Advanced Business Decision Support Systems
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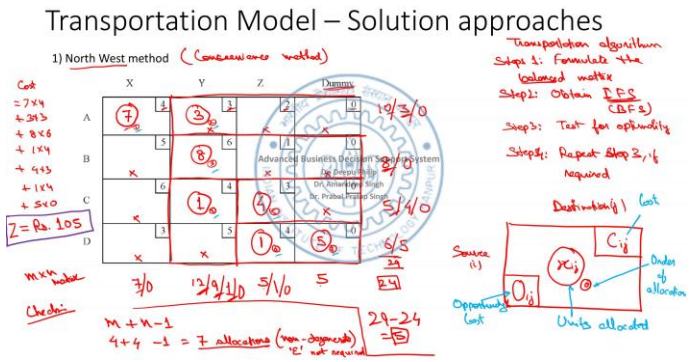
Lecture 22
Transportation Model- Solution Approaches

Welcome back to the week 6 of the course Advanced Business Decision Support Systems.



Now, North-West cathode was just a convenience method, where only Basic Feasible Solution was taken. There is a better method where we also try to consider the cost.

One of those methods is Least Cost method. When I say Least Cost method, Least Cost means, the Least Cost from the overall matrix is taken by allocation cell. When I say Least Cost method, this means the least cost among the overall matrix is taken as my cell of the allocation.

So, in the overall matrix, you can see, the least cost is 0. So, that means, we have a 'tie'. Tie between certain least cost which are available 0, 0, 0, 0. Whenever tie is available, we can pick any of the cells that is having least cost.

So, let me put the availability here 10, 8, 5 and 6 and from the demand side, it is 7, 12, 5 and 5. So, out of these 4, let me pick the least cost for the factory B. So, I allocate out of 8 and 5, 5 is my limiting number. I allocate 5 units here and I am left with 0 unit here

and 3 units here because 0 units are left in the dummy column now, so this is eliminated. I am just putting the crosses accordingly.

Now, among the remaining matrix that is XYZ markets and ABCD factories, the least cost is to be seen among all the numbers, which are given in the right corner of the cells. You can see the least cost number here is 1. This is the least cost, so I will just put the allocation here to the least cost. The Least Cost Method because the cost is taken into consideration and the purpose is to minimize the cost and least cost is taken into consideration. So, this is a better method, which goes closer to the optimal solution than what the Convenience method, the North-West method was doing.

Now, because the cost least here is 1, I will allocate number of units here. Now, for the cell BZ, the 3 and 5 units are there for the factory B and market Z respectively, so 3 is the limiting number. I will allocate 3 units here and I am left with 0 units in factory B and factory B is eliminated. Now, among the remaining unallocated cells, I need to further see the least cost. So, I could see, 2 is the minimum value, now here in the cell AZ and because 3 units were already allocated in the cell BZ, so I am left with 5 minus three 2 units here.

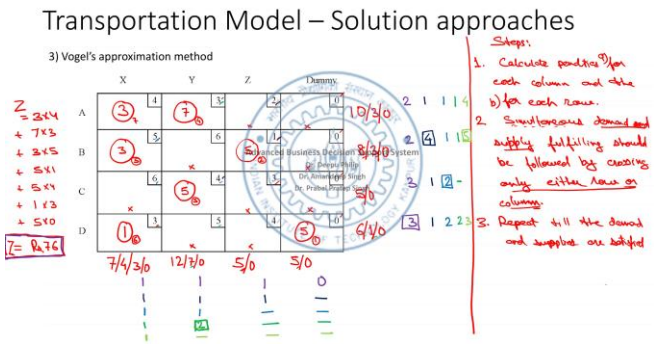
These 2 units are allocated in the cell AZ and for the factory A, 8 units are now only available and market Z demand is completed. So market Z associated cells are now put with crosses here. Now, among the unoccupied cells, we also call them empty cells. So, I would use the words occupied or filled cells and unoccupied that is empty cells. Now, the least cost here is 3 and this least cost is for any of the 2 cells between A, Y and D, X.

