Data Analysis and Decision Making - II
Prof. Raghu Nandan Sengupta
Department of Industrial & Management Engineering
Indian Institute of Technology, Kanpur

Lecture – 07
Utility Analysis

A warm welcome to all my dear friends and my students a very good morning, good afternoon, good evening to all of you. And this is the DADM-II which is Data Analysis and Decision Making II lecture series under NPTEL MOOC. And we are in the second week and as you can see this is lecture number 7 which is the second class in the second week. And, as you know this total course DADM-II is for 12 weeks and the total number of hours is 30 hours and total number of lectures is 60 lectures because, each week we have 5 lectures each for half an hour and after the each week classes we have assignments. So, we have already finished one set of assignments after assign week 1 and we will soon go for the week 2 assignments.

And, as you know my name is Raghu Nandan Sengupta from the IME Department IIT Kanpur. So, if you remember the last slide which you are discussing in lecture number 6 was basically to find out the actual characteristics of the utility function. So, we have no considering we have no information about the utility function. So, based on that I proceeded I did discuss and I also did mention in the last few minutes of the last class, that I will again come back to the same slide and try to discuss the same points once again. Maybe for some of you it may be repetition, but I think it would be good that if we discuss so, you understand at least in a very simplistic conceptual way; how this whole idea can be utilized in order to basically proceed forward and find out about the utility function.

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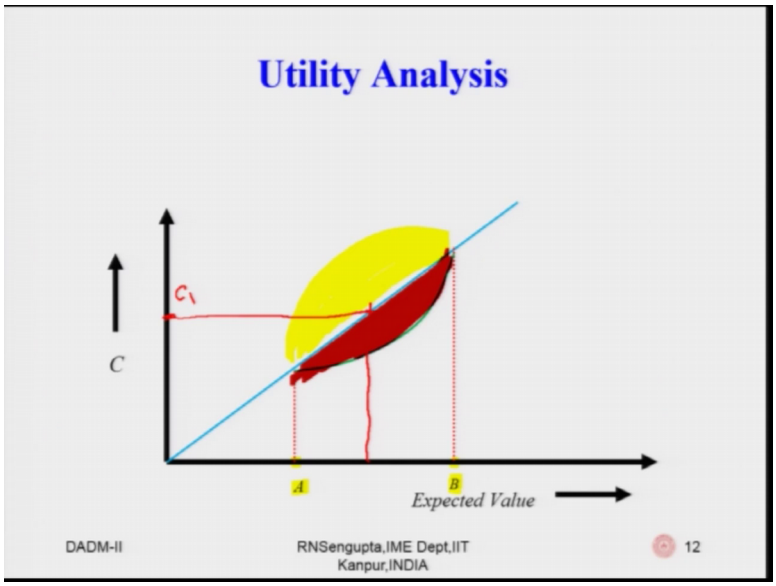

So, this is the slide which I have and along the y axis, we have values of C. Now the C values are the certainty value. So, they will change depending on the problem which we have. So, let us be aware of that point 1, point number 2 is that along the x axis, we have basically the values of the gamble which based on which we are trying to find out the C values. So, the values are A and B, A and B can be changed and we will considering a fair gamble; in the sense the probabilities of the coin coming a head out of the coin coming a tail are equal then their half. So, let us basically do the thought out experiment and whatever it is this graph signifies. Also note the blue line which is 45 degrees line is drawn in order to be make us understand that what type of utility function it is which we are going to basically draw depending on the response from the person who is taking a decision; we will come to that very soon.

