Advanced Business Decision Support Systems

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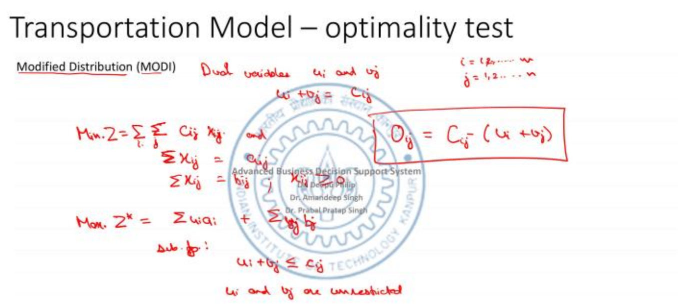
Lecture 23

Transportation Model- Optimality Test

Welcome to the week 6. We have discussed the Transportation Modeling in the first lecture where, we have only seen the methods to obtain the First Feasible Solution or Basic Feasible Solution, three methods were discussed there. Now, using the last solution that was obtained in the first lecture that was through the Vogel's approximation method, we will try to apply the Optimality test here.



So, try to understand what is Optimality test in the transportation. So, we will try to discuss only one method that is, MODI test, which is Modified Distribution test. There are certain other methods like Stepping Stone method is one which I am skipping because MODI is the best test that could be used for the optimality for the transportation models.

Here what we try to do, we try to use the concept of Dual variables. When I say Dual variables, duality I try to discuss in the lecture on linear programming. So, we add the dual variables ui and vj. So, where anyway y again varies from 1, 2… m and j varies from 1, 2…. n.

So, where,

$$u_i + v_j = C_{ij}$$

Where, $i equal\ to 1, 2 \dots \dots m\ and\ j equal\ to 1, 2 \dots \dots n$

$$Min. Z = \sum_i \sum_j Cij\ xij$$

and

$$\sum xije = Cij$$

So, what we try to do? We try to apply the Dual theory here. So, as we try to solve using the variables ui and vj majorly, we can get a dual of this.

$$\sum xij = bij; xij \geq 0$$

That is, we try to get the variables ui and vj and we try to get the Opportunity cost. What is the Opportunity cost? I denoted it by

$$Oij = Cij - (ui + vj)$$

So, not getting into the duality directly, however the duality constraints I will just mention there, how do these come and you can use these. This would be maximize.

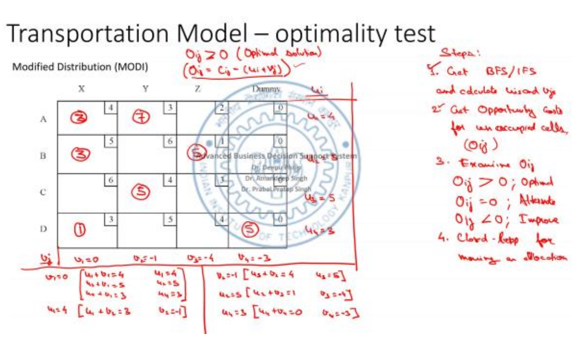
$$Max. Z^* = \sum ui\ ai + \sum vj\ bj$$

Subjected to,

$$ui + vj \leq cij$$

And, here though in the primal, so primal is the word that is the primary problem statement that we have here all xij were greater than or equal to 0, but ui and vj are unrestricted that means, they are unrestricted in sign.

They can take negative value, 0 value, positive value is anything. Let us try to take a problem statement or the problem that we solved in the last lecture and try to apply the optimality test and we will see what are these ui and vj and how do we calculate them.



So, we had this problem here in which, we have gotten the solution already. In the solution, what we had in our hand was, we had 3 units allocated in this cell and 3 units in cell BX, 1 unit in the cell Dx, 7 units in cell Ay, 5 units in cell Cy, 5 units in cell Bz and 5 units in cell D dummy. So, what we will say here, we will say, we will have ui and vj here.

So, this is my u1, u2, u3 and u4. I have v1, v2, v3 and v4 here. Now, the solution is said to be optimal, when we have the opportunity cost that is oij greater than 0. So, this is known as Optimal solution. So, what is oij? I mentioned in the previous slide, it is difference between the present cost and sum of the respective ui and vj.

So, this is Oij equal to cij - ui + vj. So, how do we go about? We need to calculate the Opportunity cost for each of them. So, before doing my iterations, let me just try to put steps for the MODI test. First step is we just try to get the Initial Feasible Solution or Basic Feasible Solution and we try to calculate the ui and vj. We try to calculate then to start with so one of the ui values is put here 0.

So, when we need to put any value of the ui 0, the value that is having maximum allocations is first selected to be put 0 from the ui or vj so that, the arithmetic calculations are reduced. So, that we will see when we will try to solve this problem here. Now, second step here is we try to get the Opportunity cost for the unoccupied cells. That Opportunity cost for unoccupied cells that is, we calculate oij whose relation is given here. Third step is, we try to examine each of the opportunity cost if oij is greater than 0 which means Optimal solution.

