Data Analysis and Decision Making – II
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Lecture – 09
Decisions and Utility Analysis

Welcome back my dear friends. A very good morning, good afternoon, good evening to all of you and this is the DADM - II, NPTEL, MOOC lecture series. DADM means Data Analysis and Decision Making – II, and this total course is for duration, this is DADM – II is total duration is 30 hours which basically means 12 weeks of 60 lectures each week we have 5 lectures, each for half an hour. And as you know that after week 1, week 2, so on and so forth we have assignments to be solved based on whatever we cover in the respective week which has just been over.

So, in the last two letters for week number 2. See if we remember that, in the last class we were discussing on the (Refer Time: 01:02) that there are other measures of ranking also. So, somewhat geometric mean methods can be considered, then you have the safety first principles stochastic dominance all these things. So, I will spend some time with these methods and then most probably we should wrap up by the 10th lectures or maximum by the 11th lecture which will be in the first class for the third week we can start with the general concepts.

So, we would discussing geometric mean and if you remember that in the geometric mean method I discussed that you have some decisions. So, decisions is 1, 2, 3, 4 and the general number of the decision is small j and small j changes from 1 to capital J. And under each decision, there are different alternative. So, alternatives for each decisions j is given by small i and small i basically can change from 1 to capital I. So, in the first decision you can have see for example 3 alternatives, in second one you can have 5 alternatives, so on and so forth. All these can be very in a very nice manner be generalized and we can basically find out the ranking system.

Now, if we remember I did mention that again I am repeating please bear with me that for finding of the mean we have basically three means, one is the arithmetic mean, one is the geometric mean, one is the harmonic means. So, generally the geometric mean are used for decisions where the financial returns are there whether financial returns have

been capital R or small r, and I have also mentioned what is the difference between capital R and small r. I have done it at least twice on thrice, I will come to that if we need to basically go into the depth of that and solve other problems.

Now, when you find out the geometric mean you basically rank them from the highest to the lowest similarly as you do for the mean value or the expected value. And if you are going to take the inverse of this geometric mean or arithmetic mean or the ratio of the variance to the mean or the variance to the geometric mean we follow the same preceded by taking from the least to the highest.

So, with this discussion I will continue further on reading in the concept of geometric mean.

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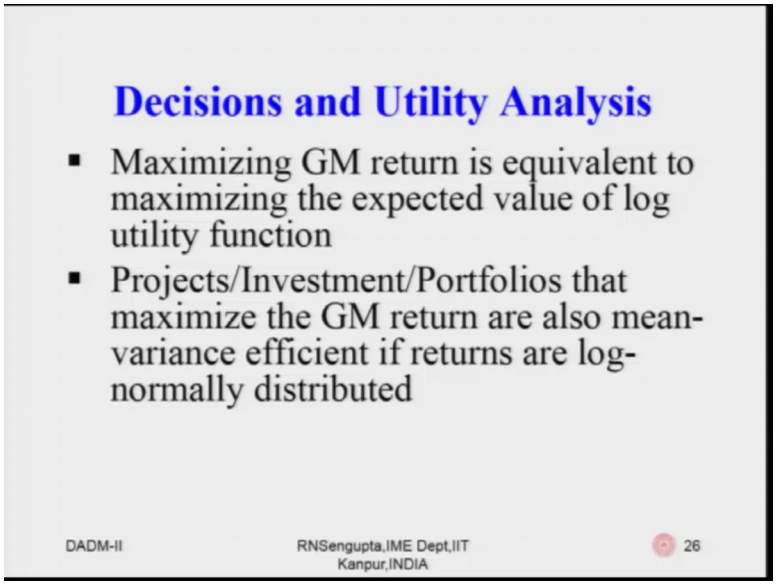

So, maximizing the geometric mean return is equivalent to maximizing these in the statements which I am going to make have theoretical proves or theoretical background but I am just mentioning these as bullet points, so which you can make a note. So, obviously, they would not be coming in the problem solving point of view for your set of information. So, let me continue reading it.

