Advanced Business Decision Support Systems

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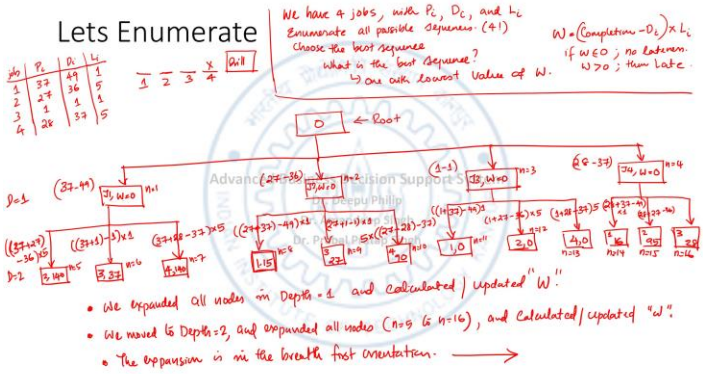
Lecture 10

Tree Search and Alternatives in Decision Making Using Single Machine Sequencing Problem (Part 2 of 4)

Good afternoon everyone, welcome to yet another lecture of Business Decision Support System which is the advanced course of Web-Based Decision Support System that was plotted in the NPTEL MOOC's program. I am Dr. Deepu Philip from IIT Kanpur and along with me this course is Dr. Amandeep Singh Oberoi and Dr. Prabal Pratap Singh. And, we have been going through the process of Decision Support System and the type of business decisions and we have also discussed what is the importance of the corporate decision making process and how the decision making process and the corporate decision why it is important to be covered all aspects of it.

And, we also seen the simple aspects of Monte Carlo simulation and how Monte Carlo can be used for making decisions, where you do not have clear values of a parameters etcetera. And, then, in the last class, I mentioned that now we will get into what we call as the decision to the Tree Search or what we call as an Algorithmic Decision making approach. So, without any further delay, let us get into the topic. So, as we discussed we were talking about the Single Machine Scheduling problem and we were going to do the total enumeration.



So, we have 4 jobs with processing time Pi due date Di and the penalty weight Li. So, enumerate all possible sequences. So, if there are 4 jobs then there will be 4 factorial sequences that is 4 factorial. Enumerate all possible sequences then choose the best sequence. What is the best sequence? The best sequence is one with lowest value of W and it is equal to completion minus due date minus Di multiplied by Li.

And, if W is less than or equal to 0 nor late, it is before time, if W is greater than 0, then penalty, that is a loss, that is late, not penalty, then we called as then late. So, that is the premise in which we are actually solving this problem. So, we start by the first node. This is node 0 or this is as our root node. And, then, the First depth of that, we have 4 jobs.

So, if you think about it if you think about the Single Machine, this is your as we said, is a drilling machine sequence position 1, 2, 3 and 4. For the first sequence position, I can put any of the job. So, we will take the first option here as, like this: J1, J2, J3, J4 or instead, I am just using J instead of just to clarify this. So, this is the depth equals 1, the First Depth. So, since, we are going to set its total enumeration in the Breadth First Search, what will happen is, we will visit all the nodes in that particular depth and then only we will go to the next depth.

And, our problem statement is, there is job 1, 2, 3, 4, the processing time Pi is 37, 27, 1 and 28 and the due date the Di is 49, 36, 1 and 37 and the lateness Li is 15, that means, if job 1 is late, then the penalty, the weight W will be calculated by the lateness multiplied by 1. If job 2 is late, the lateness multiplied by 5, 3 is late, lateness multiplied by 1, 4 is late, lateness multiplied by 5 . So, in the root node, what happen is, if you assign job 1 to the first position, then it will take 37 days to complete or 37 hours to complete. So, I am going to move this depth to little bit of a side D equal to 1. So, it will complete in 37 days, the due date is 49.

