

**Econometric Modelling**  
**Prof. Rudra P. Pradhan**  
**Department of Management**  
**Indian Institute of Technology, Kharagpur**

**Module No. # 01**

**Lecture No. # 13**

**Trivariate Econometric Modelling**

Good evening, this is doctor Pradhan here. Welcome to NPTEL project on Econometric Modelling. So, today we will discuss the concept Trivariate Econometric Modelling. So, in the last couple of lectures, we have discussed the entire structure of econometric modelling, that too bivariate econometric modelling, the importance, the model fitness, the reliability, etcetera, we have discussed in details.

So, here, so we will go little bit deep, so the when will you talk about the econometric modelling or regression modelling, because it is the counter part of, or you can say it is a same we can represent in econometric modelling. So, the starting point is with respect to two variables, but the complexity will be very high or you can say relatively very complicated. So, when will you move two variable **variable** to three variables, or you can say more number of variables.

So, now we just like to know how this problem is more complex, when will we add an extra variable in this particular system? So, what is the exact system, and then the starting procedure of means, the starting of econometric modelling is with respect to two variables.

So, with one variable you cannot do anything, or you cannot establish any relationship, so the origin is with respect to two variables, but when will add one after another variables, then the model **you know** beauty will start increasing; and in the same times, the complexity will start increasing; and you will **you will** find lots of interesting **you know** features or **you know** interesting issues when we will integrate more number of variables in the systems. Because in a particular set up, in a particular problem back ground, so there are large number of the variables which can influence each others.

So, with two variables, we cannot get a best fitted model, until unless even if you take this, you can say reliability, etcetera. So, the model will be **you know** considered as a best fit or can be for forecasting or different use, when the system is very consistent with the number of variables in the systems. So, that means multivariate model is more reliable, more practically feasible than the bivariate model.

So, bivariate model is just the beginning of this econometric modelling. So, there will start with **you know** various issues and challenges. So, now will see what, how is this challenge and how complex this problem and how **duty** is this problem when we will add one after another variable in the systems. Some of the problems we cannot detail explain when we are in the bivariate modelling, but when we will start with trivariate modeling or multivariate modelling that time, **you know** that structure can be explain in a nice way or you can say in a better way. So, we will see here is including another variable in the systems, so how is this entire structure or setup of an econometric modelling.

What we remember here, trivariate means here is, so always **you know** in dependent variable is here only one and more number of independent variables. So, means when will we start with the bivariate modeling, so we have two variables; one is dependent **dependent** variable, and another is an independent variable. That means, here dependent classification and independent **independent** classification should be **you know** very sincere and it should be must.

So, now for bivariate models, then obviously, there are two variables only; one has to be dependent, then other **other** side it is, another one is an independent one. So, the game is a simple one, but when the, when we add another will able in the system, then in the typical situation in this particular frame work.

So, we add more number of independent variables in the system keeping Y remain constant. So, that means whatever in the feature couple of lectures, so what we will see, so we will discuss so many problems, so many issues keeping one dependent variable with the **with** net addition of independent variables. That means, one after another variables will add to the system and we will see how is the setup and how is this structure, this is one **one** agenda or one structure of this **you know** econometric modelling. But **there is** there are certain problems here, there are multiple number of

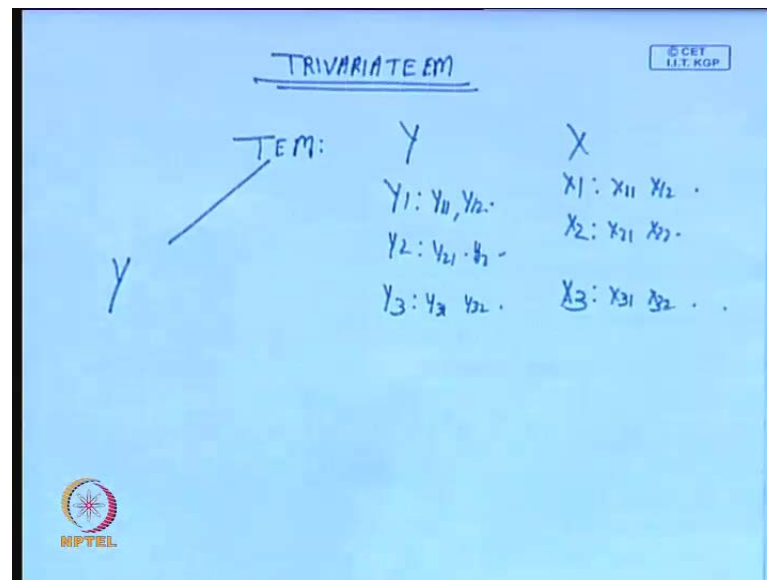
dependent variables and multiple number of independent variables, then we can also integrate each other to get a particular problem solutions.

So, that setup is something different and is a more complicated than the simple multivariate models, where only one dependent with the many independent variables and that particular concept, where the number of independent variables are many and number of independent variables are many. So, it is called as a structural equation modelling, so we will touch that structurally equation modelling in the later part of this, means after so many lectures will proceed to that particular, because that is more complicated than the present setup.

So, we will discuss with one dependent variable with several independent variables, what are the structures, what are the problems, what are the challenges, what are the **you know** short comings, etcetera. Then, after discussing all this issues, then we have to introduce or we have to enter to this, a system where there are number of dependent variables and number of independent variables.

So, now we will move to a process where there is always dependent variable, only one single dependent variable and there are several independent variables. So, let us we will assume that the **the** system is in such a way, there is only one dependent variable here and there are two independent variable. So, that means this particular system is called as trivariate econometric modellings. So, what is all about trivariate econometric modelling? Trivariate econometric modelling means, it is an econometric system where there are, means there is one dependent variable and there are two independent variables. So, that means, so let us take a case here. So, once you will talk about dependent and independent. So, every time whatever concept we will discuss in an econometric modelling. So, every time we will assume that, the Y setup, Y series are dependent series and X setup, X series is independent variables.

(Refer Slide Time: 06:50)



For instance, if it is y series here, and x series here, so with in the y series, we have **we have** number of variables Y 1, Y 2, Y 3 like this, then we have number of variables X 1 X 2 and **you know** let us say, X 3. So, within Y 1, there is the item like Y 11, Y 12, like this.

So, similarly for Y 2, Y 21, Y 22, like this, then Y 3, Y 31, Y 32, like this. Similarly, for X 1 **we will** we must have like this X 11, X 12, continue, then X 21, X 22 then up to nth term, then similarly, X 31, X 32 and so on. So, like this, that means what is my **my** point is here. So, we consider Y setup is dependent classification; X set up is independent classification. So, will say under y system, how many variables that there, and under X system how many variables are there.

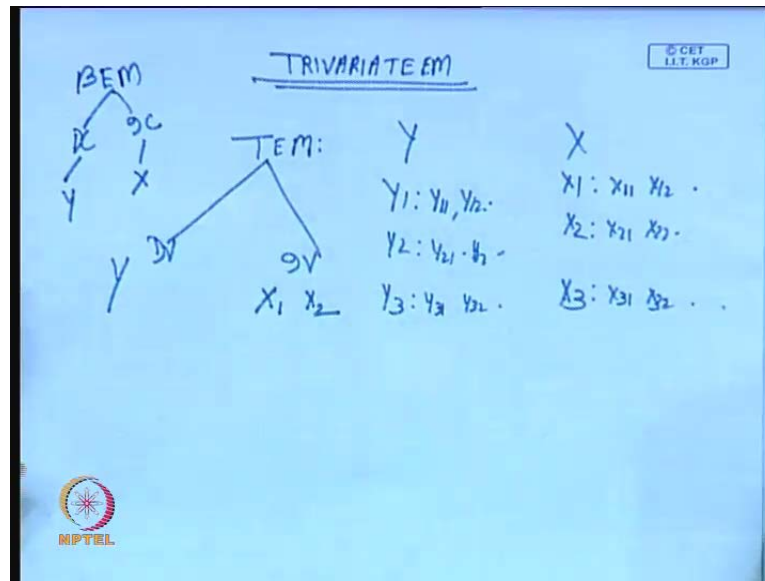
So, in the Y system if number of variables are there, we will **we will** recognize or categorize like Y 1, Y 2 up to Y n. So, every **every** item will be considered as a separate variable in the system. Similarly in the case of X, so we will assume that and X numbers of variables are there and every number we will denote as X 1, X 2, X 3, these are the variables in numbers. So, instead of putting Y 1, Y 2, Y 3, we can simply write Y i and simply will write X i or X j, there i stands from 1 to n, then that means, number of variables and Y stands from i to k, then there are Y or dependent variable.