Maximizing the geometric mean return is equivalent to maximizing the expected value of log utility function. So, if you remember in the initial case what we considered was basically they was utility function which was quadratic, and I also mentioned that the

returns based on the fact that the utility functions are quadratic would be considered normal. And this is an if and only if relationship means both way implications. So, if you come to this, so you have the geometric mean and geometric mean return is equivalent to maximizing the expected value of the log utility functions. So, it is now log utility function not the quadratic utility function.

Projects, investment, portfolio decisions that maximize the geometric mean this is a second bullet point, that maximize the geometric mean (Refer Time: 04:27) also mean variance efficient you will log if returns or log normally distributed.

So, in the case if you considering the mean variance concept which is efficient and if the distributions are log normally distributed then the decision which you going to take using the geometric mean would basically also mean that you are basically considering the mean variance efficient frontier and based on that decisions are being taken, where you maximize the geometric mean or basically take the mean variance efficient concept. And also rank them accordingly, provided the returns are log normally distributed. No, no other distributions we will consider here for the ranking.

Now, we will come to the concept of safety first principle. In this last two lectures which one we have just started the 9th lecture which is the second last one in the second week and the 10th one which will be the last one in the second week, I will first go through the concepts.

Problems whatever they would be very simple you have to only be aware of normal distribution and if you remember normal distribution, normal distribution tables on this things I had discussed in general details in DADM - I. So, I am sure people who have done DADM - I or who have the requisite knowledge about simple case of normal distributions standard normal distribution all this things will be quiet fluent in trying to solve the problems that I can assure you.

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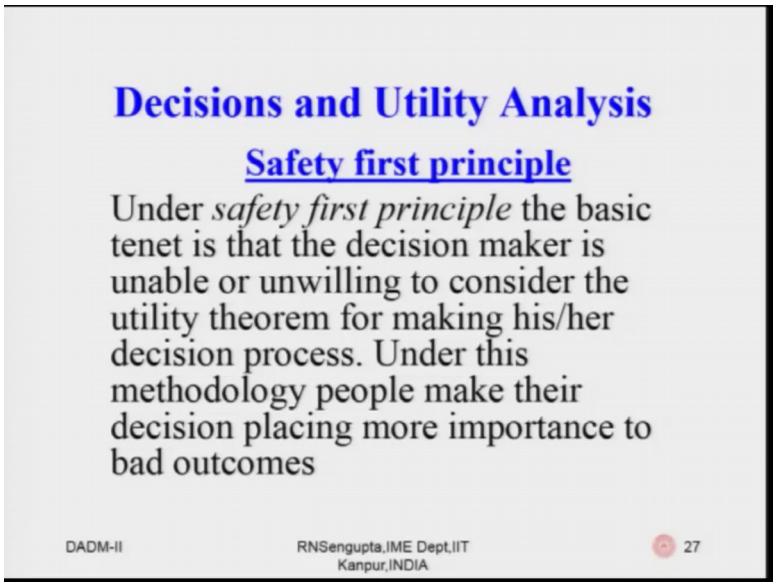

So, coming back to this safety first principle. So, under the safety first principle the basic tenant is that the decision maker is unable or unwilling to consider the utility theorem for making his or her decision process. Under this methodology people make the decision placing more importance to bad outcomes.

So, rather than looking at the good outcomes, rather than thinking of what is positive I am always think I means the person who is taking this concept of safety first principle as the guiding principle based on which he or she will make a decision. He or she is basically thinking of the negative outcome bad outcome.

So, her choice set; obviously, she has in front of her or he has in front of her the positive returns and the negative returns, but he or she would basically make the decision based on the negative returns. I am going to like come to that later on. So, technically means that say for example, I win 1000 rupees and also I lose 1000 rupees. So, I am may my main concern is not to place or make the decision based on the positive return but basically to find out the fact that what my decision should be based on the fact that I have lost 1000 rupees that means, I am only looking in the negative returns.