So, since it is completed to before time, we will say, J1, W equal to 0. There is no delay here. Similarly, for J2, it will complete in 27 hours or days but the due date is 36. So, there is also no late. Lateness W equal to 0 here. J2, W equal to 0 and then J3 will take 1 day and due date is also 1.

So, 1 minus 1 also is 0. So, there is no lateness, job will complete in time and then, the J4, the fourth job the processing time is 28, the due date is 37, 28 minus 37, it is again less than 0. So, the lateness is 0. So, no jobs are late, you can assign any one of the jobs into first sequence position and without having any lateness as part of it.

Now, the second sequence position, we move to what you call as the second depth. As we remember, how to enumerate all possible options. So now we have fixed this the sequence position, which is over. So, now, we can only put 3 jobs. So since J1 is already assigned, so I am going to do it, like this I really hope, we get time space to do this.

I am not very good at managing space but we will see . And. so job 1 is assigned now this is 2, 3 or 4. So, that means, the second sequence position or which is the third one, we can put either job 2, 3 or 4 in this case, it will be job 2 is already assigned. So, it can be 1, can be 2, cannot be assigned 3 or it can be 4.

Then, the third one is 1, 2, 4 because 3 is already assigned in the previous position and then the last sequence is, 1, 2 and 3 because 4 is already assigned. So, now, you have enumerated all leaves of the second depth. That is why, it is called as the Breadth First. You are in complete broadness, then we have done. So, now, if you look at the J to the second level for job 2, so the time required is 27.

So, it will take 37 time units to process job 1 and it will take 27 units to process job 2 . So, it will be 37 plus 27, that will be the total time, that is when job 2 will be completed and the due date of job 2 is minus 36 . So, 36 is the due date of this and if there is any dates delayed, then it should be multiplied by 5 . So, if we do that then, what happens is, we get a lateness of W, I am not going to say W, I will just write that number here 140. So, it is laid by 140 days or 140 hours and so let us do job 4 like this.

So, job 3 will be same way. It is already finished to 37 for job 1. So, it will be 37 plus job 3 will be1 and minus 1, that is the due date and the penalty is also multiplied by 1. So, 37 plus 1 minus 1. So, 37 multiplied by 1.

So, the late W will be 37 and 4 in that regard is same way of processing time of 4 is 28. So, it will be 37 plus 28 minus the due date is 37 right and times 5, that is a penalty. So 37, 37 will cancel out 28 times 5 that is,140 will be the penalty for this place. Now, same way, we calculate for this particular second node.

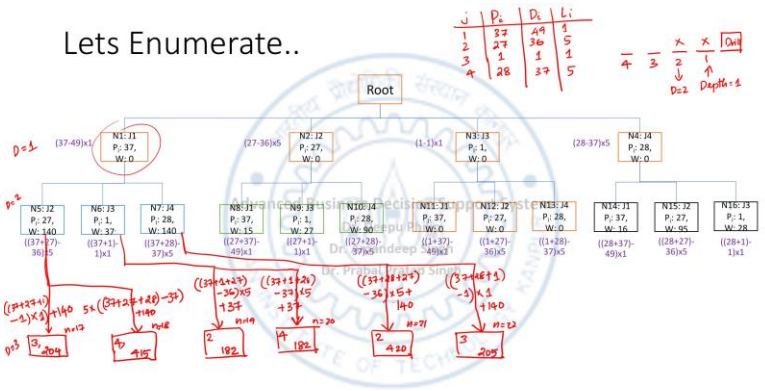
So, I am doing 2. So, it will be in this case 27 is the time before it is 27 plus job 2 is already assigned. So, this is node 2 this is 1, the processing time is So, it is 27 plus 37 minus 49 and then, that is multiplied by whatever the indices multiplied by 1. So, that value will be 15. Same way, this is 3 and for this, we calculate it as 27 plus 1 minus 1 times 1. So, that will be 27. W will be 27 and then the 4th job will be 27. For this, 1 plus the 4th job is 28 and this is the completion time. And you have to minus the due date which is 37 and the penalty multiplied by penalty. I am going to multiply this side by 5.