I will allocate 6 units to my cell D with least cost rupees 3 per unit and now, D factory is left with 0 units and 7 minus 6, 1 unit is left here. Now, this is crossed, the factory D. Now, among the available cells, again rupees 3 is there, it is the minimum cost. So, I could have picked any one of the rupees 3, which were available in the previous allocation. So, it could have been the same meaning.

Again, first allocating 6 and then coming to this allocation A, Y, 8 or first I could have allocated 8 here and then gone to the other side. So, I have allocated 8 in the cell A, Y and left with 0 units in factory A and this factory 8 whole supply is exhausted and 12 minus 8 is 4 units, which are left here. Now, out of the 6 and 4, you can see 4 is the minimum cost. I will allocate the number of units that is 4, which is being limited by the market Y demand. So, I am left with 0 units to be supplied to market Y and here it is 1.

This 1 comes here and this becomes 0. So this is my allocation. Let me try to see whether the number of allocations are again m plus n minus 1 or not. That is, 7 allocations should be there. The solution is non-degenerate.

So, let me try to now calculate the cost using the Least Cost Method. This cost should be lesser than what was with the North-West Method. Here, the cost would be 8 into 3 plus 2 into 2 plus 3 into 1 plus 5 into 0 plus 1 into 6 plus 4 into 4 plus 6 into 3 which is equal to rupees 71. So, this is my cost for the Objective function Z from the Least Cost Method. You can see, the cost which was here in the North-West Method, it was rupees 105 and for the Least Cost Method, it is rupees 71.



Let us try to see a third method as well which is a more mathematically proven method to go closer to the optimal results. This is known as Vogel's approximation method. In Vogel's approximation method, on the top of doing regular allocations that we try to do in the North-West method or the Least Count Method, we try to see the opportunity cost. Opportunity cost, wherever the maximum opportunity is there to minimize the cost that cell would be chosen for the allocation. This is known as the Vogel's approximation method.

This approximation would take us closer to the optimal results. So, how do we try to do it? What we try to do it? We try to calculate the penalties for each row. So, there are certain small additional steps here. So, in Vogel's approximation, I would say, steps number 1, we try to calculate penalties. We try to calculate the penalties for each column and then for each row.

You can put it as separate two steps or I have just put it as step 1 part A or B whatever you think. For each column, could be part A and for each row could be part B. Then, we try to adjust supply and demand and cross out the satisfied rows and columns that we did in the previous methods as well. Only one thing we need to make sure that, when we try to eliminate the column or a row that means, when we try to put the crosses there, if both the demand and supplies are fulfilled simultaneously that both of them turns to 0 simultaneously, we cannot put cross across both the column and rows because in this case, the allocation would be lesser than m plus n minus 1. So, degeneracy would also come.

So, we need to cross only the row for which, the supply is exhausted or only the column for which the demand is fulfilled. So, this is we need to keep it in mind. I think, I will put it as a step as well. Simultaneous demand and I would say, supply fulfilling should be followed by demand and supply. fulfilling should be followed by closing only either row or column.

So, let us be mindful the words I have put here demand and supply together if they are fulfilled, so only either row or column. This is to be taken care or this is to be kept in mind, when we try to do the probable approximation test. So, what we try to do, we try to cross them and the first step is we try to repeat till the demand and supplies are satisfied. So, let us try to put the Opportunity cost here. Opportunity cost, how it is calculated? We try to subtract the least value of the column from the second least value of the same column.

This is my opportunity which means, if I do not use this opportunity, I am losing these many units per unit of the supply. So, let us try to now solve the problem using these steps. Now, I need to first put my availability and the demand values here. This is 10, 8, 5 and 6 and for the markets X, Y and Z, we have 7, 12, 5 and 5 for dummy. Let us now start calculating the penalty that is mentioned in the step 1.

Calculate penalties number A, for column for each of the columns and for each of the rows. So, these penalties are what? These are the extra cost that has to be paid if decision maker fails to allot the specific allocation in itself. So, let us try to calculate the penalties. So, what is the minimum value in the whole row? We can see minimum value is there always 0, for the whole dummy column for each of the rows, the minimum values here are 0. The second minimum values here are 2 for row 1, 1 for row 2, 3 for row 3 and also 3 for row 4.

