Data Analisys and Decision Making-II

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Lecture – 53
GERT

Welcome back my dear friends, a very good morning, good afternoon, good evening to all of you wherever you are in this part of this world. And as you know this is the DA DM II which is Data Analysis and Decision Making II course under NPTEL MOOC series. Now, this total course duration which I keep repeating before the starting of the actual lecture for any week, this total duration is basically for 12 week.

And as you can see from the slide we are in the 11th week and the total contact hours is 30 which is gets converted into 60 lectures; each lecture being for half an hour and each week we have 5 lectures, each for half an hour as it should be. And we have already completed 10 weeks, we are in the 11th week. As mentioned we have already done 10 assignments for this DADM II course and my good name is Raghu Nandan Sengupta from IME department at IIT, Kanpur.

Now, if you remember we were discussing about the GERT and Q GERT concepts. So, GERT was basically the Generalized Evaluation Review Technique where the concepts of probability in which path, it will be taken, the time concepts are very important. And more so, what is very important is the looping concept, because important CPM which I had mentioned in the with this week in the 53rd lecture, in the 51st which was the 1st lecture in the 11th week I kept mentioning time and again that important CPM you will not have the concept of looping; that means, once you follow the process, you would not be coming back for rework or quality checks rework of quality checks and so on and so forth which is considered in the GERT method.

One of the important factors for the concepts of GERT or GERT was that it is based on activity on or concept not an activity and nodes, because in CPM and AND part you can basically consider the activity or arcs and activity or nodes concept.

Now, considering the, concept of and network OR networks and all these possibilities we, did discuss in the last class of which was in the 52nd class lecture the concept of basically launching a rocket were two most important work or the assignments or the operations were important. Where we considered in the first case of both operation means successful was the case where the work was successful, the completion the work was successful.

So, any combination of yes, yes was successful, any combination of yes, no, no yes and no, no for the respective first and second set of this decisions which were combined to form the whole total work was unsuccessful or a failure. And in the second example, we consider that in if any one of them is successful we will consider this as a success. So, far yes, yes, yes, no and no, yes, it was a success while for no, no it was a failure. Now, when we have a series network how would you basically find out the probability? So, it is like this.

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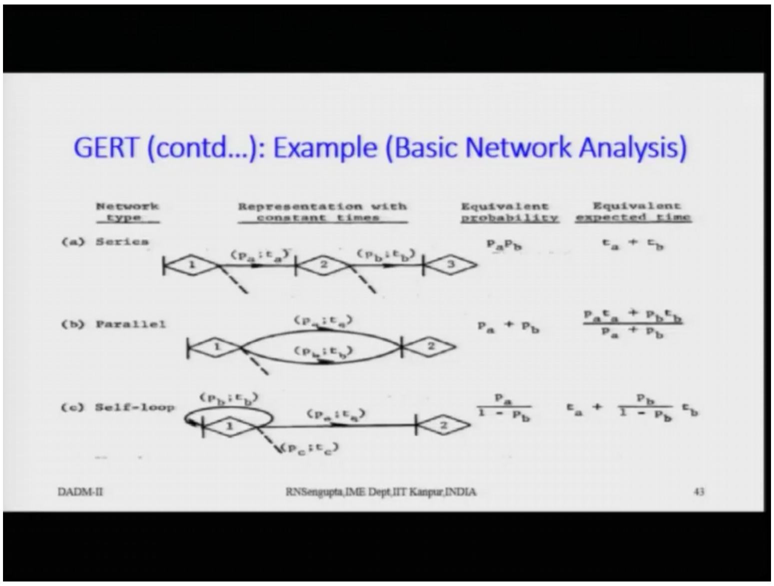

So, consider that this is a series network; that means, you are following the same concept as we as in conceptually you follow the same concept as we do in mark in electrical engineering. And then we have the concept of parallel network and the looping concept would be considered, because there is a part and parcel of the GERT or GERT and Q GERT and Q GERT processes. So, consider the series network; so what you have, again as usual I will bring the diagrams in consideration.