So, that gives you 90. So, then, similarly, if you do this, this is job 3, it is 1. So, it is 1 plus job 3 is time duration is 1 plus 37 minus 49 and if you multiply that by the lateness is 1, but you can see that, 38 minus 49, the job is not late, so W is 0 and same way for job 3, it is, we will say, it as 1 plus this is job 2, processing time is 27, 1 plus 27 minus 36 and so multiplied by the penalty is 5 since, there is no lateness, here this is also 0. Similarly, the next node is 4 again. The time is 1 plus the 4 is 28 is a processing time 1 plus 28 is 29 minus the due date is 37 and the lateness is multiplied by 5 but there is no lateness so that time is also 0 . Then, comes the 4th node, the processing time is 28.

So, it is the 4th job. So, 28 plus is 1, it will be 37 and then the due date is 49 and the weight is multiplied by 1. So, we look into that can be calculated as you get the weight value of 16 and let me redo this a little bit that is 2 and 3 . So, if you do the same process, it is 27 plus 28 is 4 plus. So, it will be 28 plus 27 minus 36 multiplied by 5 etc. So, you get the value here as 95 and then the 3rd one, the last one, you follow the same process, you get a value of 28. So, this is node n equal to 1, n equal to 2, n equal to 3, n equal to 4 and here is n equal to 5, n equal to 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16. So, these are the 16 nodes and in these nodes, we can see that some of the nodes of lateness W value. Some of the nodes does not have a W value. So, you are now gone to the 2nd depth, but we have elaborated. We expanded all nodes in depth equals1 and calculated or updated w is the lateness.

Then, we moved to depth equals 2 and expanded all nodes. That is n equal to 5, n equal to16 and calculated or updated W. So, for all the nodes, with the 2nd depth, we have completed and updated the W. So, this is the breadth. First so we have the expansion is in the breadth first orientation.

So, you are going by the breadth completely. So, I hope you guys understood the fundamentals of this much. So, we will move into the next step. We will try to move to the depth equal to 3 now.



So, let us enumerate and as I said earlier, just for quick reference, I am going to write it here. The job J1, 2, 3 and 4 and the Pi the processing time is 37, 27, 1 and 28 and the due date Di is 49, 36, 1 and 37, Li the lateness is 1515 is the problem we adopted from the industrial scheduling textbook Dr. Sood and as of now, it is a Single Machine and you have 4 sequence positions: 1, 2, 3 and 4. This one is managed by the depth equals 1, this is depth equals 2 and we already completed these 2. Now we have to move to the 3rd depth.

So, this is D equal to 1, D equal to 2. Now, we will try to do D equal to 3. So, 3 means again, we have to elaborate D equal to 3 for all these nodes. So, these are, if you look into this node 5, 6, 7, 8, 9 etc. up to node 16. This is what we have elaborate in the previous one.

So, I just summarized all of it calculations everything for your quick reference and then, from here, we will expand the next one. So, what I am going to do is, now you can only have 2 options. The sequence position 4 and 3. So, you can only put 2 jobs from the remaining. So, what I will do here is, I will do something like this.

It is little too long but let us see, I may be able to come up with a better way of doing this. These are the two from this, the first one. Then we take it 2nd. So, from all the nodes, each in N5 will have 2 child, N6 will have 2 children and N7 will have 2 children. So, this node will be, we can call it as N equal to 17,18,19, 20, 21, 22.

So, 22 nodes altogether after we expand to depth 3 from the first child of the root. So, the N17 is the option in front of us. You have already assigned the job 1 and 2. The option in front of us is to assign 3 or assign 4 here. So, this corresponds to assigning 3 to sequence position 3 or job number 4 to sequence position 4.

So, for this, N equal to node 6, where you assigned the job 3. So, 1 and 3 are already assigned. So, the option in front of it is 2 or 4. And, similarly, the last one is J1 and J4 is already assigned, so that means you have option in front of is 2 and 3.

