Data Analysis and Decision Making – II
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Lecture – 08
Utility Analysis

Welcome back my dear friends, a very good morning, good afternoon and good evening to all of you. And as you know this is the DADM II which is Data Analysis and Decision Making II course under the NPTEL MOOC series. And we are in the lecture number 8 which is in the 2nd week. And as you know this course is for 12 weeks which is 30 hours. And the total number of lectures is 60 because each week we have a 5 lectures.

Each lecture being for half an hour and an after each week we have an assignment. So, we are in the second week. So the moment you finish this that is the 10th lecture is over will have the 2nd assignment. And I am my name is Raghu Nandan Sengupta from IME department IIT Kanpur. So, if you remember we discussing about Utility Theory and the concept that how you can find out the concepts of utility theory. Then considering the different type of utility theories like the logarithmic one, explanation one, par one, and the quadratic one.

How we can find out the values of the a and a prime where a is basically the absolute risk aversion property and then also find out r and r prime? Prime means the first derivative with respect to w we are finding out where w is the wealth, where r is the relative risk aversion property. Then we saw that how these is a, a prime, r, r prime can give some information about the utility function depending on with your risk averse, whether you are in different to risk whether you love risk.

And then we also saw the concept that how we can basically utilize very simple concept of drawing in a graph paper or in a Cartesian coordinate my apologies. How you can use the concept of graph paper draw the concept of the a and b which of the gamble the values on the gamble considering the fair gamble. And also the certainty value you can find out well what type of property the person has with respect to his or her utility function whether is convoke and concave or convex.

And then we slowly went in the into the concept that if utility function is quadratic I did mention that the fittingly that if the utility function is quadratic then there is some relationship with respect to the returns being normal and vice versus.

We will come back to that those two slides again as I did mention after the at the flag end of lecture 7. So, I will again repeat the same problem, please bear with me.

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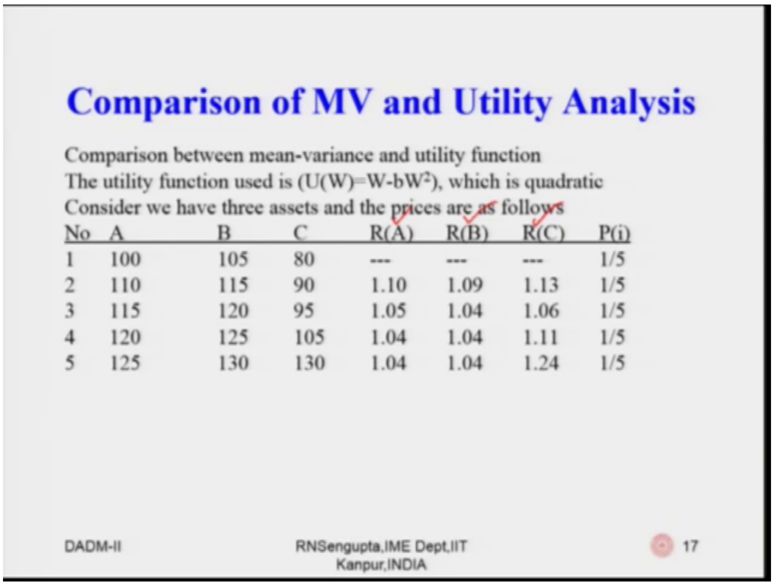

So, we are trying to compare between the mean, variance and the utility function. The utility function the utility function is quadratic which is actually which is of the form a W plus b W square plus c. And here it is given as W minus b into W square which is a quadratic function. Consider we have three asset and the prices are given assets means financial assets.

So, it can be translated into any decisions like you have three projects free decisions whatever it is, but I am keeping it very simple. The first I will again repeat few things please again I will request you to please bear with me. The first column and the numbers the second third and fourth column of the respective values of the returns and not the return the prices or the wealth whatever it you say for the three different assets or three different projects or three different decisions which are respectively ABC.

And in the then the fifth sixth and seventh column which is given as RA, RB and RC are the respective returns corresponding to the decisions or the assets or the projects ABC respectively. And on the last column here the probabilities I am taking the properties as equal in order to make RR understanding much simple. So, based on RA, RB and RC we will find out first the average return will find out the standard deviation and then proceed considering the mean variance theorem and the utility theory.

