Advanced Business Decision Support Systems
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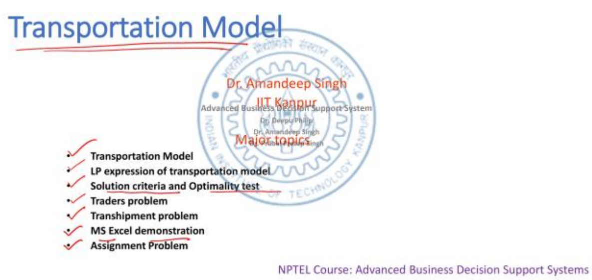
Lecture 21
Transportation Model- Introduction

Welcome back to the week 6 of the course Advanced Business Decision Support Systems. We have discussed the Decision Trees, the Types of the Models, the Classification of the Models in the first few weeks. The last week was more focused on Linear Programming, which is Deterministic Modeling. I would continue the Deterministic Modeling in this week as well. This is an extension of the Linear Programming, that we discussed in the last week and we will discuss the Transportation Models here.



Now, Transportation Models majorly would cover the introduction to them. The Linear Programming expression or the Linear Programming approximation of the Transportation Model, it is an extension or a kind of a big Linear Programming model. How is it a big linear programming model? That we will try to see. When we will try to see the equations or constraints for transportation model, but because it is a very large linear programming model, there is a separate modularity. Modularity, in a way, the separate way to solve the transportation problems, when we have the specific supplies and the demand and those are to be balanced and to be shipped to the various locations.

Solution criteria and optimality test. There are certain solution criteria, when we try to allocate the specific goods, which are to be supplied from one place to another place.

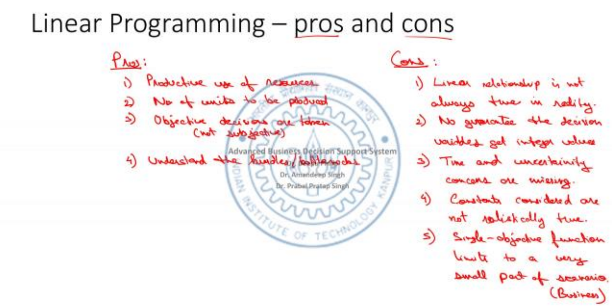
Their criteria could be very general, we just try to pick a cell based upon the ease maybe, that is known as the North-West criteria. Then, because sometimes, it is a minimization problem, at times, it could be maximization problem. Minimization problem is minimizing the cost of the transportation.

In that case, the least cost whatever it is, that is picked that is a Least Cost Method. Then, we have a little more critical method known as Vogel's approximation method which is used to go as closer to the optimal solution as possible. After having the solution criteria, we need to understand, whether the solution is optimal or not. For this, optimality tests are there, stepping toes test is there, modified distribution test is there. We will try to see how these tests are conducted and we try to reach the optimal solution of the transportation.

Another problems like, Traders problem. When I say Traders problem, traders do not only transport, they also purchase that is purchase and sell problem, when the purchasing cost is also associated, then the Traders Problem do come into play. Then is Transshipment problem, transportation and transshipment are two terms. Transporting is transporting from one spot to the destination that is, from source to the destination, there is nothing in between. Transshipment means, when the goods are transported from one place to another, but in between, there are certain stores or in between, you change the cargo.

For instance, a cargo ship from India might have to go to US. This cargo ship is directly not going to US, in between it goes to a station, where the goods are transported or shifted to another ship that goes to US or that goes to other places in between and from the specific location to US, it is again further transferred to trucks or so. Transshipment problem could also be associated with the travelling salesman's problem like, the delivery boys, who come to us, the Swiggy, Amazon people, who come to us. They try to see whichever is the lowest location, they try to make their plan or network of their movement accordingly. This is the Transshipment problem that we will try to see and also we will try to see the MS Excel demonstration of these.

In this lecture or in the connected lectures to the transportation model. In this week, I will also try to cover Assignment modelling in which, the work is assigned to the machines or certain assignments between people and machines, work and machines those things we will try to see in the coming lectures of this week.



Before coming to the Transportation modelling, let me try to see some advantages and disadvantages of Linear Programming. Linear programming, though it was talked in detail that it is a Deterministic Modelling. So, therefore, the certain advantages would be there, certain disadvantages would be there.