You can only assign 2 or 3. So, if we assign 3 in the third sequence position, then what will happen is, we will have job 1 already assigned 37, job 2 is also assigned 27. So, it will be 37 plus 27. Now, you are assigning job 3. So, that will be plus 1, that will be the total time taken to process that minus the due date of which job 3 is 1, so that is 1. So, whatever is that lateness, that should be multiplied by 1 because the lateness of job 1 is 1.

So, that will be the lateness for this level and that you have to plus, add the previous lateness 140 to this . So, with that value, your lateness value will come to 204, if you calculate that . So, now if we follow the sequence 1, 2 and 3. Assign job 1 to first sequence position, job 2 to second sequence position, job 3 to third sequence position, then your lateness penalty now will be 204. Same way, if you take job 4, then you already assigned job 1, so it is 37 plus it is already assigned second job 27.

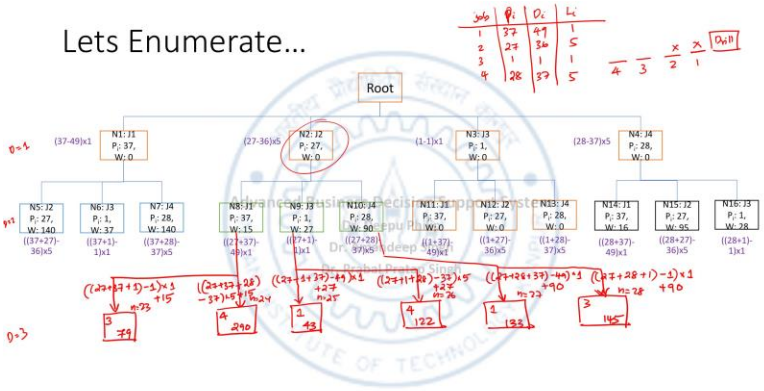
If you are assigning fourth, then that time period, that value is 28. So, 28 that is a total time minus, you need to subtract the due date which is 37 whatever that it is need to be multiplied by 5 . That number need to be added with 140, which is the previous 1 penalty and you will get value as 415. So, obviously, you know that, if you follow this path, you will definitely have much lat e of a job as such. Same way, to explain to you the next job, we will do is job 1 and job 3.

So, the processing time of 37 and 1. So, it is, we will calculate it as 37 plus 1, the previous 1 plus if you are assigning job 2, the manufacturing time is 27, then you minus the due dat e of job 2 that is 36. Whatever the delay gets multiplied by 5 then, you add this previous lateness, which is 37 to this and that will be the value of W, which if we calculate it, will come to 182 Similarly if you assign job 4 then that will be 37 for the first one, the second job will be 1 plus the fourth job. The processing time is 28 and minus the due date of it which is 37 and the delay is multiplied by 5 and plus the previous 1 37 which will give you 182 similar numbers because the processing times are exactly the some of the processing and due dates are exactly the same values. Then, the third one from this strategy. So, you have now first sequence position job 4 and third sequence position job 2.

So, that is the case we will have 37 plus 28, the first one plus you are doing job 2 processing time is 27 minus the due date of that it is 36 whatever that value is multiplied by the Li is 5 and plus you add the previous one of 140 with that should give you a total time of 420 as part of the lateness W. Same way, third sequence position depth will be same way you are assigning 37 plus 28 plus the processing type of job 3 which is 1 minus, the due date is 1 the whole thing gets multiplied by the lateness Li factor, which is 1 plus. You add 140 to it. So, all of these things put together you get 205 .

So, now you have expanded all the nodes in the first part . Now, what we have to do is,

we have to repeat the same process for the this second one then, the third one, then the fourth one. So, due to the shortage in space, we will go to the next slide and we will try to expand it for the next node for the same depth. So, now we move to so in this case. We have already seen n equal to 22.