So, I will start proceeding here. I picked a different color so that, I separate the different steps here or different repetitions that, I am trying to do to come to a final allocation of M plus N minus 1 that is 4 plus 4 minus 1, 7 allocations. So this is 2 minus 0, 2. This is 1 minus 0, 2. For the factory C, it is 3 minus 0, 3.

For the factory D, it is 3 minus 0, 3. Similarly, let me try to see for the columns, the minimum value here is 3 and the second minimum is 4; 4 minus 3 is 1. Here, in the column Y, the minimum value is 3, second minimum value is 4, again 4 minus 3 is 1. In the third column, the minimum value is 1, second minimum is 2, again we have difference that is 1. For the fourth column anyway because it is dummy column 0 minus 0 would always remain 0 here.

Now there are two options to select the maximum penalty here. Maximum penalty that is given here is 3 for the factory C and D. For both the factory C and D the penalty is 3. We

have to select 1 where though I can select any of these piece values because finally the allocations would be gotten correct whatever I pick. But still I would like to pick 1 with the maximum possibilities of the allocations.

So, this is 6 is a larger number here. So I will try to pick this 6 here. So I will pick row 4 that is I will just put this in a box. This is my key penalty that I have calculated and this row is selected where allocation would be done. Anyway when the allocation is to be done it would be first added in the least cost possible in this row.

So least cost anyway we have 0 for dummy. So let me add this least cost. So let me allocate here in the cell D dummy. So here what is the maximum allocations that could be done? 5 is my demand and 6 is my maximum supply. So demand is limiting so what I could allocate is 5 here.

So I am left with 0 that is the whole dummy column is now crossed. I am putting small crosses here and here I am left with 1 unit in the factory D. So I will put a small number here because this is the first allocation. This is just for our understanding that this was the first allocation. So I think I will put this in the key here itself.

Here also the numbers were given. I will put a small number here that is I would say if suppose m plus n minus 1 allocations are there I will put small number. I will name it as small o where this is what this is the order of the allocation. That is in which order the allocation has been made fine. So this is my first allocation here. Now comes the second iteration when the dummy column is eliminated.

Now there would be no changes in the row values here for the penalties. So I will pick another color and I will calculate penalties for the columns once again which is 1, 1, 1 and this is eliminated. I will just add dash here and for the second repetition let me try to see for the rows what are the penalties now. Now the minimum value here is 2, second minimum is 3.

So 3 minus 2 would be 1. Similarly here minimum is 1 and second minimum is 5 for the factory B. So 5 minus 1 is 4 looks to be higher number. It might enter or might be selected for the allocation. Similarly we have 4 minus 3 for the third row which is 1 and 4 minus 3 for the fourth row which is 1. So among these numbers 1 and 4, 4 is the highest number.

I select this as the maximum penalty element and corresponding row is factory B. Now minimum value in the factory B considering dummy row is not even there minimum value from 5, 6 and 1 is 1. So allocation would come here in the cell BZ. This is my second allocation. So what is the allocation value here? The value has to be between 5 and 8 and the limiting value is 5 here.

So I allocate 5 units here and in the market Z I am left with 5 minus 5 is 0 and in the factory B I am left with 8 minus 5, 3 units. Second iteration is now completed. Let me try to see another iteration for this because market Z requirement is now completely fulfilled. I will put crosses here and the column Z and dummy are now eliminated. So the third repetition of the steps 1 and 2, let me now try to calculate these once again.

So I will have 1 for market X, 1 for market Y that is the column penalties and these 2 are already eliminated. Now for the rows I have difference between 4 and 3 which is 1, difference between 5 and 6 this is 1, difference between 6 and 4 this is 2 and difference between 3 and 5 is also 2. Again there is a tie when we are having 2 values for the factory C and D which is 2 and 2. Let me try to see what is the maximum possibility of the allocations between the factory C and D.