The advantages are because the data is certain here, the decision makers can use the data and Linear Programming technique to use their resources productively. Also, this technique helps to arrive at an optimal solution of any of the decision problems by taking into account the constraints. That means, it helps us to tell the number of units to be produced or purchased or so any problem that you have. So, that means, we reach the close to the Optimal solution, but we need to keep one thing in mind, it is not sure that, whatever you are producing, would be sold in the market. It is trying to solve the specific area or specific scope of the problem that is given.

So, we are not solving the whole Business model here, only the specific situation that is given, is being trying to be catered through the Linear Programming using the Simplex or the Graphical methods that we discussed. Linear programming also helps to improve the quality of the decisions that means, the decision that is taken by the manager, becomes more objective. I would say, objective decisions are taken. Objective means, these are not subjective. In case, this is based upon the specific data that you have turned or converted into constraints, that you are trying to solve them.

Also, in Linear Programming, we are always able to highlight that, what are the hurdles in the production process. As we discussed in the last week, in the Simplex program, there was replacement ratio which was limiting the resources. Limiting resources means, the machines which have limits, the maximum number of units this machine could produce. So, it helps us to understand the hurdles or bottlenecks that, what resources are limited, where we do not have the sufficient number of units. So, what is the demand? What machine could remain idle for a longer time these are certain process? So, these are certain merits of the linear programming.

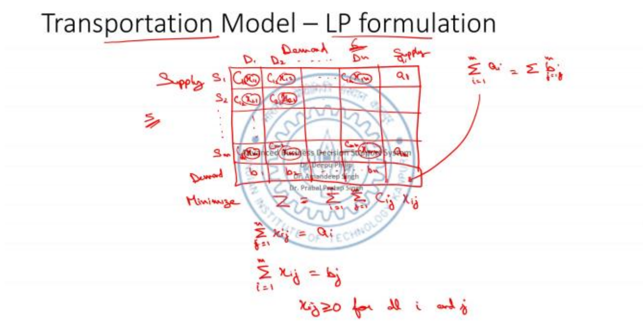
Demerits, if I say, the limitations of the Linear Programming are ascertained by the assumptions that we have taken. Because if those assumptions are not true in the realistic situations, those become our limitations. So, those who are Linearity, Certainty, Additivity, Divisibility and so I would like to put them the demerits. Here would be the Linear Programming assumes Linear Relationship always among the variables. So, Linear Relationship is not always true in reality.

So, that means, neither the Objective function nor the Constraints are linear in the real life situations. Also, one of the assumption that we talked about was the divisibility that means, it could take or decision variables could take non-integer values. There is no guarantee, here the decision variables with get integer value that which means, the man and machine which that would require to do a specific kind of a job, the non-integer value solution sometimes or in the Linear Programming becomes meaningless because number of man cannot be non-integer number of machines. So, rounding off the solution to the nearest integer takes us away from the Optimal solution. So, this becomes a limitation for the Linear Programming.

The time and uncertainty concerns are missing here which means, when we are talking about a specific situation, it becomes a Static model in which, the time is not considered with the change in time, the change of price with time or uncertainties in the production, which is similar to what we just discussed in the first limitation. The same systems might not be linear. So, these are missing in the linear programming models. Also, the parameters which we have thinking of the constant, for example, we are thinking of the cost would be constant to per unit or so, which is generally not constant as and when the time changes, the cost also do change. So, I would say, the constants considered are not realistically true.

The last limitation I would like to put here is that, we are only having a single Objective function in linear programming however, it could be a multi-objective programming that is required. Single objective function is given in which, we are also keeping in mind the assumption of the additivity or proportionality, which is not certainly all the time true. So, Regression modelling, Multiple Regression modelling, Non-linear Regression modelling could be the kind of the models which do go more closer to the realistic situations. So, here the Objective function is single, which is one of the limitations. I could say, single Objective function limits to a very small part of scenario.

When I say scenario, I am talking about the business. So, though still Linear Programming gives us the Basic Feasible solution as we discussed to understand solution would be closer to this point. At least, the specification that is given, we are able to go close to the what the results would be there for the solution and data is also deterministic here.



Let us try now to discuss about the Transportation modelling. Transportation modelling in which, the transportation is taken from one place to another.