So, now we will **we will** fix a setup here, where there is only one dependent variable. So, if there is one dependent variable time, then obviously, there is a no point to classify Y 1,

Y 2, Y 3. So, we will simply represent as a Y, so that means, trivariate econometric modelling is the system where there is only one Y, so Y, there is no subscript.

So, that means it is a simple, **you know** Y representation, that means one dependent variables at a time.

(Refer Slide Time: 09:02)



So, this is one **one** setup and Y, we have to integrate with the several independent, this is dependent variable structure, and this is independent variable structure. In this independent variable structure, since it is a trivariate, so we will take two independent variables, X 1 and X 2 and the corresponding to trivariate, when we are discussing bivariate econometric modelling.

So, bivariate econometric modelling, we have dependent classification and we have also independent classification. In that case, dependent classification is Y and independent classification is X, so there is a no X 1 or X 2. Because, so there is only one dependent variable and there is another independent variable. So, that means in dependent classification, only one variable and independent classification, there is only one variable.

So, there is no use of subscript or you can say any **any** additional **you know** supporting component. So, that means for bivariate econometric modelling, the model will be simple to understand or simple to represent if will just integrate Y to X. That means, Y is

dependent, X is an independent, but the system is more interesting or more complex in nature when we will introduce another variable. When we will enter another **another** independent variable in the system, then obviously, the X **X X** items will be start increasing. So that means, since there are for trivariate econometric modelling, since there are **two i** two variables at a time, so obviously, we will categorize or we will represent X 1 and X 2.

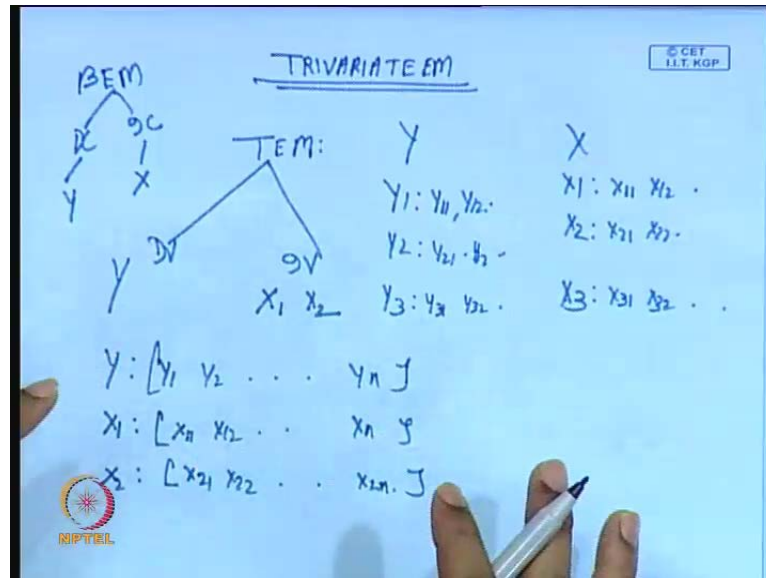
So, that means there is a need, or the it is a essential to use this subscript to even, but we can represent like this, we will take Y is another variable, X is another variable, Z is another variable. So, we can integrate Y with X Z where X and Z are independent variable. But **you know** instead of putting like that ways, it is better we will put to Y with respect to X and you subscript is then this **this** representation is much easier and much **you know** simple. For instance, you see the, what how pi can write to all this issues here.

So, what is the basic frame work of it modelling, so we have to **we have to** regress independent variable to dependent variable, just like in the case of bivariate econometric modelling, we are putting Y equal to alpha plus beta X simple, then obviously, this is mathematical form of the model and we are introducing error term, because some of the observation we cannot able to capture, because various regions that we have explained lots of lectures back. So, here what is the point here, similarly we have to fit a models first, mathematical form of the trivariate econometric modelling, then we **we** like to introduce the error, because some of the variables again we cannot identify properly because of so much complexity or so much problem.

So, obviously there is the introduction of error term sigma, then a like **you know** bivariate setup like to minimize this error component, we also do this same thing in the case of trivariate econometric modelling. In the case of trivariate econometric modelling, we will **we will** set a, we will fix a mathematical form of the model. And we transfer this mathematical form of the model into statistical form of the model by introducing the error term, because in this particular system, the starting point is we are assuming that X 1 and X 2 are influencing Y. But that is not shear that only X 1 and X 2 will influence the Y, there may be some other variables, there are some other variables which we cannot usually captures, can be consider as a influential factor for this dependent variable. For that region, it is a **you know** bound to use the error term or introduce the error term.

So, now while introducing error term or you can say a two variable, two independent variables in this particular system, then you see how is the setup.

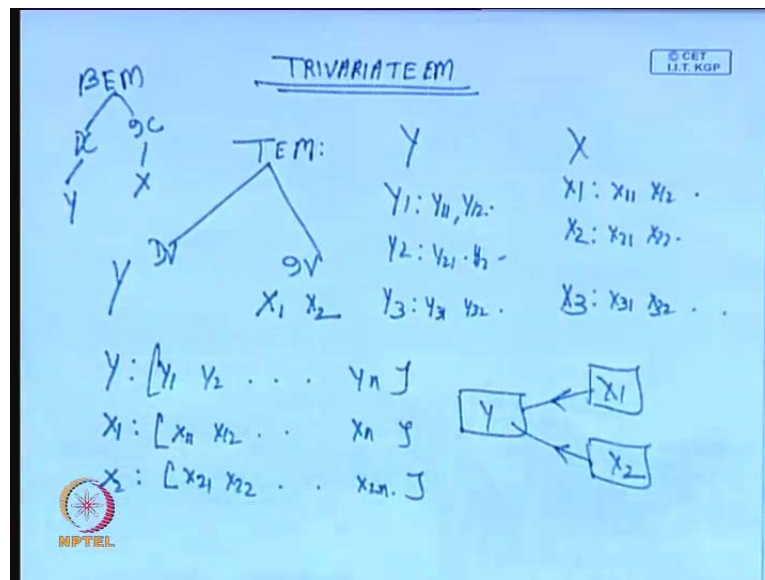
(Refer Slide Time: 12:57)



So, now let us assume that Y consist of several items, say Y 1, Y 2 up to Y n. So, this is how the Y observation. So, we assume that Y structure is like this, then X 1 is another variable which is structured as X 11, X 12 up to X 1 n. So, similarly we will introduce another variable X 2 is a **is a** variable where a representation is a X 21, X 22 up to X 2n **X 2n**.

So, this system is three variables, the integration of three variables where Y is considered as a dependent variable and X 1 is considered as a independent variable, X 2 is considered as independent variables. So, that means it is the integration of two independent variables to dependent variables.

(Refer Slide Time: 13:47)

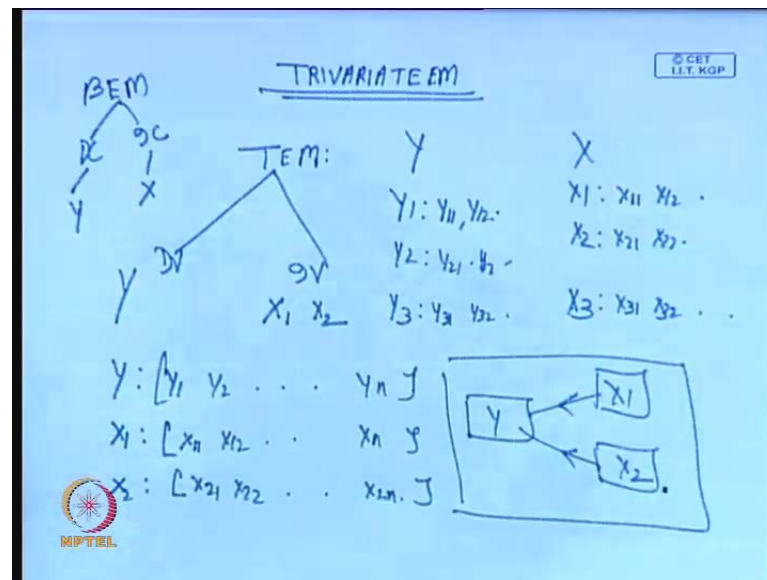


So, as a result the structure will be like this, this is Y and we have two independent variables; one is called as X 1, and another is called as X 2. So, what we like to do, we like to integrate X 1 with X 2, so this **this x x**, this process is like this.

