Data Analysis and Decision Making - II

Prof. Raghu Nandan Sengupta
Department of Industrial & Management Engineering
Indian Institute of Technology, Kanpur

Lecture – 06
Utility Analysis

Welcome back my dear friends, a very good morning, good afternoon, good evening to all of you. And this is as you know this is the DADM-II course which is Data Analysis and Decision Making. And as you all know that this course total duration is for 30-hours, which is 12-weeks, each week that means, in totality will have 60-lectures. The reason being that each week we have 5-lectures, each being for half an hour and after each week there is one assignment; we have already completed the first week.

So, we will be starting with the 6th-lecture, which is the 1st class for the 2nd week. So, hence I will just repeat it time and again, so that means you are in line with how things are proceeding. And my name is Raghu Nandan Sengupta from the IME department, IIT Kanpur. So, this is again this is DADM-II lecture-6, which is in the 2nd week for this total course.

Now, if you remember we were discussing a simple problem, and I did mention that these things I will again repeat please excuse me, and please have patience, you will understand the reason. For any utility function, we discussed four utility functions, which was the normal utility function, logarithmic utility function, exponential utility function, and power utility function for all of them before that we discussed that there were two important properties.

The important properties were one was which related to absolute risk aversion, one was related to relative risk aversion. And obviously, these two though in these two fundamental properties or assumptions of a non-satiation, and the corresponding that person can be risk averse can, person can be risk in different person can be risk lover basically comes from the second derivative.

So, the first derivative gives you the information that people, if you give more to people, more they would want which is the property or non-satiation. And the second derivative being greater than 0, equal to 0, and less than 0 gives us the properties corresponding to I hate risk, I am indifferent to risk, and love risk.

Now, those four utility functions are generally not aware or it is not available to us or the human being of the person, who is taking the decision may not be aware of what type of decision he or she is taking based on the utility function. So, in order to find out what type of utility function in what group of utility function that person is we basically take recourse or try to get the informations based on the absolute utility function, and the relative utility function.

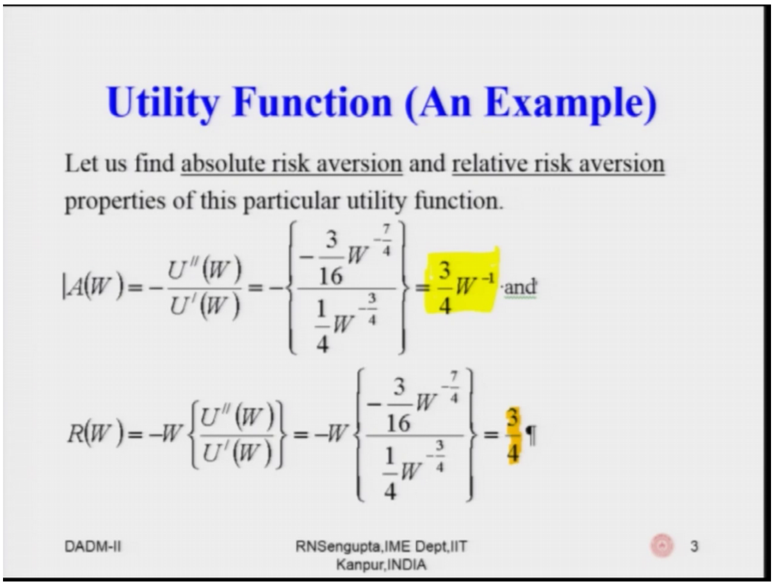
And the absolute utility function, and the relative utility function have a particular the have formulas, which were the first derivative of A, and first derivative of R. A being absolute risk aversion, R being relative risk aversion. Those properties basically depend on U double prime, because in the formulas which you see that a has a formula, where which is minus U double prime by U prime, U prime is always positive. So, the sign of A A and A prime would definitely depend on U prime U double prime.

