Econometric Modelling

Prof. Rudra P. Pradhan
Department of Management
Indian Institute of Technology, Kharagpur

Lecture No. # 17

Multivariate Econometric Modelling (Contd.)

Good afternoon, this is Dr. Pradhan here. Today, we will discuss the concept multivariate econometric modelling. In fact, we have discussed this concept last class little bit, so today we will continue from that particular point of view. The basic framework of econometric modelling means multivariate econometric modelling is that.

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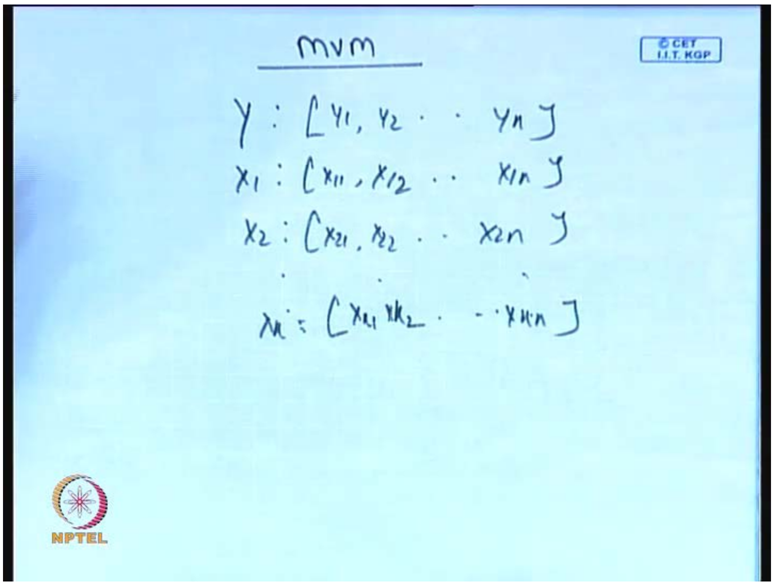

We have a system where there are dependent variables Y, which consist of Y 1, Y 2 up to Y n and corresponding variables are X X 1 say X 1 1, X 1 2 up to X 1 n then X 2 consist of X 2 1, X 2 2, X 2 n, so continue, so X k is equal to for X k it is nothing but X k 1, X k 2 X k 2 up to X k n. So that means, we are targeting here model econometric model where there is one dependent variable within it as Y and there are several independent variables that is here k number of variables are on the other sets as stated as a independent variables. So now, we like to see how is this setup in structure the complexity, the strategy of particular multivariate setup, where there are k independent variables and one dependent variable exist.

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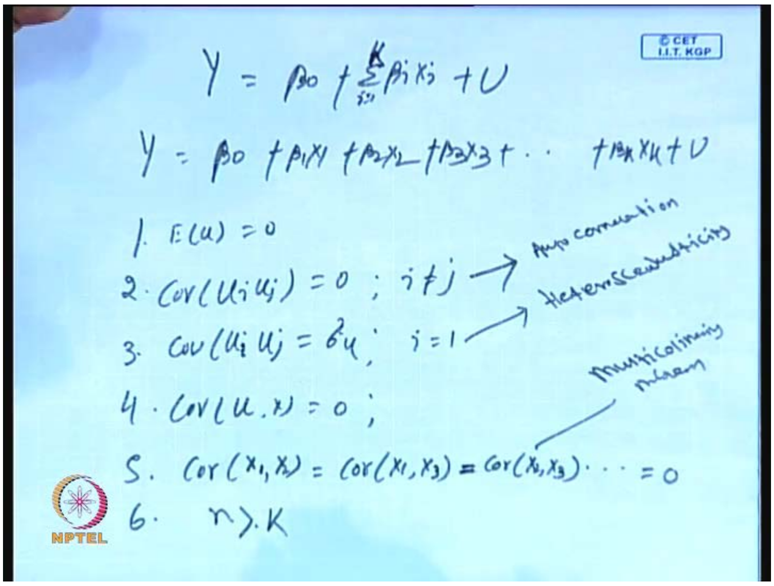

The basic framework is like that before we go to precede means to discuss about all these details about multivariate econometric modeling, so what I will like to represent here. So, here the basic framework of multivariate modelling is a like this; Y equal to beta 0 plus summation beta i X i, i equal to one to n, in fact this particular case it is equal to k then plus U, so this is this starting point of multivariate analysis. What we have to do? We will simplify this one, so this is equal to beta 0 plus beta 1 X 1 plus beta 2 X 2 plus beta 3 X 3 plus beta k X k plus U. So, these are the, this is the basic format of multivariate modeling.

So, now before we go for estimation so obviously by standard rule you have to know certain things or you have to assume certain things, so that we can able to go for this estimation, prediction and forecasting. The standard procedure is that as usual we have to apply o l s technique, because this is the standard technique through which you will get the original estimated models. Of course, we can apply what is least square methods; g l s generalize least square methods; maximum like include estimated methods. In the meantime, we are not discussing any methods here, so from the beginning and so we have you know discussing each and every models through o l s technique, so we also here present the o l s technique. So, later on we will discuss little bit about o l s technique; w w l s technique; g l s technique or maximum like hood estimators.

In the meantime by using o l s technique, we like to know what is the shape and structure of econometric modelling. Even if we will apply g l s; w l s and maximum likelihood estimators, the structure is model is of course the path is little bit different. So, here the standard assumption is that your e of error term must be equal to 0; that means, if we apply o l s technique, so before applying o l s technique you must have standard assumptions and on the basis of that standard assumption you have to estimate the models and later on the standard the assumption has to be verified, if not then we have to continuously redesign, restructure the model till you get the best fitted model, which can also satisfy the following o l s assumptions.