So, we start a like or we like to know how much X 1 influence and Y and how much X 2 influence on Y, so standard procedure as usual. So, we have to fit a regression model, so by integrating X 1 and X 2 with Y. So, that means we like to know what is the influence of X 1 and Y, and what is the influence of X 2 and Y and what is the overall fitness of the models, provided all this assumption or condition or constant should be in our hand.



(Refer Slide Time: 14:37)

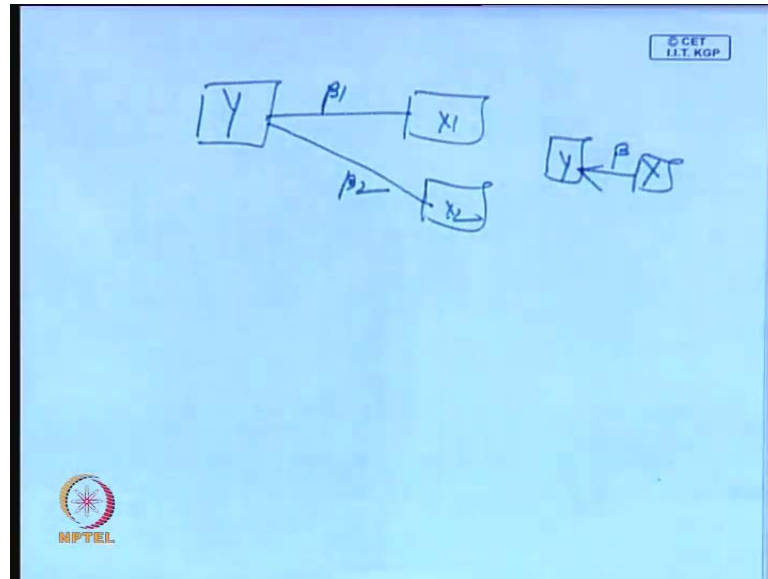


So, what is the actual process here, so before going to actual process, so we assume that **this** is the Y variables, dependent classification, these are all independent classifications. So, now the simple model of trivariate **trivariate** econometric modelling is like this. So, this is the initial setup, so where there are only three variables all together. So, Y is the classification of dependent variable and X 1 and X 2 are the classification of independent variables.

So, now if it is, so how will **how will** proceed to fit the data and two get a duty, their structure within the entire information, then as usual will go for estimation process, then again, once will get the estimated model, we have to go for the reliability of the models like with respect to parameter a significant label and other side, we have to go for the our rottenness of the model that is r square and which we have just now discussed few **few** hours back with respect to this **you know** bivariate setup.

So, now what we will **what we will** do here, so we will just **you know** follow the path of the bivariate econometric modelling, then we will little bit **you know** elaborate the concepts so that we can know, how is the system or structure in the case of trivariate where there is the two independent variable and one dependent variable.

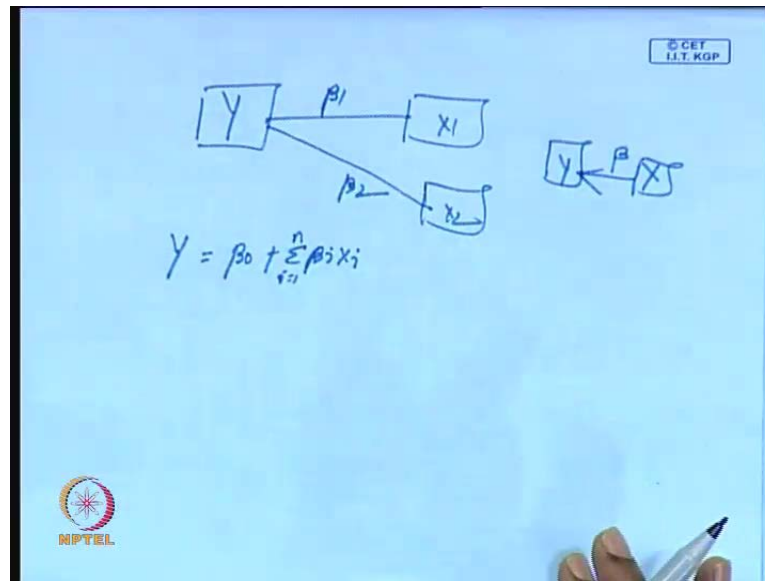
(Refer Slide Time: 16:05)



Let us say how is the enter setup, so what will do here is, so the system is a trivariate. So, Y is the dependent variable and X 1 is the independent variable and X 2 is the another independent variable. So, this, so what we will do here, so let us introduce the supporting terms called as beta 1 here, and we will introduce the term beta 2 here. So, that means in the case of **in the case of** bivariate econometric modelling, **in the case of bivariate econometric modelling**.

So, we have Y and we have X only, so the integration is only one. So, here we are using the term earlier called beta, because only one, so there is a no concept of using subscript. So, here since we are using subscriptive to X variable, so obviously, it is the supporting or parameters, so we have to also use subscript so that the **the** system will be very consistent or you can say more feasible.

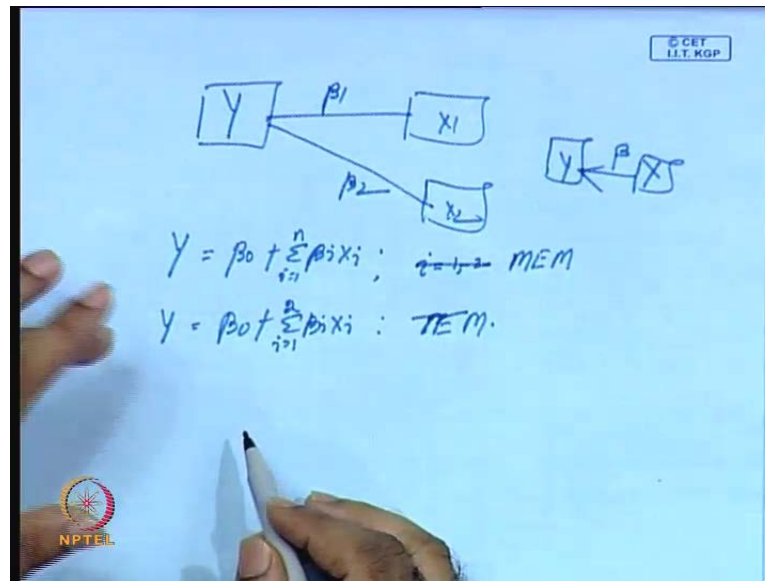
(Refer Slide Time: 17:02)



So, as a result, how we will proceed with this particular structure, so we will represent simple like this  $Y$  equal to, we will call it  $\beta_0$  plus summation  $\beta_i$  and  $X_i$ ,  $i$  equal to 1 to  $n$ , let us say this is the usual form of usual format of the mathematical form of the model. So, this is the mathematical form of the model for multivariate analysis, but we are in the process to discuss the trivariate econometric modelling. So, when will be it, when we will talk about the trivariate econometric modelling, then we will just add one constraint, then the model will automatically replace in the trivariate. So, what is that constraint, the constants is here, just you write  $i$  equal to 1 and 2 that is all.

So, 1 to 2, that means instead of ones, does you put 1 to 2 only. So, then automatically this model will be limited to a two variable setup. So, if I will put  $i$  equal to 1 to  $n$ , so then, that means there are  $n$  independent variables which can influence the  $Y$ . So, that particular setup is called as a multivariate **multivariate** econometric modelling. So, we will **we will** proceed with the multivariate modelling, let us state what in the mean times we will see, how is the setup of structure and **you know** if feasibility of this trivariate systems. So, that means, two independent variables and one dependent variable and how is the structure.