Similarly, when you consider R, R is a formula which is basically minus W into U double prime by U prime, U prime as you know again I am repeating is positive W obviously is positive, because it is the wealth. So, obviously the sign of R would depend on U double prime, similarly the sign of R prime will depend on U double prime. Now, U U A double prime, and R double prime would basically have the concepts where we can have three for A prime, and three for R prime that means, greater than 0, equal to 0, less than 0.

So, based on that we can basically have 3 cross 3 or 3 into 3 different combinations. So, those combinations being unique give us some information about the utility function, which we did which we basically derived mathematically the first derivative of A, first derivative of R, then plotted the values on an excel sheet using hypothetical values then plotted them on the graph paper and we saw on the excel sheet. And we saw that the characteristics, which we got by using the mathematical formulation were matching with the simple hypothetical example, which corroborates the fact that what we are discussing was absolutely right.

And then I just in the last slide of the last class which is the 5th class, I took a function which was W to the power one-forth. And basically found out that non-satiation was applicable, and the properties of U double prime gives us the information that it should be negative. So, further on we will proceed to solve the whole problem.

(Refer Slide Time: 05:21)

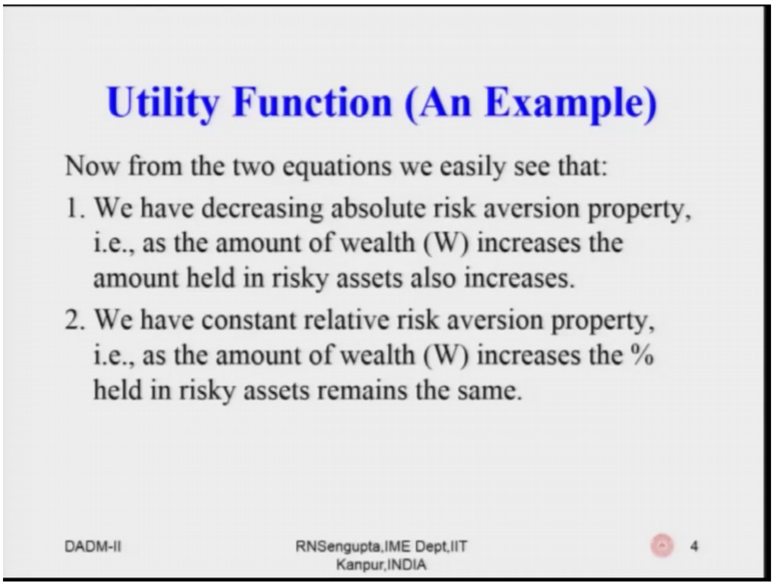


So, now let us find out the absolute risk aversion property, and the relative risk aversion property of this function, so which was the function being basically W to the power one-fourth. So, if you find A, A is basically this is so in the sign before A is nothing it is just a typo error I am sorry for that. So, A is given by minus U double prime by U, so that value comes out to be three-forth or W to the power 1 or 1 by W.

And R value as you know is basically the equation is given by minus W into U double prime by U prime, and the value when you calculate, it comes out to be three-four. So, I will just mark those values. So, this value comes out to be three-fourth of W to the power minus 1. And the other value comes to a value of three-forth. So, now you need to find out A prime and R prime to find out the properties.