First standard assumption is that E u upon equal to 0, so then covariance of u i u j is equal to 0, so then third is covariance of u i u i U j is equal to sigma square u and in this case i not equal to j in this case i equal to j, so this is the, if that this is not satisfied then it is the serial correlation problem or autocorrelation problem, which in fact we discussed earlier, so this is if not exist then it is called as a heteroscedasticity problem; heteroscedasticity problem. This is heteroscedasticity problem and fourth assumption is that covariance of u and X must be equal to 0 and fifth assumption is that correlation or covariance between X 1 X 2 equal to or correlation between X 1 X 3 or equal to correlation between X 2 X 3 like this, so many so many pairs you will find, so it must be continue is also equal to 0. This this particular structure is called as a multicollinearity problem multicollinearity problem.

So, then of course the obvious standard assumption is that, your n should be substantially greater than equal to not equal to greater than k greater than k. So, n represents number of sample and k represents number of dependent number of variables in the systems. So, last but not the least model must be correctly specified, so last but not the least model is a correctly specified. That means, before we are apply o l s technique to this multivariate mathematical model to get the estimated model, so we have to assume certain things, these are the following assumption. On the basis of this assumption, we have to take or we have to estimate the multivariate regression model. So, how do you go for that?

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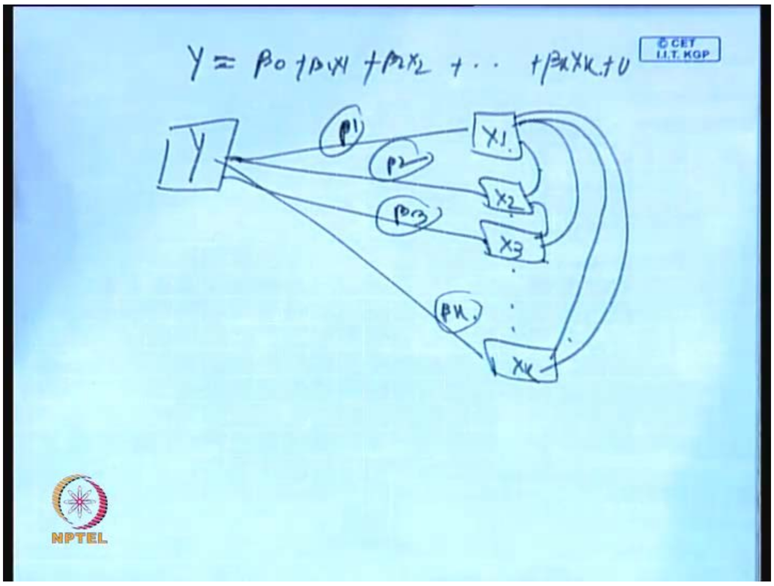

So, this is particular this particular structure is Y equal to beta 0 plus beta 1 X 1 plus beta 2 X 2 plus beat k X k, so this is how you to proceed plus of course U. So that means, if we will if we will put it in a particular format then the structure will be like this, this is X 1; this is X 2; this is X 3 and continue this is X k, so the structure will be coming like this, so this particular structure is coming like this, so followed by this is the beta 1 coefficient, this is beta 2 coefficient, this is beta 3 coefficient and this is beta k coefficient. So, by default there should not be any correlation between there should not be any correlation between X 1 and X 2 X 1 X 3 X 1 X k X 1 X 2 X 2 X 3 again X 1 X 3 and X 1 X k, so this is how the model is a fitted here. So, our our objective is here to know this particular this particular item, so this particular item. That means, we like to what is the estimated values of beta 0 hat, beta 1 hat, beat 2 hat up to beta k hat. So now, how do you proceed? So, as usual the way we have discussed in the case of bivariate setup and trivariate setup the following setup can be also apply here to get this estimate model.

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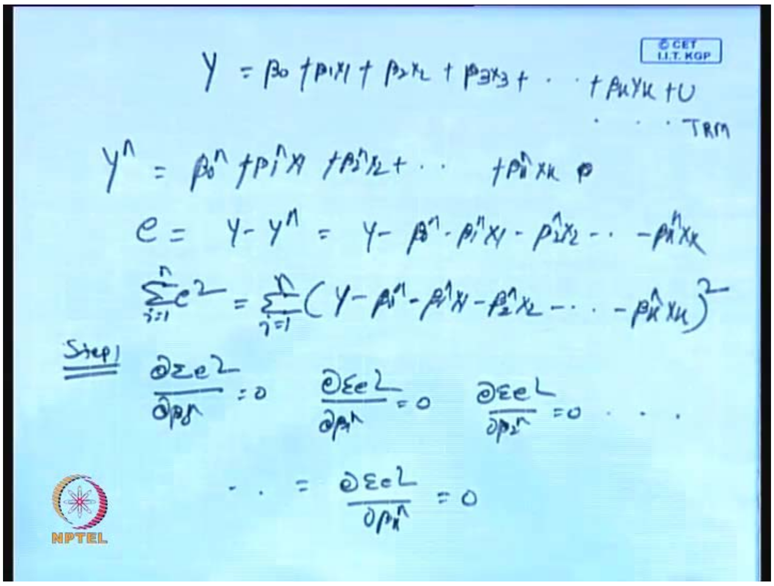

So, what is this exact structure here? The exact structure is that the exact structure is that we have Y equal to beta 0 plus beta 1 X 1 plus beta 2 X 2 plus beta 3 X 3 plus up to beta k X k plus U. So now, this is the two regression models two regression models, this is called as two regression model. Let us assume that the estimated model will be Y hat equal to beta 0 hat plus beta 1 hat X 1 plus beta 2 hat X 2 plus beta k hat X k plus; obviously, there is a no plus, so beta k hat X k. So, these are the means this is the estimated model this is the estimated regression model. So, we have a original regression model, then we have assume a estimated model, then by the by the way we have to make a difference we will get the error components. So, error is the difference between error is the difference between Y minus Y hat, which is nothing but Y minus beta 0 hat minus beta 1 hat X 1 minus beta 2 hat X 2 minus beta k hat X k.