(Refer Slide Time: 18:37)



So, now if we will put  $i$  equal to 2, then the model will represent like this,  $Y$  equal to  $\beta_0$  plus summation  $\beta_i X_i$ ,  $i$  instead of putting  $i$  equal to 1 to  $n$ , it is better to put  $i$  equal to 1 to 2 only. So, it will be automatically indicated that is a trivariate econometric modelling. So, that means this particular if I **I if I** will not write this once, then it is indication of indication that it is a multivariate econometric modelling, so M E M. So, in this particular setup, if will put like this and it is called as a **bivariate sorry** trivariate econometric modelling, this is called as a trivariate econometric modelling.

So, now this is in fact, in a implicitly format. That means, it is not clear cut, how is it the exact setup. So, we **we will it will we** will introduce in an explicitly format so that we can understand the system in a much easier way. So, what is the explicit format of this trivariate modeling?

(Refer Slide Time: 18:37)

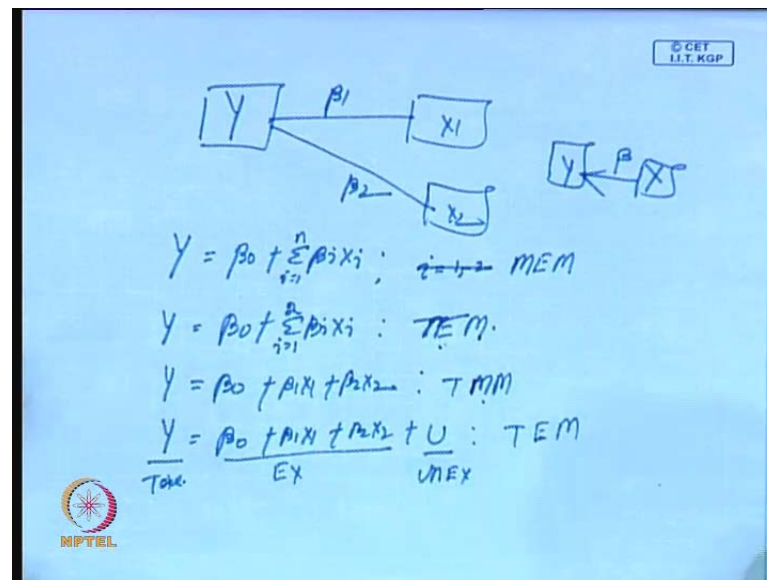
$Y = \beta_0 + \sum_{i=1}^n \beta_i X_i ; i=1,2 \text{ MEM}$   
 $Y = \beta_0 + \sum_{i=1}^n \beta_i X_i ; \text{TEM}$   
 $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 ; \text{TMM}$

So, will write like this,  $Y$  equal to  $\beta_0$  plus  $\beta_1 X_1$  plus  $\beta_2 X_2$ . So, this is more you know more you know a simple than the this one. So, this is little bit you know confusing in nature. So, when you are very smart enough, when you are hardcore statistician or econometrician, that time this particular format has a such a beautiness, but in the in the beginning, its it should be very neat and clean so that you can understand the particular system.

So, now we this is the complete structure of a you know trivariate, in fact we cannot say that it is an econometric model now, because there is a no error terms. So, the moment you will introduce the error term, and then it will be transferred to econometric modelling. So, we what we can all it right now,  $Y$  equal to  $\beta_0$  plus  $\beta_1 X_1$  plus  $\beta_2 X_2$  means, it is trivariate mathematical models. So, this is simply called as trivariate mathematical models. So, in fact this is also multivariate mathematical model, this is also trivariate mathematical model (Refer Slide Time: 20:35).

So, now this trivariate mathematical model means, so we have we have  $y$  once one part of the problem, so that is dependent and another part of the problem we have two independent variables that is  $x_1$  and  $x_2$ . So, what we will do, we will a transfer this mathematical trivariate mathematical form of the model to trivariate econometric form of the model.

(Refer Slide Time: 18:37)



So, what how we will do that, so what we will do, we will **right** Y equal to beta 0 plus beta 1 X 1 plus beta 2 X 2 plus an error terms, plus and plus and error terms. So, now the system will be represented as a **trivariate trivariate econometric model** trivariate econometric modelling. So, this part is called as explained part and this is called as an explained part, this is called as a unexplained part and this is total part (Refer Slide Time 21:20).

So, that means like bivariate **bivariate** structures, we like to know, what is the influence means X influence on Y, and what is U influence on Y. That means, what is the **explain** explained percentage and what is the unexplained percentage. So, as usual if the explain percentage is much higher than the unexplained percent, then obviously, the model fitness of the model reliability is a very high and it can be used for policy use or forecasting. But if otherwise, other way around, for instance, if unexplained i term is substantial higher than the explained percentage.

Now, in that case we, the accuracy of the model will be less. So, that is why you must be very careful, the way you will design or the way will fit so that the **the** explained percentage will be much higher than the unexplained percent, it is not simply much higher, it should be absolutely higher than that of unexplained percent. For instance, if I will put make a division between explained and unexplained, for instance, if I will take a issue like a 51 and 49 for instance.

That means, if the explained **explain** percentage is a 51 percent and unexplained percentage is 49, just **you know** simple, what will call simple majority, then in that case the model accuracy may not be perfectly ok. So, the model accuracy will be perfectly ok if we will get explained percentage is a just close to you can say 100, but it should not be 100. Even if it is 100, there is a lots of **you know** issues and complexity will start, means there are lots of questions coming into the mind, how it come, how many cities possible to get 100 percentage, because most of the instance it is **it is** various associational case.

So, the model accuracy, model reliability will be much higher if the explained percentage is substantially higher than the unexplained percent. So, if it is around **you know** close to ones like **you know** around **90 percent** a **a** 90 percent interval, then obviously, the setup is more accurate and more systematic or you can say more practical, more feasible. So, we have to see how **how** it will be done. So, we first up all we like to know what the econometric analysis behind this trivariate econometric modelling is and how we will get the estimated models.

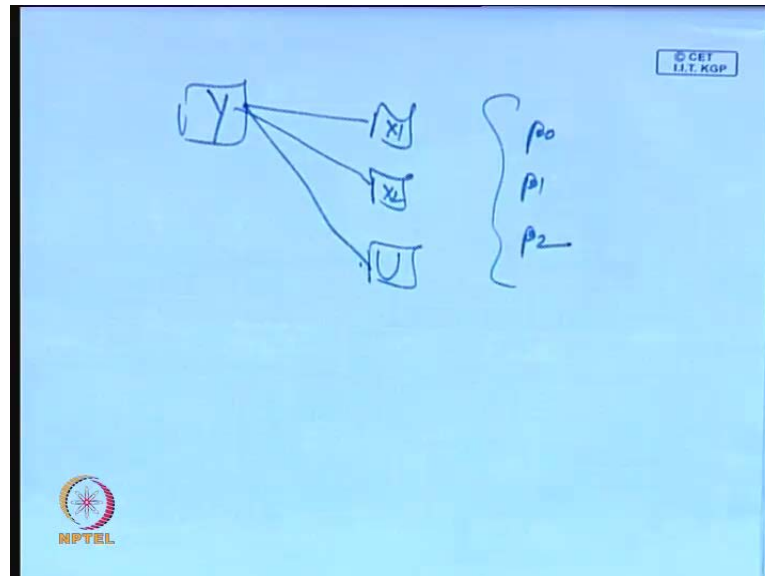
So, after getting the estimated models, so we will take examples and we like to get the, **you know** accurate estimated model and then will go for the reliability part of the model to test the significance of the data parameters and the significance of the overall fitness of the model that is coefficient of the determination. But here, there are certain interesting problems, we will discuss in a very nice way, that is **you know** adjusted r square that is the coefficient of determination and the multi collinearity issue.