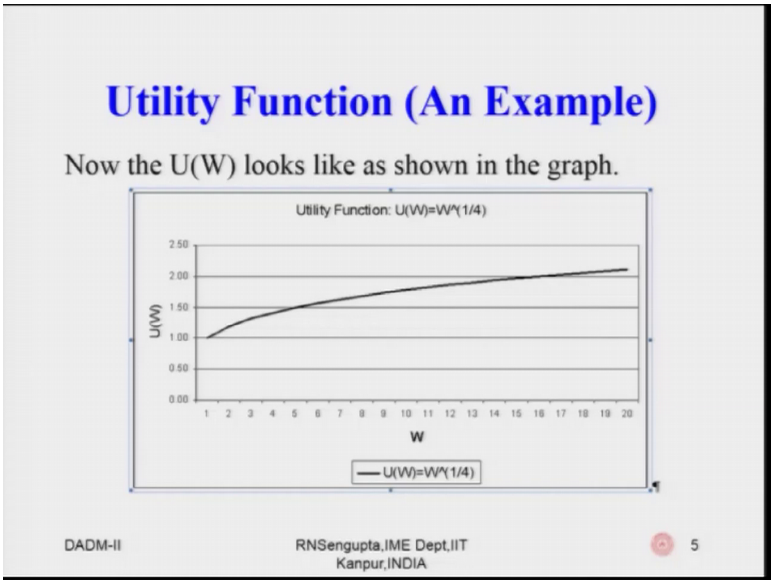
So, if you find out A prime, so obviously it will be minus three-forth by W to the power minus 2. And if you find out R prime, it will be 0. So, in the first case, it will be and the A prime is less than 0, so it is decreasing, and R prime is basically 0. So, basically it will be constant relative risk aversion property for R, and absolute risk absolute risk aversion property for A. And you can definitely see what type of functional form it is. Obviously, we do not know the utility function based on that we are we are giving us giving these set of information that means, R prime and A prime would be given whether they are greater than 0, less than 0, equal to 0 based on that we will basically try to deduce the type of utility function they are.

(Refer Slide Time: 07:13)



Now, from the two equation we can easily see that we have decreasing absolute risk aversion property, for the first one that is the amount of wealth increases as the amount held in risky asset also increases. And for the R property, we have on constant relative risk aversion property that is as the amount of wealth increases the percentage held in risky asset remains the same. So, comes to concept wise we get R and A, and then pass the judgment accordingly about the functional form of the utility function.

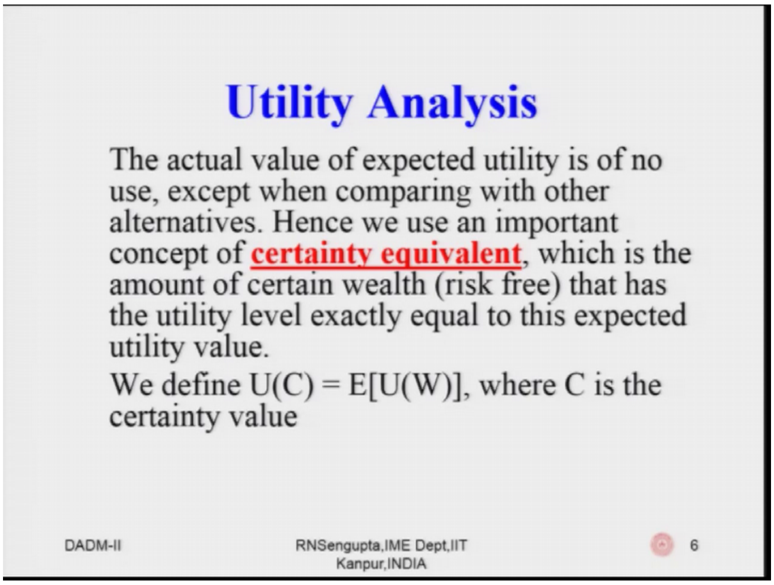
(Refer Slide Time: 07:43)



So, once we draw it, the utility function W to the bar 0.25 or W to the power one-fourth is given by this. I have not drawn the functional forms of A A prime R R prime for the other four utility functions which was quadratic power, then exponential, and logarithmic utility function. If you remember I on the same excel sheet or the same graph paper, I give the utility function A A prime, R R prime, here I am not drawn it. Because, that becomes too cluttered, you can definitely utilize excel sheet, and draw them separately to understand what actually the properties of R R prime A A prime mean. So, hence you can basically appreciate the utility functions in a much better way.

And again I am saying these drawing graphs, and trying to basically deduce the properties. And based on the properties find out what a utility function is are not for the examination point of view from the assignment point of view, they are more in order to inculcate a new that interest such that you can check these values solve this very simply. And at least get to understand, what I have been trying to discuss for the last one week in the last five class.

