Advanced Business Decision Support Systems

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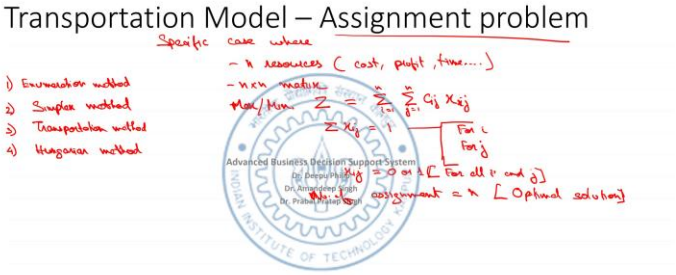
Lecture 25

Transportation Model- Assignment Problem

Welcome back to the course on Advanced Business Decision Support Systems. We have discussed the linear programming from the last week, and I have discussed the further programs of linear programming that is the transportation model in this week. This is the last lecture, where I will discuss the Assignment model, which is another kind of a transportation model. So, what is an Assignment problem? I will try to discuss here, and we will try to have a solution to the Assignment model using different methods. This is a method known as Hungarian method, which I will try to use to get the solution to the Assignment problem, and we will also try to see the solution in the MS Excel program.



So, Transportation model with Assignment problem, I have put the Transportation model itself because Assignment model is a specific case of the Transportation model only.

So, it is a specific case where the resources are to be allocated to the specific activities. For instance, a job is to be allocated to the operators. For instance, the teachers are to be allocated to the tutorial sessions. So, anything when the resources are to be allocated to specific activities, the number of resources or number of activities would always be 1 that is, the matrix that is there in the assignment problem is always square matrix.

This is one condition. And, in the Assignment problem, the availability is always 1. This

I will try to put here that is here suppose, n resources are there and n jobs are there that is, activities are there these might be some cost, some profit, time which is to be minimized or maximized and so on. So, then we get n into n matrix. And, allocation for each matrix is equal to 1.

I will just try to put this in the mathematical form where, I will say, the model becomes maximize or minimize Objective function with

$$\sum_{i=1}^{n} \sum_{j=1}^{m} Cij\ xij$$

$$\sum xij = 1$$

This is true for all the cases that is for i, for j, it is all the rows or columns and xij could attain the values binary that is, either 0 or 1. This is again for all i and j. This means the cost matrix is always square matrix as it is mentioned n into n matrix is there and the optimal solution of the cost matrix would be attained, when one assignment is there in a given row and in a given column.

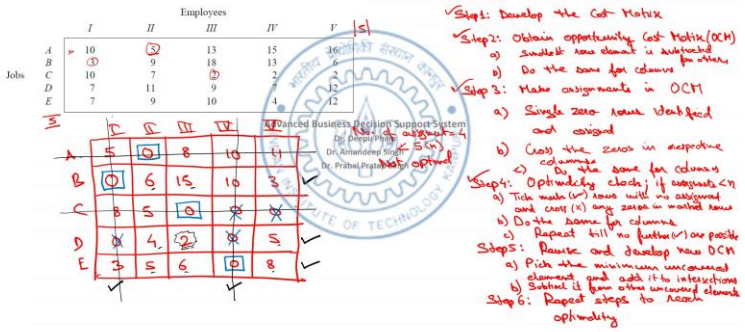
So, that is the number of assignments is equal to n this means, the solution is optimal. This is simply we are talking about the Assignment problem. There are multiple methods to solve this. So, there is method known as Enumeration method. Enumeration means, we are only trying to assign the given resource to given activities and then assignment that involves minimum cost, that is taken into consideration.

So, this is this akin to the least cost method that we discussed for the Transportation model. So, this is a simple method then also, we can have a Simplex method as I have put the constraints here. The constraints could be build accordingly and Simplex method or Simplex algorithm could be used to solve the problem.