So, please just have some patience for few minutes. So, example of basic networks I will consider. So, this utilization of this pictorial concept would be utilized time and again hence, I thought I should basically bring it here also ok.

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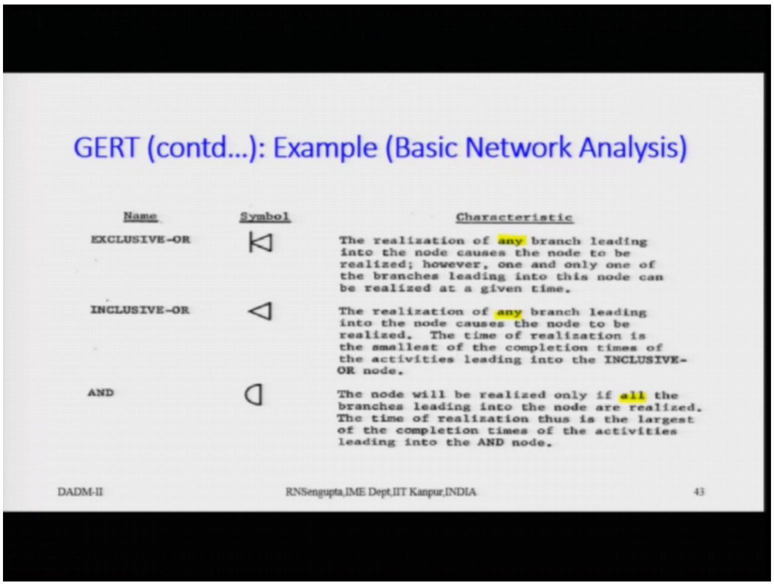

So, exclusive OR means I will did repeat it, but time and again, but I will again repeat it so that the concepts are absolutely clear to us. So, exclusively OR means that realization of any branch. So, any branch I will (Refer Time: 04:49) for any branch leading into the node causes the node to be realized any one of them. So, consider that n of them you can take any one of them.

However, one and only one of the branches leading into this node can be realized at a given time. So, if you are basically n number of branches, you will take only one of them; that means, n of them are there any one can be realized.

So, inclusive OR would be the realization of any branch leading into the node causes the node to be realized; so here also the any branch is there. The time of realization is the smallest of the completion time of the activities leading into the intrusive OR. So, in the first place we only considered the realization; now I will consider the time also. So, we will consider the any branch leading would trigger it and give the pressure positive for the time duration we will consider as the minimum of all of them taken together.

For the AND one the node will be realized only if all the branches leading into the nodes are realized. The time of realization does is the largest to the completion time of all the activities leading into the AND node. So, exclusive OR any branch inclusive OR any branch with the minimum time and being realization of OR time being the maximum of them.

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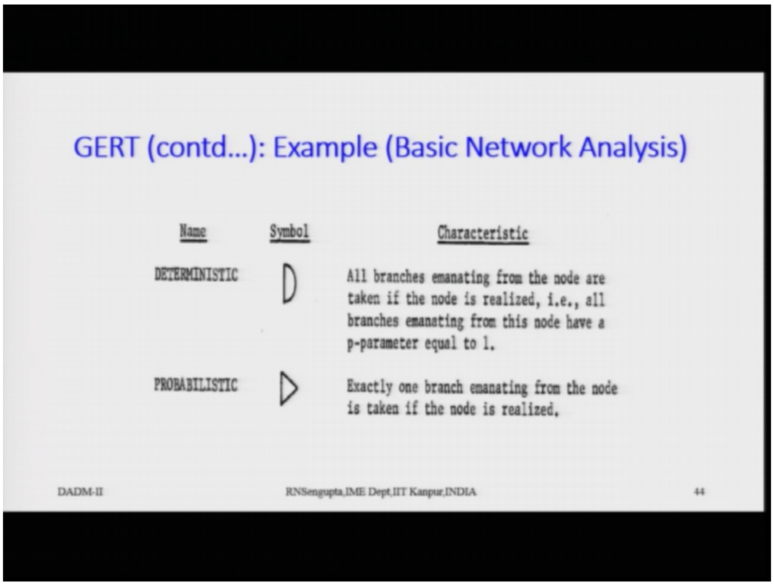

Now, with repetition deterministic would be the semicircle one, probabilistic would be the triangle one where the triangle has the verti one of the vertices pointing on to the right, right side when you are drawing the diagram. So, the deterministic one all branches emanating from the node are taken if the node is realized that is all branches emitting with this node have a p parameter equal to probability parameter equal to 1, all the branches coming out from that.