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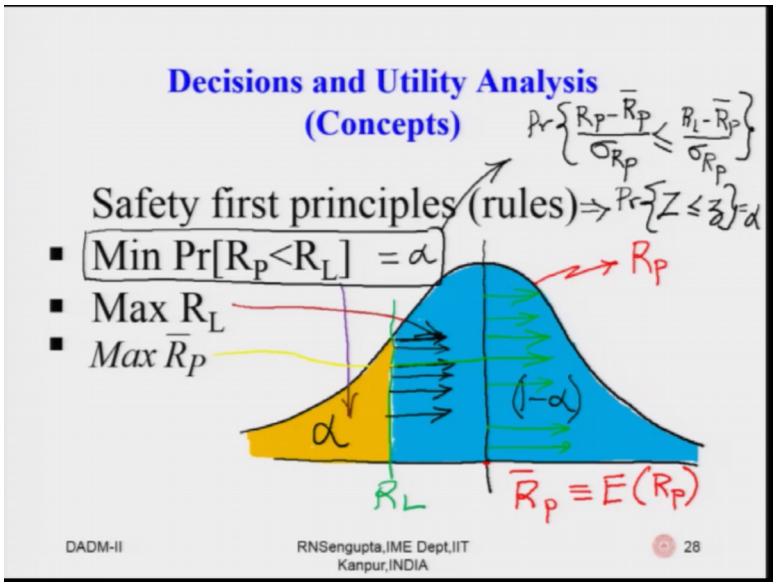

So, safety first principle the rules mathematically are given as you minimize the probability of R P is less than R L. What is R P and what is R L I am going to come to that. The second one is this basically you maximize the R L. These R P and R L, this pnl are all the suffix, and another case you basically maximize the average value which is R bar P. Now, let me draw it with a very simple concept of normal distribution, in this slide and that would be much easier for me to discuss. I will come to that later on. Just give me, let me discuss it and then I will come to that.

So, let with the discussions; that means, the theoretical knowledge let me give you and then I will come to drawing at and giving you a good feel. Now, in the safety first principle you are considering the minimization minimization probability of R P is less than R L, number 2 is through maximize the sum stimulate value of R L. So, this R L and R P are totally two different things, one is basically corresponding to the fact that what is the decision you are taking another is basically giving from the environment; that means, you have some risk concept based on that you want basically fix your returns or risk at a certain level and another one is basically you want to maximize. So, called average returns of the decisions we are going to take.

So, as I mentioned the safety first I will I think I will should draw it which I did mention few minutes back. So, it will be easy for us to understand, ok. Before I draw the and the distribution let me make few things very clear. Number 1, we will consider the normal

distribution; why I will come to that later. It can be changed to other distribution (Refer Time: 09:11) for do not worry about that. Number 2, we will consider that you have a set of decisions to make. And the set of decisions to make are in such a way that you have some limited amount of money or limited amount of resources or some constraints is there with respect to the amount of investment we can make.

So, if see for example, for any decision there are 10 different alternatives this alternatives the amount of investment which you can do in this alternatives to get a overall decision decisions would be based on the amount of money you are going to invest or amount of see for example, investment you are going to do or amount of resources you are going to do. So, that thing will denote like in the financial perspective will denote is that the portfolio. That is why the letter suffix P, point 1. Point number 2, is that this suffix L basically returns based on a certain set informations which you as a decision maker I get getting from the environment you will fix some return based on that.

So, this R L for the time being will consider this to be the say for the example the investment which you will can make when you basically invest in a risk free interest rate in the bank, which is the interest rate which is the bank is paying to you if you deposit your money and keep in a fixed deposit.

So, this R L is a set of returns which you want to beat or over that you want to basically earn the extra amount of money and R P is basically the overall returns which you are make going to make from the portfolio based on the different type of alternatives you have in front of you. And your main criteria is to find out the weights, so the investment which you going to make in the different alternatives such that you get the best portfolio.

So, I will basically draw the normal distribution. And remember this can be done for other distributions also, this is the mean value. Now, I will change the color and notify that. So, this distribution is basically for the return which you are getting from the portfolio or the different combinations of portfolios which you have, point 1. This average value which you have is basically R P bar which is actually the expected value of the return of the portfolio.