If oij equal to 0, this means Alternate solution. I will try to talk about different variations of Transportation model solutions or types. So, Alternate solutions is one that we will discuss and if oij is less than 0, we try to further improve the solution. Fourth step is, we try to make a closed loop for moving an allocation. It is either moving one allocation or we try to reduce from one part and try to go to other part.

So, how do we do that? Let us try to see from this example itself. Now, first step is get Basic Feasible Solution and calculate ui and vj for which, we try to put 0 value to any of the ui or vj there. Here you can see u1, u2, u3, u4 and v1, v2, v3, v4 values are given. So, we can see which of the columns and rows have the maximum allocations because those are the known values and based upon which we will try to calculate all the ui and vj. So we can see for the column X that is, for the market X, the maximum allocations are there which is 3, 3 and 1 for the factories A, B and D.

So, I will put this v1 as 0. If I say, v1, 0 and using this value v1 = 0. I can calculate the values for u1, u2 and u4 because we know that u1 + v1 equal to 4 and u2 + v1 equal to 5 and u4 + v1 equal to 3 putting value v1 equal to 0 we get u1 equal to 4, u2 equal to 5 and u4 equal to 3. So, this is using v1 equal to 0. Now, we have gotten the values of u1, u2, v4 and v1.

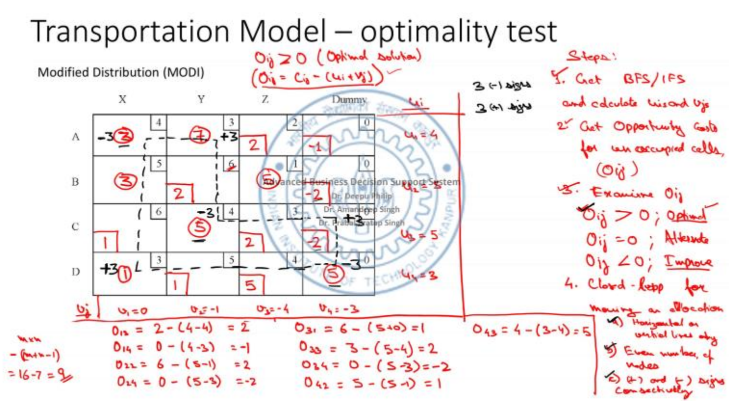
So, u1 equal to 4, u2 equal to 5 and u4 equal to 3. Let me try to now calculate further the values of v2, v3, v4 and u3. Now, I know the other allocated cells are here for which, we know the values of the sum of the ui and vj. Let me try to say, I will put the value u1 equal to 4. Using the value u1 equal to 4, I will do small calculation.

I will just pick the cell Ay and I would say u1 + v2 equal to 3 that is, 4 + v2 equal to 3, v2 equal to 3 - 4 that means, v2 equal to - 1. Now, I have got the value of v2 as - 1. So, let me try to see further. I will put value for any of the other variables which is known that is, I will put value of v2 equal to - 1. Using this value, I will now pick the cell Cy which is u3 + v2 equal to 4 which means, u3 - 1 equal to 4 which means, u3 equal to 5.

I am only left with the values to be calculated for v3 and v4. For calculating the value of v3, I have to pick one of the ui here, let me try to pick the value u2 is equal to 5. For u2 equal to 5, I can see for the cell Bz, we have u2 + v3 equal to 1 and we know the value of u2 that is 5, 5 + v3 equal to 1 that is, we get v3 is equal to - 4. So, we get here v3 equal to - 4. For the last value that is v4 that is to be calculated, I need to use the cell d dummy, d dummy equal to 4, 4.

So, we know the value u4 which is 3. I will pick this to calculate the value of v4 and we know u4 + v4 equal to 0 and we know v4 is 3. Therefore, v4 turns to be - 3. This is - 3. So, now I have gotten the values of all the ui and vj.

Now, the opportunity cost for the unallocated cells could be calculated which is step 2. Step 1 is completed now. Step 2 is to be now focused upon. To calculate the Opportunity cost simply, Opportunity cost is the difference of the sum of ui and vj from the existing cost that is cij which is given here.



To calculate the Opportunity cost I need to put Opportunity cost in the unallocated cells.

So, as per my cell notation, I will put Opportunity cost in the bottom left corners of each of the unallocated cells. And, I will examine it according to the criteria given in the step 3 here. So, calculating the Opportunity cost for each of the cells which is the difference between the existing cost and the sum of the new cost that have come through the new variables ui and vj.

So, opportunity cost for the cell Az that would be opportunity cost for the cell 1, 013 opportunity cost = the existing cost which is 2 - the sum of the ui and vj that is u1 and v3, u1 is 4 and v3 is - 4. So, which = 2 - 0, which equal to 2. So, I will put this value here as 2 in the cell Az, which is the First Opportunity Cost that I have calculated. And, for the cell A dummy, that is Opportunity cost for the 1, 4, I need to put the value of u1 and v4 and it has to be subtracted from the existing cost, that is rupees 0 - u1 is 4 and v4 is - 3. So, which = 0 - 1 = - 1.

Now, we are in a fix. We have caught one negative cost that means, there is a chance of improvement here. So, if Oi had been all greater than 0, according to that examination criteria, we could have reached the optimal solution. Now, oij = 0 alternate that we will discuss. oij is less than 0 that means, we need to improve the solution. We have to do one more iteration further and ui and vj are to be further calculated for the next iteration.