Because coefficient of determination **adjust** adjusted r square is the more the means, the feasibility or you can say the duty is more interesting when will a enter to the trivariate and multivariate systems. And in fact, a multi collinearity issue is the very typical issue, we will start here only. Because in the case of the bivariate, there is a no concept called multi collinearity, because it is the **it is the** multi collinearity is the issue of **you know** independent relation, **independent relation** independent variable relationship, since in the vibrate setup, there is only one independent variable.

So, the problem of multi collinearity and adjusted r square you can say total handicapped, so per as the analysis is a concerned. So, now, the beautiness of this problem can be analyzed in a interesting way. So, when we will introduce another or you

can say, third variables in the system, that too **you know** the starting point of trivariate econometric modelling. Let us will see how the setup is all together.

(Refer Slide Time: 25:22)



So, now in these particular systems, so our setup is like this  $Y$ ,  $X_1$  and  $X_2$ . So, this is  $Y$  and obviously, there is **error terms** error term. So, the integration will be like this, so this is how the original setup here. So, now as usual, so here there is, so to analyze the complete problems, so we have **we have**  $\beta_0$ , we have  $\beta_1$ , and we have  $\beta_2$ , these are three supporting parameters through which we have to get the best fitted model, we will get the best fitted models or we **we** will get the first estimated model, then will it go for testing, whether it is best fitted or you can say best structural form. So, before that we have to fit the model in a proper format so that we can get to know, or we can get to test the reliability part of the modelling.

So, what we have to do here, so the structure this all together **the structure is all together** here is to estimate  $\beta_0$ ,  $\beta_1$ , and  $\beta_2$ . You see, before you going to trans this **you know** or estimate this  $\beta_0$ ,  $\beta_1$ , and  $\beta_2$ , let me little bit highlight this trivariate econometric format. So, **the** we know we have discussed lots of assumption behind this bivariate setup.

So, when you will a fit an econometric models for the case of, either for the case of bivariate, or trivariate, or multivariate, then the objective behind this particular modelling is to fit a estimated model, that too get the estimated parameters and the overall fitness of



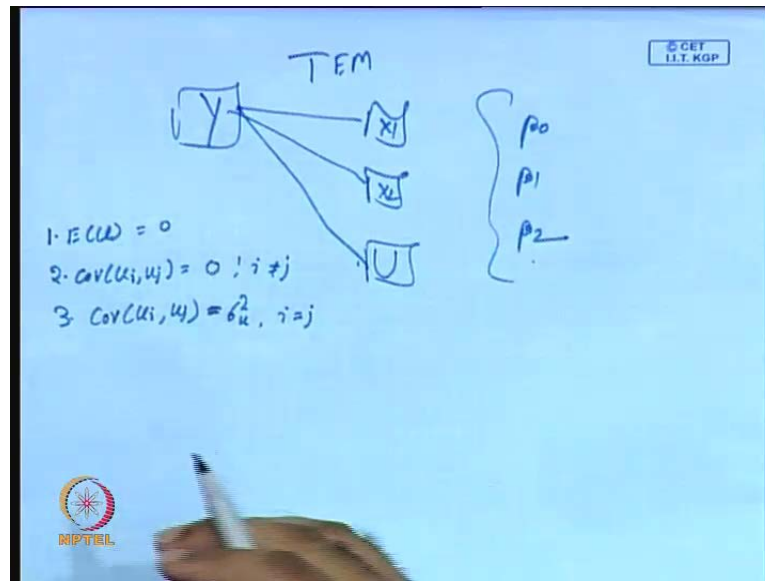
the models. So, that is how you need to have a **a** substantial, interesting problems, and typical theory behind this particular problem, and the information through which the model can be used or test or you can say check the validity.

So, now the way we have discussed in the bivariate modelling, **(( ))** so we introduce the technique called as a OLS technique. So, there are certain numbers of techniques available with you through which you can get the estimated model. So, like OLS transfer ordinary least square methods, then generalize the square methods, then what is least square methods, then maximum **(( ))** estimator method. So, like this, so many methods are there. So, by using all this techniques, so you will get the estimated models. So, that means we like to know, what is the estimated value of parameters with respect to beta 0, beta 1, beta 2.

So, now we will start with also again here OLS technique, because the other techniques like GLS, WLS and family, this more interesting when will **when will** first wear problem, then will highlight this issues in details, because at a time we cannot discuss this technique simultaneously. So, again we will start with the application of OLS technique, then we will get the estimated model, then we will go for the reliability checking. But the thing is that, you **you** use any technique to get the estimated model. So, obviously your objective must be to fit the modeling such a way that, that should be best ones.

So, even if you apply OLS technique w s w l s or g l s, still you have to go for reliability check. So, again we have to justify the best fitness of the models. So, in fact WLS and GLS is more interesting, more advance technique than OLS, still you have to start with the first OLS technique, then will be see how is the reliability structure and how it can be a substantial adjusted to get the accurate setup. So, this is how the accurate problem is all about, so now this trivariate econometric modelling setup. So, what you have to do, so we have to integrate  $X_1, X_2, U$  with  $Y$ .

(Refer Slide Time: 29:09)



So, now, how you to do, so this is how you have to establish the structures, then we have to start the estimation process. So, now one of the interesting point is here that, when will apply while s technique, there are standard assumptions, we have discussed details in couple of lectures back.

So, this one of the standard assumption is that, first is a error **error** terms, a error term should be a mean of error term should be equal to 0, this is one of the standard assumption. And second assumption is that, a covariance of error term  $U_i$  upon  $U_j$  should be equal to 0 again, should be equal to 0. Then, third covariance of  $U_i$  and  $U_j$  should be equal to units, that is sigma square  $u$  provided  $i$  is should equal to  $j$ , here provided  $i$  should not be equal to  $j$  (Refer Slide Time :30:17).

So, this is covariance of error terms, so there are couple of error terms, the moment will get error terms, then let us say, we will again categorize various other variables like **you know** once you have a error terms, we will call  $u_1$ 's, then we will create  $u_2$ ,  $u_3$ ,  $u_4$  like this, so the way we are creating  $X_1$ ,  $X_2$ ,  $X_3$  like this. So, there are series of error terms again, so the game boundary will be expanded accordingly. So, we will discuss that game boundary in different context, on difference situation when we will enter to the multivariate system **multivariate system**, or you can say more advanced problems.

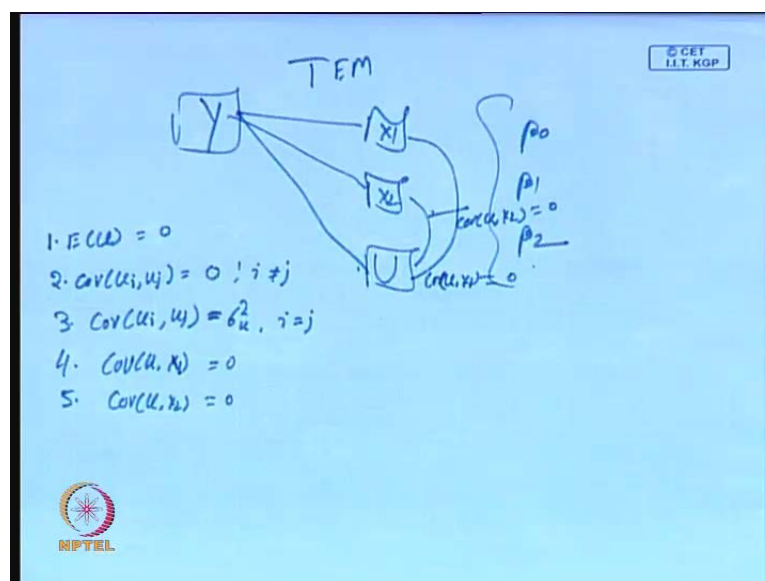
But in the means times, we will assume that there is single error term, but for in a standard procedure of OLS technique, just we are highlighting this in assumption in

every, so it will be again expanded properly or you can say analyzed in a structure when will go for the go to the particular problems.

So, this is first, a standard assumption is the mean of the error term should be exactly equal to 0, then covariance of error term should be again equal to 0. If not, then this problem called as a serial correlation or autocorrelation, we will discuss in details later part. And if the covariance of  $u_i u_j$  where  $i$  equal to  $j$  is not constant that is error way, so then there is the serious problems. So, that is called as a heteroscedasticity problem. If it is so unique, then it is called as homoscedasticity.