While in the probabilistic one exactly one branch emanating from the node is taken if the node is realized in the corresponding probability; obviously, you will know that if it is yes. So, hence the corresponding probability for that node, if it is realizes yes or else the probabilities would be given according to the distribution which you have. So, with repetation for the exclusive OR, you will take any of the branches for the inclusive OR you will take any other branches and the time would basically be the minimum.

So, let me write highlight the time with the different colour. So, the time of realization is the smallest of the completion time or the all the activities is leading to the intrusive OR.

While in this case for the AND one it is also if it is all the branches leading into node are realized and the time of the realization is the largest of the completion time of all the activities leading to the AND node.

So, semicircle one is the deterministic one where if the node is realized then all the branches emanating from this node have a parameter p value of 1. And probabilistic case being exactly one branch emanating from the node is taking if the node is realized and; obviously, if it is taking with probability would be 1 and corresponding to which it would be taken will depend on the probability distribution per (Refer Time: 08:22).

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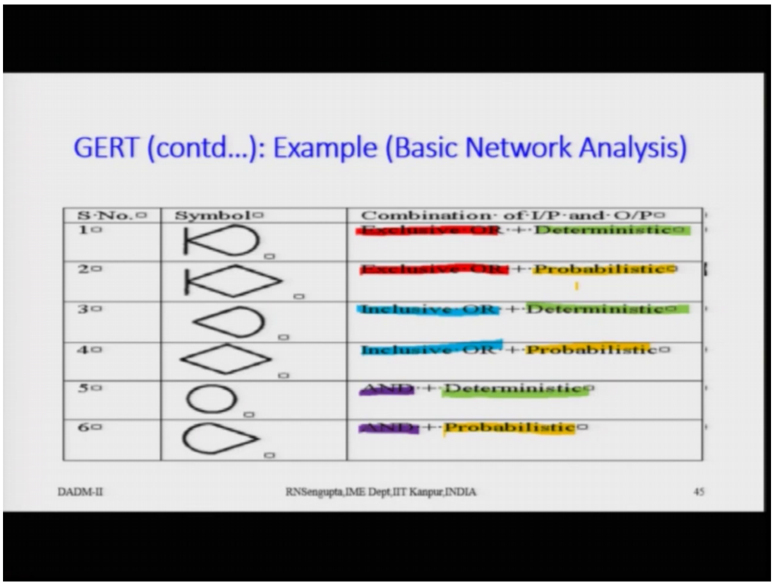

The symbols for the combinations would be you have exclusive the ORs the ANDs and the deterministic one. So, I will basically formulate 3 in to 2 6 combinations. So, the first one will be exclusive OR with deterministic case. So, deterministic case in marketers green then the exclusive OR the probabilistic case then you have the inclusive OR with the deterministic case, then you know the inclusive OR with the probabilistic case then you have the AND network.

Let me check a colour light green would do let me take the violet one AND network with the deterministic one. So, this colour would be 1 minute, it should be light one then the AND one is there with the probabilistic one; probabilistic one would be the orange colour. So, the combinations are given accordingly such that based on this exclusive OR deterministic one exclusive OR or the probabilistic one including OR or the deterministic one inclusive OR of the probabilistic one.

And then finally, the AND concept will be combined with the deterministic and the probabilistic one. You remember one thing in this exclusive OR you had basically taken any one has to be realized. In the inclusive OR, you have to basically take the concept that anyone has to realize along with the minimum time. For the AND one all have to me has to realize with the AND with the maximum time.

While for the deterministic one all of them have to be realized any parts coming out. And the probabilistic one anyone would be realized; obviously, with the probability one, but correspondingly what is the distribution of which of the paths can be taken would be given by the proper distribution.

