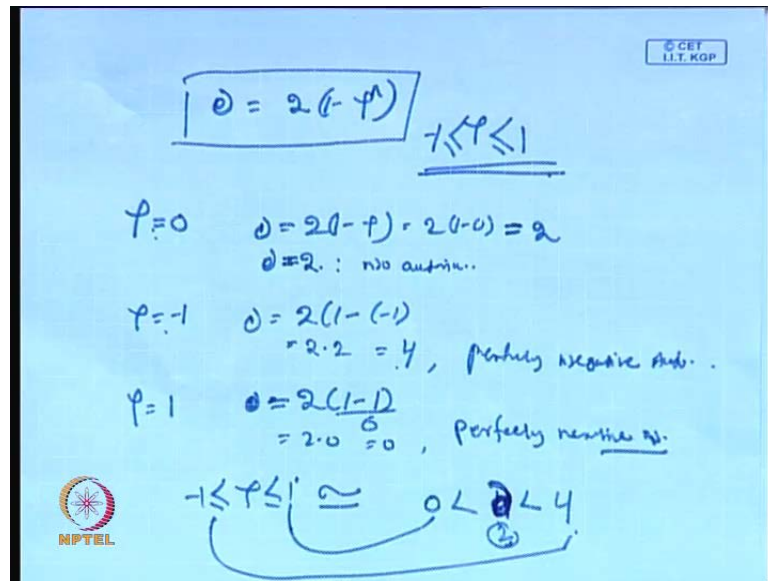
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Now, what is the situation here? So the picture will be coming like this so, I will draw here this is 0, d , and I will draw a curve like this so, this is the situation where d equal to 2, and I will take d upper limit d Durbin Watson and Durbin lower limit then, this is $4 - d$ then, this is $4 - d$ then, this is $4 - d$ so, this side is negative auto, and this side is positive auto and this side no autocorrelations.

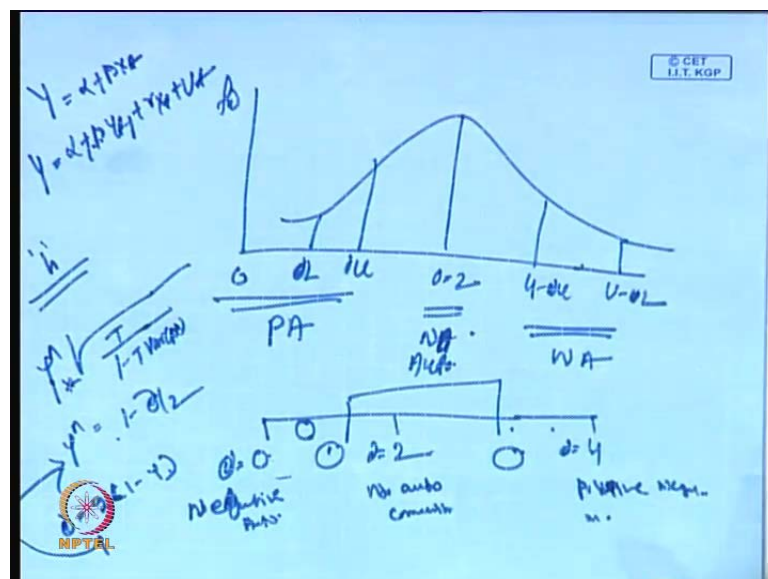
So, this is how this structure is all about. So, that means if you will simplify this one so, that means generally when we will go for any statistical software particularly you have views or you can say S P S S or you can say matlab etcetera. Then, obviously they will directly give you the Durbin Watson value statistic.

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So, now the way autocorrelation coefficient ranges between minus 1 to plus 1. Then, if you will simplify in terms of d then, d ranges from minus 1 is 2 here so, 4 then 0 so, that means 0 less than 4 4 less than 4. So, this will be d value, when it is 0 0 is called as plus 1 situation so, that means this will become like this and this will come to minus 1 it will be 4 like this. So, in between when there is 2 then, 2 will lead to zero autocorrelation.

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So, that means so, there are 3 extremes here so far as Durbin Watson statistic is concerned. So, one extreme is 0, another extreme is 2, another extreme is say 4. So, this

is 0. So, 0 means this is no auto sorry this is 0 levels so, that means it is d equal to 0 so, d equal to 2, d equal to four. So, when d equal to 0 then, obviously this particular structure is called as A, when d equal to 0, it is perfectly positive autocorrelation d equal to 0 it will be coming negative autocorrelation. So, this is no autocorrelation is positively negatively autocorrelation w d equal to 4.