Consider A and B are assumed as they are given in this slide. So, that mean just highlighted; so, this is A this is B the values are given. Now, consider that it is a fair gamble probabilities half. So, you will basically have A into half plus B into half it will be A plus B by 2. So, that will be a straight line in between midpoint between A and B. So, let me use a color let me use the red color. So, and it is bold because it is now we know the expected value. And, once we reach the 45 degree line what we do is that we go horizontally on to the left and mark this value as see for example, C 1. So now, the experiment goes like this. On table 1 on the left hand side, we have A and B and we have a coin which probability half and half and on table 2 which is on the right hand side we

have the so, called C 1 value. We do not know whether C 1 is a certainty value till now while I am using the symbol C, but that is just for the nomenclature sake. But, we do not know actually it is an certainty equivalent for that person who is going to make the decision.

So, we call that person ask him that there is a fair gamble with values A and B probabilities and half and half and on the right hand side is the value of C 1. So, what is his or her decision? If the person is indifferent between fair gamble and the C 1 value then be certain that the C 1 value which is kept in front of him is the certainty value because, he is taking a decision based on the expected value of the gamble and the certainty value. So, the obviously, that is on the straight line. So, we will assume that the person has a utility function which is linear which is U W is equal to W. Now, that is just an hypothetical case.

Consider the other two cases and which are more likely; the person says that no he is going to take the gamble. In that case it means the certainty value based on which he will take a decision whether to take that gamble which is given in front of him with values A and B. And, the value of certainty on the right hand side that C 1 is not no more a certainty value for him remember that, because the expected value which he has and the certainty value which he assumes to be would be in such a way that the certainty value is now C 2 which can be say for example, greater or less than C 1. I will come to that why can it be greater than less than C 1. So, if it is greater than an less 1 less than or less than C 1 then the values of C 2 which we are assuming would be either above this 45 degrees line.

So, I basically populate that with some ok; if it is greater so, no green would not do oh sorry I will choose so, the yellow color. So, if C 2 is greater they will be in this region and C 2 is less which is below C 1. So, the values would be here which technically means that the utility function which you are talking about would; obviously, have a first derivative as greater than 0. Because, as the non-satiation characteristics, but greater than C 1 and less than C 1 basically in the long run; as you basically play this game repeatedly would mean maybe basically the U double dash which is there for that person is either concave or convex. In the sense that U double dash can be greater than 0 or less than 0 depending on whether the person wants the risk or does not wants the risk that would basically come up.

Now, what you do is that keep changing A and B. So, change A and change B also, but do not change the probability if it remains a fair gamble so; obviously, C 1 value changes. So, which is the expected value of A and B taken and again you ask the person that whether you want to take that new value of A and then or the new value of B which is now A 1 B 1 or then the certainty or the so, called fixed value which is kept on the table on the right hand side. Consider that is see for example, C 1 dash. And, let us again ask him or her what is his or her decision. Consider the person is going to take a decision such that the certainty value for him is greater than C 1 dash or less than C 1 dash.

So, again you will have a set up value which is above the blue line 45 degree lines or below the 45 degree lines. Again change A and B; that means, now they are not A 1 and B 1 they are A 2 and B 2, again find out the expected value which is now C 1 double dash. So, these dashes do not mean derivative remember I am just giving a symbol. Again ask that person he says again that he is either greater than greater than C 1 double dash was less than C 1 double dash depending on the certain value which is true for him or her; mark that point. So, as you continue doing as you continuing taking different values of A and B you will have different values of C 1 dash or C 1 double dash and C 2 double dash so on and so forth. The certainty value of that person would also keep changing. So, if you plot them you will have either a set of points which are over the 45 degree line which is the blue one or below the 45 degree line.

And if we join them you can find out whether it is a concave function or a convex function which means it will give you these. So, this would give you the rate of change of the utility function and you can also find out the double derivative based on which you can immediately find out the important points which are; if you remember I have mentioned that there is a characteristics of A and A prime. There is a characteristics of R and R prime which are basically absolutely utility absolute risk aversion function and relative risk aversion function. Based on that you can find out what is the characteristics of the utility function such that you can say whether the utility function is quadratic whether it is basically say for example, exponential whether it is a logarithmic with this power function so on and so forth.