So now, what have to do here? So, here we have to we have to minimize the error sum, so summation e square is i equal to 1 to n is equal to summation Y minus beta 0 hat minus beta 1 hat X 1 minus beta 2 hat X 2, so minus beta k hat X k; i equal to 1 to n this is whole squares. So, this is the this is the original setup for this o l s technique. So, from this the new new dimension will come up, so that means, we have to go by this optimization principles; that is to minimize the error sum squares, so that is nothing but the we have to that means, we have to minimize the error sum scale with respect to beta 0 hat beta 1 hat beta 2 hat and up to beta k hat.

So, by default when we will apply the minimization rules, so d summation is square by d beta 0 hat equal to zero; d summation e square by d beta 1 hat equal to zero and continue like d summation is square by beta k must be equal to zero. So, this is first order condition; of course, there is a second order sufficient condition, so we are not going so much detail, because it is not mathematics completely. So, here with the basis of the first order necessary condition we have to get the process done.

So now, what is this structure? The first step is the first step is to have the partial difference. So, d summation d summation e square by d beta 0 hat is equal to zero, d summation e square by d beat 1 hat is equal to zero then d summation e square by d beta 2 hat is equal to zero and continue, so finally equal to d summation e square by d beta k hat equal to 0. This is how it is it is all about means the process of the standardization.

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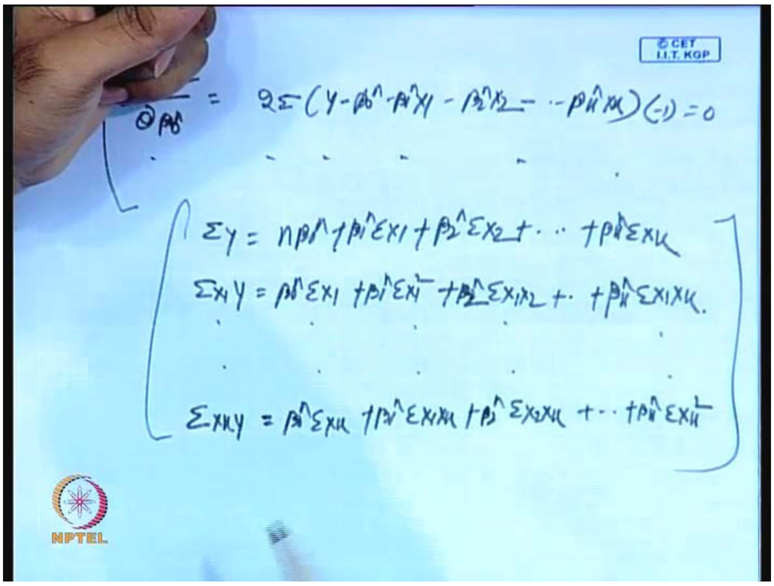

So now, what you have to do? So, we have to we have to proceed for the next step, so how will you proceed for this next step, so this is how? So now; that means, if it d summation e square d summation e square by d beta 0 hat is equal to is equal to summation 2 summation Y 2 summation Y minus beta 0 hat minus beta 1 hat X 1 minus beta 2 hat X 2 minus minus beta k hat X k, so into into minus 1 is equal to 0.

Similarly, we have to we have to go for d summation e square by d beta 1 hat, so this is 0, so d beta 1 hat d summation e square by d beta 2 hat like this d summation e square by d beta k hat. So now, if we will simplify this particular, it will continue like this. So now, the let us assume that this the second step of this particular process, means second step of this particular first structures means that is the first run necessary conditions. So, in the in the step we have to do? We have to simplify this particular equation then we will put in a simultaneous equation, so that simultaneous equation will help you to get the estimated values of beta 0 hat beta 1 hat beta 2 hat and have to beta k hat.

So now, if you will simplify this particular equation, how do you go for that? So now, it is nothing but, summation summation Y equal to n beta 0 hat plus beta 1 hat summation X 1 plus plus beta 2 hat summation X 2 plus beta k hat summation X k. So, summation summation X k X X 1 Y is equal to beta 0 hat summation X 1 plus beta 1 hat summation X 1 squares plus beta 2 hat summation X 1 X 1 X 2 plus beta k hat summation X 1 X k. It will continue. So, then summation X k Y X k Y is equal to beta 0 hat summation X k plus beta 1 hat summation X 1 X k plus beta 2 hat summation X 2 X k plus beta k hat summation X k square. So, this is how you have to proceed subsequently. So now, we have to simplify this particular structure to get this beta hat.

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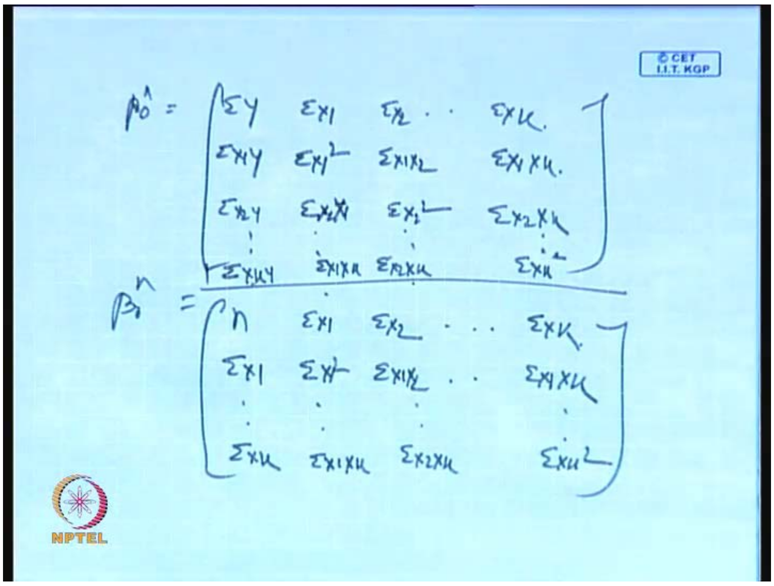

So now, by the way so, what you to do? So, if you will go by this particular process then beta 0 hat beta 0 hat will be beta 0 hat will be like this, so summation Y then summation X 1 then summation X 2 up to summation X k, then summation X 1 Y, then summation X 1 square, then summation X 1 X 2, then summation X 1 X k, then summation X 2 Y summation X 2, summation X 2 Y, summation X 2 squares, then summation X X 2 X 1, then summation X 2 squares, summation X 2 X k, then it will continue, so whole divided by whole divided by whole divided by n summation X 1, summation X 2, then summation summation X k divided by summation X 1, summation X 1 square, summation X 1 X 2, then summation X 1 X k, so it will continue up to summation X k, then summation X 1 X k, then summation X 2 X k, then summation X k squares, so this is how this structure. In the last part, this particular structures it will be summation X k Y, then summation X 1 X k summation X 2 X k summation X k X k squares. So, this is how the this is how the first beta 0 hat, so we will get beta 0 hat is a this much. This is the beta 0 hat.