So, basically we are trying to compare mean variance concept for the decision on the on one side and the utility function. Based on the factories it is a quadratic utility function remember that on the other side. Then try to basically find out the similarities and basically draw very interesting conclusions about that. I am not going to go into the proof that. I will just take it as a matter of fact, but try to utilize that for our decisions later on.

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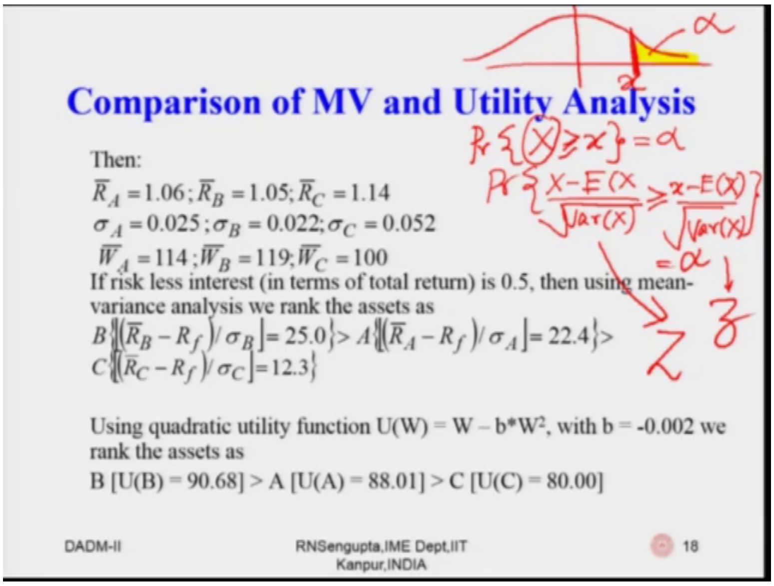

So, we find out R bar A which is 1.06, R bar B which is 1.05 and R bar C which is 1.14. And the standard deviation we are just using the same symbol as the as sigma. Technically it should have been the if you remember the concept of DADM I actually many of you would be basically interested and trying to question and rightly so that why don't we use the standard error for the sample. But I am just using symbol for the population in order to make our life simple and understanding very realistic.

So, obviously, you will be then thinking and I am coming back to the slide then you may be thinking that if you are using standard error all these things then we should be using the other three distribution corresponding the normal distribution as the t distribution chi square and the f distribution; we won’t going to that. We just simply use the standard normal distribution that is only for make you understand. So, again coming back to the slide so we have sigma suffix A sigma suffix B and sigma suffix C.

The values are 0.025 0.022 0.0252. And will consider the average width as a 114 119 100. So, the bar means the average say if risk less if risk less interest which is the bank interest which is being which is given as 0.5 then we are interested to find out that what according to the mean variance analysis how do we are how do we rank the assets. How do we are able to rank the asset means the decision. I am using the word financial assets for ABC and it can be a project as I said it can be decision whatever it is.

Now our analysis is I will write it I am sure all of you are aware of that. So, basically what we need is we need the probability of x being greater than equal to x small x now let me explain the discussion. So, I want to find out I will draw the distribution here. Now this greater than or less than would depend on what type of which side of the distribution looking for corresponding this fact whether you are looking at the asset distributions or the negative side order distribution considering it is a last distribution.

So, the concept whether you want to earn more would definitely depend on how much positive side is. And if you want to basically maintain a loss which is as low as possible. And if it is a loss distribution you will try to basically be on the left hand side of and if I am looking at from my point of view. That means, the middle value will basically give you know loss no profit, under the right hand side you have the profit more you go on to the right you have higher profit more on to go into the left is basically the negative profit which is the loss.

So, this greater than sign and less than sign would basically depend on what which side of the distribution looking at so let me consider the positive side only. So, now, if we have basically xo so I am having I am drawing x small x here. So, what I am interested in I will highlighted with yellow on to the right hand side. If I come back to the calculations very simply it is probability X minus. Now considering the normal distribution will have the expected value of x and the standard deviation of x and consider this is alpha.

So, variance of X square root which is standard deviation is greater than x minus E of x this small x is a real values remember that. So, this will basically converted into standard normal this becomes capital Z and this becomes small z so we can find out for the tables.

So, higher the value lower the value basically pass your comments or judgment accordingly. Judgment is based on the fact that how you going to like them. Now with this let me come back to the slide.