So, without knowing the actual characteristics of the utility function you can draw, do a very simple experiment find out the characteristics. And cans you may comment something about the utility function based on which you are going to work or what is the

utility fashion of the person to whom you are basically posing these questions. So, this is what the analysis is what I discussed. I will again mention it.

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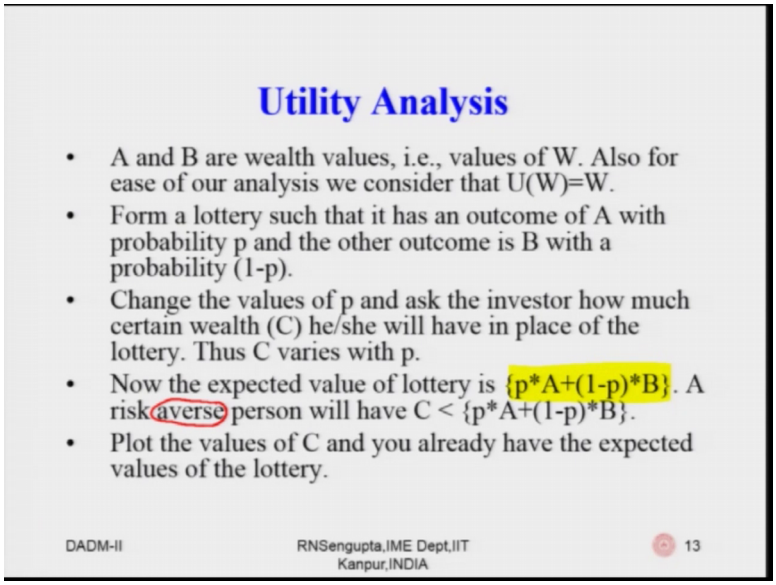

A and B are well values some amount of wealth values of the of wealth which is basically W, also for the ease of our analysis we consider that the utility function is W which is linear which we have already discussed. Form a lottery that it has an outcome of A with a probability p and an outcome B with a probability 1 minus p. In this case to make our life simple I have said that the probability is p and 1 minus p are half that is a fair gamble. So, that is there is the ease, you can change this half and half which are the probabilities to p and 1 minus p or to p and q and this p and q need not be half they can be in different values. But, we are not going to discuss that because they just will only make things a little more intense, but the overall analysis remains the same.

The third point mentions change the values of p and ask the investor how much certainty certain wealth of C he or she will have in place of the lottery. So, basically lottery is placed and on the other hand that value C 1 is placed whether that some amount delta amount has to be removed from C 1 or delta amount has to be added to C 1. So, that the person is indifferent between the lottery and the amount of value which is kept on the right-hand side of the table. So obviously, in that case the person would be would have a certainty value because, he is balancing the utility which is he or she is getting from the

fair gamble. And, what he or she is basically getting from the right hand side of the table which is basically the fixed value.

Now, the expected value of the lottery as now p and 1 minus p are different. So, let us continue with that the expected value of the lottery are p into A plus 1 minus p into B which is the utility so, I will mark it. So, this is the utility; we are considering remember we why the question may come up why we are considering A as the utility of A because, we are considering a uniform utility function U W is equal to W. If it changes; obviously, it will change a containment. A risk averse person will have the value of C is less than he is he does not want to take that he is risk averse, when let me check risk averse person will have yes, in that case the person would have a value such that just let me check yes, I think there is some it would be a let me check 1 minute. It would be a risk loving sorry my mistake a risk loving person would have where the expected value of the lottery would be more than C because, he is willing to take the risk.

So, in that case p into A plus 1 minus p into B the overall value would be get on the C because, that person is willing to take the risky decisions. Now, you plot the values of C and you have already have expected the values of the lotteries which is given. What you do is that keep plotting the expected value, keep plotting the actual value of C based on which the person will be indifferent. Once you plot them they would be either along the straight line which is the 45 degree line, a set of curve which is over the blue line and an assert or a certain of values which are below the blue line which is the 45 degree line. And, you can find out what is the characteristics of that human being who is taking decision whether he or she wants to take a risk, whether he or she is indifferent to risk and whether he or she is basically avoids wants to avoids the value of risk.