There are 5 units here and 1 unit here. There are 5 units for factory C and 1 unit for factory D. So 5 is the bigger number. I select this 2 that is for the factory C and I will allocate here. Among 6 and 4 in the factory C, 4 is the minimum number so allocation would come in the cell CY.

This is my third allocation. So here limiting number is 5 so I will allocate 5 units and this is turned to 0 and out of 12 I am left with 7 units to be still supplied to market Y. So you can see the repetitions will keep on going and we are only repeating the steps 1 and 2 which I mentioned here and we are trying to allocate based upon the penalties that is the maximum penalty would be there, if that minimum cost is not taken as an opportunity. Though the word Opportunity cost is different, that we will try to see maybe we will come to the optimality of the allocation or the Initial Feasible Solution that we are trying to get from here. So, 3 allocations have been made. Now, let me try to do another iteration or repetition here.

Before that, I will eliminate the factory C because the allocation is now already done and there are no units left. We are 0 here. So, row 3 is eliminated. Now, let me try to calculate the row and column penalties.

Now, because one row is eliminated. For the first time, in the whole exercise here, that we are doing for Vogel's approximation method. Now, these values could change, but still, the minimum value is 3 and the second minimum is 4 for the market X that means, this value is still 1 and here the minimum value now is 3, for market Y and the second minimum is 5. So, 5 minus 3 is 2. So, these two were already eliminated. For the row penalties, 4 minus 3 is 1, this won't change and 5 minus 6 is 1 and this is now eliminated and this remains 2.

So, here among the possibilities of the allocation, 7 and 1 for the two values obtained 7 is a larger number. So, I will pick market Y and try to allocate the units here in the cell A.

Y this is my fourth allocation and this would be 7 and 10, the mean lesser number is 7 out of 7 and 10. I will allocate 7 units here and for the market Y, I am left with 0 and for the factory A, I am left with 10 minus 7 is 3 units. Now, this market Y is eliminated, I am putting small crosses here.

Now, only column 1 three cells are left and some allocations would be put here anyway, we are also left with 3 more allocations. 3 allocations are left because 4 are done out of the 7, which what we made and 3 cells are only there that means, each of these cells will be allocated, so I can directly put the values based upon the Least Cost Method that is, the cost are 3, 4 and 5, first for the rupees 3 cost and for the rupees 4 cost and for the rupees 5 cost. I can do it directly, even I can try one more iteration or the repetition.

Let me try to do one more repetition just for further understanding that what we are doing. So, now this is eliminated and let me try to do another iteration as I said, this will be dash dash dash, here the difference between 3 and 4 is 1, here the difference would not be there.

These values would be directly 4, 5 and 3. I did this because this was not even required. So, now because allocations are to be made anyway and we know that 3 cells are left but from the penalty viewpoint, this is supposed to be the cell that is to be selected and from the least cost method point, this is supposed to be the cell where the allocation is to be made because this value maximum penalty, that is given is 5 and this is the cell, where the allocation should be made based upon Least Cost Method.

So, when this kind of condition comes anything that we do allocation would remain same because we are not left with any other options now. So, let me just try to follow the process that I am doing in this exercise.

Let me try to allocate first to the one with the maximum penalty. So, we have number 5 here for the factory B, so I will allocate the value here. So, this is 5 and the maximum allocations that could be done is, 3 coming from the row. Maximum here 3 units are available only with the factory B and this 3 minus 3 turns to 0 that means, this whole row is now completed and here I am left with 7 minus 3 that is, 4 units and among the costs rupees 4 and rupees 3 for the factories A and D respectively.

Let me try to pick the lower cost first that is, rupees 3 here for the factory D, I will allocate to 1 unit here. This is my 6th allocation and I am left with 4 minus 1, 3 units here and this is 1 minus 1 is 0.