Now, remember as the weights change, as the returns change returns means returns for each and every individual investment because the P has whole lot of investments under it this is a basket of (Refer Time: 12:01) your investing. These returns as they are changing

the amount of money which you are investing in different alternatives as changing this R P would be a random variable if it is a random variable, it will have a distribution that distribution we are considering as normal as I have drawn.

Now, this is given as a distribution for R P. Now, what we want is; now this one important thing I will just highlight it. This value of greater than less would depend on the type of problem you are going to solve. So, here you being risk a worse person, in the sense that your safety first principle is more important for you, you want to minimize the probability of minimize the overall coverage of the area such that you are returns from the portfolio is as much as far as possible it is less than R L, obliviously it would be but you want to minimize the distance or minimize the area. So, that we draw it.

So, let me mark R L by a different color. So, let this be R L. R L can be any way on the left, on the right. And what I want is this. The overall area of the return R P should be such that the area which I am mark marking by this light orange is as minimum as possible, which means the value is this we want to minimize as less as possible, point 1. So, this is basically, so you will try to basically have the return on the portfolio corresponding to the investments such that the area is minimum, it would not be 0 obviously, it will be as minimum as possible.

So, let me consider this would be say for example, alpha. So, the overall area is alpha and this area let me mark it with another color. This is fine. Let me do it neatly so you can understand, ok. So, the blue colour is basically the area on to the right so you want to maximize that, but you are looking at the negative value that. So, hence you are trying to minimize the probability of R P being less than R L. So, this value what you will have will be 1 minus alpha. So, this would basically with the safety first principle for the first rule.

Now, consider the second one; and let us consider on this diagram only and I will try to make you understand that using different colors. So, consider maximization of R L, means you are trying to push R L on to the right. So, technically if it is happening which means the distribution by itself will also move on to the rights hence the probability of R P less than R L would be maintained in some way. Even though the rules at R are different, different in the sense they cannot they do not run at the same time they are different rules based on which you are going to take the decision of safety first principle.

That means, what is happening is I am trying to basically push R L as far as to the right as possible.

So, if I consider so this was the first one, I think I should use different colors in order to make I want a life. So, this was; so this would basically come, the case would be this second one being for moving to the right. So, this will be this. So, these values which have moving onto the right. And the last part is maximizing the average value; that means, you are pushing the average value on to the right so the distribution will also move. So, I will use a different color if we has; so it is difficult but I will try to highlight. So, this means you have, so they are not green they are basically yellow, but blue and yellow becomes green. So, you are pushing them to the right. So, this is the value.

So, these green actually mean where I am highlighting; that means, you pushing R P bar average value which is the third rule. Maximizing R L, that means, you are trying to push R L on to the right. So, these are actually brown but they become black because the background is blue. So, these are the values. And the first principle safety first principle, the first bullet point is minimizing the probability that means, you minimizing the alpha value and try to basically maximize 1 minus alpha.

Now, issue is do would they hold it for the distribution. My answer is yes. You can do that but the beauty of the normal distribution is like this, and I will only highlight the fact based on the first bullet point also. So, I will just write the rule such that you can understand why I mentioned that you can use the normal distribution and the standard normal distribution table. So, I will use the color black.

So, this is the first one only, remember that. So, I will just highlight and come to this. So, I can write this probability as R P minus R P bar by sigma of R P which is the standard deviation for the return for the portfolio been formed is less than, I will put the less than equal to sign also minus R L minus R bar P divided by sigma R P. So, this continues to be remain as alpha.

Now, if I do that it implies this implies probability of capital Z is less than equal to small z, this also is equal to alpha. Now, you know the distribution standard normal table is z is given from based on that you can find out. So, given R L or given R P bar you can find out alpha or any of these two values being given you can find out the third.

So, hence using the normal distribution becomes very easy when you are trying to utilize and understand the concept of safety first principle the first bullet point. And this can be done for the second and third also. But if the distribution changes obviously, you have to do some simulation studies which we will come DADM – III, where you consider the different type of reliability based robust optimization when the distribution are not normal person.