This is for sure. But let us see whether this opportunity cost is maximum or not. We have to calculate all the oij here and for the cell By, let me try to see O22 which is equal to the existing cost is 6 - u2 + v2. u2 is 5, v2 is - 1 that is 6 - 4 = 2. And, I will put value 2 here in the cell By. Now, for the factory C and respective cells, where Opportunity cost is super calculated.

I will use value u3 for which, I would say firstly cell O. So, there is another cell in row 2 that is, O24 for which, the opportunity cost has to come as 0 - value of u2 and v4 which is - 3. This is - 2. Now, we have a bigger negative value that is - 2 here. Now, let us try to calculate the opportunity cost for the factory C and there are 3 unoccupied cells already in the factory C that is, I will calculate O31, which is equal to the existing cost which is 6 - u3 and v1 that is 5 + 0 = 5.

And, next is O33 which = 3 - u3 and v3. u3 is 5 and v3 is - 4 that means, this is again equal to 3 - 1 which is 2. And, for the cell O34, I have 0 - respective ui and vj that is 5 - 3 which = - 2 once again. So, we have got the opportunity cost here. So, we have got the opportunity cost for the row 3.

Now, the row 4, two more opportunity cost would come that is, for the cell Dy, O42 is equal to 5 - 5 - 1 which = 1. The last one is O43 for the cell Dz which = existing cost is 4 - 3 - 4 which = 5. This 5 is the maximum positive number, we have 1 here for the previous cell. We have obtained the Opportunity cost already now. Now, we can see that, there are negative opportunity costs and there are 3 negative values dummy cell.

Dummy cell was expected to have the negative opportunity cost. So, one thing is, how many number of unoccupied cells would be there, that is directly because we had m into n cells. So, this - the total allocations, which are there which is m + n - 1, these many allocations would be there. In this case, 16 cells are there, that is, 4 into 4, 16 m + n - 1 was 7 that means, 16 - 7, total 9 unallocated cells were there for which, 9 opportunity costs are given here. Now, the third step is completed, we have to now improve the solution for which, closed loop is to be made and we have to move the allocations accordingly.

To make the closed loop, the maximum opportunity cost to be picked up. So, we have to pick up one from the two maximum values that is, - 2 and - 2 which are in the cells B dummy and C dummy respectively. Now, which one is to be picked first that we need to see. So, when we need to now plot a closed loop. There are certain that is 6 or points to be kept in mind, I will add them in the steps here.

So, 4a is the closed loop has to be something that connects the horizontal and vertical lines. The lines has to be horizontal or vertical. We cannot have diagonal lines or so. Also the number of the points or nodes that we select should be even number even number of nodes. So, each node is having respective + and - signs conceptively.

Now, I will start my closed loop from the largest negative value of the opportunity cost. I have two opportunity cost here. So. we can see from where the loop is possible. So, I could see the loop is possible drawing from here.

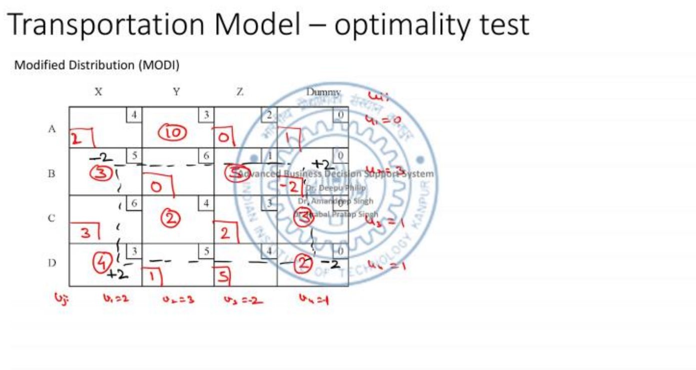
So, here some allocation is to be Now, I will start making my closed loop from the largest negative value of the opportunity cost, I have two opportunity cost here. So, we can see from where the loop is possible, so I could see the loop is possible drawing from here.

So, here some allocation is to made here, I will say + sign here, in the cell C Dummy, and I will start making my closed loop, how the closed loop could be formed I will say + here, and I would have to wherever I select a node or wherever the corner of the nodes are there, there the sign would change I would have to because we have reached end. And one allocated cell, now this is not only unallocated cell that is there, all other allocated cells have to be kept here. So, this is moving the allocations plus we have minus here then we have plus here and I go to the point from where I can take another turn because it was plus in the previous one and in the step C it is said we have to have consecutive plus and minus signs that is, consecutive addition and subtraction of the allocations or the shipments which were there.

So, I will have minus here and I go to the next cell from here itself also I can take a turn. So, here I will say plus and I will turn from here here I will also take a turn and say minus and this will be connected to the previous one. So, I have equal number of minus signs I have 3 minus signs and 3 plus signs which is by this condition even number of nodes is satisfied and all the lines are horizontal and condition 4a is also satisfied. So, that means I have got equal number of these signs and this close loop is now okay. Now what is the number of the units that I could allocate here or maybe change from here and there? One unit because the number of allocations has to remain m + n - 1 and I am going to add something where the opportunity cost is giving the maximum negative value that is rupees 2 per unit is the opportunity to improve my solution in the cell C dummy.