Here, it is just an extension of the Linear Programming as I said. Transportation literally means, physical distribution of goods and services from certain supply centers to the different demand centers. So, there are supply centers, there are demand centers, Transportation problem can also be expressed as a Linear Programming model, which we will try to see here and certain steps to solve or to get the solution to the problems that we will try to see here. So, we have Supply, we have Demand and I can put my linear programming like something a matrix in which, different supply centres and demand centres are there and supply is going from one centre to another. So, I can say, I have a supply centre S_1, S_2……S_m and the demand centres are also there D_1, D_2……Dn and this is the total supply.

I have total demand that comes here. So, here my variables or decision variables are X_{11}, X_{12}………X_{1n} with a supply I would say a_i, here it is a_1, here it is X_{21}. So, these are all decision variables X_{22} and so on. Here I would have X_{m1}, X_{m2}……..X_{mn} and i varies from 1 to m. So, this becomes Am and for the demand I have B_1, B_2…….B_n.

So, there are certain conditions that total supply should be equal to total demand. These conditions we need to see that means,

$$\sum_{i=1}^{m} ai = \sum b_{j=j}^{m}$$

So, this comes here total supply and total demand. So, how do the equations look like if I try to see the objective function and equations for my transportation model? So, to put the equations, we should also know the cost, the contributions to the Objective function. So, these are all which are given are Xij's, which are my Decision Variables.

I will put them in a round circle because I would like to put my contribution or the cost in the matrix as the major element, this is C_{11}, C_{12}. C_{11} is the cost of supplying the goods from supply center 1 to demand center 1. C_{14} would be the cost of supplying the goods from supply center 1 to demand center 4. So, similarly, we have C_{12}, C_{22}…….C_{1n}. So, these are the cost associated with it.

C_{m1}, C_{m2},……C_{mn}. So, my Objective function becomes the product of the units which are being transported from one place to another and the cost associated with them. The sum product of these would be my Objective functions. For example, if I have a lot it in the specific cells, not all the cells would be allotted, wherever these are allotted from there, the Objective function would be, I am talking about cost, so therefore,

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{m} Cij\, Xij$$

This will be more clear. I will try to come to an example and we will try to see, what does this exactly practically mean because not all the cells will be allotted and if even suppose all the cells are somehow allotted, this becomes very large Objective function.

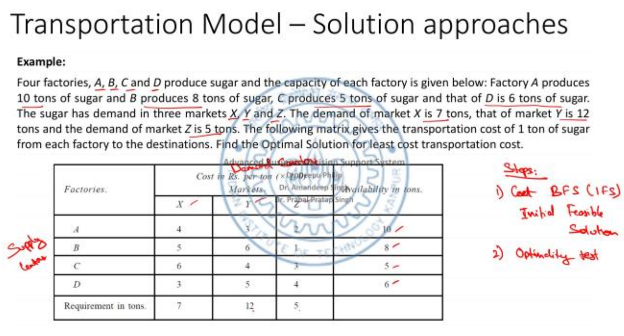
For example, if you say, there are m supply, m could be maybe 5, there could be n demands, n could be 6, 5 * 6 = 30. In the Objective function itself, we have 30 different products which are summed together. So, this becomes a very large linear program. Now, let us come to the Constraints. Constraints would be,

$$\sum_{j=1}^{n} Xij = ai$$

$$\sum_{i=1}^{m} Xij = bj$$

These two are different because in the first set, I am talking about only j varying from 1 to n and second, when we are equating it to Bj, that is, the demand is only talking about the demand that means, here i is varying from 1 to m, these are all equality constraints. So, where i and j are varying here? So, these constraints would also be very large, so we have if I say, as I said maybe, 5 supply centers and 6 demand centers, so what does it become? 5 plus 6, 11 constraints plus all the non-negativity constraints, so this is for all i and j, so this becomes a very large problem.

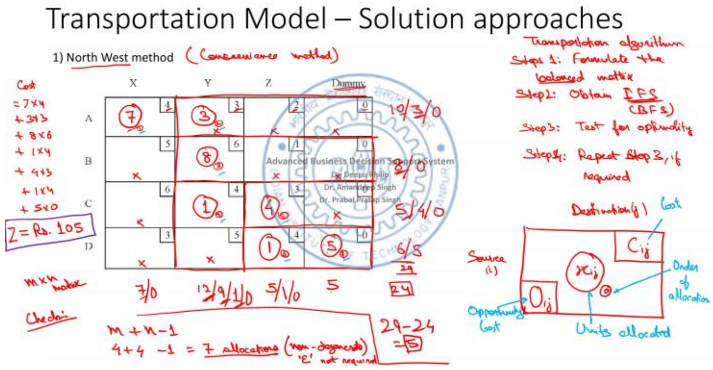
Now, let us try to see the solution of the Transportation models, here we will try to see the small techniques, which are developed by different thinkers, Vogel has given also a technique that also we will try to discuss, these are the techniques which are specific for the transportation modeling because generally, also we can solve that becomes a large linear program. So, let us with an example try to see how do we try to solve the linear program.