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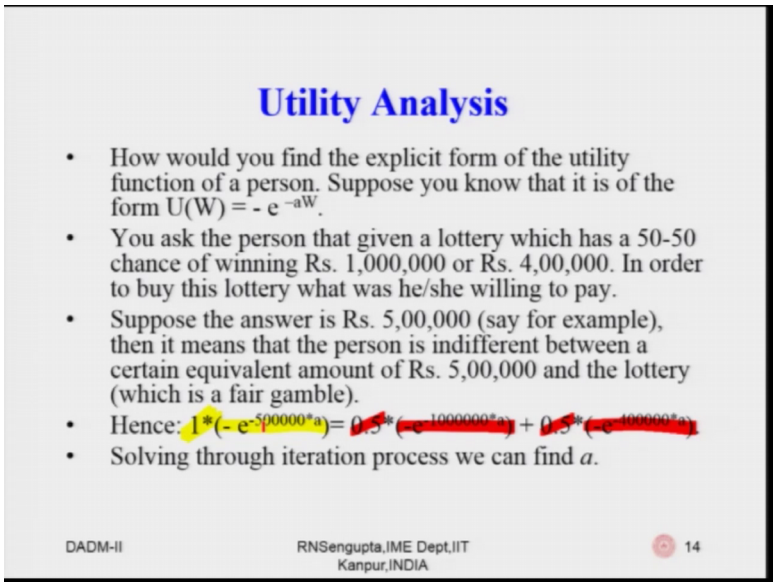

How would you find the explicit form of the utility function of a person? Suppose you know that it is of the form U W is equal to the exponential part which is minus e to the minus aW. Now, here the point is utility function is given which is exponential, but we do not know the value of a we have to find it out; you ask the person that given a lottery which has a 50-50 chance of winning which are the values are 10 lakhs and 40 lakhs; 4 lakhs sorry. So, 10 lakhs and 4 lakhs in order to buy this lottery what was he or she willing to pay. Now, you put that answer to that person and you know that person has an exponential utility function and see for example, the person says that the lottery is there 10 lakhs and 4 lakhs..

And some amount is there on you want to keep a certain amount on the right hand side and you know the person has an exponential utility function, but you do not know the value of a. Now, the person says that I am indifferent between the lottery and a certainty value and the certain value C for example, 5 lakhs which would immediately give you the information that the expected value of the lottery and the expected value of the utility should match for that person. Because, he or she is willing to either take the lottery or take the certainty value which is say for example, in this case is 5 lakhs. So, in that case what will happen the expected value of the certainty value multiplied; so, in that case expected value would be like this. I will just mark it out where it is yes so, it was 5 lakhs so; obviously, the utility is this.

I am just marking with the yellow color and I will mark the certainty value with the yellow highlighter and the probability is 1. So, the expected value of the right hand side is given which is marked in yellow, now come to the other side left hand side. So, the probabilities were half and half. So, the probabilities are given 0.5 0.5 now, what are the utilities you will ask so, one the values of the wealth worth 10 lakhs and 4 lakhs. So, the utility for 10 lakhs is given, utility of 4 lakhs is given multiply each of them with given 0.5, you get the expected value balance that and find out the value of a.

So, solving through iteration process we can find out the value of a. So, let me I continue reading it whatever is given because I repeated that, but I am still going to read it. Suppose the answer is 5 lakhs which the person says, that it then it means that the person is indifferent between a certain equivalent amount of 5 lakhs and the lottery which is a fair gamble. So, you balance the fair gamble value which is the expected value which is given in red balance with the yellow color yellow marked expected value of the utility for the certainty value. Keep repeating this experience for different values; find out different sets of ways. They would be changing because, of iteration process and you find out the expected value of that and find out the best estimate which is for value for the value of a.