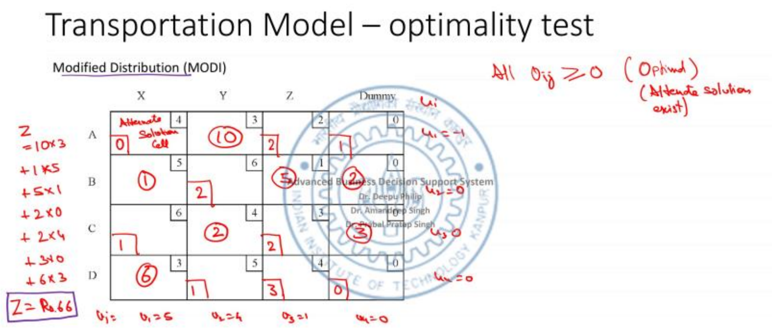
So, here allocations will be made from one point the allocation is to be taken because total number of allocations could only be m + n - 1. So, what is the minimum number of allocations that I could see from where it could be picked? Picked means there has to be negative sign we have 3 negative signs one is at cell AX second one is at cell Cy third one is at cell D dummy. The minimum value that I could see here is 3 that is allocated here. So, I would say I would do - 3 here and + 3 here and - 3 + 3 which would be new allocation to the cell C dummy and - 3 and + 3 accordingly. So, this is how the allocation is done please try to watch this video time and again so that you understand how the allocations are done and how we have selected this value 3.

This 3 is the minimum value amongst the allocation which makes one of the cells which is occupied already turn to unoccupied cell because the unoccupied cell which was there for which opportunities to be taken that is the cell C dummy the new allocation is done and total number of allocation is n + n - 1. So, this is how we do. Now the allocations are to be changed again and we need to calculate the Ui's and Vj's and reach till the point when all Oij's at the opportunity cost are positive that is only then we get the optimal solution.



If I try to reallocate it I will get something like this I will get 10 units here because 3 were added from the AX it is now - I will be remained with 2 units here because 3 were subtracted and this 3 units is the new allocation in the cell C dummy and I am left with 2 units in the cell D dummy and now this was 1 in Dx which is now 4 units because 3 were added here other cells remain same we had 3 here 5 here now total the equations are again m + n - 1 and I have already calculated the ui and vj for the current solution I have got the values as u1 = 0, u2 = 3 u3 is 1, u4 = 0. Then v1 is 2, v2 is 3, v3 is 2 and v4 is 1 and based upon this I need to now again calculate the opportunity cost for the unallocated cells which were calculated here as for the cell AX, it is + 2 for 2 for 0 here, 1 here 0 here - 2 here for the cell B dummy 3 1, 5 and 2.

Now, these values I need to correct, these were - 2 and - 2. The values of v3 and v4, I am just noting the values which I have already calculated. Now, here still we have the optimality condition not meeting so that means, rupees 2 per unit opportunity is still there. So, here some plus has to be there. I have to now again plot a loop, for this loop that is plotted is just a rectangular loop for which, this becomes plus next one we have minus, we have plus and minus and the number of units which are minimum among the 2 minus values which are the cells BX and D dummy is 2 units in the cell D dummy. So, that means, - 2 + 2 - 2 + 2. So, cell D dummy would be eliminated.

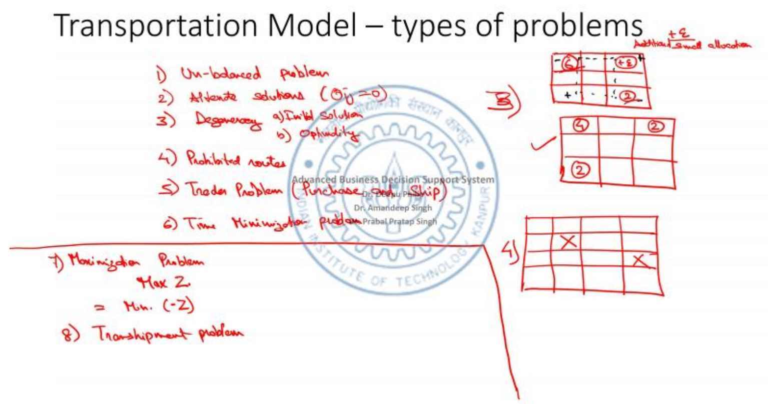


So, now the next allocation where D dummy is eliminated and we have allocations in the cell B dummy, so that would be something like this, we have 1 unit here 2 units are added here in cell B dummy and D dummy does not have anything and since, 2 units were added in the cell, dx it turns to 6. Remaining all are just like the previous one as well. I did not even calculate the cost of the objective function because I was still having the opportunity left till the point I reach the optimal solution. I have not calculated the optimal cost, so optimal cost or the final objective function cost will be calculated only when I get all the opportunity cost greater than or equal to 0 equal to 0 is accepted that means, alternate solutions would come that also we will see.