So, then, now we will have is let us say, something like this, the last one. So, the previous node, we had n equal to 22. So, now we have n equal to 23, n equal to 24, n equal to 25, n equal to 26, n equal to 27, n equal to 28 and this is depth equal to 1, depth equal to 2, depth equal to 3 and just to clarify again on the problem, we have said that job that is 1, 2, 3 and 4 the Pi is 37, 27, 1 and 28, Di the due date is 49, 36, 1 and 37 and the lateness Li the weight is 1, 5 and 1, 5.

So, in this case, if you look into it job 2 is already assigned to job 1. So, the thing that we can do is job 3 or job 4 is for the next sequence position. Remember, we already mentioned this the drilling machine, the first sequence position, second, third and fourth.

So, this already have been picked up. We are now working on this depth . So, if you look into it here, it has 2 and 1. So, the option in front of us is now 1 or job 4 and in this case, 2 and 4 is assigned. So, your option is in front of us: 1 or 3 . So, if you assign 3 here, then the idea here is that, you have already given 27 processing time for the first one 37 is for that one.

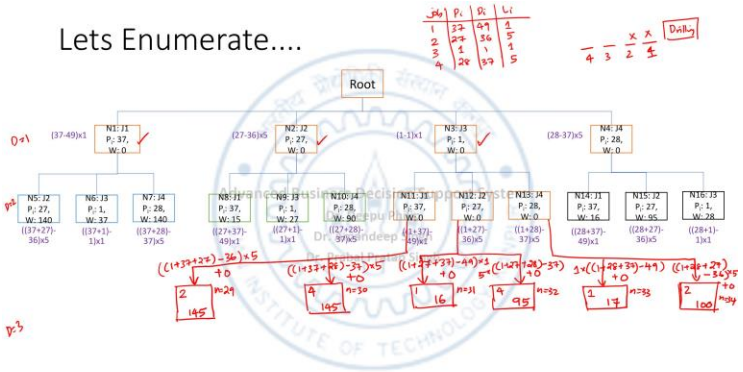
So, you will have 27 plus 37 plus job 3. The processing time is 1 plus 1 minus 1 that is the due date. That whole thing multiplied by the weight is 1 plus the previous delay is 15. So, if you add all of that, you will get 79 as the W value . And, the next job is 27 plus 37, which are the 2 previous processing times plus the fourth which is the processing time of fourth is 28 and that is minus the due date of job 4 is 37 that whole lateness is multiplied by 5 and you add the previous lateness of 15 put together and you get the value of 290 . Please check the calculations because I think I did my calculations correct, but if there is something wrong update it accordingly.

Then, comes the next one, where we will do it as the previous job is 27 for 2, job 3 is 1 plus job 1 is 37 and the due date is 49 multiplied by the delay is 1 and plus 27, which is the previous W value with this and you get the value as 43 . And, then, comes the last one, which is again 27 plus 1 plus job 4, 1 plus 4, the processing time is 28 minus the due date of job 4 is 37.

Whatever the delay is multiplied by 5 plus you have to add the previous lateness of 27 and you will get the value of 122 as the lateness here W. And, then, comes the last expansion of the node, which is the option 1 and 3. So, here you have already assigned the job 2 and job 4, so the processing times will be 27 plus 28 is already done and job 1, if you assign then the processing time is 37, the due date is 49.

All of that multiplied by the delay value is lateness is 1 multiplied by plus the previous weight is 90. If you add all of that together, you will get a value 133 . Same way, the other last part of it is, the previous job is 27 plus then, you have the 28, what 2 and 4 is already assigned. Now, you are assigned job 3, the time is 1 and the delay is 1 multiplied by the lateness is 1 plus, you have to add the previous value of 90 and you get the value of 145 . So, now you have completed the enumeration of the second branch that came out of the root node and this completes the second set.

So, now we have previously completed this one, this portion we have expanded now, we are expanding this portion . Now, we will move to the next one in the slide and we will continue again the advantage or the idea of doing this is that, we will finish this breadth completely.