The example that is mentioned here that we will address and try to understand the Transportation Modeling solution approaches.

There are 4 factories A, B, C and D that produce sugar and the capacity of each factory is given below here in the matrix. Factory A produces 10 tons of sugar, B produces 8 tons of sugar, C produces 5 tons of sugar and D produces 6 tons of it, 10, 8, 5, 6 are the availability in tons which means, these are factories, this become my supply centers. Now, there are 3 markets X, Y and Z. The demand for market X is 7 tons, for market Y is 12 tons and for market Z is 5 tons. X, Y and Z become my demand centers and their respective demand is 7, 12 and 5 for the market X, Y and Z respectively.

Now, there are certain steps when we try to solve the Transportation Model using the techniques that I am going to discuss, First step is, I try to get if we remember BFS (Basic Feasible Solution). I will also put IFS here, so I try to get Initial Feasible Solution through one of the methods and then, I try to apply the optimality test.



Now, to apply the optimality test, I need to see my problem statement in a matrix form that is usable to be applied with the techniques that I am going to discuss. The first method is the North-West method that is the top left corner of my matrix is only picked irrespective of the cost. North-West method is just not considering the costs at all that means, it is just for convenience, so it is also known as Convenience method.

So, before any optimality test to be started, let me try to talk about the steps, what are the steps to solve a Transportation model? So, I will call it as Transportation algorithm. Here, steps are formulate the balanced matrix, so it is in a matrix form, what is a balanced matrix, I will just discuss. Step 2 is obtain Initial Feasible Solution that means, we obtain some solution and that solution now taken for the optimality test and this some solution is just a starting point. This solutions that is obtained, if it is closer to the optimal results, optimality test will take lesser time. So, some solution could be taken from the any of the 3 methods, I am going to discuss.

So, we get this Initial Feasible Solution, I could also call it as Basic Feasible Solution. This is taken through any of the 3 methods, North-West corner method, Least cone method or Vogel's approximation method. Then, we test the optimality. In the optimality test itself, if we do not get it in the one go, so this is step associated with the optimality test. Only we say, repeat step 3 if required that means, in the first iteration itself, we can get the optimal results or maybe the further iterations might be required that is the step 4.

We repeat till the point we get the optimal results. This is Transportation algorithm. Now, in the North-West corner method, as I said, the North-West corner is always picked. Let me try to first see what is the Balanced matrix which I have put.

Balanced matrix means,. if I try to see, what is the demand and what is the supply, the supply centres have the availability 10, 8, 5 and 6 for the respective factories A, B, C and D sector and the markets have 7, 12 and 5 units requirement that is, demand is 7, 12 and 5. Let me try to see, whether the demand and supply is balanced or not. So, I need to obtain a Balanced matrix. Let me try to see the total demand that is there. Total demand is 7 plus 12 plus 5, this is 19 plus 5, this is 24 total demand.

Now, total supply that is available is 10 plus 8 is equal to 18 plus 5 is equal to 23 plus 6 is equal to 29, 29 units. This is not balanced which means, I have to balance it for which process is, I will add a new column or new market. This new column or new market is just a dummy market which does not have any physical significance because there is no physical significance. The cost which are put in the respective cells is rupee 0 per unit of transportation. If anything is allocated within these cells in the dummy column, that would have no meaning but major concern is the limiting factor, here is between the demand and supply whichever is lesser.

The lesser is 24 that means, my demand is 24 only. This whole 24 demand is to be covered within the factories X, Y and Z. So, now let me start with the North-West method. North-West method means Top Corner method, Top corner is my cell AX that is, from the factory A to the market X, this is the cell where what could I allocate? The supply available is 10 units, the demand is 7 units. Out of 10 and 7, 7 is a smaller number. This is the only maximum number of units that I could allocate here.