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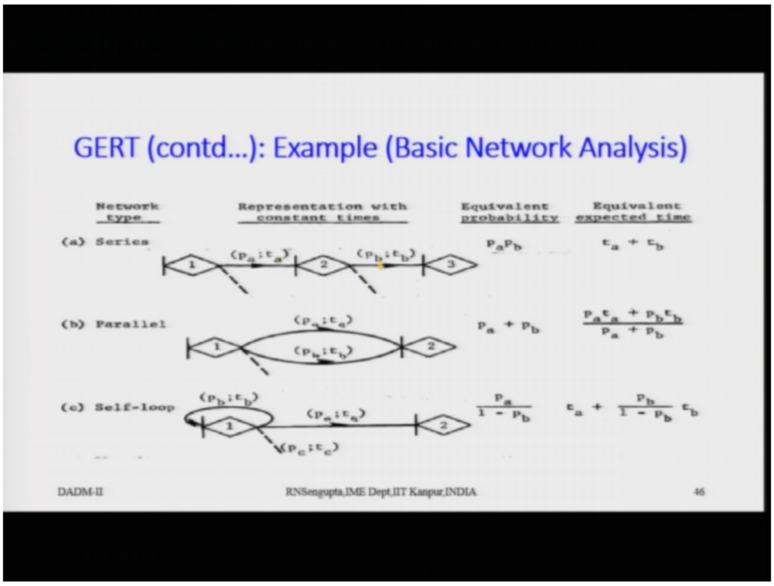

Now got let us come to the series concept; so in the series concept what we have are three components side by side; so representation with the constant time period being. So, I am going from what I will call them the nodes from 1 to 2, I go and from 2 to 3. The corresponding probability which is coming out from node 1; so if it is a diamond so, let us go to the diamond concept, this is the inclusive or and the probailistic one.

So, the probability is given as pa, time is ta the corresponding time period, another thing. So, this would be a concept of exclusive OR, because the concept of timing will be basically based on the fact that is an exclusive OR under probabilistic 1.

So, with the again the exclusive OR coming out from node 2, node 3 which is also exclusive OR one the probabilities are given corresponding as pb and time is tb.

And basically you can extend that depending on type of series which if this series continues, you will have 1 2 3 4 5 6 and the corresponding probabilities would be given by pa, pb, pc, pd so on and so forth. And the corresponding time would be ta, tb, tc and so on and so forth. Now, if we have two paths from 1 to 2 and 2 to 3; so the equivalent probability would be the multiplication of the probabilities. So, if they are three parts, it can be calculated accordingly while equal and expected time would basically be calculated by some of them.

Now when you want to basically have the parallel circuit; parallel circuits would be given that I can go from 1 to 2 in two ways, one way would be basically the probabilities of pa and ta another path can be basically the pv and tv. Hence, the corresponding equal and probability would the some of the probabilities any one of them and add them; obviously, the sum should always be addition would be less than equal to 1. And the corresponding time periods would be multiplied by the time periods multiplied by the probability.

So, you will basically pa into ta plus pb into tb divided by the corresponding total probabilities which is pa into pb plus pb. So, there are three paths so; obviously, you repeat pa into ta plus pb into tb plus pc into tc divided by pa plus pb plus pc. So, it can extend accordingly which are already discussed in the last class. And in the self- loop, I am considering that their loops are at only one end so; obviously, it can be for 1 or 2, but I will only consider or both of them, but I will consider the loop is only for 1.

So, the corresponding the loops is basically I start at 1 and come back at 1 with the probability as pa and time as tb, pb and time as tb. And when am taking the path from, from 1 to 2 am falling the probability of pa and tb and the corresponding probability which and take outside going away from this network is basically given by pc and tc. So, if I want to find out the equivalent probability by combining two things; one is the self-loop in one node and one is the linear most movement of the; or the path from p to the first to the second one.