Then, we have the Transportation method. Transportation method means, the way we try to solve the transportation model using the least cost, using the northwest collar rule or using the Vogel approximation method and then applying the Modified Distribution method for optimality transportation method could be also be applied.

The most prominently used method is Hungarian method which I will try to demonstrate while taking an example and I will also put the steps of this method and try to solve the problem.

Transportation Model – Assignment problem



So, this is the problem that is given. A department of a company has five employees with five jobs to be performed. The time (in hours) that each man takes to perform each job is given the effectiveness matrix. This is known as effectiveness matrix.

How should the jobs be allocated one per employee so as to minimize the total man hours? So, the resource here is man hours, it could have been cos,t it could have been profit if it had been cost, we have to minimize, if it is a profit, we have to maximize. Let us try to solve this minimization problem first. For this minimization problem, I will first jot down the steps. I have put the matrix here. So, the steps are, number one that is, we develop the cost matrix.

I will just put the steps first, then I will try to elaborate them further. Step two is, we obtain the opportunity cost matrix. There are certain steps to it. I am just leaving it blank. I will show you when I try to do it.

Then step three is, make assignments. Make assignment means, we try to make assignments in opportunity cost matrix I will call it as OCM. To make the assignments, there are steps that is, certain rounds of assignments are made for which, there are also certain rounds, which are there. I will just leave them blank, first round, second round and how the assignments are been made. Then, step four is optimality check.

Step five is revise and develop new opportunity cost matrix. If required, optimality check is there, if optimality is there, we need not to do it and if it is to be revised, then we keep on repeating these steps. Step six, repeat steps to reach optimality. So, first step is to develop the cost matrix. We have to develop the cost matrix based upon the the problem that is given that is, this cost matrix is given.

So, this in itself is a cost matrix that is, there we have the cost for each of the rows and each of the columns that means, we have jobs and employees. This is given already and these are also 5 and 5, so that means, the cost matrix is balanced here. So, this cost matrix, first step is there already. Obtain the opportunity cost matrix, so what is an

opportunity cost matrix that is, we try to get what is the penalty, if we do not use this opportunity, the way we did it in the Transportation model, the way we did it in the Simplex program, the opportunity cost matrix is obtained. How do we get that there are steps to do it? I will just divide them into five rows and five columns.

To obtain the opportunity cost matrix, we try to find the minimum value or the smallest element in row, I would say, smallest row element is subtracted from others that is, in this row, the minimum row value is 5 in the second row the minimum row value is 3 in the third row, the minimum row value is 2 there are three 2s here that means, more zeros would come, so I will try to subtract this from each of the values here. So, 10 - 5 is 5, 5 - 5 is 0. So, I will subtract it from each of the row elements.

I will get a matrix like this for the second row that is, for b job, 3 is the minimum value, I get 0 here 6, 10 and 3 for the third row, minimum value is 2, so I get 10 - 2 is 8, 7 - 2 is 5, 2 – 2 is 0, 0, 0, for the fourth row, the minimum value is 7, so I get 0 here 11 - 7 is 4 then, I get 2, 0 and 5 for the fifth row that is, we get here 7 - 4 is 3, 5, 6, 0, 8 then the b part of getting opportunity matrix is, we do the same for columns that is, the minimum value from the column is identified and we try to subtract it from the other values that is, minimum value in the column.

You can see, in each of the column, we have zeros so zeros to be subtracted from the other values. So, the matrix remains like this only. So, we have obtained the opportunity matrix that is, step one is done, step two is done, step three is make assignments in the opportunity cost matrix, this is an important step, where a little care is to be taken. Now, to make the assignments, the first thing is, we try to see each row one by one and any of the rows that contain single zero row.

I will just put the step here as single zero rows identified and assigned the rows that are having single zeros. So, I have single zeros in the row A, B and E these would be first assigned when I try to assign it, I just put a box over it that this assignment is now completed.