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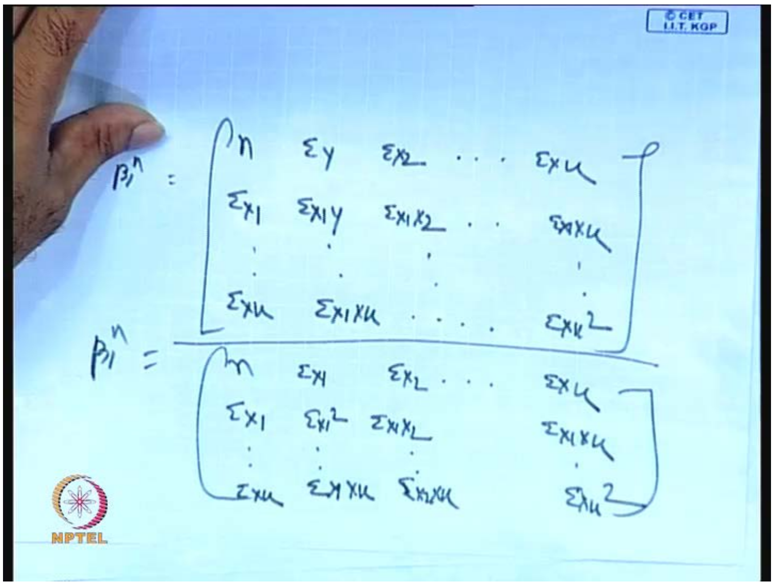

Similarly similarly, we can have also beta 1 hat. In the case of beta 1 hat in the case of beta 1 hat, so we have we have n summation Y summation X 2 then up to summation X k, then summation X 1 summation X 1, then summation X 1 Y, then summation X 1 X 2, then summation X 1 X k, so then this is summation X k, this is summation X 1 X k, this is summation X k squares divided by divided by n summation X 1, summation X 2, summation X k then summation X 1, summation X 1 square, summation X 1 X 2, summation X 1 X k continue summation X k, summation X 1 X k then summation X 2 X k, summation X k squares, this is how the entire structure. So that means, the beta 1 hat coefficient beta 1 beta 1 hat coefficient will be like this, so this is this is what beta 1 hat coefficient, so beta 1 hat coefficient is this particular domain by this particular domain.

Similarly, we have to calculate the beta 1 hat, beta 2 hat, beta 3 hat, beta k hat. So now, by the way we can again simplify in a deviation format, the way we have done in the case of trivariate econometric modelling. So, what we have to do here? In fact, it is a very implicated thing, so the way you will design ultimately by by hand or manual it is very difficult, so you have to apply some advanced technique like matrix approach to solve this particular problem very quickly. In fact, you can apply here also this is nothing but you can say matrix format even if you you like know the value of this particular matrix and value of this particular matrix then you can get the beta 1 hat. Similarly you can get value of beta 0 hat. In that process in that process you will you will continue like beta 2 hat, beta 3 hat, beta 4 hat up to beta k hat.

So now, what is the standard procedure? As usual you have to calculate beta 0 hat is nothing but Y bar minus beta 1 hat X 1 bar beta 2 minus beta 2 hat X 2 bars like beta k hat X k bar. So, first and foremost step is that, so you have to first find out beta 1 hat beta 2 hat and beta k hat. The moment you will get these beta(s) then you will put in first equation where beta 0 hat equal to Y by Y bar minus beta 1 hat X 1 bar beta 2 hat X 2 bar and so on up to beta k hat X k bar. The moment you will put all these values then you will get beta 0 hat, so no point to calculate beta 0 hat immediately, you have to calculate the subsequent items; obviously, within this with this subsequent items you can have the beta 0 hat. So, this is how you to you have to simplify this entire procedure.

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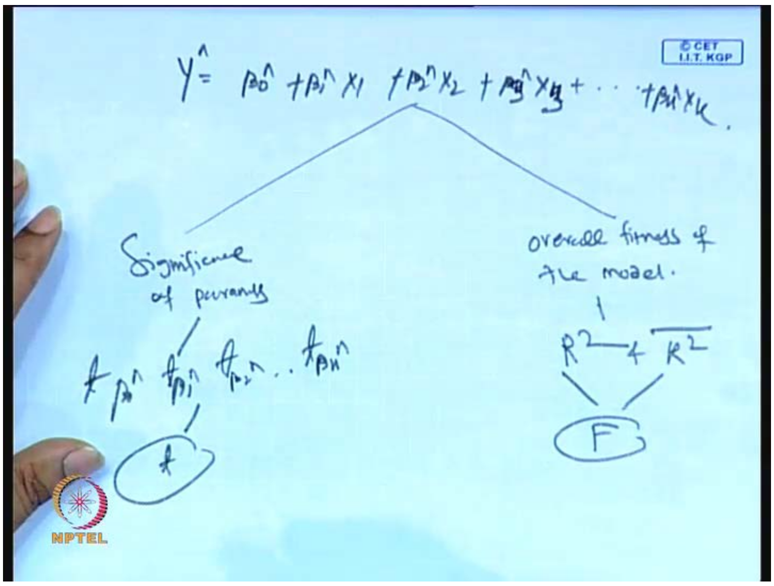

So now, what you have to do? The standard procedure is that; if you will simplify further then then it will be coming like this it can be also deviation format. So, ultimately the standard structure is that Y equal to beta 0 hat beta 0 hat plus beta 1 hat X 1 plus beta 2 hat X 2 plus beta k hat X k plus continue this is beta 3 hat X 3 and plus continue up to beta k hat X k, so this is how you have to proceed. So now, the moment you will get all these items then obviously, next step is you have to go for the reliability checking. So that means, let us assume that the standard estimated model for this particular model multivariate model is like this, so that Y hat equal to beta 1 hat X 1 plus beta 2 hat a X 2 plus beta 3 hat X 3 up to beta k hat X k.