The corresponding probabilities can be calculated to find out which is pa divided by 1 minus pb. And the time can be found out corresponding to the fact that it will be ta plus the probability from based on fact which we are trying to find out for tb which is pa pb by 1 minus p b in to tb and it can be extended for more than 2.

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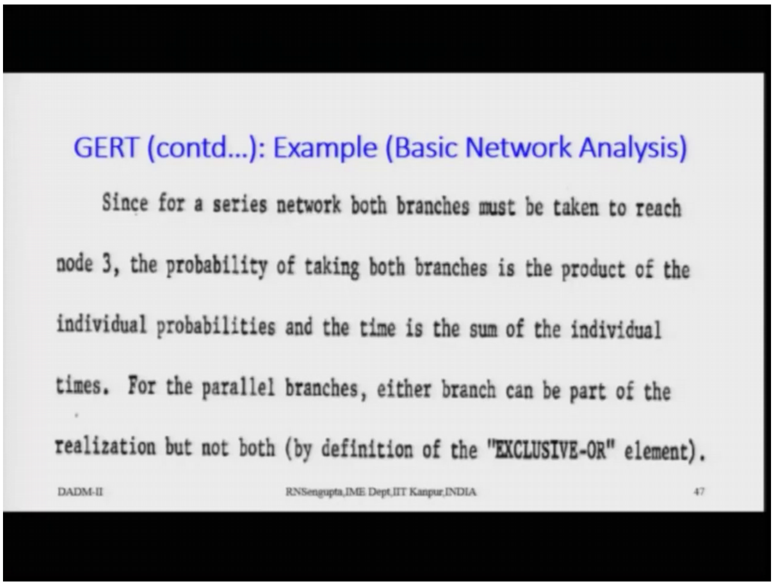
GERT (contd...): Example (Basic Network Analysis)

Since for a series network both branches must be taken to reach node 3, the probability of taking both branches is the product of the individual probabilities and the time is the sum of the individual times. For the parallel branches, either branch can be part of the realization but not both (by definition of the "EXCLUSIVE-OR" element).

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Since, for a series network both branches must be taken to reach node 3. So, node 3 what we have is basically see those for the series 1. So, 1 has to be followed then 2 has to be followed then only 3 is realized that this is what is saying. Since for a series network both the branches must be taken to reach 3 or else it is not possible. The probability of taking both branches is the production of the individual probabilities that what we have calculated.

So, it is pa into pb, if I had traversed 1 to 2 and then I traverse 2 to 3; obviously, the corresponding probability would be pa that is moving from 1 to 2 into pb which is moving from 2 to 3. And the corresponding time would be the time taken to traverse 1 to 2 which is ta and the time taken to traverse from 2 to 3 which is tb, you add them up you get the total time. So, as it is mentioned the time is the sum of the individual times for the parallel branches either branch can be a part of the realization not both.

So, any one of them has to be taken by definition is the exclusive or element. So, let us go. So, either you take the first path which is 1 to 2 falling probability of pa and ta and time of ta or else you follow the proper path of probability pb and tb.

So, the corresponding probabilities would be the one which is shown as pa plus pb and the corresponding time I have already shown which is the so called the concept of waited sums the ratios of the averages for the times where the rates are what you will basically multiply or for the probabilities would be multiplied by the time factors to arrive or the equivalent expected time.

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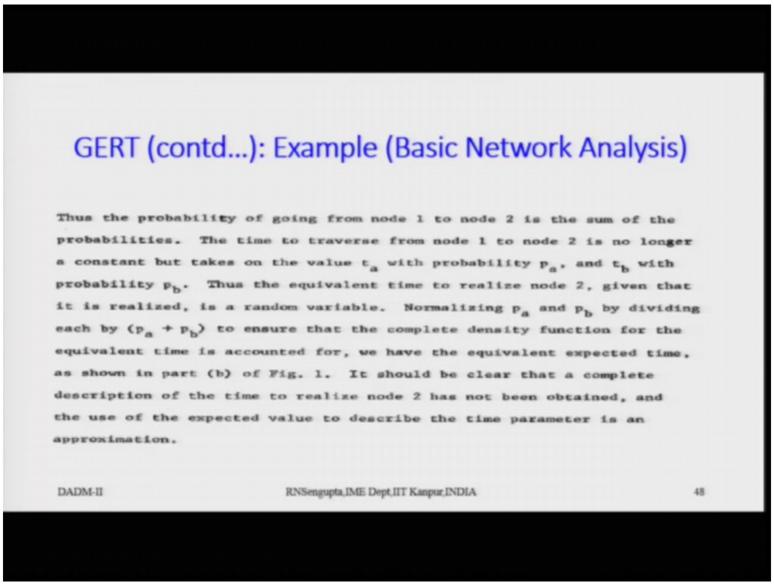