So, for here again, ui and vj were calculated and we had u1 = - 1, u2 as 0, u3 as 0, u4 as 0 and v1 as 0 and v2 as 0, v1 as 5, v2 as 4, v3 as 1 and v4 as 0 giving me the opportunity cost for the unallocated cells as 0 rupees for cell AX 2 rupees for cell Az, 1 rupee per unit for cell A dummy. It is 2 here, 1 here, 2 here, 1 here 3 here and 0 here. So, all oij that is opportunity cost is greater than or equal to 0 that means, the solution is optimal. I have put equal to that means it could also be alternate solutions exist.

However the current solution is optimal to calculate the current solution value, we have the value of Z, that is my objective function this = 10 units into rupees 3, for cell AY + 10 units into rupees 3, for cell 1 unit into rupees 5 for cell BX + 5 units into rupees 1 for cell BZ + 2 units into rupees 0 for the dummy cell. For the factory B + 2 units into rupees 4 + 3 units into 0 + 6 units into 3 which = rupees 66. So, this is my value of Z, that is the optimal value that I have gotten using the modified distribution test. This was the solution you could see, I have picked a problem statement which is a long problem statement in which, 3 different iterations were taken for the modified distribution test. This test though brought me to the optimal solution but this was a lengthy test also.

In this problem statement, there are alternate solutions that do exist. Let me try to now talk about the different kind of variations which are there in the transportation modelling that means, what kind of solutions or problems that we have here.



Different variations, if I try to talk about, I say type of the problems, first type is the unbalanced problem which means, the demand and supply is not equal. So, this was the first condition, so we had to add a dummy column here in this problem, existing problem that we just did, so this was discussed already. Second kind of the variation in the problem statement could be we have alternate solutions.

By alternate solutions, I mean to say whenever there is oij equal to 0 here, we have no opportunity in this cell AX or in the cell D dummy, D dummy does not mean anything because there is no dummy market here. So, in the cell AX, if we try to allocate something and try to bring the allocation from other cells and we try to draw a loop for the cell AX, we can get an alternate solution here. I will just write it here, this is alternate solution cell. So, if we allocate here and it is taken away from the other cells still the solution will remain optimal.

So, this is alternate solution cell. So, alternate solutions means, if we try to apply the same rule for the loop forming and try to allocate some value here, that is the number of units in the cell AX, the solution value still remain rupees 66. This is alternate solution, I am leaving it for you, I will provide it in a slide but you can try it yourself and try to find what alternate solution is.

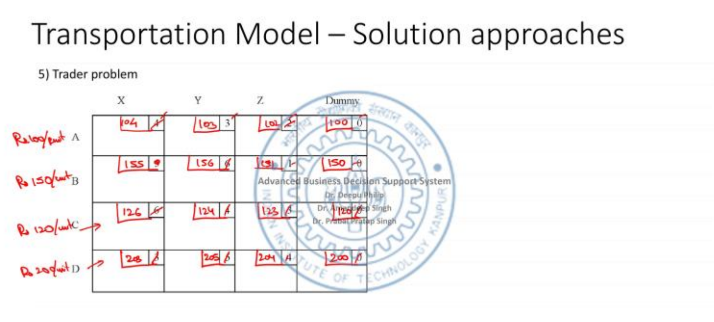
Next kind of the variation that is there is Degeneracy. In the case of Degeneracy, it means when the number of allocations is lesser than m + n - 1 total number of allocations which are there, we have to have m + n - 1 allocation. For instance, there is some arbitrary allocation which is given, I would just say, there are certain values which are given here and the allocations are lesser than the total number of allocations which were supposed to be and I have may be some value 6, here some value 2 here that the allocated value.

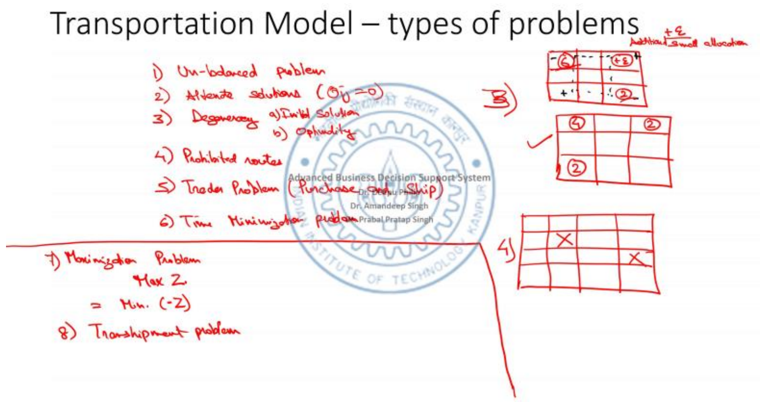
I need to add an additional value this is known as small allocation or additional small allocation. So, which I will add here let me say + epsilon here which is to be added and a close loop would be formed. So, that the number of cells now have more allocations I will have a close loop, it is plus here minus here plus here and minus here. Now, this will turn to something like this.