So, what we are considering is there is a risk free interest rate. And this risk free interest rate by the way this risk free interest rate actually would be coming into this value X minus fixed value. So, which is basically RB or RA or RC minus Rf, now RA. So, these are the r values average values. So, RA bar RB bar RC bar are non deterministic if you if you take values repeatedly; obviously, the sample averages would keep changing depending on the samples being different. So, if you find out the those values and rank them.

So, you have basically as if you find out this R bar B minus Rf divided by sigma B. Then another value would be probability would be R bar A minus Rf divide by standard deviation which is sigma a and another is R bar C minus Rf divide by sigma C. So, these values corresponding to the those three statement which I made from out to be 25 22. I am here not going to the decimal values 25 22 and 12. So, if you plant them along the x axis and then try to find out the one to one correspondence with respect to the standard normal. You can rank them immediately where B returns are greater than A returns are greater than C returns.

Now this was based on the fact that you are considering the concept of mean variance concept. Mean means this bar values were the means and the variance for basically corresponding the fact that you are taking the square root of the various which was the standard deviation which is sigma ABC depending on the suffix. Now you are rank them. Now, you want to find out the ranking which you have done based on the fact that you taken the mean variance concept whether it matches with the quadratic utility function; because the mean variance was done on the normal distribution concept.

Now, using the quadratic utility function giving a value of B as minus 0.002, when we rank them and find out the utility values of ABC. So, it comes out utility of B comes out to be I won’t repeat the second place of decimal it turns out to be 90 for A comes out to be 88 and for C comes out to be 80. Now remember here even though I have written the probabilities are 1 for all the cases we are not considering that this we can change the probabilities that would not have any effect on the calculations. So, if you do that and then the ranking again becomes higher the utility better for you so B is greater than A, A is greater than C.

So, the ranking which you have got considering the mean variance concept and the utility concept comes out to be the same. That means, B is the best A is the second best and C is basically third best. So, that the ranking system which you did corresponding to the fact that you are taking the quadratic utility function and the normality distribution which leaves out to the fact for mean variance concept are the same in the sense they give us the ranking concept and the decisions are same. This will become very important later on when we consider that different types of decision process for different type of examples.

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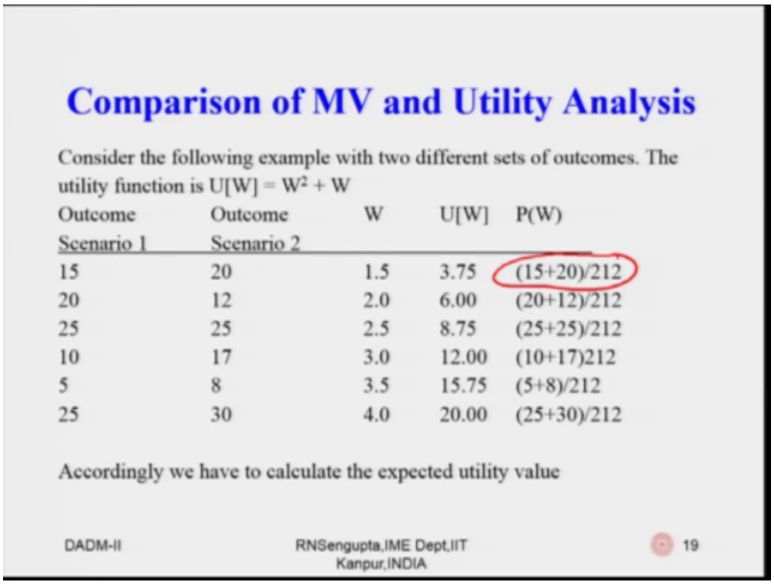

Now, let us consider further on. So, consider the following examples with two different sets of comes the utility function is again quadratic which W square plus W. So, it could have been AW square plus BW plus C anyway it does not matter. So, the outcomes are given and outcomes for scenario 1 is given as if you look at the first column onto the table when I am I am not going to market, but I am just highlighting.

This is the first column for scenario 1 this is the second column corresponding to scenario 2 and other values I will come to that later on. So, the outcomes for scenario ones are basically 15, 20, 25, 10, 5 and 25 which is the first column. The outcomes corresponding scenario 2 has 20, 12, 25, 17, 8, and 30. And the corresponding values of

W which is the wealth UW which is the utility function based on the fact you are going to consider the quadratic utility function and probability are given.