(Refer Slide Time: 09:09)



Now, we will basically consider a certain concept, which is known as certainty equivalent. And what is a certainty equivalent value, it will become clear to you why we mean certainty equivalent; so, the actual value of no use. So, generally giving us the value there the expected utility value is 25 or 35 or 252 would not make any sense to us; until unless we have some base or datum based on which you are trying to compare that why it is required. So, we will discuss that what is the concept of certainty equivalent or technically the expected, value which you have been discussing, and why it is important.

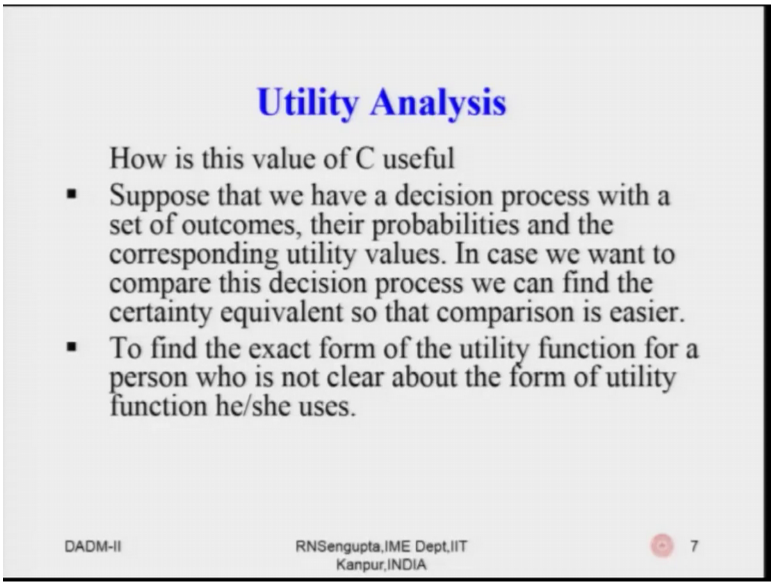
The actual value of the expected utility is of no use, except when comparing with other alternatives. Hence we use an important concept of certainty equivalent, which is the amount of certain wealth, which is risk free that has the utility level exactly equal to this expected utility value which is there for us. Now, we defined certainty equivalent is a value, it is not an utility function as such.

So, what we actually mean in the consider there is a value C, whatever the value it can be rupees, dollars, yens in wealth in some amount, and based on that certainty equivalent we find its utility function. Because, we put it in the utility function value find out U of C and that U of C it exactly matches the expected utility of a non-deterministic value, then we will say that C value based on which we are trying to find out the expected value exactly matches. Since hence, we are certain that there is some value for which we are we are equally disposed both for the gamble or the non non-deterministic event as well as the certainty value. That means, we are trying to find out 1 to 1 equality based on the fact that the expected values are same. So, our main task is to find out a C such that the expected values both on the left hand side and the right hand side matches.

So, now what is on the left hand side? Left hand side would basically be the gamble or the non deterministic event, so that means that would be a summation of the utilities for h each and every step multiplied by the corresponding probability, we sum them up that is why I have said summation. And on the right hand side, there would be value of C which is the certainty value such that the utility of the certainty value multiplied by 1 because the probability is 1 remember that, so that would is the expected value; so, that both of these value should match.

So, if you have a certainty value for a decision we can basically compare the decision with respect to other decisions such that the certainty values can be different or can be same based on which we can take the or all that which is the best decision for that human being or the foreperson who is taking that decision. Obviously remember certainty value would change depend for the same situation on the gamble which is there on the left hand side would change depending on the utility function which is the person has. It is not unique, the certainty value once fixed it would definitely not be the same for all the people who are taking the decision. So, I will come to that.