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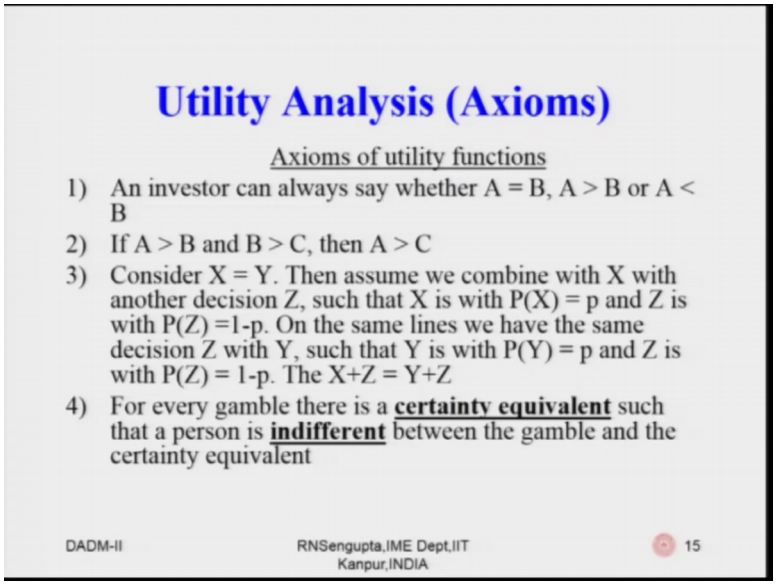

Now, there are few axioms of utility functions and investor can always see whether A is so, consider I am in investor, I am a decision maker and there are many different

decisions in front of me. So, we will assume always that I am able to put my decision in such a way that I either I am indifferent between A and B whether I am I am willing to take A and not B and vice versa number 1. Number 2 is that if I have three decisions and in excuse me and in case if A is better than B; better in the sense what is the utility or the value which is coming to me, B is better than C then A would always be better than C. Now, here I would like to pause in many of the nonparametric decision-making process which you are going to consider it may not be true.

In the sense that my decision for favoring A with respect to B is more towards A, my decision for favoring A with respect to C is more towards A, but it may have its sorry mean the case let me put it as B and C. So, my decision for taking B with respect to C is more towards B so; obviously, from the first information I know A is greater than B. From the second information, I know that B is greater than C, but it may so, happen that the decision that A is better than C may not happen in many of the practical situations where, nonparametric decision making and to be taken in practical sense. But, to overcome that we will continue considering this axiom that A is better than B and B is greater than C would always imply A is better than C, we will continue doing that and where there is a change I will notify that.

So, this is an important point which I wanted to mention. So, these problems will be coming up if you remember I mentioned about data envelopment analysis, analytical hierarchy process, analytical network process ELECTRA, TOPSIS, MACBETH. So, all these processes which are nonparametric different type of multi credit criteria decision making this type of issues would be coming, but I will mention that accordingly so, we can handle it. Consider the third action which says that X is equal to Y. Then assume we can combine X with another decision Z, such that X with the probability of p and Z with the probability of 1 1 minus p such that on the same lens we have the same decision with Z with Y.

So, in that case if both of them the expected values are same then in that case it will be X plus Z is equal to Y plus Z. In the sense that the expected value which are getting from combining X and Z and the expected value which you are getting by combining Y and Z would give me the same utility based on which I am going to take a decision. That means, if there are two different sets of decisions for which the expected value are same then I would be definitely be indifferent between them. So, these decisions can be totally

different say for example, in one case I want to build a factory in in the city of say for example, Lucknow and another factory I want to be building say for example, in city of Vijayawada in near the cities.