So, now, how do you find out the probabilities? We are trying to basically find out the outcomes both outcomes corresponding to utility one divided by the total number of outcomes. So, the total number of outcomes if you add up for the scenario 1 and scenario 2 comes out to be 212; 212. Now the probability corresponding to the fact if you only concentrate on the first row the probability corresponding to the fact that he have the U utilities 3.75 corresponding to the fact that the wealth value is 1.5 comes out to be this one; 15 plus 20 which is 35 divided by 212.

And if you follow the last column the probabilities are given. So, for the second row that is for the utility of the 6 it will be 32 by 212 for the third row which is the utilities 8.75 the value of probability comes out to be 50 by 212 and so on and so forth. So, accordingly we have to calculate the expected the utility value and basically rank them accordingly. So, they are there are there can be two different utilities they can be same you to do with the corresponding fact that you are trying to use the mean variance concept whatever it is.

In all the examples we will see that I am again repeating it if the utility function is quadratic and this distribution corresponding to the utility utilities that I am I am basically investing some money and getting utility I am getting a return also. So, utility as I mentioned it is quadratic and the returns are normal. Then to one to one corresponding based on the fact that how the ranking system would be done would be same for both these decisions.

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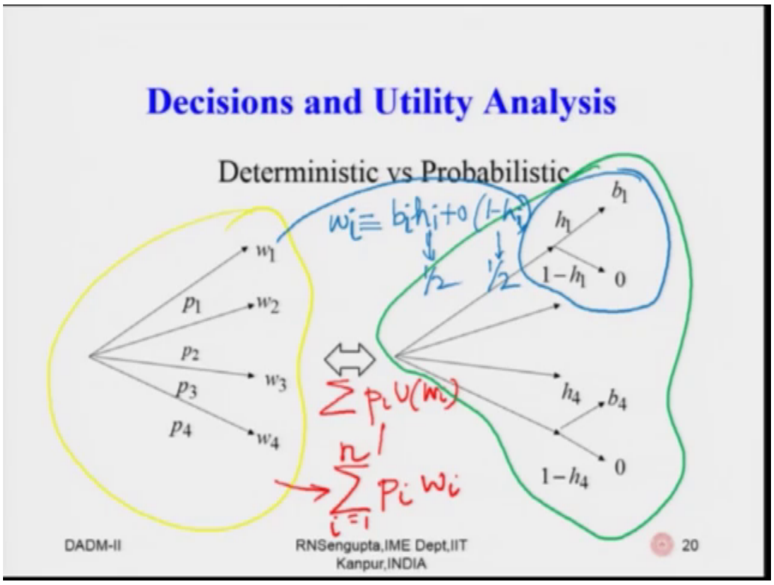

Now, whenever you will consider a utility always remember our main aim is you have a gamble need not be fair point 1. Point number 2 is that they need not only two outcome they can be multiple outcomes. So, let us consider the example which is given in the slide. So, concentrate on the figure which is on the right left hand side where I am just hovering the pen now so this one so let me highlight it. So, these value now if you look here what do you have we have we have probabilities P1 and the outcomes are given as W1, P2 W2, P3 W3, P4 W4 there can be more outcomes also.

Now remember two or three very simple things which are correspond which are proof corresponding to the fact that we are considering these examples. Point 1 the some of the probability should be 1, point number 2 the values which are give it as W1, W2, W3, W4 remember they are the outcomes. So, if they are based on the fact that those are the wealth; obviously, you to have convert those wealth or those decisions into the utility based on the fact that what is the utility function.

So, if you have the utility function as quadratic. So, for W1 you will basically find out the quadratic utility function based on the fact that the wealth is W1 or it or the general returns is the W1 on the decision is W1. Similarly for W2 it will be quadratic functions based on W2 similarly for W3 W4 W5 so and forth. Now, if the values are given as it is that is W1 is the utility function and W2 is the utility function you do the calculations accordingly.

So, for the left hand side; if I ask you or if I ask any one of the decision makers to find out that what is the average value. So, what you will do? Considering the fact that the W1 W2 W3 W4 are the corresponding utility functions you will basically multiply P1 into W1 plus P2 into W2 plus P3 into W3. So, your actual value for this decision I am using a different color would be summation of Pi Wi; i is equal to 1 to n.