This is first step and we have obtained three assignments that is, in the cell A2, B1 and E4. Now, we need to examine each of the columns and wherever there are zeros in the assigned rows, those zeros are to be crossed, so I will cross this zero because assignment is already made for the employee one and I will also these two zeros. So, second part is, cross the zeros in respective columns.

Now, further we need to examine each of the columns. Here, for each of the columns starting from column one, if there is any column having one zero, that column would be given an assignment that means, I can give an assignment to the column three here and wherever is a zeros in the respective rows, those zeros are crossed.

So, the next step is, I will put here as part C, do the same for columns so that means, we have obtained the assignments and all these zeros are now taken away, zero means where the cost is minimum. In this case, we are talking about time, the time is minimum. So, here the minimum values are subtracted from all other row and respective column elements, so zeros are the best opportunities which are available.

So, there are no zeros left now and the number of assignments here are four which is less than five that is, n that is not optimal. Now, we are talking about step four, we are trying to see the optimality can check, so the solution is not optimal.

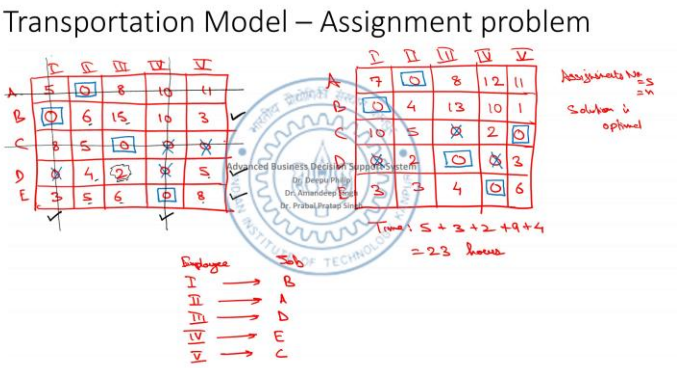
So, that is, we need to now work to get the optimal solution that is, we try to move to the next step and we try to draw the minimum number of lines, how does this go? We try to mark the row where there is no assignment. So, row D has no assignment. I will tick mark this row. Then, we will try to mark the columns where the row that is tick marked here has zero elements. So, column four has zero element.

This is tick marked, so column four is marked and column one is marked because these have zero elements in the unassigned row. Now, amongst the marked columns, wherever assignments are there, these columns we need to see and we will try to mark these columns, tick mark row B because it has assigned zero for column one. Also, we tick mark row E because it has assigned value in the column four.

Now, there could be no further ticks, they got no zeros. Now, we try to draw the lines that means, we are still on the optimality test here, all the unmarked rows and marked columns are drawn with lines.

Unmarked rows are row C and row A. So, I draw straight lines here, row C and row A and for the marked columns that is, column 1 and column 4. Now, we try to identify the minimum value from the unmarked elements, unmarked values are here: 6, 5, 3, 4, 2, 5, 6, 8. The minimum value that is thereis two. So, this value is now my key value that is to be taken into consideration.

Now, comes the next step, to get the revise matrix, to get the revise matrix and develop new opportunity cost matrix we need to subtract this value from each of the uncovered elements that is any of the elements which are not covered by lines and these are to be added to the intersection points. I am not writing all of these steps here, I will just give you that in the notes later that you can read it from here. Now, how do I go about?



Now, I have brought this matrix here into the next slide, where intersection points would be added with the minimum value from the uncovered elements. Uncovered elements minimum value is 2. So, I will obtain another matrix here which is the revised opportunity cost matrix.

Here, at each of the intersection points, I will add the minimum value that is, 2 that is 5 turns to 7, this 10 turns to 12, 8 turns to 10 and 0 turns to 2 and I will subtract it from each of the uncovered elements that is, in the element B2, it is subtracted and this becomes 4, B3 becomes 13, B5 becomes 1, D2 becomes 2, D3 becomes 0, D5 becomes 3, E2 becomes 3, E3 becomes 4 and E8 becomes 6.