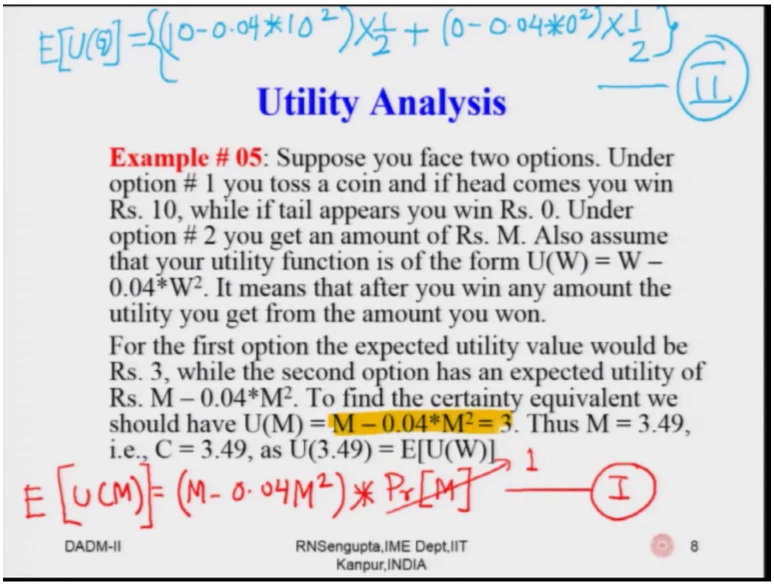
(Refer Slide Time: 12:15)



Now, the question is that how is the value of C useful? Suppose that we have a decision process with the set of outcomes, what are the whatever the outcomes are, their probabilities and the corresponding utility values are known to us or given to us. So, in case we want to compare this decision process we can find out the certainty equivalent, so that the comparison is easy. So, given C as I mentioned just few seconds back we can compare the decisions and make our choice that which is the best decision of, which is the second best decision and so on and so forth.

To find the exact form of the utility function for a person, so yes certainty value would also be important to find out the utility function based on which the person is taking decision. So, obviously, the value of a, a prime, r, r prime are important, they give us some notion or some information about the utility function. But this certainty value would also help us in trying to find out what type of utility function it is.

(Refer Slide Time: 13:11)



So, let us consider an example. Suppose you face two options. So, under option 1, you toss a coin and if a head comes you win 10, 10 rupees, 10 dollars. So, let us consider rupees 10, while if a tail appears you win 0 rupees, none. So, under option 2, now consider another option. So, it is basically a coin. So, we would consider is an unbiased coin. So, if the 10 comes with the probability half, 0 comes with the probability half. Under option 2, you get an amount of rupees m.

So, also assume that your utility function is of the form that U W is equal to W minus 0.04 into W square. So, we are considering an option 2, you get an amount of M, and there is no probability so obviously, the probability is one. So, under the third question we want. So, the first question was at a gamble. Second was an certainty value M. And the third question which is given to us information let us put with this the information is basically what is the utility function. So, utility function is W minus 0.04 into W square. So, it means that after you win any amount of the utility you get from the amount you win would be calculated based on this utility function.

So, if I consider the value of M, so the utility would be M minus 0.04 into M square. So, that is the utility that would be multiplied by 1, because the certainty equivalent, so that would give me the expected value of the certainty equivalent which is U of M. Now, let us go to the gamble which is when you are tossing a coin, the value of 10 would be calculated. So, what is the utility of that? So, utility would be ok, let me write down all the values because I mean talking, and if me if I missed something you may not be able to catch that, so that is that is why I will I will do the calculations. So, I will use red color for the certainty value. So, certain value is given as M.

So, the utility of M is given by M minus 0.04 M square. So, I put a bracket, because there is a probability. So, I need to find out the expected value of the utility. So, I input a expected value here. This would be multiplied by the probability of M occurring, so that value is basically given by 1. So, your utility would be because if it is 1, so utility would be M minus 0.04 into M square. So, that is for the I will put equation 1 because that is corresponding to the certainty value.