So, thus the probability of going from node 1 to 2 is the sum of the probabilities. In this case, what we have is the sum when am moving from 1 to 2 either I take path 1 or path 2, if there are more than 2; obviously, I can take 1 or 2 or 3, but 1 has to be realized not more than 1.

Thus, the probability of going from node 1 to 2 is the some of the probabilities, the times to traverse from node 1 to node 2 is no longer a constant, but takes on a value ta with the probability pa and tb takes the probability pb. So, if there is another third path for time tc, it will be multiplied on the corresponding probability would be pc.

Thus, the equivalent time to realize node 2 given that you have started at 1 that it would definitely be a random variable. However, normalizing pn pb by dividing each by pa plus pb would ensure that the complete density function for the equivalent time is accounted for and we have the equal and expected time as given which will be, be pa into t a plus pb into tb divided by pa plus pb.

It should be clear that a complete description of the time to realize node 2 has not been obtained and the use of expected value to describe the time parameter is an approximation based on which we are trying to find out the calculations.

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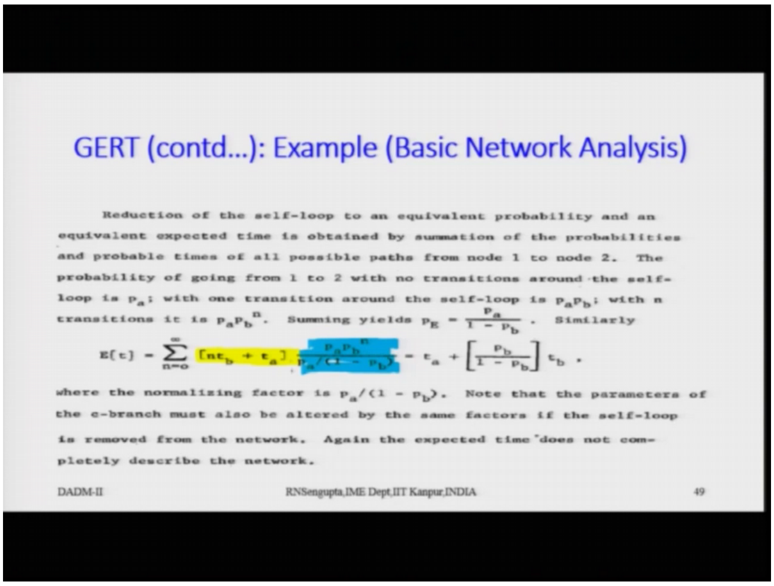

So, now we will consider the rejection of the self- loop to an equal and probability and an equal and unexpected time is obtained by summation of the probabilities. So, if I continue doing this, I will add up the probabilities series of the parallels which we have. The probability of going from 1 to 2 with no transition around the self -loop is basically given as pa and probability from going from node 2 to 3 would be given by probability pb.

So, with one transition on the same loop, it will be given by pa into pb with n number of transitions would be basically pa pa traverse 1. So, if there are n numbers are traversing for the probability pb, it will be pb multiplied by pb n number of times. So, it will be pa for the first loop and the second loop I am going to traverse n number of times it will be pb to the power n.

Summing basically if you find out you find and find out the expected value and the equivalent values of probability which is p. Similarly, we can find out the expected value. So, expected value of time would be the corresponding probability multiplied with the time.