So, minimum number between 6 and 2 is 2. So, we will subtract 2 and add 2 accordingly. So, this will become + 2, this will also be + 2 and this will be subtracting 2 from here this will be 4. You can see there were only 2 allocations here and we have now had 3 allocations here. One allocation is increased in the improved cell, this is to deal with Degeneracy. Degeneracy could come in the initial solution as well it could even come in the optimality. As well as the optimality.

Next comes the Prohibited Routes. Prohibited routes means for instance, there is a big problem statement and they say, the cars cannot travel on these roads and the shipment is only to be made with cars only or there are certain government obligations, where the goods cannot enter that city. For example, there was some time when in Haryana, alcohol was banned for few days. So, alcohol trucks could not enter that place. So, there would be some Prohibited Routes, where we might have to put crosses. For example, row 2 and column 2, there is prohibition and may be row 3 and column 4, there is a prohibition.

We try to add this. So, this is regarding the Prohibited Routes. The previous two were regarding the degeneracy. Next comes the Trader problem. Trader problem means when the trader has to purchase and ship the goods. So, in this case, the cost that is there for the goods, he does not even only have to do the transportation, he also has to purchase the goods that means, the cost or purchase cost would be added.





For instance, in this problem statement itself, let me say, the cost of the good is rupees 100 from the factory A and rupees 150 from the factory B, rupees 120 for the factory C and rupees 200 from the factory D.

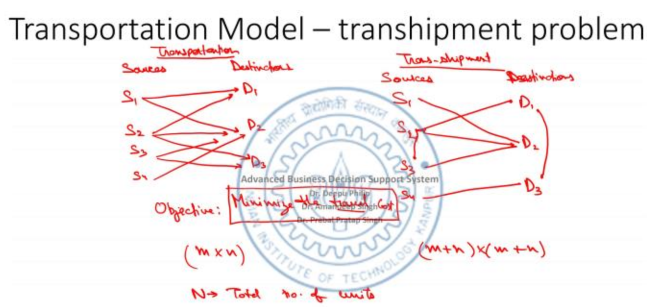
Now, we have the cost per unit for the transportation only here respectively. These cost would not be now 4, this cost would be 4 + per unit cost if number of units are to be transported. So, this is rupees per unit. These cost would also include the cost of the purchase of the goods. Let this cost is not 4 but this becomes 104 and this becomes 103, this becomes 102, this becomes 100 and so on, this becomes rupees 155, that is I have added 150 to this.

This becomes rupees 156, 151, 150 and so on. So, these costs are to be added here, this is known as the Traders problem. Variation that I would put here is the Time Minimization problem. When you say Time Minimization, it is similar to the Cost minimization what is the point is that the variable name of the item. Also, there could be Maximization problem. When I say Maximization problem, simple way to deal with Maximization problem is, we turn the Maximization problem to the Minimization problem by multiplying it by - 1 and apply the same rules of transportation for the Minimization problem that we did.

However, in Maximization problem because the profit is to be maximized, we could directly use the Maximization rules where the rules would just be contrasting to the minimization rules. For instance, the difference between the minimum two costs were taken for the Opportunity cost, for the Vogel approximation test. Here the difference will be taken for the maximum and the second maximum cost. For instance, the opportunity which was taken in the ui that was the all opportunity cost that oij was supposed to be greater than 0 for having the optimal result in the optimality test, that is the modified distribution test. In the case of Maximization problem, the thing would be completely conversed, that is, all oij would be required to be less than 0 to have the optimal test.

Simple way is, we try to convert a Maximization problem that is, maximum Z. What we try to do, we try to minimize - Z. This is how we deal with Maximization problem. The last variation I would like to discuss here is the Transshipment model. If I put Maximization problem as one of the variations in transportation, I have a Transshipment problem.

Transshipment problem, as I said, certain distribution centers are there. Sometimes, there is a central point from which, as a warehouse the things are collected there and those are taken to the various points, where it is distributed to and from the factory or from the production point, it comes to the warehouse.



Here, there are sources and destinations. Suppose, if I have may be four sources and three destinations, what my transportation problem is trying to solve is, when I go from source 1 to destination 1, source 1 to some destination 2 or from source 2 to destination 1, to destination 2 and destination 3 and so on, it is trying to solve this kind of a problem. This is my transportation. I have put arrows which means directions.

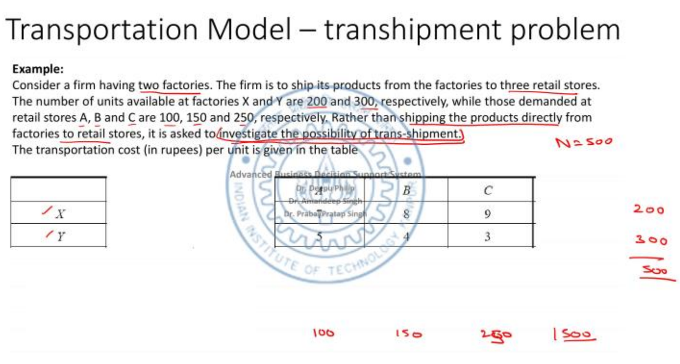
Now, in the case of transshipment, that is shipments between so sometimes, one s is just kept as silent as single word transshipment is put. So, that is how it is spelled though grammatically the spelling should be trans-shipment and I have sources and destinations. So, for the same sources s1, s2, s3 and s4 if destinations are there then the same source is there. So, it can go from any direction for example, it can go from one source to another destination, it can go from this source to this destination, it can even go from source to source.