And now let us change the color, and let me use blue one. So, I am going for the gamble. So, I will do it somewhere here. So, for the gamble I need to find out the expected value, and so there I will use expected value for the I will use the word G for the gamble there is no in a such meaning just for our convenience. So, the utility when it is 10 rupees, so it will be 10 minus 0.04 into 10 square multiplied by 10 square. This is the utility multiplied by the probability which is half because in unbiased coin. The second arm or the gamble would be 0 minus 0.04 multiplied by 0 square multiplied by half. So, this is the utility corresponding to the gamble, and I will basically mention it as equation 2.

So, you will basically have you want to basically equate equation 1 in equation 2. Equation 2 is everything is known; in equation 1 M is not known. Put it in the equation, find out M, that M is the certainty value. So, let me read it the last portion of this problem. For the first option, the expected utility value would be 3. So, the first portion for the is for the gamble, so the equation 2 value is 3. While the second option has an expected utility value of M minus 0.04 and M square as it is given in equation 1. To find the equivalent, equate them. So, we equate them the equation is this one, thus M comes already 3.49. So, the C value certainty value is 3.49.

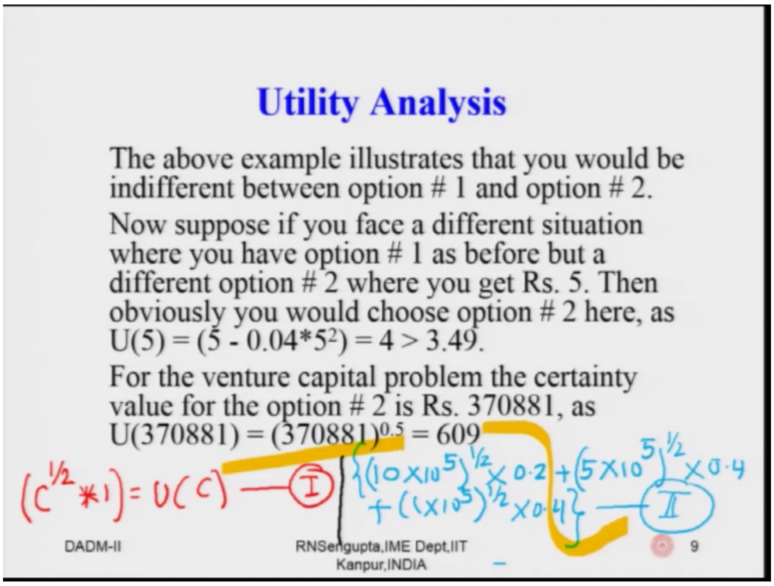
Now, the question would be how does it help you. So, say for example, I have a gamble on the left hand side, I have a 3.49 on the right hand side the person has a utility function as given as a quadratic one which is given here. So, if I place this inform set of information on the gamble and the certainty value on the table, considering the utility function on that person exactly as it is given, he would he or she would be indifferent.

Now, consider that the value of 3.49 is change, it is decreased and it becomes say for example, 3. In that person technically the if the person is rational, the person would should take the gamble because the expected value of the gamble is 3 while the value of the certainty value if calculated put into the equation would be less than 3. So, the person would be more inclined to take the gamble. And if the value is more than 3.49, say for example, it is 4.49, in that case the value of the gamble expected value of the gamble would be less than the utility based on the value of C which is now different which is 4.49, and then hence the person will take the sure event.

Now, in that case that value which I said that in the first case you decrease 3.49 into 3 or in the second case you increase the value of 3.49 to 4.49, those are not known as certainty equivalent. They would be just values given for that that sure event. Certainty equivalent is basically the concept which you are using where the equation just balances in the sense the expected value of the gamble and the expected value of the sure event exactly match.