So, let us continue to enumerate but before we enumerate this, just quickly summarize the problem job 1, 2, 3 and 4, the processing time Pi is 37, 27, 1 and 28 the due date. Di is 49, 36, 1 and 37 and the lateness Li is 1, 5, 1 and 5 and we are again, as I said earlier, is

a drilling machine, single machine. Now, sequence position of 1, 2, 3 and 4 and we already assigned these two are working on the depth 3.

So, this D is equal to 1, D equal to 2 and we are working on the D equal to 3 depth and we have completed this one and this one, now we are working on this the third one is what we are working on and we are going to take the white space below because this is not very easy to enumerate all these things. So, I am going to work on from here. So, this way and we will do it somewhere here, so the two children that come out of it. So, you already assigned the job 3 and the job 1. So, now the only option left out to in front of you is job 2 or job 4 that is in this branch.

Now, we do is, this branch and it has already assigned job 3 and job 2. So, in front of this option is 1 or 4 and then comes the last job. I am taking in white space purely because we need to have some computations and since here is job 3 and job 4 already assigned, so option in front of it is 1 and 2. There are remaining two jobs you can assign. So, now looking at the number again, calculating it so you have already assigned the job 3 and job 1, so job 3 processing time is 1, so it is if you write here, it will be 1 plus job 1 processing time is 37 and now you are assigned job 2, the processing time is 27 minus the due date of job 2 is 36. So that is 36 okay and if it is laid by anything you are multiplying by the factor of 5, so it is multiplied by the factor of 5 and then all of this value plus you have to add the previous lateness which is 0.

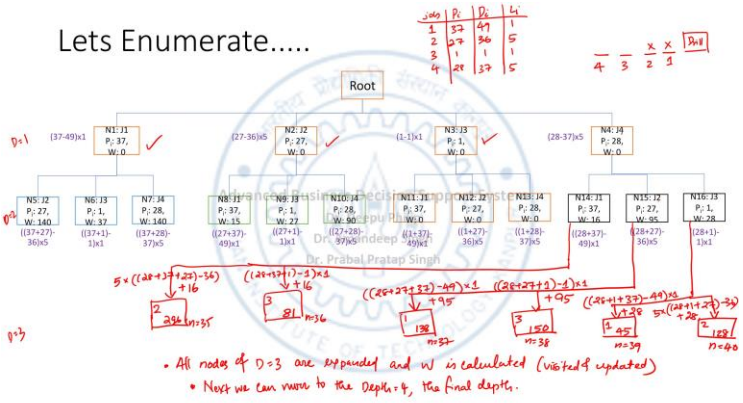
If you add the lateness so then, you will get the value all of this put together as 145, same way for the next one, it is you are assigned job 3 and job 1, so job 3 and job 1 is 1 and 37 is the processing time and then you are assigning job 4, the processing time is 28 minus the due date of job 4 is 37. Whatever be this lateness multiplied by 5 and add the previous W value of 0. All of this put together, you get the value of 145, so you get two of them at the same value at this point. Then, similarly, job 3 and job 2 is the next option that you are assigned. So, now you are assigning job 1, so if that is the case, you have job 3 is 1 job 2 is 27 is the processing time.

Now, you are assigning job 1 in which 37 is the processing time minus the due rate of that it is 49, you are multiplying that with 1, which is a penalty plus you are adding the previous value of 0. So, all of that put together, you will get a value of 16 here as the W. Same way, the next node of this one that is 1 plus 27, you are assigning job 4, the processing time is 28 the due date is 37, whatever that is multiplied by 5 and plus the previous value of 0 and you get that value close is as 95. Then, comes the third node and the two children of the third node.