Factory D is now completely exhausted and the conditions would be completely satisfied, when these 3 units for the factory A and market X are added here. So, I will put 3 units here and this is my 7th allocation 3 minus 3 is 0 for the both factory A and market X. This is Vogel's approximation method, where we are trying to use logical

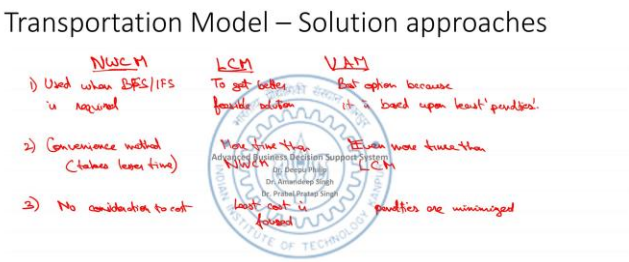
mathematical penalty based allocation because this is based upon some mathematical calculations, where logic is the penalty has to be minimized, minimum costs differences are taken.

So, this should be more close to the final Optimal solution that we will get, but this is not always the case. Let us try to see what is the final value coming for the first Feasible solution or Basic Feasible Solution using the Vogel's approximation method.

So, we see that here, Z value is equal to for the cell AX. It is 3 into 4, 3 units into rupees 4 for the cell AY, it is 7 units into rupees 3 for the cell BX, it is 3 units into rupees 5 for the cell BZ, it is 5 units into rupees 1 and for the cell CY, it is 5 units into rupees 4 and for the cells, for the factory D, we have 1 unit into rupees 3 to market X and 5 units into rupees 0 to the dummy market.

This value is equal to, if I calculate everything 3 into 4 is 12 plus 7 is 21, if I take some product of these 3 into 4 is 12 7 is 21 plus 15 plus 5 plus 20 plus 3. This is equal to rupees 76, this is value of Z. Now, the value that was obtained from the last two methods, let us also try to have a look over that as well. So, for the North-West Method, the value was rupees 105, that is the minimum cost was rupees 105 but still we had at least a Feasible solution then, we had the Least Cost Method.

So, where the minimum Objective Function value came as rupees 71 and for the Vogel's approximation method, we have rupees 76 which is higher than Least Cost Method. So, that is not always case that, this gives us the minimum value but this is only the purpose that we get the Initial or the Basic Feasible Solution.



So, though we have discussed these three methods, let me try to see what is the major difference between the North-West Corner Method, Least Cost Method and Vogel's approximation method. North-West corner method is just, when method based upon the convenience, when we try to get the Basic Feasible Solution or Initial Feasible Solution on which, the optimality test would be applied to. Least Cost method is one, when we try to give some emphasis on the cost that is, the cost to be minimized.

So, this is Least Cost Method, we will try to discuss the maximization problems as well. So, how to deal with that? There is a simple rule that welled say, but that could even be here, the Least Cost is there, the maximum cost also could be seen there. Vogel's approximation method is a method based upon the criteria of minimizing penalties. So, this method is used generally when Basic Feasible Solution or Initial Feasible Solution or first Feasible Solution is required. Least cost method is used to get better feasible solution because when we will try to apply the optimality test, so this will reduce our efforts.

There to get to the optimal solution early. Vogel's approximation method is, I would say, best option because it is based upon least penalties. What are these penalties? This definition is discussed in the last two slides. So, other differences I can just put that, it takes lesser time, it is convenience method that is takes lesser time. However, the basic feasible solution is nearer to the optimal solution in the Least Cost method, but time taken is more than the North-West Corner method. Here, maximum time is taken, here I would say, even more time than Least Cost method.

There are multiple other characteristics that I could put. For instance, anyway, there is no consideration to cost. So, here, Least Cost is focused and for the Modi's test we are more dependent on the penalties or I would say, penalties are minimized. These penalties are actually opportunities or Opportunity cost could also be one of the terms, but we will better try to explain opportunities cost when, we will try to discuss the Optimality test. I just gave an introduction to what is Transportation Modelling, when supply centers are there and demand centers are there and there is a cost associated with the number of

units that we could supply and the limiting constraints are for the number of units that could be supplied, that is required in the specific markets. We looked at the Linear Programming representation of the transportation model, but they were small techniques that we tried to see the Nort-West Corner method, the Least Cost method and the Vogel's approximation method.

This is not yet completed. This is just to get the Basic Feasible Solution. We will see the solution optimality in the next lecture.

Thank you.