Now, let us go to the example which you have guessed about the government securities app with respect to the non-deterministic securities. In the government securities, if you remember if I can recollect you had 6 lakhs with certainty value of with the probability as 1; and in the second case there were basically values of 10 lakhs, 5 lakhs, 1 lakhs with probabilities respectively as 20 percent, 40 percent, 40 percent. So, I need to find out certain equivalent for that gamble such that I am indifferent between the gamble. Gamble means the non-deterministic event which is there and the certainty value.

(Refer Slide Time: 20:31)

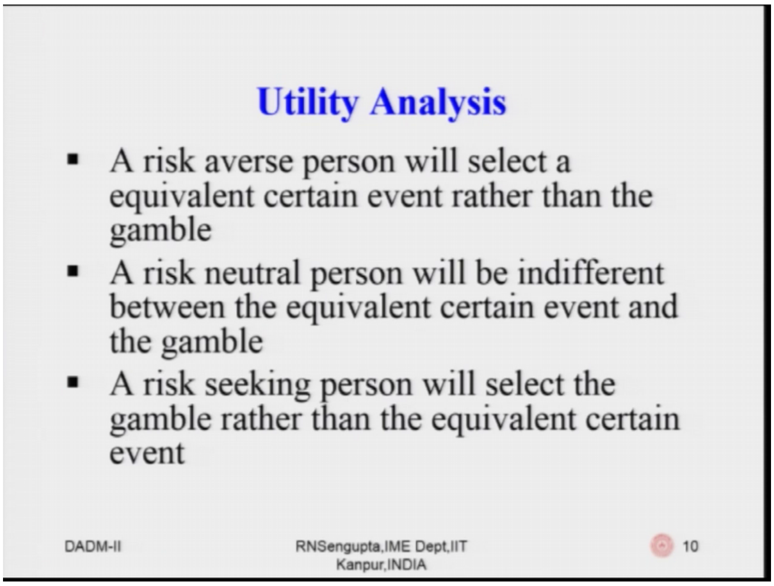


So, let with that let me read the problem. The above example in illustrates that you would be indifferent between option 1 and option 2. So now, suppose if you face a different situation where you have, so let me first finish the example which we have discussed that M value ok. Now, suppose if you face a different situation where you have an option 1 as before, but a different option 2 where you get rupees 5, then obviously your actual value of the expected value of rupees 5 would be 5 minus 0.04 into 5 squared and the probability is 1. If you put then that value comes out to be 4. So, 4 is greater than 3.49. So, obviously, you will take that that sure event where the expected value is higher.

Now, coming back to the venture capital, in the venture capital case consider the expected value for the non-deterministic event as you know was basically with the utility function was W to the power half. So, it was basically 10 lakhs to the power half into 0 point 0 0.2, the 0.2 is basically the probability. So, you had basically I will use the color scheme again the same red and blue. Red for the certainty value; so, the certainty value was C, C to the power half the probability is 1. So, this is the expected value. So, this is equation 1.

And equation 2, I will write it on the side. So, you had 3 options. So, it was I 10 to the power 10 to the power into 5 to the power half into 0.2 plus 5 into 10 to the power 5 to the power half into 0.4 plus I am just writing the equations as it is 1 into 10 to the power 5 to power half into 0.4 put a bracket here find out. So, this is equation 2. Balance equation 1 and 2 find out the value of C that is what the main cons actual idea is. For the venture capital problem, the certainty equivalent value of option 2 is 370881 as the value of utility which is 370881 to the power half which is basically coming out to be 6. If you see here these values, so this value of C this is matching. And why this 609 is the value which you cannot calculated using equation number 2. So, they should match and then you can find out the certainty value which comes out to be about 370881.