So, these are the changes which have come. All other elements remaining same that is, this remains 0, 8, 11 then 0, 10, 5, 0, 0, 0, 3 and 0. Let me try to apply or get the opportunity matrix once again. We have 0s in all the rows and if these are subtracted from the other elements of the rows, we will get the same values.

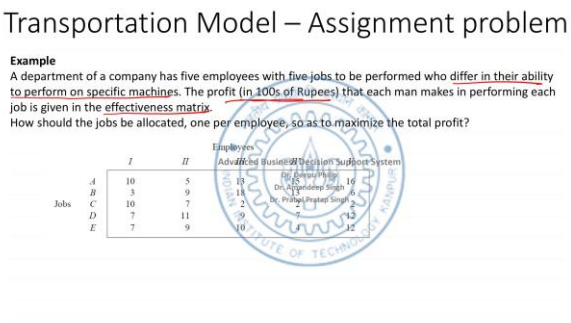
Similar is the case of the columns that means, each of the columns and rows are now having 0 values. Let me try to start assigning the values in the similar fashion. I will repeat this step 2 here that is, I will try to see the rows with single 0s. I will try to assign them. So, now I have obtained my opportunity matrix here.

Because we have 0s in all the rows and 0s in all the columns and if I still apply this step 2, where the minimum value amongst the rows is to be subtracted from the other elements because the minimum value is 0, the matrix remains same. Now, the next step is to assign the values. To assign the values, first let me identify the rows which are having single 0s. This row is having single 0 and this row is having no respective 0. Second row B is also having a single 0 and other 0s are crossed.

Now, row E also has a single 0. This is also assigned and other 0s in the respective columns are crossed. So, 3 assignments have been made. So, this step is done now. Now, let us try to see the columns which are having single 0s. So, I am only left with column 5 which is having single 0 because column 3 has two 0s in the rows C and D.

So, column 5 is now assigned and corresponding 0s in the assigned rows are crossed. So, you are left with only one 0 that is, element D3 which is assigned. Now, we have number of assignments equal to 5. Assignments number, this is equal to 5 which is equal to n. Therefore, solution is optimal and each row is assigned with 0 and each column is having 0 that means, the jobs and the employees are assigned.

Now, if I try to calculate the cost which is given in the first matrix here. So, these costs which are given, those costs come to be for the respective elements, this is actually time. So, time here would be 5 plus 3 plus 2 plus 9 plus 4 which is equal to 23, the units given here are hours and the solution is to be written in such a way that, employee is given a specific job that is, employee 1 is given job B, employee 2 is given job A, employee 3 is given job D, employee 4 is given job E and employee 5 is given job C. This is the optimal solution to the given assignment problem. I will also like to take a maximization problem.



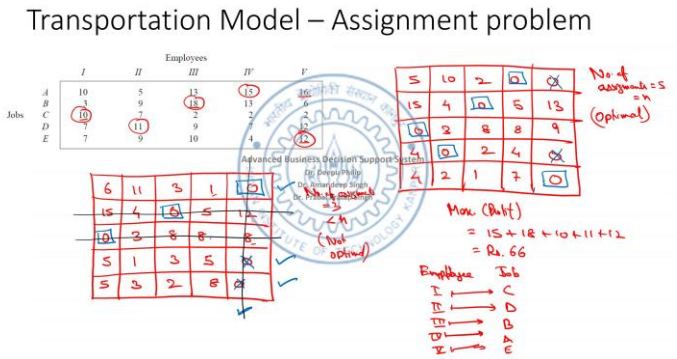
Now, I have taken the same data, but for the maximization, I have redraft this statement and it is converted into profit now. Now, it is drafted as a department of a company has 5 employees with 5 jobs to be performed who differ in their ability to perform on specific machines that means, 5 employees are there and 5 machines are there.