So, the probabilities are given here. So, probabilities are here and the corresponding realized values are given here, you multiply the time with the factor of probabilities and you get the total time would be ta plus tb where tb would be multiplied by the corresponding probability of pb divided by 1 minus pb, it can be extended to more than 2 also.

So, note that the parameters of the c branch, for the third case must also be altered by the same factors is the self- loop is removed from the network. Again the expected time does not completely describe the network, because it is just an approximation depending on the number of combinations which we have.

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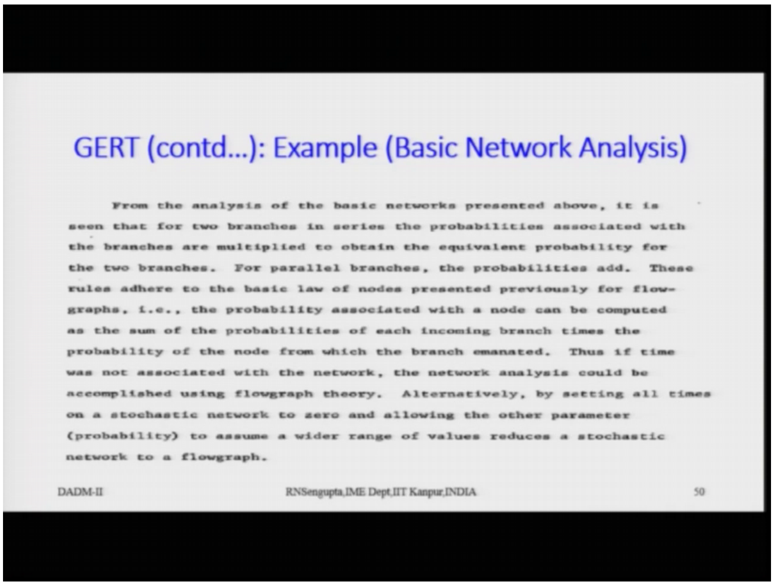

From the analysis of the basic network present a dub it is seeing that for two branches in series the probability associated with the branches are multiplied to obtain the corresponding value. So, if I go back to this. So, if am going in a series sector, I keep multiplying path 1 to 2 into path 2 to 3 and so on and so forth. While if I follow the concept of basically going in the parallel circuit with 2 loops anyone has to be followed, it will be sum if 2 has to be followed I will sum them up, but correspondingly remember the sum of the probabilities is 1.

And if I follow the concept of self- loop. So, if there are two self – loops, I will basically have to multiply the corresponding probabilities. In this way in the first there is a self-loop which is pb and in the second we are not considering self loop so; obviously, this would not be considered in the example here. From the analysis of the basic network represented above, it is seen that for two branches in series the probabilities associated with the branches are multiplied to obtain the equivalent probability for the two branches.

For parallel branches the probabilities add. These rules adhere to the basic laws of nodes presented previously in the flow process concept of the flow diagrams. That is the probabilities associated with the node can be computed at the some of the properties for each incoming branch times the probabilities of the nodes for which the branch have been connected.

So, you are trying to basically multiply the probability and the time. Thus, if the time was not associated with the network; the network analysis could be accomplished using flow graph theory only, but now as time and probabilities are there you have to use the different concepts definitely. Alternatively by setting all times on a stochastic network to 0 and allowing other parameters to assume a wider range of values; it reduces a stochastic network to flow diagram which is basically a deterministic one; that means, you are doing away with the probability as required..