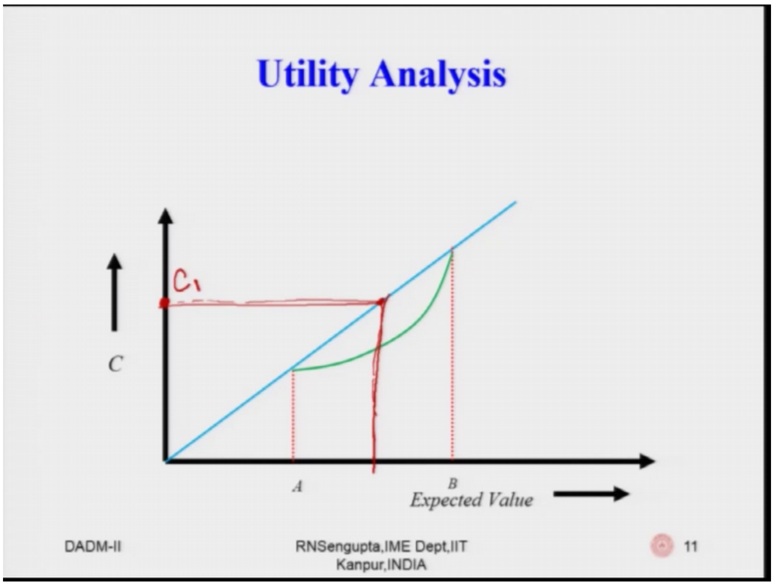
(Refer Slide Time: 23:47)



A risk averse person will select an equivalent certain event rather than the gamble. A risk neutral person will be in different between the equivalents certain event and the gamble, because the values are given to him. So, he or she is indifferent. Risk seeking person only select the gamble rather than equal certainty equivalent because for him or her the decision of taking the gamble would be much worthy than long run ok.

Now, we will consider some why the certainty equivalent is important. So, first have a look at the graph then I will explain. It will take a little bit long time to for me to explain, I will go slowly. So, what you are doing is that on the x-axis, you are you are measuring or noting down the expected value or the values of the gamble whatever the gamble. And consider the gamble is for the time being a fair gamble which is a fair coin being tossed probabilities being half and half. And there are certain values when the probability is half, there is certain value consider it is A. When there is a tail, there is a certain value which is known as B. So, A and B can be changed. We are not going to change the probabilities half enough. So, that is being measured A and AB. And as and also the different values of A or different values of B are measured along the x-axis.

(Refer Slide Time: 25:09)



Now, along the y-axis we basically measure and write down the certainty value; certainty value based on what type of gamble which you have, so those are the 2 x axis. Now, draw a forty 5 degrees line which is the blue line as shown on the graph paper or on the slide. Now, let us basically think about the example which we are doing. Consider you take a value of A and B whatever the value is. And the value of A is taken in such a way that the expected value of them considering ok, another thing which is important for me to mention is that consider the utility is linear which is utility or U of W is W, there is no change. So, if U of W is W, so the expected value which you will have when you basically combine A and B would be A into half plus B into half, so it will be midpoint.

So, the midpoint would be I will use the color, say for example, not red, the brown one, hopefully you can differentiate. So, the value of the middle part of A into half plus B into half would be a value which is here. You extend it meets the 45 degree line blue line. And then you go on to the left hand side, where it meets that value. Now, as the person that whether he or she with for respect to the gamble, she would like to have a value of some C star or not let us not mention C star, C 1.

So, whether the person is willing to take a value of C 1 or take a value higher or take a value of lower. If the person is willing to take a value of C 1, that means, that C 1 is the certainty column value for the gamble. If the person is willing to swap the gamble for a value of C which is higher than C star, then which means that the person is risk averse, if the person is basically not willing to go for that C 1 value, but go for a lower value, then the person is he wants to take the risk. And based on that you can find out for different values of A and B, you can find out to which portion of the of the utility would be on the lower side or the higher side, that means, lower side or the higher side means whether it is below the blue line or above the blue line, whether it is concave and convex.

And based on that you would basically find can find out the utility functional for some characteristics whether it is concave, convex, whether U, U double prime is increasing or decreasing or constant and we can basic pass a judgment accordingly. I would come back to this slide again in the 7th class and discuss a little bit more about that and continue the seventh class accordingly. So, with this I will end the 6th class. And thank you for your attention. Have a nice day.

Thank you.