So, that means it is just in opposite direction so, there are means the basic idea is that the Durbin Watson statistic is a three different results; so, d equal to 0, d equal to 2, d equal to 4. So, that leads to the autocorrelation coefficient lies between minus 1 to plus 1. So, now 0 2 is one extreme range and another extreme range is 2 to 4.

So, now it means it cannot go beyond 0, and it cannot go beyond the concept where that is so far as d variance value statistic so, what is the conclusion here why we need to transport these autocorrelation coefficient to Durbin Watson statistic because, autocorrelation has the negative dimension and positive dimension. But here we are putting all these transformation into the positive dimension. Of course, the interpretation is little bit different with different value. So, but we are in the positive side now, we have to see what is the positive sequence? That means if it is moving from 0 to 4 where the turning point is at the 2.

Now if it is close to 2 this side or close to the 2 that side then, obviously there is no such autocorrelation problem or it is in decreasing turned about autocorrelation towards the optimality. Now if it is close to 0 it is serious problem of autocorrelation, if it is close to 4 then it is problem of the autocorrelation So, ultimately the optimum range is in between something so, that means you have to find out its midpoint 0.2, and here you find out 2 to 4 the midpoint ok.

So, in between these two if the autocorrelation lies then, that you can say consider as the best value or best association in that association, in that range. Then the model can be considered as the best. But if it is less than that or more than that then, obviously it will lead to serious problems.

So, in that case you have to redesign the estimate till you get the better fitted models. This is about the way of the Durbin Watson statistic. However this statistic has a one interesting limitation. One interesting limitation is that this particular statistic cannot be

perfectly us when there is a distributive lag model, where there is dependent variable, and independent variable, and you can say lag dependent variable.

So, in that context it is a term called as the Durbin Watson statistic. Durbin f statistic is generally defined as like this so, in that case Durbin h statistic is calculated the value is rho hat which we have already derived square root of t by 1 minus t variance of beta hat, and provided the model will be like this. So, $Y_t = \alpha + \beta Y_{t-1} + \gamma X_t + U_t$. This is the model which we have derived, but ultimately our starting model is $Y_t = \alpha + \beta X_t$.

So, you can put it here, beta you can put it here no problem at all. Just we are interchanging so, this is rho hat equal to $1 - d$ by 2 which we have derived here, because, we have derived d equal to $2 - 2 + 1 - \rho$. So, similarly, if you will calculate simplify the rho hat equal to $1 - d$ by 2. So, rho multiplied by t in minus 1 by t is say number of sample observations, and variance of beta hat will be like this.

If we will calculate then, we will get Durbin h statistics so, then again Durbin h statistics will give you signal represents about correlation in fact the table is also there to know the optimum range. So this is how, we have to detect or you can say we have to know the degree of involvement of autocorrelation in the econometric modelling.

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Handwritten notes on a blue background:

- Top right: © IIT, KGP
- Equation 1: $Y_t = \alpha + \beta Y_{t-1} + \gamma X_t + U_t$
- Equation 2: $Y_t = \alpha + \beta Y_{t-1} + \gamma X_t$
- Equation 3: $U_t = U_t$
- Text: ESS, RSS, TSS
- Text: R^2, F, DW
- Text: $\rightarrow y$
- Text: $\rightarrow 0$
- Equation 4: $U_t = \delta_0 + \delta_1 X_t + \beta U_{t-1} + \beta^2 U_{t-2} + \dots$
- Text: R^2
- Equation 5: $(n-p)R^2 \approx y^2$
- Text: p -Lagrange
- Text: $\rightarrow 800$
- Bottom left: NPTEL logo

So, now we get to know what is the Durbin Watson statistic and there are certain other test also like Q test then, von Neumann ratio test then, b g test so, many things are there for instance if we will take b g test then, your starting point will be like this Y_t equal to $\alpha + \beta X_t + U_t$. Then, what you have to do after estimation then, we will get \hat{Y} equal to $\hat{\alpha} + \hat{\beta} X_t$.

So, by the way we will get error term e_t or you can say U_t . So, now what you have to do so, by any chance you will get $\hat{\alpha}$ then, $\hat{\beta}$, standard error of $\hat{\alpha}$, standard error of $\hat{\beta}$, t of $\hat{\alpha}$, t of $\hat{\beta}$, then R^2 then, F then we will add another component now Durbin Watson statistics. Particularly, for this autocorrelation issue so, R^2 F and DW provided there must be ESS and RSS then TSS . These are all the other side of the problem in spite of addition to this $\hat{\alpha}$ and $\hat{\beta}$, t of $\hat{\alpha}$ and t of $\hat{\beta}$. So, you need to have this is our assignment.