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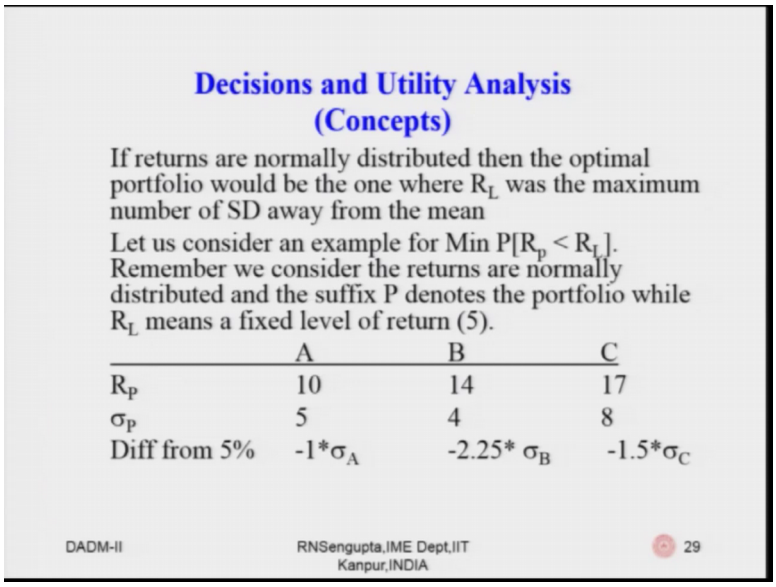

So, if returns are normally distributed then the optimal portfolio will be the one where R L which is the stipulated value which we have set for our self is basically would be the maximum number of standard deviation away from mean value. So, whether it is on the light of the left we can decide whether you are trying to take the positive returns on the negative returns. So, I will come to all those things later on.

So, let us consider an example of for minimization of probability R P is less than R L. Remember we consider the returns are normally distributed as I mentioned and the suffix P denotes the portfolio while R L means of fixed level of returns which we for case are considered as 5, 5 percentage just I am taking the value of 5.

R P value which are given based on the fact you have invested some amount of money in different above investment and the overall combined returns for the decisions A B C are respectively 10, 14, 17, as you can see is in the first row for this table. The standard

deviation for corresponding to those investment or the decisions which we mean by the portfolios for A B C are given by 5, 4, 8, respectively.

And if you find out the difference from 5 percent, this 5 percent if basically the returns which have considered R L they come out to be, if you are taking on the negative side on the left hand side they come to be minus sigma A, they come to be minus 2.25 sigma B and they come to be minus 1.5 sigma C. So, this A B C suffixes are corresponding to the decisions A B C. So, you can normalize them and find out for which the overall alpha value is reduced the maximum, and you can take the decision accordingly.

Now, again I am repeating this is very nice to solve, very easy to understand and you can use standard normal table, solve different type of problems if the distribution is normal. In case it is not obviously, as I said you have to use different type of simulation and mathematical techniques for that.

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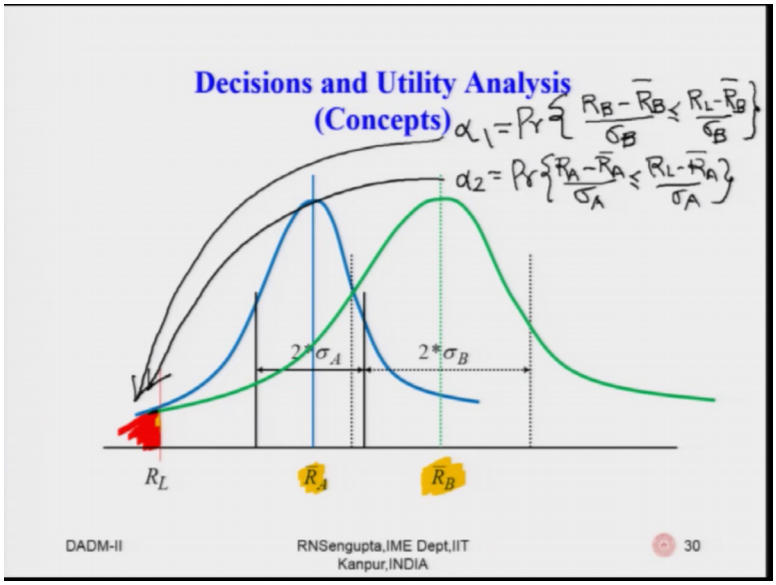