So now, you know what is the first objective here? So now, we we have to assume that we have to means we have to observe that these parameters are statistically significant and second thing is the overall fitness of the model will be statistically significant. So that means, we are not going so much complexity to derive all these beta coefficient simultaneously since in the case of bivariate modelling, so we have to components beta 0 hat and beta 1 hat in the case of trivariate, it is a beta 0 hat beta 1 hat and beta 2 hat. So, it is it is mandatory that you need to have a information about beta 0 hat beta 1 hat and beta 2 hat, but in the case of multivariate up to when there is a k k independent variables are there and k you can say k number of parameters are there then obviously in that context it is very difficult to go manually and it is it is not practically feasible to calculate independently the beta 0 hat beta 1 hat beta 2 hat and up to beta k hat.

So, we remember one thing, when we have multivariate problems like this; either you go directly to this software else software or else you have to go, you can say by matrix approach. If you will apply the matrix approach you can have the results also very quickly. I will have highlight little bit here about these matrix approach how this multivariate complexity problem can be very simple way can be analyzed in a very simple way, so this is means I will discuss this concept later.

Let me first highlight here the practical feasibility of this particular estimated model. So, we have to different objectives here once the estimated model is with you, so first objective is to know the significance of the parameters, so that is the that is the significance significance of parameters and second is the overall fitness of the model overall fitness of the overall fitness of the model. First is significance of parameters and second is the overall fitness of the model. Overall, fitness of the model of course it is depends upon R square and adjusted R square and it is followed by F statistics, so this is significance of parameter means all beta 0 hat beta 1 hat beta 2 hat up to beta k hat. Also means, we need to have a t beta 0 hat, t beta 1 hat, t beta 2 hat, t beta k beta k hat. So, this has to be statistical significant; so that means, it is the through t statistic we have to we have to follow up the significance of the parameters and through F statistics we have to signify the overall fitness of the models. So, we need two different tables altogether, so I am briefly highlighting here. So, if you do not go for this tabular form then it is better, so what is the usual format of estimation process?

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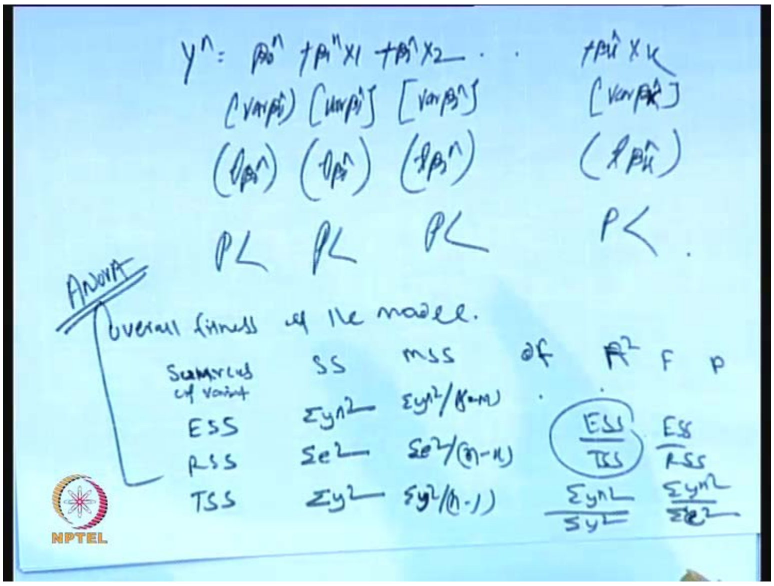

So now, we have you know Y hat equal to beta 0 hat plus beta 1 hat X 1 plus beta 2 hat X 2 up to beta k hat X k. So, we need here variance of beta 0 hat, we need here variance of beta 1 hat, we need here variance of beta 2 hat, we need here variance of beta k hat variance of beta k hat variance of beta k hat, followed by t of beta 0 hat, t of beta 1 hat, t of beta 2 hat and t of beta k hats; t of beta 0 hat, t of beta 1 hat, t of beta 2 hat, t of beta k hat, so then followed by probability level of significance probability level of significance probability level of significance probability level of significance. That means, the standard rule here is that like bivariate format and trivariate format, so we first get the estimated value of beta 0 hat, beta 1 hat, beta 2 hat and beta k hat; that means, estimated value of all this parameters k number of parameters and followed by you have to calculate the variance of all parameters, so beta 0 hat, beta 1 hat, beta 2 hat and you can say beta k hat. The moment you will get variance of all these parameters then you have to go for standard standard errors, so that is square root of all these variance variances, so that is variance of beta 0 hat, variance of variance of beta 1 hat; variance of beta 2 hat and variance of beta k hat.

So now, the moment you will get the standard error of beta 0 hat, beta 1 hat, beta 2 hat and beta k hat; then obviously, you have to set the null hypothesis; that means, you have to set the null hypothesis for beta 0 hat, beta 1 hat, beta 2 hat and beta k hat, so that means, your null hypothesis h 0 such that beta 0 hat not equal to 0, beta 1 hat not equal to 0, beta 2 hat not equal to 0 and beta k hat not equal to 0. Then, the alternative hypothesis it means against the against the all hypothesis the alternative hypothesis is that beta 0 hat not equal to 0 beta, 1 hat not equal to 0, beta 2 hat not equal to 0 and you have to k hat not equal to 0. So, that is how the fitting of null hypothesis and alternative hypothesis.