So, now if by any chance you if Durbin Watson statistics is say close to 4 or you can say close to 0. Then, obviously one needs to find out some solution and then, accordingly you have to test where that exact process is. For either you have to calculate this one or you have to be careful about this. That means by any chance if there is such autocorrelation then, you have to. This is how Durbin Watson will give indication but, another way once you have U_t and Y_t then, what you have to do? So, you have to integrate with U_t equal to $\alpha_0 + \alpha_1 X_t + \gamma_1 Y_{t-1} + \gamma_2 Y_{t-2} + \dots$

So, you set another regression equation then, you have their R^2 value. Here R^2 square before the situation and R^2 square after the situation. So, that means let us say you do not like to use Durbin Watson statistics so, what you have to do usual procedure. So, you have estimated model \hat{Y} equal to $\hat{\alpha} + \hat{\beta} X$ provided, a t of $\hat{\alpha}$, t of $\hat{\beta}$ then, you get to know what is the significance of this particular parameter whether it is 5 percent or 10 percent or 1 percent similarly, for $\hat{\beta}$.

Then other statistic for overall fitness of the model is ESS , RSS , TSS then R^2 adjusted R^2 and F statistic. But we are not discussing anything about the Durbin Watson statistics. But suppose this specification or statistical significant this side and statistical significant of the other side that is overall fitness of the model. Still the model

cannot be considered as the best fitted model until and unless you check this autocorrelation issue and now, we like to know whether this problem has autocorrelation or not? If autocorrelation at what extent or what degree.

So, Durbin Watson is one of the statistic which already given the idea how you have to detect the data correlation problem? And what is the optimum range where the existence of autocorrelation is not serious problem to best fitness of the models. So, now here what we have to do. Another way to detect the autocorrelation is the once you get U_t then, you fit another regression equation U_t as a function of X_t , U_{t-1} , U_{t-2} , and you check the exact number of lag length.

So, there is two statistic which we can detect that exact lag length that is called as AIC statistic, akaike information criteria and similarly, there is SDC criteria, a prediction criteria. So, like there is lots of criteria like this so, it will choose the lag length on the basis of lag length. Then, we have to fit a regression equation, and have the R square value.

So, if then accordingly you have to find out a statistic $n - p$ into R square followed by chi square distribution. So, that means p is orr here, and n is the sample size. If this chi square statistic is statistically significant then, there is auto autocorrelation is a serious problem. If that particular item is not significant then, that means there may be autocorrelation but, it is not a serious problem.

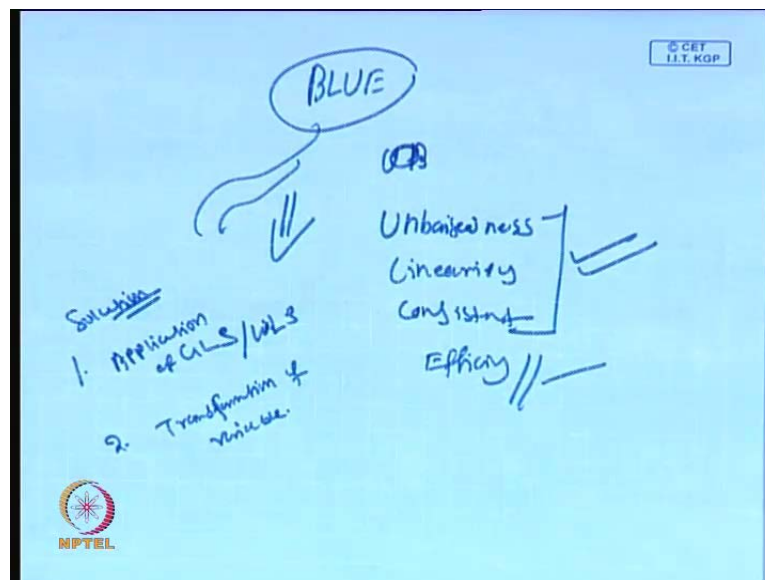
Ultimately, when the degrees you go by this way, and if you are not interested further we know Durbin Watson strategy but by any chance if you will get R square this chi square is significant here. Then, obviously you have to redesign, and reformulate but, when it will give you indication significant then, you are not sure what the exact range is? Whether by the criteria of Durbin Watson strategy, if the Durbin Watson strategy value is close to 2 or you can say 1.9 or 2.2 something like this. Then, in that case it is not a problem. So, autocorrelation is there, but that problem cannot effect the goodness fit of the model.