That means, homoscedasticity is the **good for** good fit for the econometric modelling, but heteroscedasticity means it is there is a huge problem **problem** for econometric modelling. So, in fact we briefly highlighted it little bit during the bivariate econometric modelling, but right now we will **a will** just again highlighting the issues, but the detail discussion about this problems in the later stage.

(Refer Slide Time: 29:09)

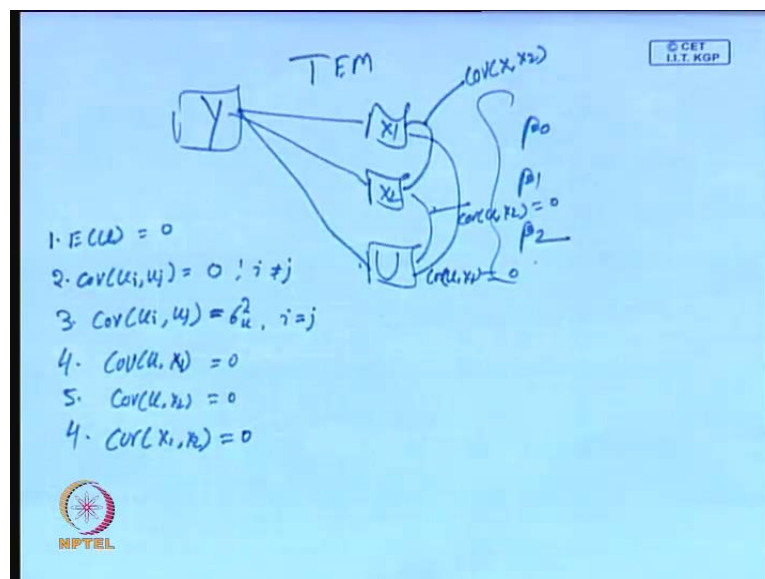


So, now, this is sigma square  $u_i$   $i$  not equal to  $j$ ,  $i$  equal to  $j$ , then another assumption we have put that covariance of **covariance of**  $u$  and  $x$  should be equal to 0. So, this is bivariate assumptions, so that means, when there is a bivariate assumption, bivariate models, so that time there is only one independent variable, that too  $x$  only. So, there is no way to represent  $x_1$  and  $x_2$ .

So, but here we are in the trivariate system and that too here, two independent variables in the system. So, obviously we will represent  $x_1$  and  $x_2$ . That means, the standard assumption will be modified accordingly. So, what we will do that; that means, covariance of  $u$  upon instead of  $x$ , we will put  $x_1$ . So, it will be equal to 0, then another assumption we will add here. So, covariance of  $u$  upon  $x_2$  again equal to 0, so that means, there this particular **this particular** setup and this particular setup should be equal to 0, this particular setup is called as a covariance of  $u$  upon  $x_1$  and this particular setup called as a covariance of  $u$  upon  $x_2$ , this should be equal to 0 and this should be equal to 0 (Refer Slide Time: 32:58). If that is so, then **you you know** the use of OLS technique can be a more practical or more feasible.

So, otherwise system will be in consistent, we will **will** discuss details when will go to this particular **particular** setup.

(Refer Slide Time: 29:09)



So, now what we will do, the standard problem will discuss is here about this multi covalent issue, that multi covalent issue means the existence of linear relationship between or among various independent variables. But here there is no question of various, because there are only two independent variables. That means, just to like to integrate  $x_1$  and  $x_2$ . So, this particular setup is called as a covariance of  $X_1$ ,  $X_2$  or correlation between  $X_1$  and  $X_2$ . So, that means covariance of one  $X_1$  and  $X_2$  should be equal to 0 **should be equal to 0**.

So, that means **so that means so that means**, they should be completely independent. So,  $u$  and  $x_1$  should be independent,  $u$  and  $x_2$  should be independent, then  $x_1$  and  $x_2$  independent. So, that means whatever variables in the left side, whether it is with respect to independent variables or error terms, this should be completely independent, this should not have any relationship, or you can say covariance or correlations.

So, between them, so you mean if there is such a relationship, then **you know** it will affect the model reliability and I am very serious **you will not** you will not receive any best fitted model or you can say it will affect the reliability part of the models. So, that **that you know** assumption or that condition should be **you know** tested and should be required for this model fitness. So, these are the standard tricks before you go for the estimation process, this is one of the serious problems we can at least detect in the **trivariate setup** trivariate system, it is usually not possible to discuss in the bivariate setup.

So, now covariance of  $X_1$  upon  $X_2$  should be equal to 0, then as usual the standard, other information must be there like **you know what to** what we have discussed in the couple of lectures back, in the beginning of the bivariate econometric modelling. That means, since there are three variables in this particular setup  $Y$ ,  $X_1$  and  $X_2$ .

So, in all the cases, the information should be **you know** consistent. So, that means if it is  $Y$  is consist of 10, then obviously,  $X$  should be 10,  $X_1$  should be 10,  $X_2$  should be 10. If there is inconsistency between all these variables with respect to their sample size, then obviously, we cannot go for estimation and you cannot go for getting the estimated model. So, this must be very **very very you know** means urgent requirement for before going to estimate the model, so this is must.

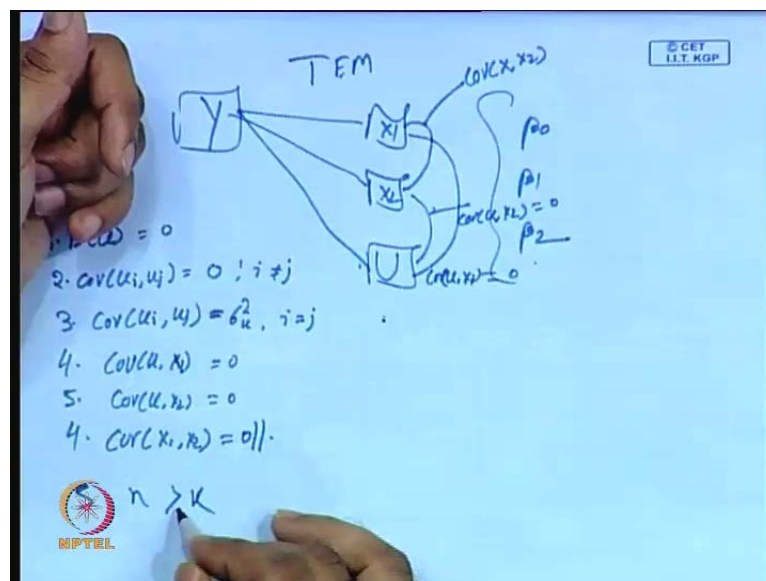
So, first thing is here in sample information must be very **very** uniform with respect to each and every variables in the system. That means, there are three variables in the system, so your sample size should be very **very** consistent with respect to all these three variables. So, this is another assumption before going to apply the OLS or **you know** estimated models.

Now, another thing is that, so here your sample observation should be consistently high. For instance, in the case of bivariate with little bit sample you can proceed, but here the sample size, in fact, it should be substantially **it should be substantially** high. For

instance, **for instance** since, there are three variables in this systems. So, what we can represents in that particular system is called as or denote as items called as k, k represent number of variables in the systems, so number of parameters in the system.

Since there are three variables in the system, so obviously, there should be three parameters, by default it will coming three parameters. So, now here k represents, means k indicates 3. So, obviously k equal to 3 there means, there are three variables in the systems. So, obviously n should be substantially greater than to 3, but in the case of bivariate setup n should be greater than to 2, because there are two variables in the setup. So, now here there are three variables in the systems. So, when if when n **stands to** stands for sample size and k stands **k stands** for number of variables in the system, then the system will be more consistent or the model **model** can be fitted. So, if n is substantially greater than to k, so that means, n should be greater than to k.

(Refer Slide Time: 29:09)



So, n should be greater than to k, it should not be equal to k, when it is equal then there will be lots of problem. So, that is y I am not highlighting n greater than to k, it n greater than k, it is better to put n greater than k that too high in number.