So, here you are assigned job 3 and job 4. So, that is job 3 is 1, job 4 is 28 and now you are assigning job 1 is 37, the due date of that it is 49 and the penalty is 1, so multiplied by 1 and the previous value is 0. Add all of that put together, you should get a value of

17 then is the second, where the last node of this side, where again you are assigned the job 1 and job 4. So, it is 1 plus 28, now you are assigning job 2, the processing time is 27 minus, the due date is 36 and the penalty is multiplied by 5 and previous value is 0, if you add all of those things together, you get a value of 100. So, you can see that the previous you have calculated until node 28. So, now your node will be 29, equal to 30, node equal to 31, node equal to 32; node equal to 33, node equal to 34.

So, you have now gone through the third one also we have expanded now the only thing that is left out for us is the fourth one to finish the expansion of the D4.



So, we will continue with the fourth level expansion. So, again for quick reference, I am adding the problem statement here quickly, the processing time which is 37, 27, 1 and 28 the due date is 49, 36, 1 and 37 and the lateness is 1, 5, 1 and 5 So, now we have already completed this depth equal to 1, depth equal to 2 and are working on depth equal to 3 and we have said that, this is your drilling machine, you have your 1, 2, 3, 4 as a sequence position. These two are already assigned as part of the previous one.

So, we have completed this, this and this enumerating depth and breath. Now, we are doing processing of enumerating the last node, the fourth node. So we will try to enumerate in such a way that, we use the white space here. So, I am going to bring it, you have one node here and we have the other node here. Then, same way, this will also have two nodes. So, I am doing just this, so that, I have enough white space to move further and I am not squeezing this anywhere.

So, in this case, you have assigned job 4 and job 1 already, the only option in front of you is, you can assign this part, job 2 or job 3 because 4 and 1 are already assigned. In this case, second one, you have already assigned job 4 and job 2. So, the funda of it is assign, it is 1 or 3 and last one you assigned is job 4 and job 3, so the funda is same of 1 and 2.

And, we have seen already that n equal to 34, so n equal to 35, n equal to 36, n equal to 37, n equal to 38, n equal to 39, n equal to 40. So, in depth equal to 3, when we elaborate all these things together, you will get what we called as 40 nodes.

That will finish off the entire third depth. Now we need to calculate what is the W value for each. So, just to continue the calculation, so here, you assigned job 4 and job 1. So, it is 28 plus 37 plus if you assign job 2, the processing time is 27. And then that is minus the due date of that it is 36, and you are multiplying that by 5 as a lateness plus the previous lateness, which is 16.

You add to that and you get a value of 296. Same way, the next one, because now you are assigning job 3 to the third sequence position. So, that will be 28 plus 37 plus the next one is, you are assigning job 3. Job 3 processing time is 1 minus 1 multiplied by the lateness is 1 plus value of 16 and you get that as 81. Similarly, for the one you already assigned job 4 and job 2.

So, that is 28 plus 27 and you are assigning job 1 that is 37. The due date is 49 multiplied by 1 plus the lateness, previous one is 95. You add all of that put together, you get 138 as the W value. Similarly, the next one is, you add 28 plus 27 plus you have job 3, the processing time is 1 minus 1 multiplied by 1 plus the previous lateness of 95.

ou add all of that put together, that will come to 150. So, then comes the last two nodes. So, you have assigned job 4 and job 3. So, that is 28 plus 1 plus, now you are assigning 1 right here. So, the 1, processing time is 37 minus 49 and so on multiplied by 1 plus the previous lateness of 28, you add of all of that together, you will get what you call as 45. Then, is the last one that is again 28 plus 1 plus, now you are assigning job 2. So, processing time is 27 minus 36, the whole thing multiplied by 5 plus the previous lateness is 28 add all of that put together you get 128.

So, now this completes. So, all leaves or not we do not want to call it as leaves is the terminal node all nodes of D equal to 3 are expanded and W is calculated. So, visited and updated. Now, next we can move to depth equals 4, the final depth.

We will take a small break here, because I know it is a very large number of computation we will take a quick break here and then we will continue in the last enumerations in the next class.

Thank you.