So, this is two decisions depending on many things; it may be depending on type of labor I get, type of raw materials I get, transportation cost, land acquisition cost, building cost and all these things. And, another case considering I am able to build up another factory on another set can me near Pune another one say for example, near Bhubaneshwar. So, in this both the cases which is basically Pune plus Bhubaneshwar and other cases between the first example; we will consider these decisions in such a way such that the expected value or the general utility of the decision based on which you are going to take a decision of where, to go for the first combination the second combination would be such that the overall expected value overall utility of the decision for the person who is going to take the decision would come out to be the same.

So obviously, if I take a decision and you take a decision for the same thing it not may not give you the same ranking of the decisions. Because, it may be possible that I like A and then I prefer B less the A, but your case may be that you prefer B more than A so; obviously, the decisions would not change. Because, they would depend on the type of utility function the person has, the age of the person, the type of family background the person is, by the type of financial background on the person is, what is the risk taking ability of the person. So, it basically depends on many intrinsics and not very objective criterias, but will try to combine them as far as possible in the different type of decision making process which we have.

There would be questions from your end which may not be very satisfactorily answered depending on when you solve the problem, but they are generally used in or in a different scenario cases such that we are able to tackle problems in a in a decently realistic manner. For every gamble there is a certainty equivalent as I mentioned that. So, you have a gamble fair gamble I would consider but, if it is a gamble where the probabilities are different still we will have a certainty value. For every gamble there is a certainty equivalent such that a person is indifferent between the gamble and the certainty equivalent. So, as that the expected values for both the gamble which is the fair gamble in our very simplistic case and the certainty equivalent the expected value would be the same.

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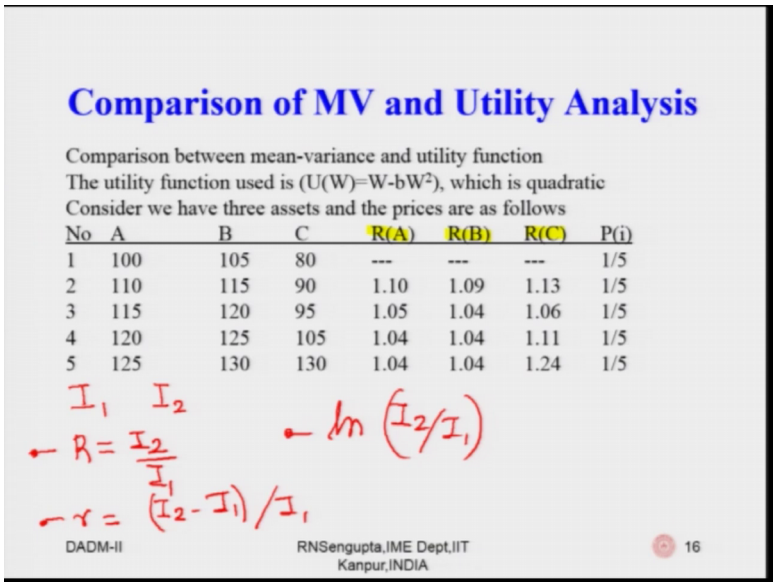

Now, consider comparison between the mean variance and the utility theory. So, what is the mean variance concept which you are considering? In the mean variance concept we are considering that the mean which is the first moment variance of the second moment; we will always try to basically put the ranking on the decision based on the mean. That means, for the maximum to the minimum of the variance from the minimum to the maximum. So, if you take a ratio as I mentioned in the first class or a first lecture, it will basically be the ratio of the mean to the basic variance and you will rank them from the highest to the lowest. If it is the ratio of the variance though who the mean, we will rank from the lowest to the highest and take the decisions accordingly.

And, the utility concept would be that we will see later on that mean variance concept would be true and it will basically give us the same type of answers considering the utility function is quadratic; I will come to that later on this is very important. So, comparison between the mean variance and the utility function.: the utility function used is basically quadratic which is U W is equal to W minus bW square and there is no constant term for the time being. It is actually the quadratic function would have been aW minus plus bW square plus c. So, A B C would be the parameters whether positive or negative that is a different question. The utility function is used which is a quadratic function and quadratic function would have some significance later on. Consider we have three assets and the prices are as follows. Now, here if you look at the chart, what we are doing? This is again a hypothetical example; we are taking the numbers.