So, I put number 7 here and put it in a circle. In a circle means, I have allotted the units. This is the notation that I am going to use throughout the presentation that means, whatever is to be allocated, I will put them in circle. So, if this is a cell, I have my destination j and source i. So, in this, the number that is there in circle is my units which are allocated which are my Xij and the number on the right top corner is my cost associated Cij. Here in the cells, you can see these values 4, 3, 2, 0, these are the cost on the right top corner of the cells, which are my cost of the cell j and on the left down corner or left bottom corner, I will put opportunity cost when I come to the optimality.

This is Oij Opportunity cost. Let me just try to label them. This is Opportunity cost. Xij in the round is the number of units allocated in the specific cell ij and Cij is my cost. So, I have allocated 7 units here in the top left corner and you can see, I am left with 0 demand by factory X. I will put small crosses here in the column X because this is eliminated.

Now, I am left with only columns Y, Z and dummy and all the factories are still available. Now, out of the remaining matrix that is this matrix, now the top left corner is the cell AY in which, the maximum number of units that is the limit number of units that could be all allocated is determined from the number of units available. Now, when I have allocated 7 units to my factory A out of the 10, I was left with only 3 units here. So, 10 minus 7 is 3. So, now out of 3 and 12, which one is the limiting number? 3, I can allocate maximum 3 units in the cell AY and I am putting it, in a circle because these are allocated and the factory A is now having 0 units and market Y is left to be supplied by 9 units.

12 minus 3 is 9. That means, factory A is also eliminated. I will put small crosses here. Now, the remaining matrix is this within the red lines. Now, the top left corner is BY. So, what is the limiting value here between 8 and 9? I can only supply 8 units from factory B to the market Y and in the factory B, I am left with 0 units. In the market, Y 9 minus 8, I am still left with 1 unit to be supplied because factory B is now completely exhausted.

So, I will put crosses here as well that is, no allocation could be given here and now this lower 6 cells are left. And now, here the top left corner is my corner CY. From factory C to Y, the limiting number is between 1 and 5 that is, 1. 1 unit could only be allocated here and because 1 unit is allocated, I am left with 0 units in market Y to be supplied to now and market Y is now completely eliminated.

Now, only these 4 cells are left out of these 4 cells. Northwest corner is now corner CZ. In CZ because I was left with 5 minus 1, 4 units in factory C, so maximum 4 could be allocated here. So, 5 minus 4 is 1 and 4 minus 4 is 0. Now, this is eliminated my factory C.

Now, out of these 2 cells, I will allocate 1 unit here. If 1 unit is allocated I am left with 0 units here and 6 minus 1 is 5. I missed to put very important point here when I added a dummy cell, the purpose of adding dummy cell was only to balance the demand and supply. So, demand and supply difference here was 29 minus 24, which is equal to 5. So, these 5 number of units what to be put for the dummy market that means, 5 units be allocated here. Now, if you see the total number of units, 7 plus 12 plus 5 plus 5 is 29 and 10 plus 8 plus 5 plus 6 is also 29.

So, this is how we balance it out. Now, in the last cell, 5 units are there, so 5 units are allocated here and we are now available with our Initial Feasible Solution or Basic Feasible Solution. There is always one check. Whenever we obtain a solution, whether the solution is feasible solution or not. To obtain the Feasible solutions, if there are m rows and n columns, the number of allocations that should be there, should be m plus n minus 1.

This is a check because we have m into n matrix always. So, the check is number of allocations should be m plus n minus 1, here m is 4 and n is also 4. So, 4 plus 4 minus 1, which is equal to 7 allocations. So, if you count them 1, 2, 3, 4, 5, 6, 7. So, when 7 allocations that is, m plus n minus 1 allocations have been made.

We can now say, the solution is a Feasible solution. So that means, the occupied cells or the allocations should be m plus n minus 1. So, that means the solution is non-degenerate. Otherwise, the solution could have come or become degenerate solution and we could have given a dummy number or dummy value that is generally given by Epsilon. We will put it here as Epsilon, this value is given.

So, this is not required here. So, this is the Northwest method. So, for the Northwest method, what is my solution value now? Solution value is the number of units allocated in the associated cost that is, I would say, my solution value or the cost is equal to 7 into 4 plus 3 into 3 plus 8 into 6 plus 1 into 4 plus 4 into 3 plus 1 into 4 plus 5 into 0 which is equal to rupees 105. My cost through the Northwest method that I have obtained here is rupees 105. I will put z is equal to rupees 105. We will see the solution in the next lecture. Thank you.