For example, in the machine shop, one employee is more proficient in working in a lathe machine, another employee is more proficient in drilling. So, the worker who is more able to work on the lathe machine is also able to work on drilling but the profit that it generates while working on lathe machine is higher. So, these are the associated profits which are given.

So, profit in hundreds of rupees, each man makes in performing each job is given in the effectiveness matrix here. How the job should be allocated one per employee so as to maximize the profit. So, the variations of assignment problem are similar to what we did

in the transportation problem. The unbalanced, if it is unbalanced, that is the number of rows are not equal to the number of columns, we add additional row or column so as n cross n matrix is obtained.

This is one part then, the solution that is to be maximized, this is a criteria for that, I will try to just do then, alternate solutions could also come. Alternate solution means, whenever in a solution, the ways to strike off a certain number of zeros is more than one, then alternate solutions could also come.



So, in the maximization problem, major criteria is, we try to just pick the maximum value of the matrix. I will just try to do it here. So, the criteria in the maximization problem is we try to pick the maximum values in the respective rows and columns. So, maximum values means, here 16 is maximum value, 18, then I have 10, then 12 and 12.

These are the maximum values among the rows. I try to the step is, I will subtract all other elements from this maximum value and obtain my opportunity cost matrix. So, how will I will try to do here? So, how is this obtained? Let me try to quickly perform the same method for the maximization problem.

So, here maximum value is 16, so 16 - 10 is 6, so I have obtained this value as 6, 11, 3, 1, 0, 15, 4, 0, 5, 12, maximum value for the row C is 10, 10 - 10 is 0 then, 10 - 7 is 3, 10 - 2 is 8, 10 - 2 is 8, 10 - 2 is 8, the maximum value for row D is 12, 12 - 7 is 5, 12 - 11 is 1, 12 - 9 is 3, 12 - 7 is 5 and 0. Similarly, for row E, I will put the values.

So, then we do the same criteria for the assignment. I am missing one step here, I am not doing the same iteration for the columns. If I do this, value would also turn to 0, 0 here because here the maximum value is 8. So, let me try to miss that criteria and try to solve. Now, let me try to see, I will assign the zeros wherever in the rows, we have only one 0.

So, this 0 is first assigned in row A, row B, row C. When row A was assigned, these two zeros were crossed. So, there are no zeros left. Now, again I will try to solve the same

criteria. I will first try to mark the unassigned rows. So, wherever assigned 0 is there in the respective column I will try to mark this column and wherever the marked column has an assigned 0, this row is also assigned.

So, there is no possibility of further tick marking anything because all these zeros are already covered. So, I will try to now draw lines that will intersect at points, at lines through the unmarked rows and marked columns. You will see only very few lines have come, I will just try to quickly draw the matrix once again here.

The minimum value amongst the given numbers here that is the uncovered numbers, the minimum value here is 1, this is 1. So, this is subtracted from all other values that is, the uncovered values and these are added to the intersection points that means, this 12 at B4 turns to 13 and this 8 at C4 turns to 9 and this is subtracted from all the uncovered values this is 5, 10, 2, 0, 4, 0, 2, 4, 4, 2, 1, 7 all other values in the elements remaining same, this is 0, 15, 4, 0, 5, 0, 3, 8, 8 and this is 0 and 0.

Let me try to apply the criteria for assigning zeros once again. So, I will just pick the rows with single zeros, row B is picked, row C is picked and row E is picked. There are two other zeros in the column corresponding to row E that is, the column 5, 2, 0 are crossed.