So, there are actually the accuracy of sample size is very interesting issues. For instance, for **you know** simple problems your sample size should not be less than to 30. If it is less than to 30, then **you know** model reliability is very **very** in a other way around. So, it is

very difficult to predict something else, even if you are getting the, **you know** reliable model also or best fitted models, still it cannot be used for forecasting.

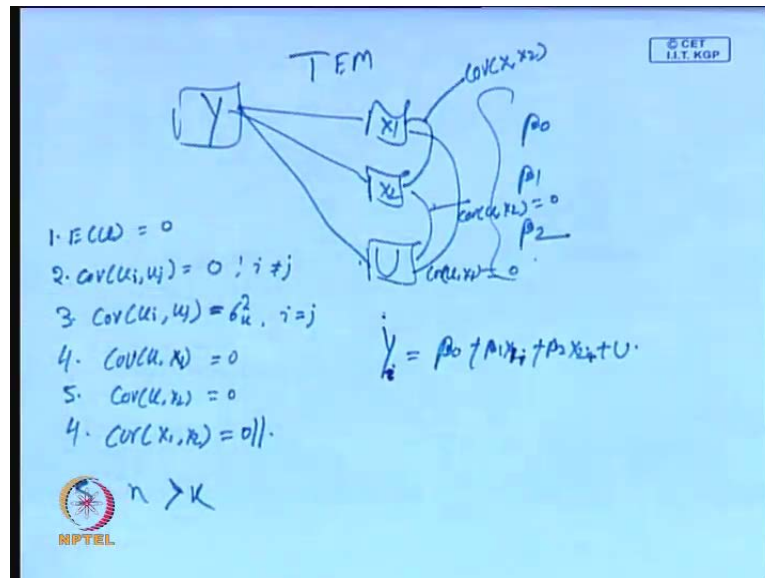
Because your sample size is not substantially high, sample size should be substantially high, that means  $n$  should be greater than to 30 provided, means it is should be an bivariate structures, but in that trivariate structures, it should be substantial again have, this is the minimum requirement of the sample size before estimation. If that is not the case, then it is very difficult to use thus model for forecasting and obviously, there will be serious problem on the reliability testing.

So,  $n$  should be substantially greater than to  $k$ , then obviously, the model will be correctly specified, because we are using here  $Y$  equal to  $\beta_0$  **beta** plus  $\beta_1 \times 1$  plus  $\beta_2 \times 2$ . But this particular formalities in a linear setup, what there may be some non-linear relationship or there is a some different mathematical form of the models, because it may be in a non-linear setups like **you know** algorithm format or **you know** exponential format or any different steps like **(())** or you can say cubic.

Then in that particular structure, the game plan will be a all together different, but in the mean times, we **we** are representing the simple structure or we are assuming that they are linear in natures. Even if **there is** they are non-linear in natures, we can transfer into linear format, that transformation we will discuss in details in the later part, not right now. But in the **in the in the** meantime **will** we will assume that the **the** association between this two variables with  $y$  should be linear natures, so that means, it is linear in parameters.

So, that means  $\beta_0$ ,  $\beta_1$ ,  $\beta_2$  should be linear in parameters. So, that is also one of these standard assumptions, before you go for estimations. So, now with this basic background, so we have to set the, you can say we have to fit the econometric trivariate econometric modelling.

(Refer Slide Time: 29:09)

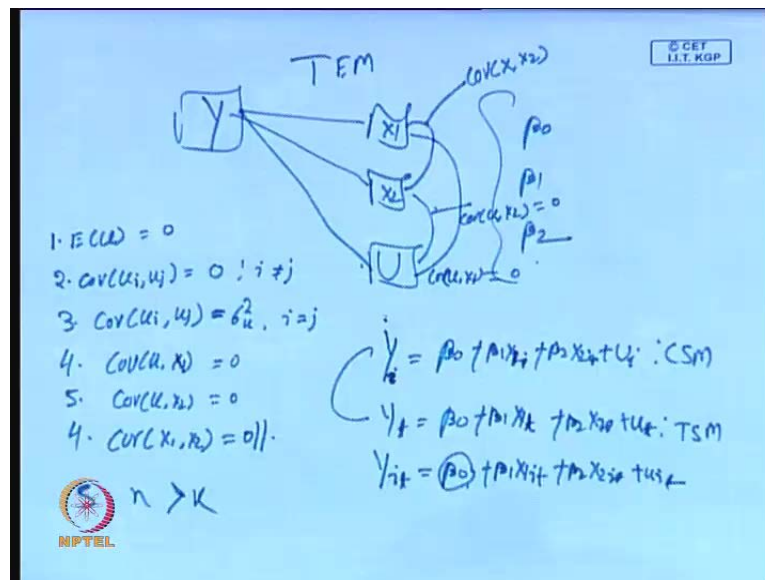


So, the trivariate modelling setup is  $Y$  equal to  $\beta_0$  plus  $\beta_1 X_1$  plus  $\beta_2 X_2$  plus  $U$ . So, now you can also what you can do, you can use also  $i$  here,  $i$  stands for or indicate this sample size structures. So, obviously  $X_{2i}$ , so this is  $X_1$  infect  $X_1 i$  this is  $X_2 i$ . So, now as usual, I have discussed couple of lectures back, that the particular model can be a represented **represented** in three different formats, that is two cross sectional modeling, and times series modelling, panel data modelling, so this particular when we **when we** use this subscript  $i$ , then it becomes a cross sectional modelling.

So, this particular structure or the particular identification is called as a cross sectional modelling, but when I transfer **transfer**  $i$  to  $t$ , then it becomes, the representation becomes you can say is a **times time** time series format, or you can say time series modelling. Then, when we introduce both you can say  $Y_{it}$  together with respect to  $\beta_1 X_{1it}$  like this, so this is called as a cross sectional modelling.



(Refer Slide Time: 29:09)



So, similarly we will write bit Y equal to beta 0 plus beta 1 X 1 X 1 t, then beta 2 X 2 t, then U t, obviously there is U t. So, here there will be U i, so this side is called as a time series modelling. So, similarly if will integrate this two, then if will represent the situation, this is called as a panel data modelling.

So, what we will do, this is  $(Y) Y_{it}$  equal to beta 0 plus beta 1 X 1 i t plus beta 2 X 2 i t plus u i t. Obviously, this **you know** when will go for hardcore panel data modelling, then the intercept term also all can also vary with respect to sample setup and that too in **you know** cross sectional observation and times series observations.

So, we will discuss in detail when will go for pure panel data modelling. So, in the **in the** means times, there are three ways we will represent trivariate structure that too cross sectional observations, times series observations, and penal data observation

So, now accordingly these there are three different models which we have represented here in a nice way. So, now, you see here, so how do you go for the this particular econometric setups.

(Refer Slide Time: 43:18)

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + U \quad \text{--- TEM} \quad \text{①}$$
$$\rightarrow \hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 \quad \text{--- ②}$$

So, now so  $y$  equal to  $\beta_0$  **beta 0** then plus  $\beta_1 \times 1$  plus  $\beta_2 \times 2$  plus  $U$ , then as usual by bivariate objectives, so we must have **we must have** very strong objective here. So, means similar kind of objective, so your first objective is to get the estimated model. So, that means this is a econometric model, trivariate econometric model. So, now we will transport this to a estimated models, so we will transfer it to estimated model quality  $\hat{y}$ . So,  $\hat{y}$  equal to  $\hat{\beta}_0$  plus  $\hat{\beta}_1$  plus  $\hat{\beta}_1 X_1$  plus  $\hat{\beta}_2 X_2$ , but that is all.

So, this is what the estimated trivariate econometric modelling **estimated trivariate econometric modelling**. So, we will call it equation number 1 and we will call it equation number 2. So, that means the objective of this particular process means, we first process is a, we must have a trivariate mathematical form of the model and will like to test that particular model, then you transfer the trivariate mathematical model to statistical form of the model. So, that is what we will call it a trivariate econometric modelling. So, then once you have trivariate econometric modelling setup, so our objective is to transport this particular setup into a particular estimated format.