Then by the by the process since it is equal to 0 then obviously t of beta 0 hat is nothing but, beta 0 hat Y standard beta 0 standard of beta 0 hat then t of beta 1 hat is equal to beta 1 hat by standard error of beta 1 hat, then t of beta 2 hat equal to beta 2 hat by standard of beta 2 hat, so you will continue like this up to beta k hat. So that means, ultimately you like to know what is the t of all these parameters; means t statistic of all these parameters. The moment you have all these means t statistic of all these parameters; that is that is your that is you know calculated t statistics then again you have to compare the tabulator statistics with respect to the number of observations and particular the degrees of freedom. So that means, here we like to see again you have to go by this significance level to 1 percent, 5 percent and 10 percent.

So now, for instance; for beta 0 hat you have to see whether the calculated t of beta 0 hat is substantially greater than tabular t beta 1 hat and if it is so then what level? Is it significant at 1 percent level or it is significant at 5 percent level or it may significant at 10 percent level? So that means, you have to go individually for each and every parameters and the requirement is that each parameter has to be statistical significant. So, that one; that means, beta beta 0 hat must be statistical significant means most probably we always we always expect that this should be significant at the most at the highest level. So that means, at the most at the one percent level. So, we are expecting that beta 0 hat, beta 1 hat, beta 2 hat; all these parameter should be statistical significant at 1 percent.

So, this is our assumption or you can say expectations. It may not happen, so if it happen then is very fine. If it is happen then we have to see the other side of the problem that is the overall fitness of the model. That means, if all the parameters are highly significant at 1 percent then obviously by default R square should be statistical highly significant; otherwise, it will go other way around. So that means, if all these parameters are all these parameters are statistically significant; in other side R square is not statistical significant. Means in typically you can say let us say all this parameters are statistically significant at 1 percent level, but in the case of R square it is statistical significant at 10 percent.

So, then still the problem can be fitted or cannot be considered as the best model for forecasting or policy use. So that means, the standard structure is that, if all these parameters are statistically significant at 1 percent level then in the other side R square should be statistically significant at 1 percent level. Otherwise, if it is 1 percent level and if it is 5 percent this is okay, but still the model cannot be cannot be called as a best fitted models. But in reality it is very difficult to find a situation particularly for multivariate problems, where all these parameters are highly statistically significant; that means, it is very difficult to find a situation where all these parameters are statistically significant at 1 percent, so there is a lots of ups and downs that means, if 1 parameter few parameters few parameters are statistically significant at 1 percent, few parameters are statistically significant at 5 percent and few parameters are statistically significant at 10 percent or there may be chance that few parameters are also not statistically significant.

The model may be very accurate, may be practically I cannot say practically flexible, but it will it will be more justified if few parameters means this type of structure must be there in the model, means most of the variable are statistically significant at higher level then followed by few are at the medium level; that is at the 5 5 percent level and few are also at 10 percent level; that is at the low level and few parameters say one or two should be should not be statistically significant.

If such type of model is in front of you then you know it will be considered or it can be practically true, but in other case if all these parameters are statistically significant then this model is no doubt about it is considered as best model and can be used as forecasting, but still there is a strong doubt about this type of models and further before using that model for forecasting and policy use, you have to go through very carefully once again with respect to problem setup, data setup, data entry and estimation and also the significance choice means criteria whether this is okay or not, so then you have to say that yes this is the correct model and it can be considered as the best model and it can be used as a policy use and you can say forecasting. But if majority of the parameters are not statistically significant, but R square is you can say highly significant then the problem has a lots of problem and that is must natural sometimes.

For instance; R square is usually calculated with the ratio between explained sum square by total sum square. So, total sum square is always same, whether you are in bivariate format or trivariate format or multivariate format. If you will include one independent variable whatever you know P S S you will get it; for instance, we have Y information, then the first problem we have Y X 1 in the second problem same Y with X 1 X 2, so Y X 1 then in the second case the same Y X 1 then we add another variable X 2 then you take a third case, so same Y X 1 and X 2 then we put another variable X 1 X 3. So, this is how you to proceed one after another.

So, if you will proceed like this you know Y with the 1 X 1 and Y with 2 X 2 X and Y with the 3 X like this; then obviously, in case the summation Y square is always constant, but we are we are copying and pasting the same Y value everywhere in this particular setup. So obviously, if you compare in all cases then summation Y square is always equal whether it is bivariate problem, whether it is a trivariate problem or whether it is a multivariate problem. So, that is why there is a possible chance that a few parameters are not statistically significant or you can say most of the parameter are not statistically significant or significant at the very lower level, but R square will be statistically significant at the higher level.

In this particular context, the modelling is very interesting, because by means you have to assume that this model is not considered as the best model. So, as a result you have to again go for rotation or reformulation etcetera till you get the best fitted models. Of course, it is very continuous process and in fact in the multivariate case, the complexity or the process is very volatile or you can say very flexible in nature depends upon the situation setup and problem information. So now, accordingly you have to see the situation and find out the best model for you. So that means, the degree of decision making is very high in that case of multivariate models multivariate model econometric modelling. So in that case there are number of constraints with respect to X and various side problems are there before handle this particular model or before you call that model is a best fitted model. So, this is one part of the story with respect to specification of the parameters.

The second part of the story is that, we need to have this means it is the question of overall fitness of the model overall fitness of the overall fitness of the model that is as usual. So, you have sum squares, means first is sources of variations sources of variations, then sum squares, then mean sum squares, then residual then you can say degree of freedom, then of course F statistics a path first will be R square then F then will be probability value of significant. This is how you have to proceed, so this is nothing but explained sum square, this is residual sum square, this is you can say total sum squares, so as usual the structure is more or less same, so this is otherwise called as a summation Y hat square; this is summation e square; this is summation summation Y square.

So obviously, this is summation Y hat square by k minus (( )), so here if it is k then obviously k minus equal to 0. So obviously, there will be some gap here, so summation Y square is also divided by number of freedom and summation Y square by obviously it is n minus 1. This should be n minus k. So, you have to see some degrees of freedom here, because we are considering here k as the total number of variable, so obviously, it will be little bit complex, but let us say there is model up to you can say m m then obviously you will find out k minus m, so this is how the degrees of n beta. That means, that means we have to see, what are the degrees of freedom for this particular context means in this particular contest degree of freedom is very important, because it is a multivariate problem. So, this is R square ESS by TSS and obviously F is ESS by RSS.