So, this is not a serious issue but in that case it will dictate the autocorrelation but it will not give you the degree of involvement of autocorrelation. That is why Durbin Watson statistic is considered as the best statistical tool to detect the autocorrelation problem. So, this is how you get to know the structure of this autocorrelation problem, causes and

detection criteria. In fact, there are multiple criteria but one of the interesting criteria is the Durbin Watson statistic, and it is a extreme range and optimum range.

So now, next question is what are the consequence if there is a high autocorrelation? Last class we have discussed that if there is multicollinearity again there is a problem whether you have to go ahead with multicollinearity or you have to solve the multicollinearity. That means that depends upon objective specification if your objective specification is you can go for prediction and obviously high R square with low significance t ratio you can go ahead. But if it is vice versa for instance, if for second objective for reliabilities constant then obviously, multicollinearity is a serious issue. So, you have to remove first then you have to go for forecasting or policy use; similarly, in the autocorrelation.

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So, autocorrelation obviously, when we will get a estimated model. It will go through this theorem best linear unbiased estimator that means what are the parameter you have received that should be considered as a best. So, now when there is autocorrelation it will go against the blue property. So, that means it cannot be biased, it may not be linear, it may not be unbiased (()).

That means it will give you high variance. If it is high variance then, obviously by default this statistical means this parameter will be low significant. So, to get high significant, high weightage and high goodness fit of the model. Then, obviously the variance of that particular parameter should be less and less.

So, if but, the default if there is autocorrelation problem then, obviously the variance will be very high for this particular parameter so that, what you have to do ultimately? So, you have to redesign or restructure the system till you get the best or optimum structure where you can use that model for forecasting or policy use. So, this is how the so, that means most probably when will go for, when you have autocorrelation problem. So, it will not affect all these blue property it will affect only few property. For instance, you have a property called as unbiasedness. Then, there is linearity then, there is consistent then, there is efficiency.

So, that means when there is autocorrelation these will be always there but it will not affect much. But it will affect the efficiency property that means, what is efficiency? That means it is the cluster of minimum variance, and unbiasedness. Unbiasedness property may be there but, it will lead to high and high variance so, that is how it can be means it can be in that case that variable, that model cannot be considered as the best fitted and that cannot be also used for forecasting or policy use.

So, that means you need to have a solutions so, that means if the particularly by Durbin Watson criteria if the value of autocorrelation is less than 1.5 or you can say more than 2.5 then, it is a serious problem. In that case there is a redesigning or re-estimation of that particular model.

So, now one of such interesting that means obviously problem is how to get the solution. So, what are the criteria to get the solution? So, there are two standard techniques to solve this autocorrelation problem. So, that is solutions issue. So solutions issue one standard of problem is use application of GLS technique that is generalized list square method or sometimes, we apply WLS technique. We will discuss detail when we will discuss the heteroscedasticity issue.

So, there is application of GLS technique, and another you have to go for transformation of variable. So, these are the two best solutions to solve the autocorrelation problem. So, that means if there is a autocorrelation problem instead of OLS you use different techniques like GLS and WLS, even if you will apply other techniques like maximum likelihood estimator that may solve the problem. But the best technique (()) to solve the autocorrelation problem; in addition to that so, you can go for transformation rule so, that variable can be transferred into shape then, after transformation if you use that particular

variable for modelling and obviously, it will satisfy the minimum variance criteria. That means altogether the model will follow the blue theorem best linear unbiased estimator; so in comparison which the previous situation after transformation situation.

So, now but, if you will use transformation immediately, you should not use the transformation rules, until and unless there is some serious problem. So, first you detect the problem then, you go for transformation otherwise, without any such problem it will go for transformation then, you will lose the originality.

So, that is why you first detect the problem if that is the problem is there then, you have to go for transformation, sort out the solution. This is how the autocorrelation all about so, we get to know what is the exact problem of autocorrelation? What is its impact? What are their futures? What is its nature? How it comes to the econometric modelling? That means what are the causes behind all these autocorrelation issue, the detection criteria and also its consequences and finally, what are the procedural measures through which we can solve the autocorrelation problem. With these, we can conclude this particular session thank you.