So, here I have again drawn two different distributions. So, the one which have drawn was basically the normal distribution. Again the same normal distributions are drawn but remember here there are two things number one the average values are changing. So, I will basically use the yellow highlighter, because the colors are blue green, so it will be easy for me to highlight using. So, the orange one, light orange.

So, this is R bar P average value is the green line and the distribution is given by this green normal distribution, while for the case for the investment A, I have not drawn C, only A and B are drawn. So, for investment A, investment in the sense the decision A based on the investment which you are doing. The average values R bar A and the corresponding values for standard deviation, so plus minus sigma; that means, you are going minus to on to the left if I am looking at the diagrams, on to the left and plus sigma on to the right the values are given as 2 sigma suffix B, 2 sigma suffix A.

And if you look at R L, R L is somewhere on the left. So, what I aim to find out is the area for the green distribution what is the area on to the left, similarly I will find out the distribution the corresponding to the blue one what is the area on to the left and based on that we will find out the values and rank them accordingly. So, if I basically use the formula. So, probably of I am just writing the final one for B and A separately. So, this would be R B minus R B bar divided by sigma B is less than equal to R L minus R bar B divided by sigma B.

So, this will give me say for example, I write it on the left hand side alpha 1, similarly find out alpha 2. So, alpha 1 and alpha 2 basically coming on from here diagram. So, this is probability of R A minus R A bar by sigma A less than equal to R L minus R A bar by sigma A, find out alpha 1 and alpha 2 and you can solve the problem accordingly. So, you just use the standard normal.

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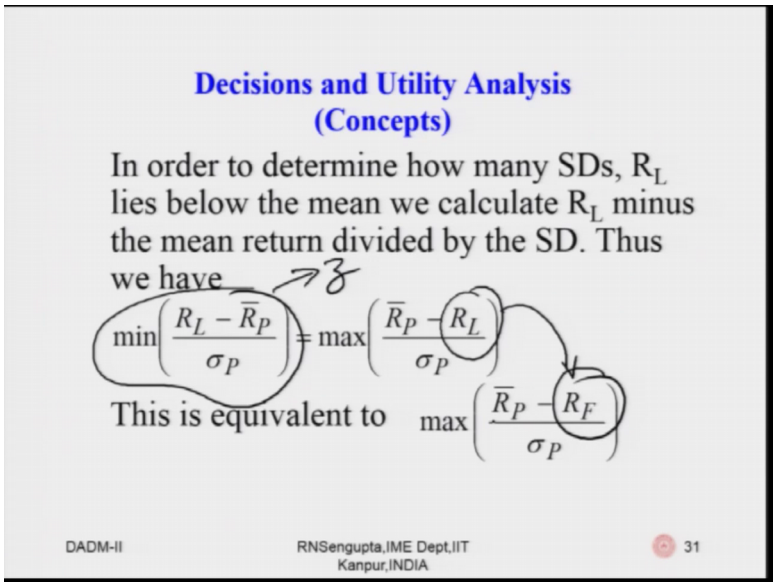

In order to determine how many standard deviations R L lies below the mean we calculate the R L minus the mean returns which you have done the left hand side if you remember. This which is basically what you find out with the small z.

So, minimizing R L minus R P by sigma P is same as trying to basically put the negative values, and then you get the in place of minimum you get the maximum. So, this will be maximum of R P bar minus R L divided by sigma P. So, the probability the optimization problem if you want to solve remain the same. So, this is also equivalent to the fact that I am basically replacing R L by R F. Now, if you remember this R F is basically the standard on this risk free interested based on which you will basically rank or peg your decision that how good or bad it is.

So, with this I will close this 9th lecture and continue more discussions about the safety first principle in the 10th lecture.

Have a nice day and thank you very much.