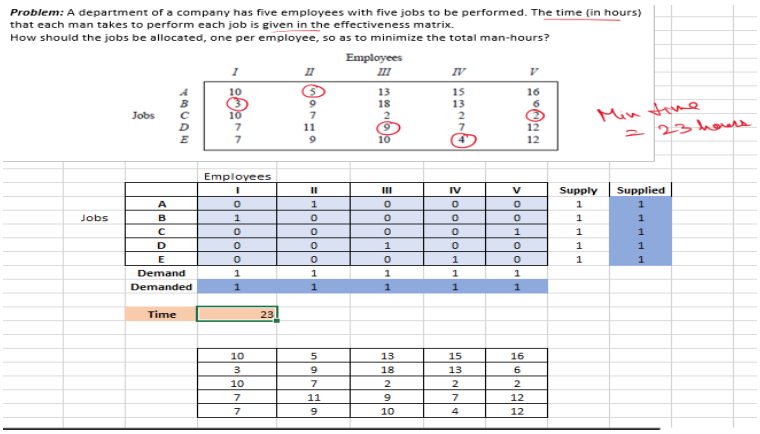
Now, for the column as well, let me try to assign column 2 has single zero, column 4 has single zero. Now, the number of assignments are 5 here, previously number of assignments were 3 which is less than n.

Therefore, not optimal In the next step, number of assignments is equal to 5 which is equal to n therefore, optimal. Now, let us try to see the cost for the respective workers, this is the maximization problem, the maximum cost would be coming, I would see better use the word profit maximize, that is profit.

So, the maximum profit that is coming for the respective cells that is, for A4, B3, C1, D2 and E5, you can see almost the maximum values are picked from each of the rows. So, this becomes 15 plus 18 plus 10 plus 11 plus 12 that is equal to rupees 66 and the assignments are in such a way that is, employee and job for the employees 1, 2, 3, 4 and 5 the jobs, which are given are C, D, B, A and E respectively.

So, this is the maximization problem. So, this was just a quick solution to the assignment problem, for the minimization and the maximization, both of them were discussed. Hungarian method was used. Now, I will take you to the actual program, where we will try to see assignment problem, how it is solved.

Now, I have opened the actual program here to get the solution to the assignment problem using the solver analysis tool in MS Excel is easiest amongst all the methods that we have discussed in the solver.



Problem: A department of a company has five employees with five jobs to be performed. The time (in hours) that each man takes to perform each job is given in the effectiveness matrix.
How should the jobs be allocated, one per employee, so as to minimize the total man-hours?

Employees

Jobs	I	II	III	IV	V
A	10	5	13	15	16
B	3	9	18	13	6
C	10	7	2	2	2
D	7	11	9	7	12
E	7	9	10	4	12

Min time = 23 hours

Employees

Jobs	I	II	III	IV	V	Supply	Supplied
A	0	1	0	0	0	1	1
B	1	0	0	0	0	1	1
C	0	0	0	0	1	1	1
D	0	0	1	0	0	1	1
E	0	0	0	1	0	1	1
Demand	1	1	1	1	1		
Demanded	1	1	1	1	1		

Time	23

10	5	13	15	16
3	9	18	13	6
10	7	2	2	2
7	11	9	7	12
7	9	10	4	12

So, because assignment here is only one and directly the simple constraints would be covered as we did it in the transportation models.

So, only I need to copy the data. The problem is already given here. This is the same problem that we did it on the slides during the Hungarian method. So, here number of employees are there and jobs are there, I will just try to put this data here directly jobs employees.

Okay, now the data is entered here. I will just label them. So, the way it is given, this is employee 1, employee 2, employee 3, employee 4 and employee 5 and these jobs are A, B, C, D and E. Simply, it is given just setting the matrix to look it more clear. This is the time matrix. So, time matrix, I will just pick this data and copy it down here because I need to get my decision variables here and I would just say these values are 0. This is just a problem formulation that is, data cleaning to be entered into the solver program.

So, I would say, here both the supply and demand is 1, supply and demand. This is 1 each and demand is also 1 each. So, again as we said, in the previous slides supplied and demanded, this would be put here, this is equal to sum of the respective column cells. I will just drag the formula here to copy it.

Now, this is equal to summation xij for the rows, enter. I will put this formula here. The only point that is left is to put the time that is my objective function. So, this is equal to sum product of the decision variables with my number of hours respectively. So, I will just color the variables that are going to change here.