So, similarly this is nothing but summation Y hat square by summation Y square. This is summation Y hat square by summation e square. So, this is how you have to proceed. So, this particular structure is like this. This is summation e squares. So, this is this is the structure of particularly called as overall fitness of the model and otherwise it is called as ANOVA test analysis of variance, it is called as analysis of variance it is called as analysis of variance, where there are three standard components is very important; that is, explained sum square, residual sum square, total sum square and followed by summation Y hat square, summation e square, summation Y square and corresponding with the help of the degrees of freedom we have to calculate the R square and f statistics.

So, most probably in the case of multivariate model R square should not be very less by default it will be very high, because number of X variables are substantially high. So, as a result most of most of the multivariate cases, we will find R square is a substantially high, but the major difficulty is that or major problem is that; you may not get all these parameters are statistically significant, but reverse is always true. That means, there is enough chance that R square will be highly significant and you can say parameters are not all parameters are not statistical significant.

But, by default R square will get high and obviously it can be there is enough chance that it may be significant, but in the same time in the same time you can say your significance of parameter may not be true. So, there may be few are significant, there are there are few may not be significant at lower level or there are certain situations there are statistically insignificant. That means, there is also chance that all all parameters may not be significant, so R square will be very high, but R square high does not mean does not mean that it is significant, so it has to be checked through F statistics, because when R square is high with respect to the environment of independent variables; then obviously, we need to calculate first adjusted R square, so which is which is which is considered as the best indicator rather than R square.

But whatever components you will use R square or adjusted R square it has to be tested, so obviously you have to go for this F statistics. So, the moment you will go for f statistics, so f statistic is always followed by degrees of freedom; that means, we have not mentioned here degrees of freedom there is a degrees of freedom here and there is degrees of freedom here. So, we have to find out the degrees of freedom then with the basis of this degrees of freedom, the significance of R square may be you can say it may affect, but but by enlarge R square will be substantially high, whether it is significant or not that depends upon the sample observations, because of course, the situation is very interesting when number of number of samples are also very high, number of variables setups are very high, then in that case there is a enough chance that R square will be substantially high and in the same time F is statistically significant, this is ok, but in the other site majority of the parameters are not statistically significant.

So, in this particular setup it is okay, if the sample observation is true enough than this involvement of variables; means particularly independent variable. So, you will get both higher square and both high value of F statistics, but in the same time means it is not mandatory that if you will get high R square and high f value that all these parameters in this side will be statistical significant, but the reverse is always true reverse is always true mean that, if you means if all the parameters are highly statistically significant then there is means possibility of 99 percent chance that you will get this R square and F statistical means both are statistically significant.

So, in that context you have to find out means you to formulate problem such so that both can go simultaneously. That means, all this parameter has to be statistically significant and it is not mandatory that it should be significant means all parameters should be significant at 1 percent level, even if you are 1 percent level, if you are 5 percent level, if are 10 percent level does not matter, but should be majority of the parameters should be statistically significant and R square will be statistically significant. But, if you will find a particular situation where few are statistically significant at the higher level, few are statistically significant at the lower level and few are non-significant and other said R square is high and F is statistically significant. So, here there is means there are two methods or two criteria(s) here to chose the best fitted model.

There is a technique called as a step wise regression. So, step wise a regression if you will go one by one then means step by step you will get the model, which is absolutely best for this forecasting or policy use. Means the problem is here that means, the moment you will get a model estimated model where few parameters statistically significant and few parameters are not statistically significant then in that context what happens? the variables, which are not statistically significant may be or can be dropped, in the present context.

So now, let us assume that there are there are 10 variables in the system; that means, if k equal to 10, so let us assume that there is a few variables X 7 or X 6 is it not significant, so what you have to do, you first drop X 6 then you estimate the model and you check whether after dropping X variables other variables are statistically significant and R square is high and also it is a highly statistically significant, then if it is okay then its fine. Otherwise, you drop X 7 then you have again estimate the models and you check all this parameters, well even parameter should be highly statistically significant and R square will be high then followed by F is also statistically significant. If you will get the all these all these parameters are highly significant after dropping X 7 and in the same time R square and F are very high, then that is considered as the best model for forecasting and policy use. But by the way if you will drop X 6 or X 7, if other variables are insignificant in the same time then there is again problem; the problem is more complex.

For instance, some variable may not statistically significant, but it has a substantial integration or you can say connection to other variables. That means, some variables may indirectly influence and that has a significant impact on Y. So now, if you drop the particular variables then obviously, the impact of the additional variable means previous variable say X 3 will be get hampered. So that means, X 3 may not be significant, but if you will add if you will include X 6 then obviously X 3 will be active and it will influence on Y. So, this type of problems basically or you can say you have to face in the case of multivariate models. So, it is how multivariate problem is very complex problem and it is very it is in fact very serious challenge to analyst researchers particularly you know those who are estimating or analyzing the particular problem or you can say analyzing the data setup.

So, we need a situation or we need a problem or information by the way we have to get the estimated model and that model means the way we will estimated model must most of the cases all these parameters should be statistically significant and you can say R square and F should be substantially very high, but if few out of which if few variables are highly significant and few variables are not significant, then in that case you have to check cross check the by dropping this variable whether there is any kind of important in the rest of the models. For instance; if you will be dropping X 6 then rest of the item X 1 up to X 10 means without having the X 6 involvement and obviously R square and F then you check the models so that means like this.