So, that is what we call it  $\hat{y}$  equal to  $\hat{\beta}_0$  plus  $\hat{\beta}_1 \times 1$ , so  $\hat{\beta}_2 \times 2$ . So, the way we will move equation number 1 and equation number 2, so that is very, you can say interesting. So, what is your agenda here, so we like to know what should be our objective, we will like to know what is  $\hat{\beta}_0$ , what is  $\hat{\beta}_1$ , and what is  $\hat{\beta}_2$

hats. So, we like to know, so you like to means, we like to our objective should, our objective must be to get y hat. So, y hat equal to beta 0 hat plus beta 1 hat x 1 similar beta 2 hat x 2. So, that means the way we will move transformation from equation 1 to equation 2. So, it will be by default or assumption that we are minimizing the error term. So, that means the way we will minimize the error term. So, the equation 1 will turn into equation number 2, that is how we have to apply these OLS techniques.

(Refer Slide Time: 43:18)

© IIT KGP

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + U \quad \text{--- TEM} \quad \text{①}$$

$$\rightarrow \hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 \quad \text{--- ②}$$

$$\hat{\beta}_0 = ? \quad \hat{\beta}_1 = ? \quad \hat{\beta}_2 = ?$$

$$Y = \hat{Y} + e$$

NPTEL

So, now the standard idea is to apply OLS techniques, first **you know** you get the error term. So, what is exactly error term, so now as usual. So, y **y** can be represented as a y hat plus e **y hat plus e**. So, that means this is y hat exactly it every times whether it is a bivariate format, or trivariate format, or multivariate format. So, y hat is the summation of, or influence of all independent variables together. So, this is the **this is** otherwise called as the explain **explain** percentage, and whatever not explained, that is represented in the error terms. So, that means the total **total** y is typical influenced by the explained item which is represented in y hat, and another is represented in e format, that is error format.

So, now **you know** when we will go for reliability, it is also again this same structures, total sum square equal to explain some square and residual some square. So, by the way you will get again y hat square y summation y square equal to summation y hat square summation a square. In fact, the **the you know** formula and the derivation is little bit

complex and something different, then the bivariate setup, what the structure or you can steps are moralisms. So, now what will to do, so the y the total percentage, total variation y is influenced by the explain variance, explain percentage and unexplained percentage, so now so we get to know what is e.

(Refer Slide Time: 43:18)

© GET  
I.I.T. KGP

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + U \quad \text{--- (1)}$$

$$\rightarrow \hat{y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 \quad \text{--- (2)}$$

$\hat{\beta}_0 = ? \quad \hat{\beta}_1 = ? \quad \hat{\beta}_2 = ?$

$$y = \hat{y} + e$$

$$e = y - \hat{y}$$

$$e = y - (\hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2)$$

$$e = y - \hat{\beta}_0 - \hat{\beta}_1 X_1 - \hat{\beta}_2 X_2$$

NPTEL

So, e is the difference between e is the difference between y minus y hat e is the difference between y minus y hat. So, that means what is what is Y hat here, so Y hat is here? So, Y hat here is represented beta 0 hat plus beat 1 hat X 1 plus beta 2 hat X 2. So, what you have to do, we will just represent here. So, y minus instead of writing y minus y hat, we can write y minus beat 0 heads plus beta 1 hat X once plus beta 2 hats X 2 beta 2 hat X 2, put it here bracket. So, this is what we call it e, so e is defined as the difference between y minus the sum total of beta 0 hat plus beta 1 hat X 1 plus beta 2 hat X 2. In fact, when in the case of bivariate setup, we use the parameters called alpha and beta.

Since, there is a only one X, we are using beta intercept, but you can start with like you know beta 0 and a beta 1 x 1. So, it can be possible, but you know since there is only one variables in that particular setup, so we are not introducing extra intercept terms. So, that is we are representing simple X. So, since we are using simple X, so that is why we are introducing the beta concept only. And in the on the contrary for intercept, we are using

another constant called as a alpha heads, but the game is a very interesting more **more** **you know** eye catching when will go for trivariate modelling.

So, in that case of trivariate modelling, we are not using any separate, we are not using different parameters all together. Because the moment we will be using different parameters of together, **the understood** there may be some understanding problem for beginners and in the same times and there may be lots of confusions and **you know** the model beautiness will also get a affected, here **here** the **the** model beautiness will be interesting and once we will set the system with respect to only beta, so since we have two variables;  $x_1$  and  $x_2$ .

So, we are introducing beta 1 there and for  $x_2$  we are introducing beta 2. So, obviously we can start with the concept called as a beta 0, that is the supporting component. So, beta **beta** 0 is this supporting that is intercept on and **you know** the beta 1 is the supporting component of  $X_1$  and beta 2 is the supporting component of  $X_2$  **alright**.

So, now **so, now** the process of integration, so we will get the error term which is the difference between the true value  $y$  minus the estimated value  $\hat{y}$  here. So, by default the error term is nothing but  $y$  minus, if I will simplify further and  $e$  equal to  $y$  minus  $\hat{\beta}_0$  minus  $\hat{\beta}_1 \hat{X}_1$  minus  $\hat{\beta}_2 \hat{X}_2$ . So, this is how the entire system all together **all together**, so what is our objective here **what is our objective**? As usually in the bivariate structure, we have two strong objectives, what is this two strong objectives, let me highlight here once again.

(Refer Slide Time: 50:47)

$$e = y - \beta_0 - \beta_1 X_1 - \beta_2 X_2$$
$$\sum_{i=1}^n e_i^2 = \sum_{i=1}^n (y_i - \beta_0 - \beta_1 X_{1i} - \beta_2 X_{2i})^2$$

OLS  
Ordinary  
Least Squares

© CET  
I.I.T. KGP

NPTEL

So,  $e$  equal to here  $y$  minus  $\beta_0$  hats minus  $\beta_1$  hat  $X_1$  minus  $\beta_2$  hat  $X_2$ . So, now our objective is to get the  $\beta_0$  hat,  $\beta_1$  hat, and  $\beta_2$  hats. So, how do you go for that, so now, since our objectives is to get the all the parameters, that is with respect to  $\beta_0$  hat,  $\beta_1$  hat, and  $\beta_2$  hat. So, we like to minimize the error some squares, so as usual the bivariate setup. So, what we will do, so we like to know what is the error sum squares, then we have to apply the OLS techniques to minimize that error sum squares.

So, what is the error sum squares, the error sum square will be represented as a summation  $e$  square, obviously  $i$  is there. So,  $i$  equal to 1 to  $n$ , and is equal to summation  $y$  minus  $\beta_0$  hat minus  $\beta_1$  hat  $X_1$  minus  $\beta_2$  hat  $X_2$ , so obviously, it will be squares. So,  $i$  equal to 1 to  $n$  here and obviously, this is  $X_1$   $i$ , this is  $X_2$   $i$  and this is  $y$   $i$  this is the **this is the you know** starting procedure of estimation. So, that means we have to apply here OLS techniques that is ordinary least square methods ordinary ordinary least square method standard ordinary least square methods to minimizing this error sum.

So, again we have to go for optimization technique **you have to go for optimizations technique**, because here since we like to minimize the error sum, then obviously, we have to go for minimization principal, by standard format again we have to follow the necessary condition and sufficient condition, but we have to see how you to minimize the

systems. That means, here the minimization system is with respect to  $\beta_0$ ,  $\beta_1$ ,  $\beta_2$ .

So, the way we will minimize, so we have to differentiate with respect to summation a square with respect to  $\beta_0$ , summation a square with  $\beta_1$ , summation a square with  $\beta_2$ , then all together we will simplify then we **we** will all together get a system, simultaneous equation systems where there are three equations with respect to three variables. So that means, if we will solve this particular three equation which is derived from this OLS technique from this standard error summation squares, so we will get the estimated value of  $\beta_0$ , estimated value of  $\beta_1$ , and estimated value of  $\beta_2$ .

So, now all this details we will discuss in details, that is in the next class; in the timing it is not possible to discuss, we will start and within one minute we cannot finish this job. So, we will conclude the session today here. So, we will discuss detail this session in the next class. Thank you very much, have a nice day.