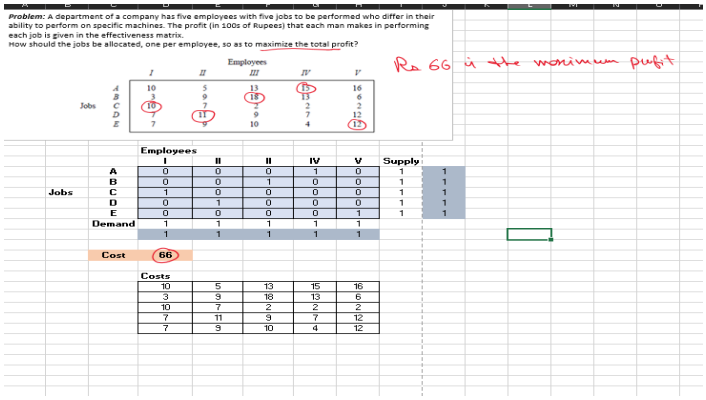
The decision variables are colored, this would be equal to 5. Anyway, this is a balanced matrix, if it has been not balanced, we might have to balance it. This is all good now. I will just say, center align everything. Now, I will directly come to my solver program and go to data, go to solver.

I will just reset all and say ok. One additional thing that you will learn from here that is, in the assignment problem is, we need to add a constraint known as Binary. That is the decision variables correlate values 0 and 1 that is, xij is either 0 or 1. Now, objective function is to be first selected that is, this cell D23 enter. Then, we have to minimize this time and change variables are given in my decision variable matrix that is entered.

Constraints, I need to add constraints. The constraints, first two constraints are there that is the total xij for respective rows is equal to the bj. Then xij that is the demand is equal to the ai, these are added. One constraint is missing that these should be only able to take the binary value. I need to add a constraint that the decision variables need to take only binary value. So, among the expressions which is the central drop down menu here, I need to select bin which is binary, it has given here the constraint would be binary.

I say ok. So, this is only additional part that we need to add a binary constraint into the solver to get the solution. Now, the program that is to be selected is simplex LP and I try to solve. So, I have got the solution 23. So, I have got the time as 23 hours.

So, this is the same time that we have gotten for the minimum value. Time is equal to 23 hours and assignments are given in such a way that we have assigned job A to employee 2, job B to employee 1, job C to employee 5, job D to employee 3 and job E to employee 4. So, this was the minimization using the MS Excel program for the assignment model that we try to solve using the Hungarian method. We can see the solution is coming same. Ok, let us now also try to see the maximization. If I only turn this minimization to maximization. Let me only try to see this once.



So, I have again copied everything here. So, here what I will try to do is, I will just try to put the values for the objective function and I will say maximize not minimize. I am selecting maximize and change variables are selected. The maximum value that we have seen there was 66 rupees.

It is the same problem that is put here. The profit was 66 rupees. So, I need to add the constraints. The constraints are for the respective xij equaling to pj and ai and a binary constraint, all the decision variables could take only binary variable that is 0, 0, 1. Ok, now the problem is maximization and I select Simplex linear programming once again and I try to solve.

You can see the profit that is obtained here. This is the same problem. We were to maximize the profit and that is obtained as 66. So, we can say rupees 66 is the maximum profit. When we have assigned job A to employee 4, job B to employee 3, job C to employee 1, job D to employee 2 and the job E to employee 5. This was the assignment problem. In this week, we have discussed the transportation modeling and its variations, the different variations where we also discussed the transshipment model, the assignment program.

So, these were all discussed and this is covering the deterministic modeling which was part of this course. Next we will discuss the python programming. Dr. Prabal will come in the next week. He will give you brief introduction to what python program is and how it is used, from where it has emerged and almost everything that Dr.

Deepu Phillip and I have discussed that would also be tried to be solved using the python coding language. Let us meet in the next week where Dr. Prabal will take the lectures. Thank you.