So, everything is open here. So, even it can go from may be destination 1 to destination 3 as well. So, any of the connections could be made so as the cost is minimized. The purpose is minimize the travel cost.

Here, when I say travel, it is the shipping of the goods. So, this is the purpose. This is objective here. In the case of a transshipment kind of a model, when there are m sources and n destinations, so we are not left with only m into n matrix. So, I would say, it is (m * n) matrix is for transportation, but here in transshipment, the matrix turn to (m + n) * (m + n) because each of the sources become also the destinations, if destinations also become the sources.

So, m + n becomes a variable. So, I get a square matrix which is m + n by m + n. So, this matrix is obtained. Now, the point is, what should be the demands or supply at each destination? So, this is the total number of the units which is to be transported from one place to another that is termed as N. N is total number of units. Now, what happens, when capital N, we know because we take the total of the supply and total of the demand which should be equal, which we say is equal to N. Then, this capital N is added to each of the sources or each of the destinations and we try to get a normal transportation problem.

So, this is how Transshipment problem is converted into a transportation problem. So, when we say that for a given n number of the units, which is the total number of units the demand at source and at the supply at each destination because it is n, the problem can be solved by a simple MODI method that is Modified Distribution method for transportation problem and in the final stations, we can try to ignore the units which are transported from a point to itself may be. So, may be for example, diagonal cells because they are not having any physical meaning or no transportation is required. So, how do I formulate the problem? Let me take this with an example and try to explain.



Let us take this example. Consider a firm having two factories. The firm is to ship its products from the factories to three retail stores. There are two factories X and Y, for which the available number of units are 200 and 300 respectively and demanded at the stores is A, B and C are the stores for which 100, 150 and 250 units are demanded. So, my supply is 200 and 300 here for the factories X and Y and my retail units have a demand of I am putting it for 100, 150 and 200. First thing is, 200 + 300 this = 500 also, 100 + 150 + this is 250, 100 + 150 + 250 = 500.

That means, first condition is being fulfilled that the demand and supply is balanced. So, now what is the total value that is, I am having value of N as 500. So, since this value N is 500, I will convert this transshipment problem because it is written that rather than shipping the products directly from factories to retail store, it is asked to investigate the possibility of transshipment. So, whether the transshipment is even possible or not, how do we try to work upon that?

Transportation Model – transhipment problem

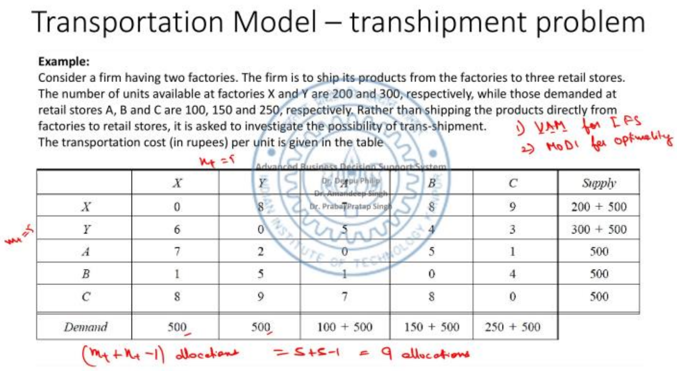
Example:
Consider a firm having two factories. The firm is to ship its products from the factories to three retail stores. The number of units available at factories X and Y are 200 and 300, respectively, while those demanded at retail stores A, B and C are 100, 150 and 250, respectively. Rather than shipping the products directly from factories to retail stores, it is asked to investigate the possibility of trans-shipment.
The transportation cost (in rupees) per unit is given in the table.

N = 500

	X	Y	A	B	C	
X	0	8	7	8	9	200
Y	6	0	5	4	3	300
A	7	2	0	5	1	
B	1	5	1	0	4	
C	8	9	7	8	0	
			100	150	250	

So, let us try to see how do we solve this problem, I have now put these values here it becomes something like this. So, it is given that the transportation cost per unit here from factory X to X anyway because this diagonal matrix is always there, this is always 0 because from X to X, there is no shipping of goods that happens. So, this diagonal is always 0 and from X to Y that is, from factory X to factory Y rupees 8 per unit is still a cost from factory S to definitely the destination units, that is a retail store.

There are the cost associated like for example, from factory X to A, B and C, 7, 8 and 9 rupees per unit are the cost. So, this is just a cost matrix that we used to also have available in the question itself as in the transportation model. So, now here, what we are having is 200 units and 300 units which is for X and Y and here what we had was 100 units, 150 units and 250 units for retail stores A, B and C. Now, my capital N that is, total is 500 to convert this into a transportation model, what I will do, I will add these total supply pass demand that is, 500 to all the columns and all the rows.