And if this is not valid corresponding to the fact that W1 W2 W3 are not the utility function you will basically replace that with summation I am not write writing the limits. Probability Pi into U of Wi whatever the U for functional values. Now if you consider the diagram on the right. So, what you are doing is this and use the blue one yes.

So, what you are doing is that for each of the values of W1 we are replacing them with a fair gamble in the sense W1 can be replaced by two outcomes probabilistic. Probabilities are h1 and minus h1 and the values being b1 and 0. Now h1 and 1 minus h1 if it is the fair gamble the values would be half and the corresponding values of b1 and 0 would be based on the fact that whatever the utility function your utilizing for W1.

If at all it is a utility function then you will use the same utility function on the right hand side for the diagram which is given which have basically circled using the green color. Now, the fact remains that if W1 is by itself a utility then you will basically find not what is the utility function. And use that utility function to convert b1 into its corresponding utility and 0 into its corresponding utility. So, what you are trying to do is that you are trying to find out the one to one correspondence between the value of W1 and b1 into h1 plus 0 into 1 minus h1 such that the expected value on the left hand side and the right hand side are the same; which means W1 in general should be equal to b1 into h1.

I am using the same blue color in order to make you understand plus 0 into 1 minus h1. So, this if I replace for all the values it will wi bi hi and hi. So, if the probabilities are same so hi becomes half 1 minus hi becomes half and you can do the corresponding calculation. So, this can be made as you go more on to the right b1 can be further broken down into say for example, c1 and 0. And the corresponding probabilities can be made accordingly. And if you go on to the left here then; obviously, the overall decision is basically the expected value is you will basically have uncertainty value can use the certain value to match the gambles.

I am not talking about the fair gamble they can be gamble with probabilities being different on to the left hand side and right hand side. So, you have find able to find an unique value of C which would basically be able to replace the overall different type of non deterministic decisions. But remember the utility function which you are going to utilize throughout for all the decisions should be the same.

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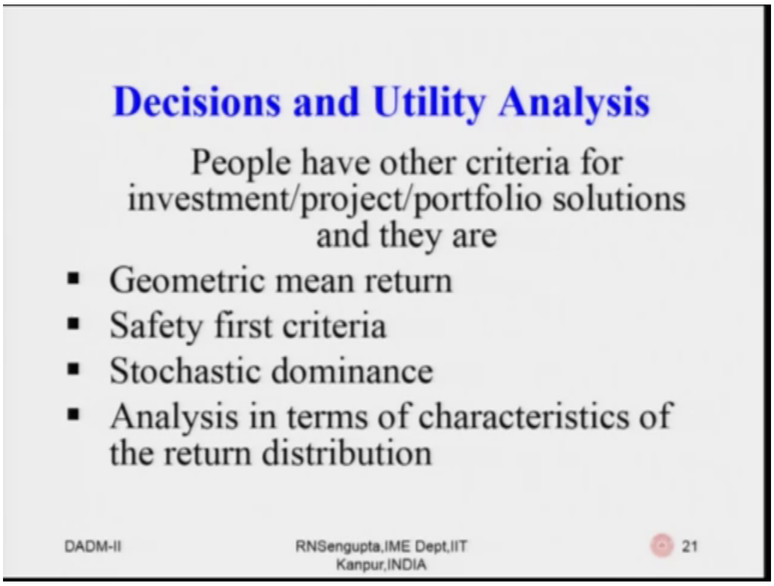

So, decisions and utility analysis; so people have other criteria for investment, projects and portfolio selection for decisions. So, they can be geometric mean returns safety first criteria stochastic dominance and there are the analysis in terms of characteristics of return distributions. So, we will basically go one by one from corresponding considering that.