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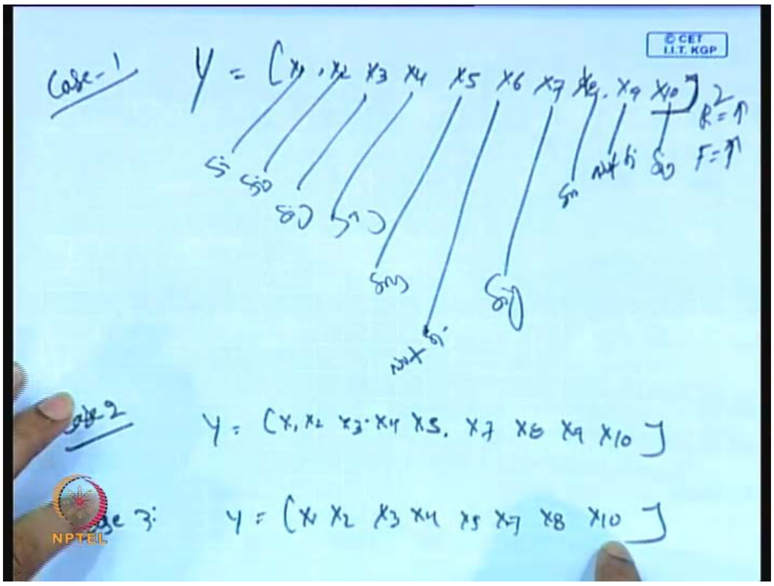

So, what you will do here? You have three different, let say Y is X 1 up to X 2 up to X 10. So now, by the way I will write it here X 3, X 4, X 5, X 6 X 6 this is X 7 then let say this is X 5, this is X 6, we will take X 7 here X 7 here, it is better I will right it once again. Y equal to X 1, X 2, X 3, X 4, X 5, X 6, X 7, X 8, X 9, X 10. Let us assume that the model consists of 1 dependent variable Y and 10 independent variables independent variable that is X. So now, you go by estimation process, by the way this significant, this is significant, this is significant, this is significant, this is significant, this is significant and this is not significant. Let us assume that this is not significant, this is significant, then this is significant and this is not significant and this is significant. R square is default is very high and F is also default very high this is by default its very high.

So now, this model can be practically can be considered as a best models particularly considered as a best models still for flexibility part of the concerned or you can reliability part is concerned, so what you have to do, you make an experiment. So, this is you can say original case and you call it case 1. So, what you to do? You go to case 2, what have to do, you take models Y equal to X 1, X 2 up to X 1, X 2 then X 3, X 4, X 5, dropping X 6, then X 7, X 8, X 9, X 10, take it the models status. Then case 3 case 3 you take another case Y equal to X 1, X 2, X 3, X 4, X 5, X 7, X 8 then X 9 is dropping here, then it is X 10.

So now, there are three different cases. So, what have to do? This is already in front of you, so you have to go for this estimation and you check the R square value and this significance of these parameters. So, by dropping this variable if the significance of these parameters are relatively very high high level of significance and R square is relatively high and F is relatively high then that will best one considered as a best. Otherwise; if it is affecting the significance level at the lower level, then it is better to reject this particular case 2, then you proceed to case 3. By case 3 you have to again re-estimated the model then, by the drop means by the process of dropping X 9 if all these parameters significance level is relatively high and R square F is a relatively high then again, means it is consider as the best (( )) to case 1.

But by the, means by the process of dropping X 9, if X 1 the significance of all other variables will be drastically reduced and R square F is drastically reduced; obviously, R square will not R square will be reduced because we have already dropped one variable, but we have to check whether this R square is adjusted R square is relatively okay and F is statistically significant and other variables level of significance is also relatively high. If it is okay then it is considered as the best fitted model than case 1. So, if it is not okay means after dropping this variable, suppose few other variables are its significance level is lower level than the original case, then it is better you remove this case 2 and case 3 you go ahead with the case 1; that means, even if in the case 1 two variables are not statistically significant that will be considered as the best model and can be used for forecasting.

So, this is how the basic problems you have to face in the multivariate models, so these are lots of complexity, so that complexity you have to work out there are there are various or trailer error and methods you to apply to get the model best fitted. So, either you redesign the models, restructure the models then reformulate the data setup, reformulate the problem setup till you get the best fitted model. So, whatever structural change you have to do, so all these cases the model estimation information may be different. Most of the cases it will be unique means very rare case it will be unique, but most of the cases you will find the difference of results, so if you will change the setup. So, ultimately we are going for experiment to get the best fitted models.

So now, in addition to that you will face lots of different complexity like various variables inclusion and exclusion, so it will be it is very interesting game, so far as a regression technique called as stepwise regression. So, by this technique we can get to know means if you go by stepwise technique then it is first you have to track, which particular variable is the most important variable; that means, that you have to target with the help of t value. So, we we quickly make a look look with respect to all these parameters coefficient and we have to find out the t statistics then if it t statistics of particular variable say X (( )) is substantially higher than the other t statistic then we have to include (( )) first step.

So, that is how the process of stepwise, then you to collect R square and F. Then you you enter the next next highest t value. So, that variable has to be included. So, again you have check R square and F. R square will be very high substantially. So, once you move on one by one, but in the sometime you have to check the F statistic, but in the same time you have to see the variables should be significant equally. Then, you add third variable third highest variables then then by the way all these variables should be significant and R square will be high, F will be substantially high. So, you continue this process till you get till you get the best fitted model and you have end in that particular point where the inclusion of a particular variable inclusion of a particular variable will not you can say improve anything about R square F or significance of the parameter. So, that is the criteria, through which we have to we have to judge best models in a multivariate case. So, multivariate case is very complicated case very in fact, in sometimes this is very interesting, because it gives lots of structural frameworks, through which you have to go here and there to get the best fitted models.

With this, we will conclude this session. In the next class, I will discuss the matrix approach of this multivariate modelling. The way we have discussed here; in fact, it is a it is very much related to very much related to simultaneous equation format without having means direct touch of matrix, but you know in next class, I will discuss a format, where by the beginning we have to apply matrix and we can conclude matrix, so that it will be very easy and you can get the answers very quickly. Even if by the matrix approach multivariate problem in the class itself we can solve that particular problem, so it is very interesting game, so we will discuss this detail in the next class. Thank you very much, have a nice day.