And that means the serial number depending on what is the outcome and the decisions are A B C and the prices are given as prices of the values are given like in the second column, third column, fourth column corresponding to the fact that we are trying to find out the outcomes for A B C respectively. So, in the fourth, fifth and sixth we are trying to find out the returns for A B and C; the returns are given because it will basically be returns can be if we know. So, in one case I will use as I 1 is the input which is happening for time period 1 and after some time period I get an output which is I 2. So, I do not want to go to the exact values.

So, they can be to to time returns return capital R is I 2 by I 1 and small return would be I 2 minus I 1 divided by I 1. Another one in the case when you are considering the financial returns we consider so, this is 1 2 ln of I 2 by I 1. So, we are only we will consider anything and then proceed, we are considering for the time being capital R. So, the capital R values are given for A B C, the probabilities we consider is one-fifth, one-fifth, one-fifth for all the cases; this is a simplistic case.

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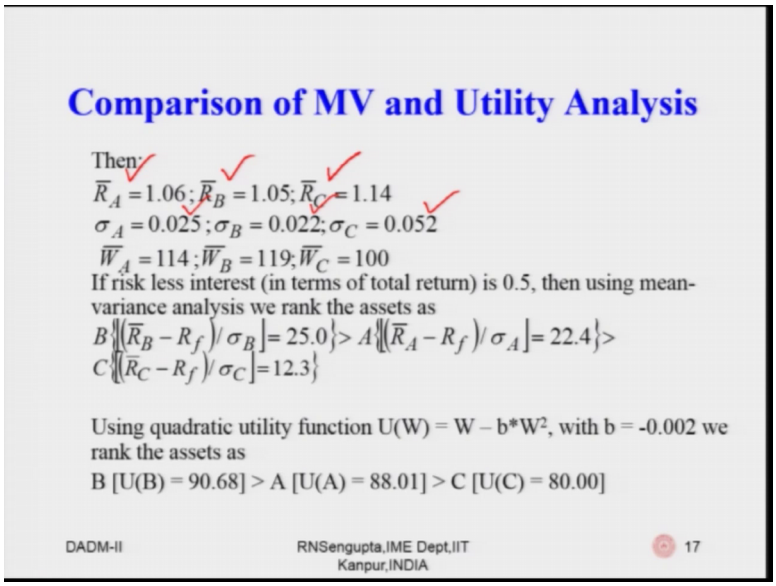

Based on that we find out the returns average returns; that means, add them up divide by the number outcomes. So, those are given R bar A R bar B R bar C and the sigma which is the standard deviation or the standard error for this sample for R A R B R C returns are given, again calculated. I am just putting it on tick mark so, we can understand. Now, I I consider the weights average to be given which is 104 the wealth given as 114 119 and

100. Based on that we use the concept that, what is the value that the if the risk free interest rate is given as 0.5 then what is the mean you using the mean variance ranking system I rank them such that the values are greater than 0.5. So, I find out the values. Now, the issue is that if you use the utility ranking based on a quadratic concept and if you use the ranking system based on the normal distribution such that you rank them over and above a certain probability; the ranking would always be the same.

Reason being in a very simplistic sense quadratic utility function and normality of the returns going hand in hand. So, if the utility function general is quadratic your returns would be normal and vice versa. So, we are in a position where the normal distribution can be utilized, its properties can be utilized in a very big way considering the utility function is quadratic. So, this slide which you are seeing I will again consider this slide and the example, in the next class. And, with this I will end the class considering the 30 minutes are almost over and I would like to go into the details of these problems once more. So, that you can understand and tackle such problems in the assignments which are going to come. Have a nice day and.

Thank you very much.