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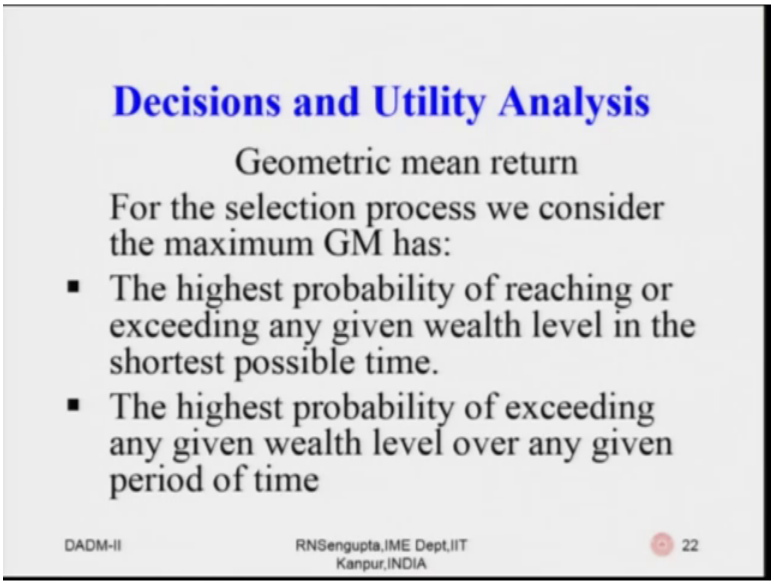

The first one being been basically be the geometric mean returns. For the selection process we consider the maximum geometric mean returns which is basically it means that this has the highest probability of reaching or exceeding any given wealth levels in the shortest possible time. So, you want to basically define on the highest probability corresponding the fact for a certain time period you want to basically exceed that.

And another concept would be the highest probability of exceeding any given wealth level over and any given period of time. So, the fact remains that in one case the time is fixed and you want to find the highest probability. And another case that time may not be fixed still you want to find out the. So, called expected value based on the fact that the values are the same, but the probability; obviously, should be as highest possible.

So, I will come to that concept when we solve very simple problems. Now remember these are just to initiate the interest for utility functions. And that if the two utility functions are same that how are we able to take a decisions. Whether decision one or decision two which one is better will just utilize them from the from the multi attribute decision making concept or non parametric decision making concept.

You would not going to the mathematical details, but try to utilize the concept wise the logic when you are trying to basically make different type of decisions. So, considered the returns; returns being given by the suffix i and j ith is possible return on the jth portfolio.

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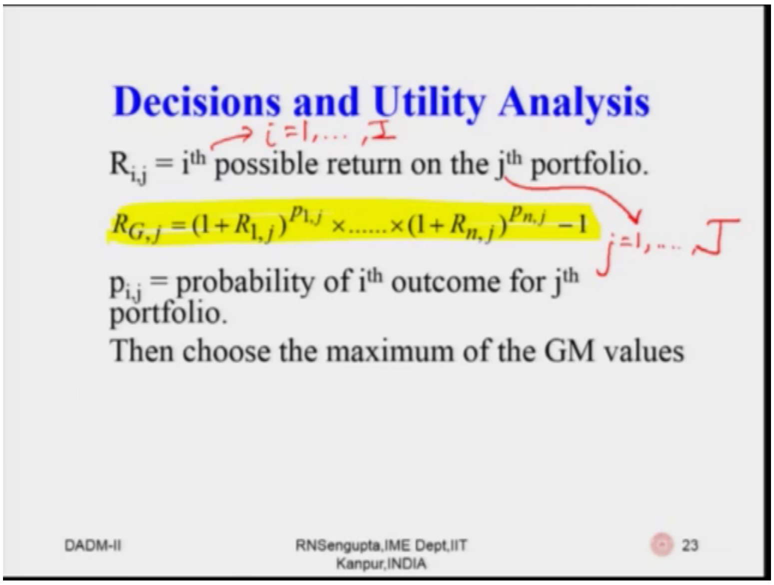

And the; obviously, they would be for each portfolios each decisions j can be basically from 1, 2, 3, 4, 5, 6 till capital J. So, I will consider that let me use the red color. So, j can basically be capital J. And similarly i can be basically possible to capital I. And the probabilities are given by P i j. Now, if you remember for any decision the probabilities from of i 1, 2, 3, 4 till capital I.

If you sum them up for that corresponding value it will be the probability corresponding to the decisions small j. Similarly j can be 1, 2, 3, 4, 5, 6 we will find out the probabilities. Now P i j is and; obviously, will also check the corresponding fact that the sum of the probabilities should be equal to 1 I will come to that later. So, the probability of the ith outcome for the jth portfolio is basically P i j and we should basically choose the decision for which the geometric mean value basically gives us the maximum so called return means capital R it can be capital r also.