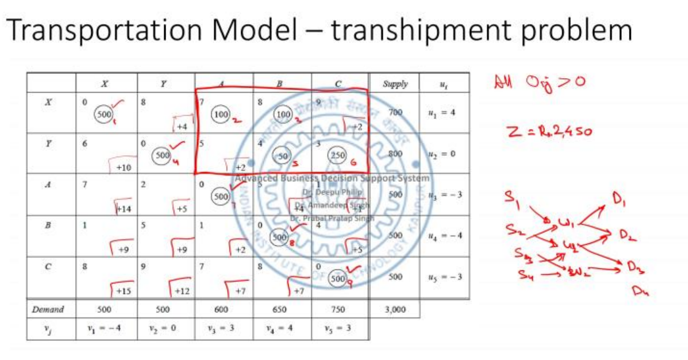


Transportation Model – transhipment problem

Example:
Consider a firm having two factories. The firm is to ship its products from the factories to three retail stores. The number of units available at factories X and Y are 200 and 300, respectively, while those demanded at retail stores A, B and C are 100, 150 and 250, respectively. Rather than shipping the products directly from factories to retail stores, it is asked to investigate the possibility of trans-shipment.
The transportation cost (in rupees) per unit is given in the table

	X	Y	A	B	C	Supply
X	0	8	7	8	9	200 + 500
Y	6	0	5	4	3	300 + 500
A	7	2	0	5	1	500
B	1	5	1	0	4	500
C	8	9	7	8	0	500
Demand	500	500	100 + 500	150 + 500	250 + 500	

So, then my table becomes something like this. Now, this is my demand 500, this is 100 + 500, 150 + 500, 255 + 500. So, this is how my table is obtained, so simply, what we do, this transportation matrix, we have obtained, now we apply simply the VAM method that is Vogel's approximation method, this is for having I of S Initial Feasible Solution, then we apply Modified Distribution method for optimality. Now, this all is solved in a matrix and you can see the number of allocations which are there, so this is 5 into 5 matrix that is (m + n) * (m + n)where m is 2, n is 3, m + n is 2 + 3, 5, 5 * 5 = 25 cells are there, if I say, m + n, I would say, m (total) + n (total) so that is mt that is 5 and nt that is 5, so total number of allocation should be 5 + 5 - 1 that is, mt + nt - 1 allocations that means, 5 + 5 - 1 = 9 allocations.



Transportation Model – transhipment problem

	X	Y	A	B	C	Supply	u_i
X	0 (500)	8 +4	7 (100)	8 (100)	9 +2	700	$u_1 = 4$
Y	6 +10	0 (500)	5 +2	4 (50)	3 (250)	800	$u_2 = 0$
A	7 +14	2 +5	0 (500)	5 +4	1 +1	500	$u_3 = -3$
B	1 +9	5 +9	1 +2	0 (500)	4 +5	500	$u_4 = -4$
C	8 +15	9 +12	7 +7	8 +7	0 (500)	500	$u_5 = -3$
Demand	500	500	600	650	750	3,000	
v_j	$v_1 = -4$	$v_2 = 0$	$v_3 = 3$	$v_4 = 4$	$v_5 = 3$		

Now, this solution is already given, you can see 1, 2, 3, 4, 5, 6, 7, 8 and 9 allocations were made using VAM method, then to see the optimality, we try to apply the MODI method in which ui and vj were given here in which maximum allocations were given for the row Y, so u2 was taken as 0 and each of the ui and vj values are calculated and opportunity cost are also calculated.

Unfortunately, in the this problem statement itself in the first iteration that is the first VAM method that we have applied, we have got the first or the opportunity cost in the first MODI iteration as all positive, so I would say all oij is greater than 0, you can see these are the opportunity cost here which are given on the bottom corners, these are all positive that means, the cost that is for the total transshipment problem, that is there would be optimal, here the cost could be calculated again.

This total cost would be the number of units times the cost per unit in the cell. One interesting thing is always to be noted in the transshipment point is that, the 500 units that we have added to each of the columns and the rows, those are always turning to 0 how does that happen? you can see the diagonal matrix has always the allocation.

There are 5 allocations in the diagonal matrix, so because in the diagonal matrix, the cost would always be 0, so these cost would always be 0, so the total cost if even I include the diagonal matrix is only given for in this problem statement, the original matrix, which was there this one, so that would be 7 * 100 + 8 * 200 + 4 * 50 + 3 * 250, so this total cost would be 700 + 1500 + 200 + 750, 1500 + 200 = 1700 + 750 = 2450.

But this is a simple transshipment problem that we have discussed now and the transshipment problem can also be converted to the transportation problem and we will try to solve it using the same MX action program separately.

There are transshipment problems which sometimes are not able to be converted to the transportation problem because sometimes, a center or the warehouse point is there from which, all these sources send the things to the warehouse. There are maybe, I would say, this is destination 1, 2, 3 and 4, but there could be in between warehouse 1, 2 and 3.

So, these become an additional transportation problem, so we do not know what connections would be made, so these kinds of problem could also be there. Now, I will discuss the transshipment problem in the coming lecture and also do small MS Excel demonstration on the transportation and the transshipment models. There we will see how do we use the solver and other MS Excel techniques to solve the large transportation and transshipment problems. Thank you.