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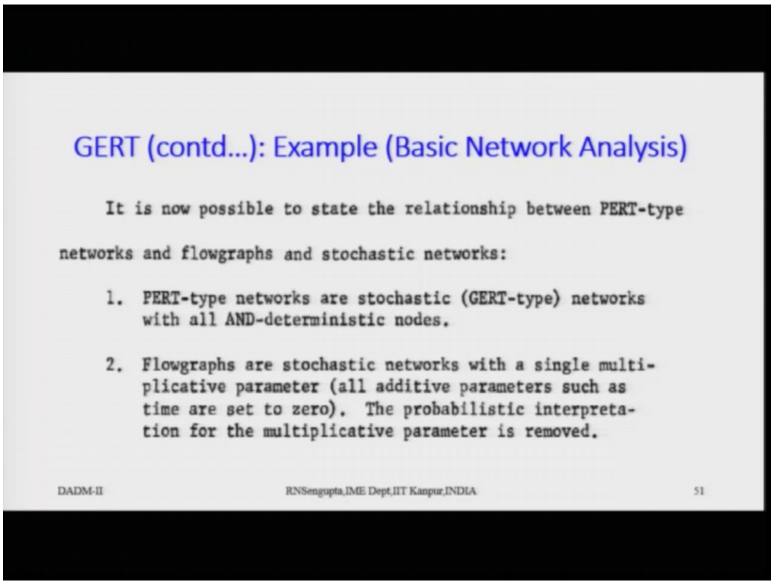

So, it is now possible to state the relationship between the pert type of networks which I have been mentioning time and again in the flow diagram in the stochastic network PERT time and it was a stochastic that is they are GERT provided the distribution is known. If you remember we had considered the most probable time, most pessimistic time and the most optimistic time.

So, PERT time networks a stochastic networks with all and deterministic nodes flow graphs has two stochastic diagrams with single multiplicative parameters all additive parameters such as time are set to 0. So, whatever they are to be multiplied you will set them as non- zero others additional to be 0, the probabilistic interpretation from the multiplicative parameter is removed. So, you would not be considered it accordingly.

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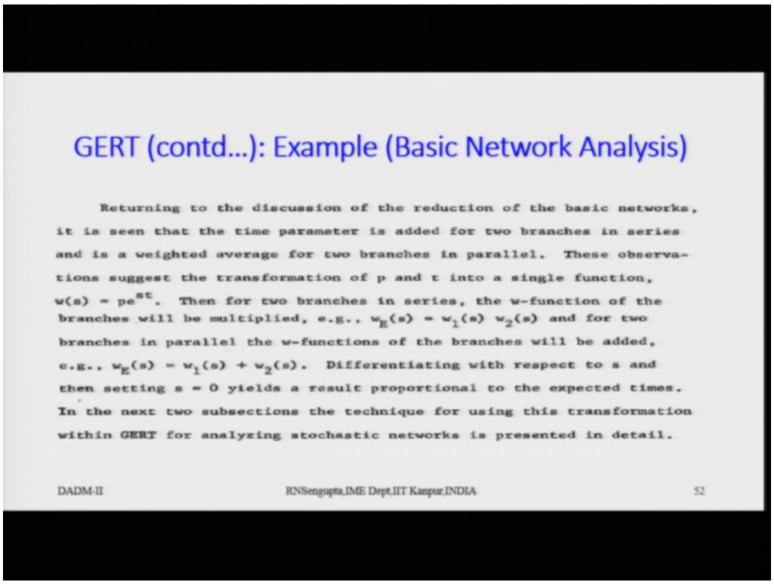

So, returning to the discussion of the reduction of the basic networks, it is seen that the time parameter is added for two branches in series and it is weighted average for the two branches in parallel, 2 is just a symbolic representation, it can be 3 4 5 6 7 so on and so forth. These observations such as the transformation of p and t into a single function such that we use the so, called moment generating function concept which is given by p e to the power sg.

Then for the two branches if I have anyone to basically add them about and multiply them depending on what type of series or parallel connections, it is there I will be utilizing this so called functional form of p e to the power st repeatedly depending on the way the calculations would be done. So, thus I will basically have the equivalent of 1 and 2 as W 1 s into Ws 2 and we will basically have W 1 s W suffix 1 and W suffix 2 would be given by p 1 into e to the power s 1 t and p 2 p to the power s 2 t p 2.

So, differentiating with respect so, and for the additional one, you will basic and add them. So, differentiating with respect to s and then setting the as value as 0 is a proportional result which is equal to the expected value. In the next two sub sections the techniques of how they can be calculated and be found out equivalently. With this I will close the 3rd lecture for the 11th week and consider and the discussion of our different topic as it will require in the concept of DADM II corresponding with this and have a nice day.

And thank you very much.