Now let us come back to the formula if you remember I will just I won't digress I will just come back to one example. For investment if you remember the reason why I am saying that in the in the investment purpose will always use the geometric mean. Now let us step a little bit thicker not a D2 steps side wise. If you remember when we are considering the concepts of in DADM-I. I did mention that the general characteristics or the mean values can basically be considered using the concept of average value which is mean.

Mean can be arithmetic mean, mean can be geometric mean, mean can be harmonic mean. Now in the geometric mean example I did mention that when you are trying to consider different type of returns for decisions we general take the geometric mean. So, that is the basically the simple history based on which why we are taking this geometric mean. And for any decisions financial facts being very important they would all always consider that geometric mean concept to be true in order to make the decision accordingly.

So, P i j are the probabilities of the ith outcome for the jth portfolio. Then we choose the maximum of the geometric mean value. So, what will find out? If that for any portfolios any decisions we find out the probabilities P i j we have already form found out we will find out the corresponding. So, called average value which is RGj, j is basically for each and every port decisions. So, what will do you will find out 1 plus R1 to the power P i j and multiply them corresponding to the fact and then subtract the value of 1. Because in the geometry case you are trying to utilize this geometric mean.

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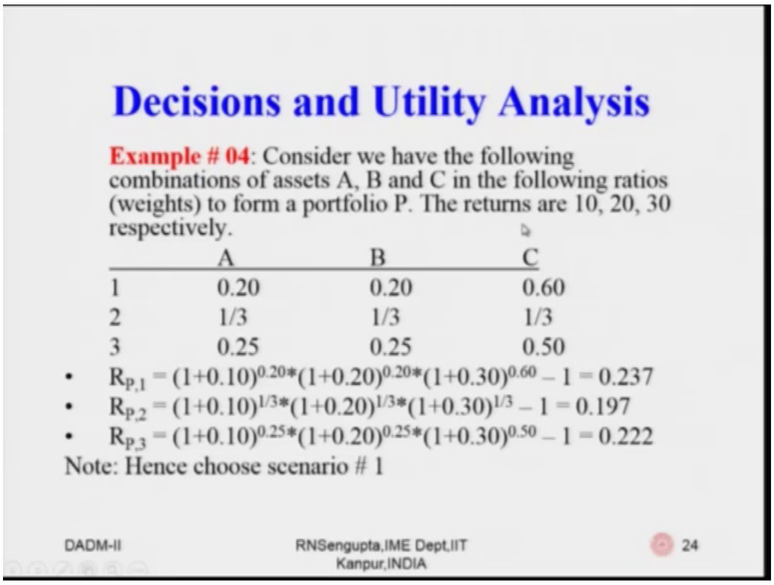

So, if you consider that let us corresponding focus our attention to this problem. Consider we are the following combinations of assets; assets again I am using in every general sense it can be any decisions any project whatever are ABC. In the ratios when the returns are 10 20 30 respectively. And we consider the A has this combination the assets which is ABC for portfolio 1 or decision 1 is 20 percent 20 percent 40 60 percent which adds up to 1.

For portfolio 2 his decision 2 or project 2 their equivalent to the fact that each is one third, one third, one third, which is 33 and one third and 33 and 0.33 value. And when you come to portfolios 3 which is this decision 3 or project 3 the in investments for ABC are 25 percent 25 percent and 50 percent. If you utilize that you can find out the RP1 the portfolio value for decision 1 comes out to be 23.7 for 2 comes out to be 19.7 for 3 comes out to be 22.2.

So; obviously, we will choose the decision one. And these calculations can be done for different type of probabilities corresponding the fact the geometric mean is the mean deciding criteria based on which you are going to take a decision. And you will see that these different methods gives a different ranking system it does not mean that one ranking system for a decision is worse than the other on one is better is basically how you analyze the problem from your point of view. And what is consider your utility function point 1.

And also these distributions would come out to be important later on we will see to that come to that later on in the fact can of this course. And obviously, some of the examples will be considered in the DADM-III course also which will lead on ask to the fact that how we are going to utilize the optimization concept in decision making. And that those would be parametric optimization I will come to that later on only.

But let us basically concentrate on DADM-II. So, this I will end this 8th lecture and continue discussing further on depending on the utility functions. And then most probably by the 10th lecture or the 11th lecture which we 11th lecture would be the third week will start the actual concepts of different type of non parametric decision making. With this I will end this lecture and have a nice day and.

Thank you very much.